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Exploring The Field-Independent Student's in Understanding Derivative Concepts: A Case of Commognitive Perspective

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ABSTRACT

Background: The derivative is an important concept in mathematics and therefore understanding the derivative concept play an essential role. However, it has been found that students' understanding of derivative is still below expectation. The reason is that they cannot explain parts that has not been understood. They has just stated that they forgot or did not understand without any detail explanation. This shows that the provided explanation which is given by students does not support the cognitive process. To overcome this situation, we need to understand how the cognitive processes that lead to such understanding. One simple tool that allows for this purpose is the theory of commognition. **Objectives**: This study aims to describe how the field-independent students understand the derivative concept viewed from the perspective of commognition theory. **Design**: This type of research is descriptive with a qualitative approach. **Setting and Participants**: Three participants were selected from 41 students of an undergraduate mathematics education program in a state university through a cognitive style test. **Data collection and analysis**: Task-based interviews and a focused group discussion were used for data collection. **Results**: The analysis results show that not all commognitive were arising during the students' understanding process. The Keywords Subject arise in the use objective phase. Writing symbols, mentioning symbols and showing symbols with hand movements are all as the visual mediators. Definition and theorem of limit as well as definition and theorem of derivative are used in routine procedures. Students tend to use ritual routines instead of exploration routines discourse. On the other side, deeds routines do not appear. Furthermore, the forms of commognition, such as gestures and semiosis, are figured out. **Conclusions**: The exploring the subject's cognition and communication during the discussion is a challenge in this research. Further research is needed to develop this kind of research.

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Keywords: commognition; derivative; cognitive style; understanding.

INTRODUCTION

Calculus is the scientific basis for the development of advanced mathematics. The research about how students understand Calculus has been conducted by Bergqvist (2007) who said that 70% of calculus tasks can be solved by imitative reasoning. Park (2013) discussed the derivative as a function based on the concept of a function at a point and a function at an interval. Tallman et al. (2016) used the Exam Characterization Framework (ECF) in classifying 150 calculus final exams based on cognitive demands, explanations of answers and problem solving. Moreover, Ng (2018) discussed the Calculus that was mediated with the help of a touchscreen dynamic geometry environment (DGEs).

The commognition theory has been applied by Nardi et al. (2014) and Ng (2016) in the derivative discourse. In addition, the discourse functions have been investigated by Nachlieli and Tabach (2012); Tabach and Nachlieli (2016); Robert and Roux (2018). Commognition becomes an alternative to explain how humans develop to build their knowledge. This discourse cannot be separated from one's thinking activity. Therefore, to understand how someone thinks mathematically can be conducted by understanding the discourse.

Cognition and communication are the two inseparable terms. The combination of these forms called commognition is known as the commognitive framework. The characteristic of every communication act is then called discourse (Sfard, 2008; 2020). Commognition assumes that learning is an individualization of patterned collective activities (Sfard, 2008). These become the initial interests of the Commognition theory. According to Sfard (2008) thinking is interpersonal and interactive communication in which someone plays the roles of all interlocutors. Sfard (2008) assumes cognition as communication with oneself whose activities are in groups. Furthermore, Sfard (2008) state that cognition could be defined as an individual communication activity. In other words, cognition and communication are two processes that cannot be separated because communication is a communication activity to yourself. According to Berger (2013) commognition is essentially a participationist theory; learning only occurs through individual participation in mathematical discourse. According to Sfard (2008) mathematics is a discourses-about-discourses that has a multilayered recursive structure because mathematics is an autopoietic system, which is creating its object.

Furthermore, Sfard (2008) states that the discourses are the type of communication that is different from commognition by engaging several individuals and excluding others. In other words, the different types of communication are called discourses. Moreover, mathematical discourse is characterized by keyword, visual mediators, endorsed narratives, and routines (Nardi et al., 2014; Sfard, 2008; Ben-Zvi & Sfard, 2007).

The use of keywords or certain words is characteristic of discourse, and the words used can express the quantities or shapes. Sfard (2008) states that the keywords are essential because these are the meaning of the word. Visual mediator is objects which operated by students to recognize object of communication in mathematics discourse (Arcavi, 2003; David & Tomaz, 2012; Nardi et al., 2014; Robert & Roux, 2018).

According to Sfard (2008), the endorsed narrative includes those series of utterances that are called true and those that are described as felicitous. As a result, it can be said that narratives are often labelled as true. In the mathematics discourses case, endorse narrative is recognized as mathematics theories. Moreover, this includes a discursive concept that is definition, proof, and theorem (Nardi et al., 2014; Sfard, 2008; Robert & Roux, 2018). Routine is a recurring pattern that characterizes discourse, namely regularity using keywords and visual mediators in the narrative (Sfard, 2008; Robert & Roux, 2018). Routines can be in the form of procedures, exercises, such as generalizing, justifying, or endorsed (or rejecting) mathematical narratives (Berger, 2013). Moreover, Park (2013) states that routines describe a discursive pattern which indicates a meta-level rule (metarule) involving when to start or end a particular action. He emphasizes that the patterns can be seen as actions.

Semiotic is one of the studies in communication theory. Semiotics emphasizes the use of signs in the process of thinking and communicating. According to Peirce (1998), there is a triadic relationship in semiotics: object, sign, and interpretant. A sign is a triadic thing: 1) signs are always about, 2) something (object/to), 3) somebody (interpretant). According Sfard (2009) a gesture is a nonverbal communication that reveals spontaneous expression conveyed by face, arms, or hand, fingers following the speech. Furthermore, he stated that gestures are body movements fulfilling communication functions, also an inseparable part of the thought process.

