**Capturing Strategies and Difficulties in Solving Negative Integers: A Case Study of Instrumental Understanding**

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**ABSTRACT**

**Background**: Until recently, many elementary school students made mistakes in solving negative integer problems, especially for students who had instrumental understanding. Studies that focus on strategies and difficulties faced by students can be a solution to finding student weaknesses so that students achieve a better level of understanding. **Objectives**: The objective of this study was to classify cases of 5th grade elementary school students who have instrumental understanding in solving negative integers tasks with a focus on strategies and difficulties faced with number line models. **Design**: This study uses a case study research design. **Setting and Participants**: 5th grade elementary school students who have studied the material on negative integer operations in Sukodono, Indonesia. One of the students that has the most varied instrumental understanding and representation strategy was chosen as a case study. **Data collection and analysis**: Qualitative data collection is done by providing a number line model task and interview instructions. Data analysis was performed by means of comparative analysis, namely by comparing all data collected, including the transcribed audio and video recordings. **Results**: Researchers found various types of strategies that experienced difficulties to cognitive completion by students with instrumental understanding, along with difficulties in solving number problems. **Conclusions**: The implications of this study are very useful for further research and lifelong learning practices, especially in the process of how to deal with elementary school students who have difficulty in constructing knowledge and the concept of negative integer arithmetic operations.

**Keywords**: strategy and difficulty; representation; instrumental understanding; negative integer; case study.

**Capturando Estratégias e Dificuldades ia Resolução de Números Inteiros Negativos: Um Estudo de Caso de Compreensão Instrumental**

**RESUMO**

**Contexto**: Até recentemente, muitos alunos do ensino fundamental cometeram erros ao resolver problemas de números inteiros negativos, especialmente para alunos que tinham compreensão instrumental. Estudos que enfocam as estratégias e dificuldades enfrentadas pelos alunos podem ser uma solução para encontrar as fragilidades dos alunos para que eles alcancem um melhor nível de compreensão. **Objetivos**: O objetivo deste estudo foi classificar casos de alunos do 5º ano do ensino fundamental que possuem compreensão instrumental na resolução de tarefas de inteiros negativos com foco nas estratégias e dificuldades enfrentadas com modelos de linha numérica. **Design**: Este estudo usa um projeto de pesquisa de estudo de caso. **Ambiente e participantes**: alunos do 5º ano do ensino fundamental que estudaram o material sobre operações de número inteiro negativo em Sukodono, Indonésia. Um dos alunos que possui as mais variadas estratégias de compreensão e representação instrumental foi escolhido como estudo de caso. **Coleta e análise de dados**: A coleta de dados qualitativos é feita fornecendo uma tarefa de modelo de linha numérica e instruções de entrevista. A análise dos dados foi realizada por meio de análise comparativa, nomeadamente comparando todos os dados recolhidos, incluindo as gravações de áudio e vídeo transcritas. **Resultados**: Os pesquisadores encontraram vários tipos de estratégias que enfrentaram obstáculos para a conclusão cognitiva por alunos com compreensão instrumental, juntamente com dificuldades na resolução de problemas numéricos. **Conclusões**: As implicações deste estudo são muito úteis para futuras pesquisas e práticas de aprendizagem ao longo da vida, especialmente no processo de como lidar com alunos do ensino fundamental que têm dificuldade em construir conhecimento e o conceito de operações aritméticas inteiras negativas.

**Palavras-chave**: estratégia e dificuldade; representação; compreensão instrumental; número inteiro negativo; estudo de caso.

**INTRODUCTION**

Elementary school students understand the concept of negative integers is not as easy as understanding positive integers. According to (Bofferding, 2014), students' difficulties in solving integer problems are due to their lack of prior knowledge of negative number knowledge. When the concept of the first negative integer is given it will seem abstract to students, as a result it will conflict with previous knowledge of integer arithmetic operations (Cengiz, Aylar, & Yildiz, 2018). Relates to solving problems involving negative integers, Bofferding, Aqazade, and Farmer (2017) stated that students who are not accustomed to using negative numbers or students who rely on the concept of positive integers will often ignore the use of negative signs. Likewise, solving problems that involve number operations often overgeneralizes based on previous students' experiences in operating positive integers. This results in a misconception, where students think that addition is always bigger, while subtraction is always smaller (Whitacre et al., 2012).

Rabin, Fuller, and Harel (2013) said that student misconceptions continued, students would face many challenges when facing new mathematical ideas that came from previous schemes. Therefore, several studies have tried to bridge students to make it easier to understand the concept of negative integers, including through mental models (Bofferding, 2014), through the problem sequence (Bishop et al., 2014), through problems have different cases (Aqazade, Bofferding, & Farmer, 2017), and through the opposite model (Cetin, 2019). However, this research has not emphasized the difficulties faced by students in solving the representation-based problems used. Tambychik and Meerah (2010) explains the understanding of educators or researchers about students 'difficulties in solving problems which can be seen from the representation of answers which is very important before developing students' thinking skills.

The number line model provides visual assistance so that students can check the relationships between integers with each other (Kent, 2000; Whitacre et al., 2011). In addition, Wessman-Enzinger and Bofferding (2014) stated that number lines were introduced to students to facilitate the transition of students' understanding of negative numbers. Therefore, in this study the number line is used as an intermediary for students to solve integer operation problems. Through the number line, the representation of students in adding and subtracting positive or negative numbers will be seen, so that the researcher is able to classify the types of strategies used and identify the difficulties that occur in students in developing strategies

The solution to a problem depends on the student's understanding. Skemp (2006) argues that there are two types of understanding, namely instrumental and relational understanding. The difference between instrumental and relational understanding is the difference in the measure of mathematics and the reasons for the decision making why the action is taken. Students who are classified as instrumental understanding are students who are able to perform mathematical steps, but students do not know why the measuring instrument is used. Conversely, students who are classified as having a relational understanding are students who are able to take mathematical steps and know the reasons for doing these actions. If the teacher does not find a solution why students who have instrumental understanding or students who have less relational understanding tend to fail in math problems, then this will hamper students' critical attitudes in solving problems (Anderson, 1996).

Several previous studies by Bofferding et al. (2017) explain the strategies students use when facing problems there are differences. Bishop et al. (2014) characterized students' strategies that were classified as relational understanding through a number line model and a series of open sentences. However, previous studies have not characterized the strategies and difficulties used by elementary school students who have instrumental understanding in solving the problem of addition and subtraction of integers through the number line model.

**Problems of Study**

This study classify the strategies students use with instrumental understanding, along with the difficulties that occur with negative integers to solve problems through number lines and open sentences. This fact leads researchers to eliminate gaps that occur through research questions:

* What are the strategies used by students who have instrumental understanding in solving negative integer arithmetic operations on number lines and open sentences?
* What are the difficulties of students having instrumental understanding in developing problem-solving strategies?

This is so that the results of research findings can contribute to science in determining what strategies are owned by students who are classified as having poor understanding. These findings are expected to be useful for long-term research, namely how students understand development or transition who have instrumental understanding to use relational understanding through settlement strategies.

**METHODOLOGY**

**Research Design**

This study tries to explore the strategic cases of students using instrumental understanding in solving the problem of negative integer operations through number lines and open sentences, as well as to identify the difficulties experienced by students in solving these problems, because the case study is used as the research design (Cresswell, 2012). The types of cases in this study include intrinsic cases of instrumental students who are prone to cognitive problems which are interesting cases to study (van de Walle, 1998).

**Participants**

A total of 23 students of 5th grade elementary school in Sukodono, Indonesia were involved in this study. These students have studied the material on negative integer operations. Among 23 students, four students were classified as having instrumental understanding. A student named Yulia (pseudonym) was chosen as the research subject because she was the only student who had good initial knowledge of negative numbers, had poor understanding of number lines, but had a representation strategy that was generally more varied. However, after being confirmed, Yulia could not explain the mathematical arguments in completing the numerical count operation on the number line compared to the other three students. Regarding the involvement of all participants, including Yulia, in this study. Researchers have received ethical clearance from the school committee which the students' guardians or parents also know.

**Data Collection and Instruments**

To answer the research questions, qualitative data collection was carried out by giving the number line model task instrument and interview instructions. The line model task in this study consisted of negative numbers task, lift task, and open sentence task.

***Negative Numbers Task***

The negative number task aims to task students' initial knowledge of negative numbers and number lines. The results of this task are used as one of the considerations for finding research subjects, so that it can be ascertained that the subjects obtained really have good initial knowledge. This task was modified by the instrument researcher by Bofferding (2014) in terms of identifying students' knowledge of negative numbers.