In exploring students' derivative discourses in the cognitive process is divided in seven subcategories (Anderson & Krathwohl, 2001), namely (1) interpreting, changing the form of representation one to another, for example from the form of images to the form of words or otherwise, and from the form of words to the form of words again; (2) exemplifying, i.e. giving an example of being able to make an example of a functions that have derivative and has no derivative; (3) classifying, with stimulus giving examples of functions that have derivatives and functions that do not have derivatives, students are able to classify into functions that have derivatives and are not; (4) summarizing that is abstracting / generalizing the main characteristics of functions that have derivatives and functions that do not have derivatives; (5) inferring that is presents information or provides a collection of information, based on the information provided can make conclusions; (6) comparing, which compares including finding correspondence, which determines the similarities and differences between two things in a problem; (7) explaining is building a causal model or a particular system, for example, with certain known functions, then using the relationship of derivative functions with other functions. In this study, seven categories of understanding were used.

A student has an individually different characteristic to understand the course of the subject. This characteristic is known as a cognitive style that reflects an individual in information processing. Cognitive style is an individual characteristic that is consistent in organizing and processing information (Tennant, 1998). Cognitive style refers to someone in processing information and using strategies to respond to a task (Godfrey & Thomas, 2008). Moreover, cognitive style is an individual characteristic in understanding, solving problems, the consistency of thought process and reflects in the individual process when individual processes information (Liu & Ginther, 1999; Witkin & Goodenough, 1981). Based on several definitions, cognitive style affects commognition because of influencing a person behavior. This study discusses students' commognition which is influenced by cognitive style. This paper reports the students' commognition in understanding derivatives based on the independent field style. Therefore, the research results outlined in this article will be more easily understood and used in helping someone learns mathematics.

METHODOLOGY

Research Design

This type of research is qualitative because it is relevant to the purpose of study, which describes students commognition in understanding derivatives in a homogeneous group of cognitive styles.

This research has received approval from Tadulako University, Palu and was ratified on December 23, 2020, with opinion number 138. This institution is located in the city of Palu, Central Sulawesi, Indonesia. This research was approved to be carried out by involving students (humans), as stated in the research data collection permit.

Participants

Students that are considered as subjects are 3 out of 41 students in the third semester with the criteria of students who have an independent field cognitive style which means an analytical individual who can choose a stimulus based on the situation. On the other hand, subjects are taken based on relatively similar mathematical ability with 70 score and the same gender, namely feminine. Selected research subjects are presented in Table 1.

Table 1

Research Subjects based on Cognitive Style, MAT and Gender

Subject's Initials	Score GEFT	Cognitive Style Category	MAT Score	Gender Category
Meri	15	field-independent	76	Feminine
Siti	10	field-independent	74	Feminine
Lidia	12	field-independent	73	Feminine
Feni	13	field-independent	70	Feminine
Hani	15	field-independent	71	Feminine
Mila	10	field-independent	73	Feminine

From Table 1, there are six students who meet the requirements, three students were selected as subjects because they met the given criteria that provided by purposive sampling technique. In this study, four supporting instruments were used. Firstly, for the Mathematics Ability Test (MAT), we used 5 questions taken from the Calculus book by Verberg et al. (2010) and 5 questions from the 2018 SBMPTN Basic Ability Test for Science and Technology (TKD Scientech). Furthermore, the questions were modified into the 10 questions of essay test form. The reason for choosing essay test is to determine the subject's mathematical ability because by this test, the problem solving process conducted by the subject can be evaluated. Second, the cognitive style GEFT (Group Embedded Figures Test) is used to test students' ability in order to find simple patterns hidden in an intricate image, with the characteristics of grouping students who are easily influenced and who are not easily influenced by the elements of the distractor in processing information from image adopted from (Witkin et al., 1971). The third is a gender questionnaire. Fourth is the Derivative Understanding Task (Task), namely, Task 1 and Task 2. These tasks consist of seven questions, respectively. Furthermore, in this study, seven cognitive processes are used in the understanding category (Anderson & Krathwohl, 2001).

Data Collection

The data collection technique was conducted by giving the task of understanding derivatives to the subject in focused group discussions (FGD). FGD can be defined as a method and technique in collecting qualitative data in which a group of people discusses a particular problem or topic guided by a facilitator or moderator. The discussion process was recorded using a SONY DSC-H300 camera recorder, then made observations when the students worked on understanding derivatives. In this process, the students commognition was observed. The next process is to transcript the video data of each group discussion into text and data in field notes.

Data Analysis

The credibility verification of the research results is conducted as follows: (1) prolonged engagement with the subjects, getting acquainted with the subject and conveying the activities' aims and objectives to be carried out. Furthermore, the researcher asked for some information about the subject's self-identity. As a result, communication will be more open and trust each other. (2) Persistent Observation, researchers continually read, study, investigate and analyze the research data obtained. Researchers are also diligently and consistently looking for theories related to commognition that are suitable and following the data obtained. The theory of commognition obtained is then used as a theoretical basis in research. (3) Methodological triangulation, this step is conducted after condensing the discussion transcript data from each group. The credibility of data is determined using the criteria "meaning convergence." and (4) member check, asking the research subject examines the triangulation result data with careful regarding the suitability of the subjects' knowledge with the researcher interpretation. Data analysis was carried out in three stages, namely data condensation, display data, and conclusion drawing and verification (Miles et al., 2014).

RESULTS AND ANALISES

The main finding in this study is the students commognition in understanding derivatives. The data was collected using a focus group discussion technique, where students sit together to discuss the task of understanding the given derivatives. The tasks given are based on seven cognitive categories, namely interpreting, exemplifying, classifying, summarizing, inferring, comparing and explaining. Students are given two kinds of tasks to understand derivatives, namely the task of understanding derivatives 1 (Task 1) and the task of understanding derivatives 2 (Task 2). As a result, the number of tasks that was discussed is 14 questions.

This section presents the research findings. The findings of this study were obtained based on the results of discussions conducted by the field-independent group which consists of three subjects. The findings show four aspects of cognition, namely keywords, visual mediators, endorsed narrative, routines. Semiotic and gestures, as other aspects that are obtained in this research, are also discussed. Interpretations of the obtained data are presented in Table 2.

Table 2

Interpretation of Meaning Convergence

Cognitive Category	Meaning Convergence
Interpreting	- keyword, visual mediators, endorsed narrative, gesture
Exemplifying	- flexibility, gesture
Classifying	- keyword, visual mediator, endorsed narrative, applicability, semiotics, gesture
Summarizing	- keyword, applicability, gesture
Inferring	- applicability, semiotics, gesture
Comparing	- visual mediators, endorsed narrative, gesture
Explaining	- visual mediators, by whom the routine is performed, gesture

Keywords, visual mediator and Aspect endorsed narrative in interpreting

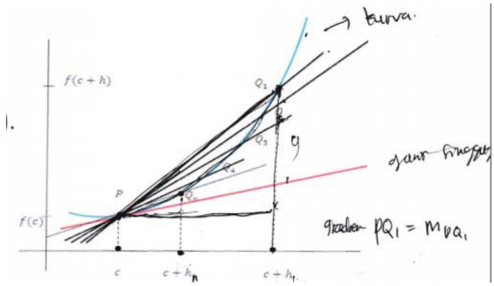
Interpreting is a changing one form of representation to another, for example from an images into the words and conversely, and from the words to the words. The form of representation used in this study is an image form in Task 1 and derivative formulas in Task 2. Here, the subject has been able in defining the derivatives well.

Task 1, the subjects (MRFI, SIFI and LIFI) were asked to mention the elements contained in the picture. They could reveal which one the "curves", "secant line (PQ_1)" and “tangent line." Furthermore, the gradient of the line $\overleftrightarrow{PQ_1}$ was symbolized by the subject with m_{PQ_1}. The subject provided good responses by saying $h_n \to 0$ after given the direction with $n \to \infty$ and $Q_n \to P$. Consequently, we can reveal that the subject is able to use endorsed narrative. On the other side, the subject was "raises her hand" to show “accent” when she was reading $f'(c)$. In this case, the subject has been able to do it as shown in Table 3.

Table 3

Transcript of field-independent subject group discussion for interpreting category

Interview	Discussion Transcript
Researcher	Based on the picture in question number 1, what can you get?
MRFI	graph
SIFI	Curve
LIFI	Graph
Researcher	which curve? The blue curve (*students answer together*)
Researcher	The red one?
MRFI	straight line
LIFI	cut line
	yes line *(answer simultaneously)*
MRFI	The red one is a tangent line *Subject marks on the question sheet*



Researcher	There is a point P and a point Q_1then these two are connected, what is the name of the line?
MRFI	*look confused (while holding his nose)*
LIFI	*look confused while holding his head*
SIFI	secant line
Researcher	Is it true a secant line?
LIFI	*yes (while nodding)*
Researcher	The coordinates of point P?
SIFI	$(c, f(c))$ (*all student shows what they write*)
Researcher	coordinat Q_1?
LIFI	$(c + h_1, f(c + h_1))$, (*followed by Meri and Siti*)
Researcher	the distance from point c to $c + h_1$?
MRFI	h_1
Researcher	Next, we consider $Q_1, Q_2, \cdots Q_n$
	students immediately mark the intended points on their question sheets.
Researcher	does the line PQ_1 have a gradient?
MRFI	yes
SIFI	*usually symbolized by m*
SIFI, MRFI	(*mentioning notation*) mPQ_1
LIFI	*directly write*
	$m_{PQ_1} =$
Researcher	in this figure which one is the gradient of the line PQ_1?
MRFI	*immediately look at the picture in front of him*
LIFI	*pointing to the picture using the pen*
SIFI	*pays attention to Lidia's hand (then write)*
	Each student can write the gradient PQ_1
MRFI	*write*
	$m_{PQ_1} = \dfrac{f(c+h) - f(c)}{h_1}$
LIFI	*write*
	$m_{PQ_n} = \dfrac{f(c+h_n) - f(c)}{h_n}$

Researcher for n large enough ($n \to \infty$) and $Q_n \to 0$ then ….

MRFI limit h is close to 0

Based on the information you have provided above, state the definition of derivative!

LIFI The derivative of f at $x = c$ (*stop and show what he wrote to his friend*)

Researcher Please continue Lidia

LIFI When the function value at $x = c$ (*silent*.….)

SIFI the slope of the tangent can be determined

SIFI The derivative is the gradient of the tangent

LIFI The derivative of f at $x = c$ as the gradient of the tangent to mPQ_1 that $(f(c+h) - f(c))\,/h_1)$

MRFI,SIFI *pay attention to what Lidia said*

MRFI The derivative is defined as the gradient of the tangent obtained from $f(c+h) - f(c)$ divided by h_1 (*silence*…..) then pointing to the picture on the question sheet)

LIFI Definition:

The derivative of f at $x = c$ is the gradient of the tangent to mPQ_1 which is obtained from $(f(c+h) - f(c))\,/h_1)$

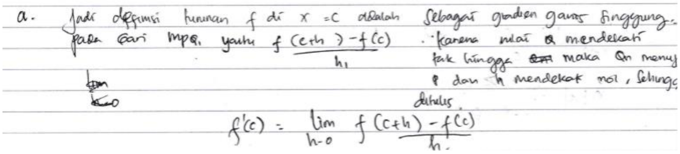
Researcher what is the symbol for derivative?