**Table 1**

*Indicators and Questions from the Negative Numbers Task*

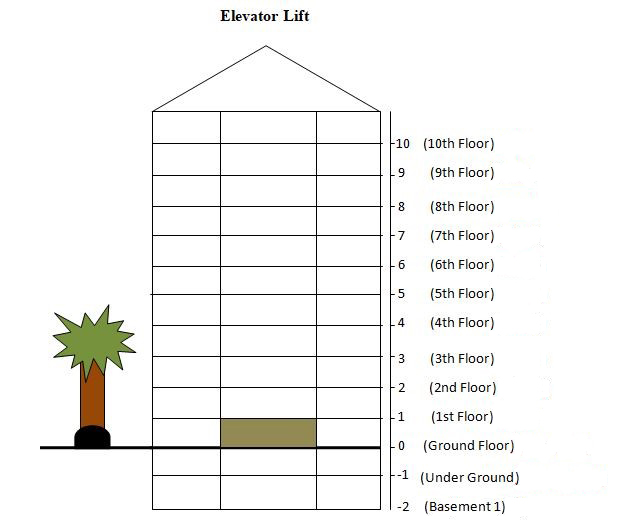
|  |  |
| --- | --- |
| **Indicators** | **Questions** |
| • Finding positive and negative integers on the number line | • Find the 10 numbers before 5 on the number line below. |
| • Determining the number of negative numbers that appear on the number line | • How many negative numbers are there on the number line? |
| • Comparing the number of integers | • Which is greater between 0 and -6? |
| • Summarizing the existence of an integer on the number line | • So what can you deduce from the number line? |
| • Drawing arithmetic operations on a number line. | • Draw the addition and subtraction of 0 + 2 + 5 - 3 on a number line. |
| • Describing arithmetic operations of positive integers on a number line, starting at zero. | • How many steps to move from number 0 to number 2? In which direction is it moving? |
| • Stating the number of steps for the displacement of each jump in the direction of the line number. | • How many steps to move from numbers 2 to 7 and numbers 7 to 4? In which direction is it moving? |
| • Defining other forms of arithmetic operations with results similar to previous arithmetic operations. | • Are there any other forms of addition and subtraction that have the same result? If there is, write down and draw the number line below, if not, give a reason! |
| • Drawing arithmetic operations for positive integers on a number line that does not start at zero. | • Draw the addition and subtraction of 2 + 5 - 3 = ⎕ on the number line. |
| • Describing the displacement steps nd direction of the number line. | • Describe the steps of displacement and number line's direction. |
| • Summarizing the number line at this number and the previous number. | • What is the conclusion after solving problems number 4 and 5? |
| • Declaring deficiency must draw a line starting at zero. | • Then is it possible to draw the form of addition and subtraction on number line that is not preceded by the number 0? |
| • Drawing negative integer arithmetic operations on a number line. | • Draw the addition and subtraction of 3 + 4 - 9 on a number line. |
| • Explaining understanding after completing arithmetic operations on a number line | • Try to explain what you understand after seeing the form of addition and subtraction. |

***Lift Task***

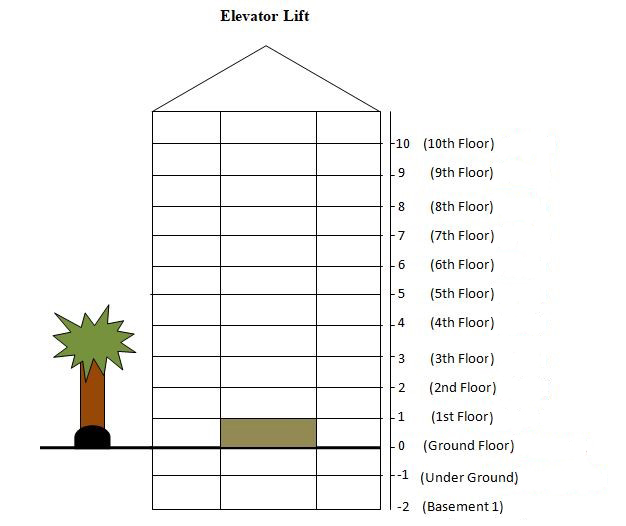
The lift task aims to identify student problem solving from negative number operations related to the elevator context. This assignment is modified by Bofferding (2014) which also trains and cultivates students' instrumental understanding of number lines in everyday contexts.