LIFI f accent c (*his hand is raised while demonstrating the intended accent*)

Each subject reads the definition of the derivative they wrote down

LIFI The derivative of f at $x = c$ is the gradient of the tangent to the line mPQ, namely $\frac{f(c+h)-f(c)}{h_1}$, because n is close to infinity then Q_n is close to p and h close to zero, so it is written by

$$f'(c) = \lim_{h_1 \to 0} \frac{f(c+h)-f(c)}{h_1}$$

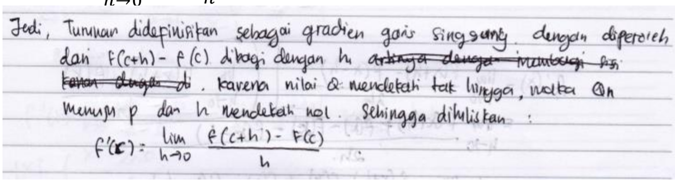


a. Jadi definisi turunan f di x = c adalah sebagai gradien garis singgung pada garis mPQ, yaitu $f \frac{(c+h)-f(c)}{h_1}$. karena nilai Q mendekati tak hingga ~~...~~ maka Qn menuj p dan h mendekat nol, sehingga ditulis

$f'(c) = \lim_{h \to 0} f \frac{(c+h)-f(c)}{h}$

MRFI The derivative is defined as the gradient of the tangent to the given by $f(c+h) - f(c)$ divided by h. Because the value of n tend to infinity then Q_n close to p and h approaches zero, so it is written by

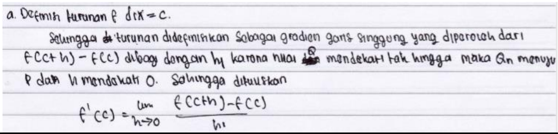
$$f'(c) = \lim_{h \to 0} \frac{f(c+h)-f(c)}{h}$$



Jadi, Turunan didefinisikan sebagai gradien garis singgung dengan diperoleh dari f(c+h) - f(c) dibagi dengan h ~~artinya dengan membagi fisi kanan dengan di~~. karena nilai Q mendekati tak hingga, maka Qn menuju p dan h mendekati nol. Sehingga dituliskan :

$f'(c) = \lim_{h \to 0} \frac{f(c+h) - F(c)}{h}$

SIFI	*Wrote*


a. Definisi turunan f di x = c.

Sehingga ~~di~~ turunan didefinisikan sebagai gradien garis singgung yang diperoleh dari $f(c+h) - f(c)$ dibagi dengan h_1 karena nilai Q mendekati tak hingga maka Q_n menuju P dan h mendekati 0. Sehingga dituliskan

$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h_1}$

The words and symbols spoken by the subject in expressing the derived definition can be made in the form of signifier-realization. The relationship between signifier and realization can be seen in Table 4.

Table 4

Pair Signifier – Realization to Define the Derivative at Task 1

Subjects	Signifier	Realisation	Realising procedure
SIFI	Secant line	lines PQ_1	
		line PQ_2	Connecting the P
LIFI	Secant line	line PQ_3	and Q points
		$\vdots$	
		line PQ_n	
LIFI	Coordinate (P, Q_1)	$(c, f(c)$, $(c + h_1, f(c + h_1))$	
SIFI	Tangent line	line through point P	tangent line at P point
Partisipant	Distance point c to $c + h_1$	h_1	
MRFI	Gradient line PQ_1	$mPQ_1 = \dfrac{f(c + h_1) - f(c)}{h_1}$	
LIFI	Gradient linePQ_n	$mPQ_n = \dfrac{f(c + h_n) - f(c)}{h_n}$	
	for $n \to \infty$ and $Q_n \to 0$	limit h to 0	Define
		$f'(\text{c})$	

The last part of the realization is the derivative symbol. Subject mentions the derivative of f at $x = c$ to pronounce $f'(c)$. On the other side, the subject "raises her hand" to show "accent" when she was reading $f'(c)$. Derivative of f at $x = c$ is the gradient of the mPQ tangent line, namely

$f'(c) = \frac{f(c+h)-f(c)}{h_1}$, because the value of n approaches to infinity then Q_n approaches p and h approaches zero. As a result this writes as $f'(c) = \lim_{h \to 0} \frac{f(c+h)-f(c)}{h}$. Taks 2 changes the representation form of derivation formulas into the words to express the definition of derivatives. They read the symbol of $f'(c)$ as "*turunan* $f(c\ )$" and "*turunan* f aksen c." In the meantime, it should be read as a "derivative of f at c" or "first derivative of f at c.

The research findings of keyword that mentioned by subject are secant line, curves and tangent line. These words are important and useful for constructing the definition of a derivative. The keywords mentioned by the subject can be explained in a concrete form or objectified. Keywords in discourse are grouped into four phases, namely passive use, routine-driven use, phrase-driven use and objectified use (Sfard, 2008; Roberts & Roux, 2018).

In this finding, colloquial terms that are always used in the nature of limits, such as “approaching” and “toward to” are in line with (Fernández et al., 2012; Radford & Barwell, 2016). They said that students' mathematical concepts about the limit of a limit function at a point are influenced by the use of everyday terms such as "to approach," "to tend toward," "to reach," and "to exceed". During the dialogue, it will be seen the relationship between everyday discourse and mathematical discourse with the term "informal" and "formal" language (Radford & Barwell, 2016).

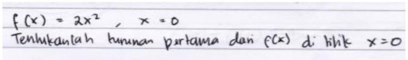
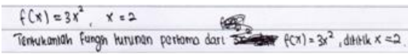
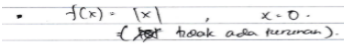
Objects such as symbols, graphs, algebraic formulas that associate relationships and operations with mathematical objects used in interviews are called visual mediators (Sfard, 2008; Arcavi, 2003; David & Tomaz, 2012). In this aspect, the subject in general used more symbols, for example when they mention the gradient PQ_1 then write it as mPQ_1. Then the subject was asked "based on the picture in the question, which is the slope of the line PQ_1"? The subject immediately saw the image in front of him and pointed to the picture using his "pen". In this case, the subject used a physical object as a visual aid which can be called a visual mediator. Another example, in Task 2, students were confused to pronounce $f'(x)$, to demonstrate an "accent" sign by imitating it with his hands. This is also called a visual mediator (Berger, 2013; Nardi et al.,2014; Zayyadi, et al.,2019). In the interpreting category, to define a derivative, several process sequences are needed. Subject mentioned supporting narrative h_n towards zero ($h_n \to 0$). These supporting narratives are grouped into mathematical definitions.

Exemplifying Category

Exemplifying means subject being able to create examples of functions that have derivatives and no derivatives. In this case, the subject has been able to do it as shown in Table 5.

Table 5

Transcript of field-independent subject group discussion for exemplifying category

Interview	Discussion Transcript
Task 1:	
Researcher	Give two examples of functions that have a derivative at one point
LIFI	$f(x)$ = two power x
MRFI	$f(x)$equal to two power x
Researcher	Where at point the function have derivative? Lidia shows what she wrote to Meri while Siti still looks confused
MRFI	*Wrote* 
LIFI	*Wrote* 
SIFI	at value is 12 (mentions the value she gets and shows what she did
SIFI	*Wrote* 
Task 2:	
Researcher	Give an example of a function that has no derivative
LIFI	absolute value function at $x = 0$
Researcher	Try to write
LIFI	*Wrote* 
Researcher	One more example function have no derivative
LIFI	$f(x) = \frac{1}{x}$ and $f(x) = \sqrt{x}$
Researcher	Have derivative?
MRFI	There is Mam
MRFI	*Wrote*

	$f(x) = \frac{1}{\lvert x \rvert}$, $x = 0$
LIFI	*Wrote* $f(x) = \frac{1}{\lvert x \rvert}, di\ x = 0$
Researcher	Can you prove the function has no derivative?
LIFI	can not Mam

In Task 1, SIFI, MRFI, LIFI give the correct answer even though at first the given function was incomplete. However, the subject can complete it at the point where the function has a derivative. The MRFI wrote that $f(x) = 2x^2$ at $x = 0$, LIFI wrote $f(x) = 2x^3$ at $x = 1$ and SIFI wrote $f(x) = 3x^2$ at $x = 2$ are functions that have derivative. Furthermore, in Task 2, the subject also gives the correct answer, namely the function $f(x) = |x|$, at $x = 0$ and $f(x) = \frac{1}{|x|}, at\ x = 0$ is a function that has no derivative.

The subjects can provide the derivatives function examples that are similar with what other subject written in a group. Subject replaces only the variables or recycles those that have been written by other subjects. For example, the function $f(x) = 3x^2$ at $x = 2$, the subject simply replaces it with $f(x) = 2x^3$ at $x = 1$. This shows that the subject is still fixated on themself assumptions. They think only “answer is correct”. The three subjects have answered correctly. However, it is still difficult to explain why the function has no derivative. In this category, subject flexibility is very stiff or there is no variation in providing the requested sample. The findings are consistent with Verschaffel et al. [29] using the term “routine skills”, namely being able to complete mathematical tasks quickly, accurately and without understanding.

Applicability in Classifying, Summarizing and Inferring Category

In the classifying category, they were asked to determine which function has a derivative from several functions given. The questions are presented in Table 6.

The results show that they could use formal keywords such as "absolute value function" and "greatest integer function." Moreover, one student could differentiate from other forms of functions. The subjects were more familiar with using the symbol $f(x) = [\![x]\!]$ by mentioning it as greatest integer function and then the symbol $f(x) = |x|$ was declared by the subject as an absolute value function. Subject mentioned that the functions in Task 1 has no derivatives and the functions in TMT 2 has derivatives. The subject has difficulty to give reasons for each function. Not all are the given functions were described by the subject. The subject only gave an explanation of questions c) on Task 1 and questions e) on Task 2. The explanation of the subject is given in Table 7.

Table 6

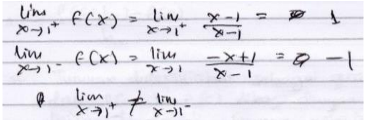
The questions Task 1 and Task 2 in classifying category

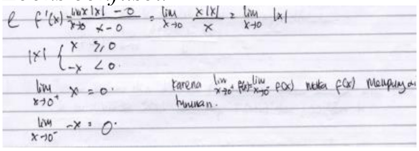
Task 1	**Task 2**
Pay attention to the following functions a. $f(x) = \lvert x \rvert$ at $x = 0$ b. $f(x) = \lceil x \rceil$, at $x = 0$ Note: $\lceil x \rceil$ is the greatest integer number $\leq x$ c. $f(x) = \lvert x - 1 \rvert$, at $x = 1$ Which of the above functions has a derivative at a point?	Pay attention to the following functions $d.$ $f(x) = x^2$, at $x = 0$ $e.$ $f(x) = x\lvert x \rvert$, at $x = 0$ $f.$ $f(x) = x^2 - 1$, at $x = 0$ Which of the above functions has a derivative at a point?