**Figure 1**

*Descriptions of Lift Task*



Rani was at the mall with her brother. Those on the 3rd floor are buying clothes. After that they went up 4 floors to buy food. After being full, they went home and went down 9 floors to the parking lot.



Rudi entered the lift on the 5th floor, in the elevator he met a workmate and he was asked to help deliver a letter to Susi on the 8th floor. When she arrived, Susi was in the parking lot taking something. Rudi went down 9 floors to meet Susi.

**Table 2**

*Indicators and Questions of Lift Task*

|  |  |
| --- | --- |
| **Indicators** | **Questions** |
| • Illustrating the problem of the story in a more contextual situation, namely in an elevator | • Describe the position of the related problem presented in the lift. |
| • Determining the final position according to the problem presented | • What floor is the parking lot located on? |
| • The arithmetic operation determines the form of the presented problem process | • Determine the arithmetic operations according to the form of the story. |
| • Illustrating the problem of the story in a more contextual situation, namely in an elevator or elevator78 | • Describe the position of the subject as the problem presents the story. |
| • Determining the final position of the story subject | • What floor do the two people meet? |

***Open Sentence Task***

This task aims to classify the strategies used by students who have instrumental understanding in completing a series of open sentences in negative integer arithmetic operations. These tasks are formed in two types, eg open sentences such as ⎕ + 6 = 2 and comparison tasks.

**Table 3**

*Indicators and Questions of Open Sentence Task*

|  |  |
| --- | --- |
| **Indicators** | **Questions** |
| • Illustrating negative integer operations in the form of an open sentence at the end. | • Illustrate the counting operation  5 - 9 = ⎕  -9 + 6 = ⎕ |
| • Illustrating the operation of a negative integer in the form of the initial open sentence. | • Illustrate the calculation operation  ⎕ + 6 = 2  ⎕ - 3 = -5  ⎕ + 3 = -6 |
| • Illustrating a negative integer operation in the form of an open sentence in the middle. | • Illustrate the counting operation  7 + ⎕ = 3  8 - ⎕ = 11  6 + ⎕ = 4 |
| • Declaring true or false statements of negative integer arithmetic operations. | • True or False of the calculation operation?  • Try to prove the arithmetic operation by drawing the number line.  • What is the reason you answered right or wrong?  9 + (-7) = 2  9 + -7 = 9 - 7  7-2 = 7- -2 |

***Interview Instruction***

Interview instructions were used to explore students' thinking about strategies and instrumental student difficulties in solving number line problems. This interview was conducted 2 times at different times, this was also intended as data triangulation to ensure the validity of the data. Students are asked to explain problem-solving steps to see the results of tasks that have been done previously while recording audio and visual of student activities. After completing the entire task and interviewing the researcher screening of the video whether it is clear or not. If there are answers that are different from the results the students wrote, the researcher will clarify this. Questions will be developed according to the answers expressed by students about the problem-solving process carried out by Polya in making plans and carrying out the analysis of the completion plan.

**Data Analysis**

All interview data were collected, audio recordings, and videos were transcribed and copies of student work were combined with each transcript. Data analysis procedures involving open, axial, and selective coding processes for continuous and qualitative data involve permanent comparative analysis between each category and new categories emerge (Cresswell, 2012).

Researchers conducted interviews and compiled transcripts of students' answers. To analyse the interview data and transcripts were reduced to fragments containing explanations of the students' main ideas. Data were coded, sorted, and read repeatedly to answer research questions. Data validity was ensured by determining accurate and complete data collection by managing written tasks and interview transcripts after recording.

**RESULTS AND ANALISES**

The results of the study were shown by students' learning on initial knowledge, namely Yulia's understanding of the context of the situation, strategies, and difficulties in solving problems involving the number line model.

**Yulia Understanding of The Context Situation**

Sometimes students still don't know the existence of negative numbers that actually exist in real situations, for that in the next task more researchers want to identify students' understanding of negative numbers in real situations through contextual problem elevators. Through contextual problems, they will understand more easily so that they can solve problems properly because they are related to everyday life. After the researcher knows the understanding and knowledge that students have, the students will be introduced to problems related to everyday life, in this case, negative integers that contextualize these problems in the real life of students. Students are introduced to the existence of negative numbers in real life students are the tools that usually make it easier for someone to get to a location quickly in a building or mall which is often called an elevator. Students illustrate problems that make up a story into a picture that resembles an elevator. Students illustrate by marking by shading in the appropriate position on the elevator problem image. Then students add a curved line next to the picture as a sign of displacement.