Table 7

Transcripts of field-independent subject group discussions classifying categories

Interview	**Discussion Transcript**
Task 1:	
Researcher	How about part c?
LIFI	present absolute value
MRFI	No have derivative?
MRFI	Because if enter x it becomes zero
SIFI	*Looks confused*
Researcher	For functions that have this absolute value sign, what do you usually do?
MRFI	Removed the sign, Mam?
LIFI	For problem c Mam. wrote $\lvert x-1 \rvert \begin{cases} x-1 & \geq 1 \\ -x+1 & \leq 1 \end{cases}$
MRFI	Yes, have derivative
MRFI	This Mam (show her working) $F'(x) = \lim_{x \to 0} \frac{\lvert x-1 \rvert, \cancel{dx} = 0.}{\cancel{x-x} \; x-1.}$



MRFI	no have derivatives Mam
SIFI	because left limit and right limit not equal
Task 2:	
Researcher	How about part e?
LIFI	Have derivatives
MRFI	no Mam
SIFI	*Looks confused*
MRFI	
SIFI	*I don't understand, still look part c*
LIFI	Problems have derivative because left limit equal right limit

The MRFI and LIFI said to remove the absolute sign, using the definition

$|x-1| = \begin{cases} x-1 & x \geq 1 \\ -x+1 & x < 1 \end{cases}$ for part c) dan $|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$ for part e).

The subjects stated that a function has a derivative if the left limit equals the right limit in the classifying category. They have used different interpretation which is used formally in general. They were then asked to write the left and the right limits that they were intended. The process of work done by the subjects was correct. Moreover, the emphasis on to use of the left and the right limit symbols is needed. Even though they performed the correct process, they could not use the term widely and formally. As a result, this can be classified as an "applicability" ritual, meaning that the procedure can only be applied depending on the situation.

The "applicability" ritual occurs in the summarizing category. The subjects here could not distinguish the definition of continuous functions from functions that have derivatives. In this category, one of them said that the continuous function is a function that has the left limit equal to the right limit. In consequence, the term used by the subject could not be used in general. In this case, we must consider what subjects mean about "the left limit equals to the right limit."

In the Inferring category, confusion also occurred due to subject statements. They said that a function that has a derivative showing that its limit and function is continuous. In this case, the subject mixing up the term "limit exists" to determine the existence of a function that has a derivation and a continuous function. As a result, the subject must emphasize the difference between the function characteristics, which has derivative and continuous functions. However, the subject could finally have the correct conclusions from the provided information. This category includes the "applicability" ritual. The ritual of "applicability" arises because there are differences in terms and symbols used by the subject to define a function that has a derivative. The findings show that the process of working that conducted by the subject is correct and the terms used are acceptable. However, this limited to the process of working on that problem only. The suggestions which given by the participant by encouraging the subject tries to better understand the use about derivatives were responded positively by the subject.

Based on three types of routine, only the ritual type arises in this study and will then be discussed. Applicability is the part of the ritual that arises in several categories of routines aspect. Ritual "applicability" is defined as something that is produced depending on the situation (narrow). In this study, the applicability can be found in classifying, summarizing, and inferring.

Based on these three categories, it is necessary to interpret what is meant by a sign resulting from the process of determining the nature of the derivative. In 'good' learning, semiosis continues until the learner is able to use mathematical signs in a way that is meaningful to himself and commensurate with his use by other mathematicians (Berger, 2010).

By whom the routine is performed on comparing and explaining category

In the comparing category, question on Task 1"What is the relationship between the derivatives of the sum of two functions and the sum of the derivatives of each function at the same point? and TMT 2 "What is the similarity between the limit of the sum and the derivative of the sum of two functions at one point?". Explanation of the subject is presented in Table 8.

Table 8

Transcript of subject discussion field-independent category comparing

Interview	Discussion Transcript
Task 1	
MRFI	two functions are formed then combined and derived?
MRFI	Let's two functions f(x) and g(x)
LIFI	are derived simultanoes
LIFI	wrote symbol $f'(x) + g'(x)$
SIFI	*attention and listen*
MRFI	$(f+g)'(x) = f'(x) + g'(x) \rightarrow$ sama² memiliki turunan
LIFI	there is a relationship that is both have a derivative at the point
Task 2	
SIFI	Use limit notation, Bu?
LIFI	previously, there is an f accent x and g accent x?
SIFI	*Wrote* $\lim_{x \to 0} f(x) = \lim_{x \to 0} g(x)$
MRFI	Like this (Lidia and Siti show their work) $\lim_{x \to 0} [f(x) + g(x)] = \lim_{x \to 0} f(x) + \lim_{x \to 0} g(x)$

In Task 1, the subjects can only partly interpret the given question "derivative of the sum two functions and the sum of derivatives of each function" which is symbolized by $(f + g)'(x)$ and $f'(x) + g'(x)$. Furthermore, the subject writes down the notation $(f + g)'(x)$=$f'(x) + g'(x)$ by adding the explanation "both have derivatives". The subject concludes that "the relationship between the derivatives of the sum of two functions and the sum of the derivatives of each function at the same point" is that both have derivatives at the point. Moreover, in Problem 2 Subjects can only write the notation symbols of limit. In this category, the subject discourse is less developed because there is no explanation about what they have written. In the Explaining category, a causal model or a particular system is developed. For example, by knowing certain functions, the relationship between derivative functions with other functions can be applied Task 1: Assuming that f is an odd function and has derivatives everywhere. Could you explain why the derivative of this odd function is an even function?.

MRFI said that is based on the problem, she assumes that f is an odd function, that is, f is negative $x = f(x)$. LIFI said that the function is odd, i.e. f negative x equals negative f(x). MRFI writes $f'(-x) = f'(x)$ and notes that besides symbols with odd functions Moreover, LIFI does the same thing. Furthermore MRFI said that our goal is to determine $f'(-x) = f'(x)$. The subject was silent for a while then they said "confused, ma'am". Subjects are directed to write the first derivative formula. MRFI writes the first derivative formula $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

Furthermore, MRFI writes $f'(-x) = \lim_{h\to 0} \frac{f(-x+h)-f(-x)}{h} = \lim_{h\to 0} \frac{f(-x+h)+f(x)}{h}$ and he said "we are confused and can not continue anymore". Subjects then were given guidance, and asked to continue, namely $f'(-x) = \lim_{h\to 0} \frac{-f(x-h)+f(x)}{h}$. SIFI said "exclude the negative" then continues its steps, namely $f'(-x) = -\left(\lim_{h\to 0} \frac{f(x-h)-f(x)}{h}\right) = f'(x)$. SIFI said the part in brackets is the same as - $f'(x)$. They were asked again to explain, but they said "don't know". According to MRFI, "we only see the purpose, ma'am". SIFI writes $f'(-x) = f'(x)$. Furthermore, MRFI gives an example of $f(x) = x^3$ an odd function, its derivative is an even function.

In TMT 2, the subject said that she had not studied symmetric functions. The researcher asked the subject to see the symmetric function. MRFI writes the definition of a symmetric function, namely $f_S'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x-h)}{2h}$. Furthermore, LIFI describes it as $f_S'(x) = \frac{1}{2} \lim_{h\to 0} \frac{f(x+h)-f(x)+f(x)-f(x-h)}{h}$. SIFI asks why it can be written $-f(x) + f(x)$. LIFI gives an answer by saying that later it will form the first derivative of x.

Furthermore, MRFI writes $f_S'(x) = \frac{1}{2} \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} + \frac{1}{2} \lim_{h\to 0} \frac{f(x)-f(x-h)}{h}$. SIFI says the first part equals $\frac{1}{2} f'(x)$. MRFI writes as $f_S'(x) = \frac{1}{2} f'(x) + \frac{1}{2} f'(x) = f'(x)$. The researcher asked whether the second part is also equal to $\frac{1}{2} f'(x)$?. MRFI said the second part is the same, ma'am, but we cannot give an explanation. SIFI said, so the derivative of a symmetric function is the same as the first derivative of the function.

Subjects have not used endorsed narrative, namely the definition of odd functions and they cannot connect the two functions. At each step, they have not based on their thinking yet. They still need someone else or need schafolded. In the ritual aspect "by whom the routine is performed in every process that carried out by the subject in conducting the task 1 and task 2, the participant must always provide scaffolded. In addition, the subject also said that they had never discussed a similar problem. This is because the subject could not show a causal relationship from the given problem, in this case the relationship between the derived function to other functions. This finding is in line with Hmelo et al. (2007); Margulieux and Catrambone (2021) who stated that the less initial knowledge of student will certainly require instructions to build their knowledge.

Other findings in research

From the "applicability" ritual as previously described, this study also finds semiotic. This can be seen based on the response that the subject given when saying that a function that has a derivative means that it has a limit. The subject also states that a function is said to be continuous, meaning that it has a limit. The response given by the subject is wrong because in theory, a function that has a derivative if it satisfies the left derivative equals to the right derivative. Moreover, the function is said to be continuous if the limit value of the function is equal to the value of the function. After the participant investigated further, it is found that the Subject had misunderstood about symbols. In fact, the process they have already done is correct.

Semiotics

Furthermore, the procedure carried out by Subjects is defined as a process of thinking and communicating, which is conveyed by using a sign known as semiotics in line with Duval (2017) states that a sign is very important to use in communicating. Semiotics is a process of triadic. The triadic theory of signs has three main elements, namely, object, sign, and interpretant. As discussed in the routine discourse at the classifying category, the subjects were asked to determine whether the function has a derivative or not.

The function in the question is as follows: $f(x) = |x - 1|,\ at\ x = 1.$

The process conducted by students is given below:

$$f'(1) = \lim_{\mathrm{x}\to 1} \frac{f(x) - f(1)}{x - 1}$$

$$= \lim_{\mathrm{x}\to 1} \frac{|x-1| - 0}{x - 1} = \lim_{\mathrm{x}\to 1} \frac{|x-1|}{x-1}$$

Moreover, the subject determined the value of the right and the left limits as follows:

the right limit,

$$\lim_{\mathrm{x}\to 1^{+}} \frac{x-1}{x-1} = 1,$$

the left limit,

$$\lim_{\mathrm{x}\to 1^{-}} \frac{-(x-1)}{x-1} = -1\,.$$

The subject concluded that the right limit unequal to the left limit from the obtained results. Consequently, the function $f(x) = |x-1|$ has no derivative.

Another example given is $f(x) = x|x|$, at $x = 0$.