Yulia responds to the number line problem, that when someone uses the elevator and the process goes up, Yulia assumes it includes addition, while when the elevator goes down, Yulia assumes it includes subtraction. Through Yulia, contextual problems are easier to solve and she can also write down the form of arithmetic operations that occur according to the problem presented. The focus of this research is how Yulia's strategy in solving problems related to negative integers by solving using a number line. Questions are designed using open sentences. The researcher hopes that he can solve the problem with a strategy that suits the understanding of each student.

The strategy used by Yulia for this problem tends to use signs or lines as the position and direction of the perpetrators of movement in the elevator. Students successfully solve the problem about the story presented and students can write the form of addition and subtraction according to the problem. When students are able to solve the problem, the researcher hopes that students can solve the problem of counting negative integers using number lines.

**Yulia Strategies In-Line Numbers Problem**

This point will be explained in detail about Yulia as a research subject. Knowledge about negative numbers is categorized as Yulia. Students can determine 10 numbers under the number 5 in sequence and right on the number line, students can also say the number of negative numbers that appear on the number line if there are five numbers -5, -4, -3, -2, -1, students are also able to compare between 0 and -6 which is greater than 0 on the grounds that the number 0 has no negative sign, and students can also conclude that the right of the number that lies on the number line, the greater the farther to the left and vice versa the number that lies on the line numbers getting smaller and smaller.

The next problem is almost identical to the previous problem, it's just that the set of questions before the operation starts with a count of zero while in this problem it doesn't start with zero. After completing this and previous questions, students are expected to conclude. Students have been able to solve the problem well and are able to conclude between this problem and the previous question is the same result, students also conclude that the drawing operation on the number line does not have to start with the number 0. This last problem is an evaluation of the previous questions about how students understand the addition and subtraction of negative integers on the number line. Students can answer questions and draw number lines correctly, but when students are asked to write down what the students understand the problem is, they cannot be concluded.

**Yulia Strategies and Difficulties in Solving Numbers Line Problems**

In the previous negative numbers task and lift task, the students tended to model operations involving negative integers vertically on the number line. In the open sentence task, students are asked to illustrate horizontal number lines. Yulia already understands the problem, seen from the correct steps to solve it, but when asked to make addition and subtraction forms that are almost the same as before with the same results, she cannot solve them. It appears in the blank on the student answer sheet. This is one of the reasons researchers found that Yulia is classified as instrumental because she can perform mathematical steps but has not been able to connect previous concepts with new concepts. In this case the researcher deliberately started the operation of counting with the number 0 so that the future Yulia could compare with the next question. When conducting an interview with Yulia, the researcher submitted an task sheet that had been done to the previous students in the form of a series of open sentences. Students are interviewed about how and why students solve problems so that researchers know the strategies used.

The first two problems raised involved addition and subtraction of negative numbers with open lines as a result of addition or subtraction operations. The first task is task 5-9 = ⎕ and followed by -9 + 6 = ⎕ students must complete the task correctly. Yulia's first response resolves 5-9 = ⎕ by jumping 9 times from number 5 to the left or towards the smaller number value. The researchers assumed the students used a countdown strategy. Students' understanding of this problem assumes the reduction will move to the left. If the task before the student moves to the left then the task -9 + 6 = ⎕, Yulia completes the task by jumping to the right of the number -9 jumping 6 steps, the student assumes that because it is added it will move to the right. Students' strategies regarding the two problems were the same when students were interviewed a second time. Students use the definition of a number line and the notion of addition as a move to the right (forward) on the drawn number line and reduced as a move to the left (backward). Researchers think so because of the plus it will move to the right. Students' strategies regarding the two problems were the same when students were interviewed a second time. Students use the definition of a number line and the meaning of addition as a move to the right (forward) on the drawn number line and reduced as a move to the left (backward). Students think that because it is added, it will shift to the right. Strategy = students regarding the two same problems when students are interviewed for the second time. Students use the number line and the notion of addition as a move to the right (forward) on the number line drawn and reduced as a move to the left (backward).

The second problem lies in the open sentence in the middle of the task such as 7 + ⎕ = 3, 8 - ⎕ = 11 and 6 + ⎕ = 4. Students have completed all three tasks correctly. In the fourth task, students were asked to prove the correctness or error of tasks such as 9 + -7 = 2, 9 + -7 = 9-7, and 7-2 = 7- -2. The first problem students solve it well, the second problem the answers are wrong but the strategy is right, the third problem students answer wrong.