The subject used the first derivative formula, that is,

$$f'(1) = \lim_{\mathrm{x}\to 0} \frac{f(x) - f(1)}{x - 0}$$

$$= \lim_{\mathrm{x}\to 1} \frac{x|x| - 0}{x - 0} = \lim_{\mathrm{x}\to \mathrm{o}} \frac{x|x|}{x} = \lim_{x\to 0} |x|$$

Moreover, the subject determined the value of the right and left limits as in the following:

the right limit,

$$\lim_{x\to 0^{+}} x = 0,$$

And the left limit,

$$\lim_{\mathrm{x}\to 0^{-}} \mathrm{x} = 0,$$

The subject concluded that the left limit equals the right limit. As a result, the function $f(x) = x|x|$ has a derivative.

Based on the subject's working process from the two given functions, the confusion and misperception of concluding occurred because of equal value of the left with the right limits by meaning that a function has a derivative. The subjects should use the left and the right derivative terms to make a conclusion as presented below:

$$f'_{+}(1) = \lim_{x \to 1^{+}} \frac{x-1}{x-1} = 1$$

$$f'_{-}(1) = \lim_{x \to 1^{-}} \frac{-(x-1)}{x-1} = -1\,.$$

In the second question, the subject did misinterpretation by providing right-hand derivative

$$f'_{+}(0) = \lim_{x \to 0^{+}} x = 0,$$

and left-hand derivative

$$f'_{-}(0) = \lim_{x \to 0^{-}} -x = 0.$$

In the classifying category, the triadic process that researchers can conclude as follows: Firstly, Sign; The sign used is verbal because the questions are written in sentences instead of mathematical symbols. Secondly, the effect of the sign; Object can be seen from the subject process in order to determine the right and left limits. Thirdly, when the sign is interpreted or is understood as that arises from cognition and communication which is called an interpretant. This is a symbol of right and left derivatives.

Furthermore, in the summarizing category, the subjects revealed that the function could be continuous if it has the left limit equal to the right. Confusion also occurred here due to the use of the term. Subjects have not distinguished the definition between the characteristics of continuous function and functions that have derivatives. So, it is necessary for emphasizing the difference between the function features having derivative and the continuous function by distinguishing their symbols. Here, the researchers have not seen the process of semiosis, in which the sign can be seen from the given questions.

Gesture

Subjects were often not being confident to respond to the given questions. For example, in the interpreting category, subjects have mentioned that there existed a secant line in the curve picture. However, Subjects were appearing confused when the researcher asks, is what line? The subjects spontaneously started to do some movements such as holding her nose, holding her head, and playing her pen. Researchers categorized these movements as positive responses because the subject can still determine what is asked from the problem. Moreover, the subjects wrote and then read the questions in a low voice and looked down at the same time when the question from the researcher urged them to answer. Subjects prefer to write and then read what they have written on their paper rather than directly answer them. Their hesitation arose when they had to discuss, for example: playing a pen, reading the questions by grinning, less attention, reading the questions over and over, and keeping quiet for a while. Gesture that subject performed for example talking to hisself and moving hands will be able to help students (Radford & Barwell, 2016).

Commognition can be said as a discursive theory because of its usefulness for describing the learning process. Collective activity that carried out in this study is a social effort which can encourage students to build knowledge that comes from outside them self. Studying mathematics means having a unique routine, for example thinking, doing, seeing and communicating. Mathematical discourse is a process of individualization in mathematics learning.

One by one the task of understanding derivatives is done by the subject in groups, that's where the students commognition could be seen to be description. The analysis in this article only focuses on the student discourse during the discussion and the results showed that students are more likely to use ritualistic discourse. The results of this study showed that not all students perform perfectly when they expressed their thoughts and explained what they have mentioned.

CONCLUSIONS

According to the main commognitive assumption, thinking is defined as an activity of communicating with himself which is carried out in groups. Mathematical discourse is the main object of commognitive research and this is what distinguished it from other studies. The analysis in this study only focuses on student discourse during the discussion. Moreover, the result is that students are more likely to use ritual discourse.

The results showed that not all the characteristics of students' discourse arise in each cognitive process category. During interpreting, classifying, and summarizing process, the keywords of noun and formal words were found.

Moreover, during interpreting, classifying, comparing, and explaining the visual mediators were arose by the students used symbols as a communication medium. Routines appeared in the category of exemplifying, classifying, summarizing, inferring, and explaining. Furthermore, the form of commognition in the semiosis term appeared in the category of classifying and summarizing. Furthermore, the commognition form i.e., gestures, appeared in each category of cognition.

From the description, it can be shown that there is a triadic process which is a sign, object, and interpretant. The first triadic process, namely the sign, is the task of understanding the derivatives given to the subjects. Every process that carried out by the subject in solving the problem is said as an object. In this case the object is a symbol because there is a sign that has been agreed upon. Interpretant is a sign that is interpreted in the communication process. Here the interpretant is presented as an interpretation of the symbols of the left and right limits. The sign is also a tool to facilitate communication and can be applied in life. Signs make us think, communicate with others and can give meaning to what is around us.

AUTHORS' CONTRIBUTIONS STATEMENTS

R.L. conceived the presented idea, developed the theory, built the task 1 and task 2, collected the data, and analysed the data, design, drafting manuscript. T.Y.E.S and A.L. developed the theory, built the task 1 and task 2, collected the data reviewed and approved the final version of the work.

DATA AVAILABILITY STATEMENT

The data presented and supporting this research results are available at a reasonable request to the first author, R.L. Anyone who makes a reasonable request to the first author of the article will be provided with the data that support the results of the study.

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