The third task is task ⎕ + 6 = 2, ⎕ - 3 = -5 and ⎕ + 3 = -6. Students must complete the task ⎕ + 6 = 2 correctly. Students start the strategy from the results and go back 6 steps because the open sentence in front of the student's understanding is slightly different from before. When there is addition and students start from the results of the backward movement. On tasks ⎕ - 3 = -5 and ⎕ + 3 = -6 students still find it difficult and finally the students' answers and strategies are still wrong. Students start from the middle to the task ⎕ - 3 = -5, which is from number 3 then left to number -5, to find out the results of students calculating the distance between 3 and -5. Students start from the results then add 3 moves to the right. In this task students forget the concepts that have been worked on in the point ⎕ + 6 = 2. Researchers have differentiated Yulia's answers based on the right and wrong strategies when interviewed (Table 4a and 4b).

**Table 4a.**

*Yulia's Correct Strategy and Response*

|  |  |  |  |
| --- | --- | --- | --- |
| **No** | **Problems** | **Strategies** | **Interview responses** |
| 1a | 5-9 = ⎕ | Count down | • "From the number five, if you lower it to the left, counting from steps five to nine, the result was negative four" why to the left? "Because if it was reduced to the left". |
|  |  |  | • "Five minus nine" refers to student number five then moves left nine steps, the result was negative four "Why moved left?" minus. |
| 1b | -9 + 6 = ⎕ | Calculating Forward | • "From negative nine plus six, one, two, ...., six. The result was negative three." "Right to maturity plus" |
|  |  |  | • “From negative nine plus six. The result was negative three. "Why step right? Because added" were you sure you answered correctly? "Certain" |
| 2a | 7 + ⎕= 3 | Behind Start | • The student "Three plus seven results in a negative four" pointed to the three then shifts to the left seven steps. Then the result was negative four. The student's concept changed at the beginning when the student stated that if the addition moved forward, in this case the student actually moved backward. |
|  |  |  | • "Three minus seven negative answers four" Why did he go left? "Due to minus". |
| 2b | 8 - ⎕= 11 | Count down | • "From the number eight added eleven, one, two, …., eleven. The result was negative three. ”The student's concept changed at the beginning of the student stating that if added it moved forward but in this case the student actually moved backwards. |
|  |  |  | • "Eight minus eleven, becomes negative three" why gone left? "Due to minus" |
| 2c | 6 + ⎕= 4 | Behind Start | • The student "four plus six results in negative two" points to the number four then moves left six. The result was negative two. The student's concept changed at the beginning when the student stated that if the addition moved forward, in this case the student actually moved backward. |
|  |  |  | • "Four minus six was negative two." Why gone left? "Because it was reduced." |
| 3a | ⎕ + 6 = 2 | Behind Start | • "Because the numbers were almost non-existent, the numbers start behind". Student pointed to number two then moved to the left for six steps and finally stop at negative number four. So the result was negative four. |
|  |  |  | • "From number two the result was negative four minus six" why was it reduced? |
| 4a | True or false:  9 + -7 = 2 | Behind Start | Student answered correctly.  • From number two student began to draw a line number from the number two rungs to the left of nine rungs and ended at a negative number seven. But unable to explained why the student had answered correctly "didn't knew, grade was calculated beforehand" |
|  |  |  | Student answered correctly.  "From the negative number seven plus nine the result was two" why did you answer correctly? "Because the result was the same as and because I had calculated it" |
| 4b | True or false:  9 + -7 = 9-7 | Calculating Forward | Student answered incorrectly.  • Solution 9 + -7 "of negative seven. The result was two" student starting at the negative number seven and then progressing to nine steps and ending at two.  • 9-7 completion, "nine" refers to student number nine then taken seven steps back and ended at two. The answer should be corrected because the number line represented both. But the student answered incorrectly. "Because I had counted but I slipped" |
|  |  | Count down | Student answered incorrectly.  • Solution 9 + -7 "of negative seven plus nine, a result of two"  • The solution was 9-7 made 2.  Why were line numbers 9 + -7 going to the right and line numbers 9-7 going to the left?  • "Because 9 + -7 was added, while 9 -7 was subtracted.  • Then the reason you answered wrong? "Because I have counted" |

**Table 4b.**

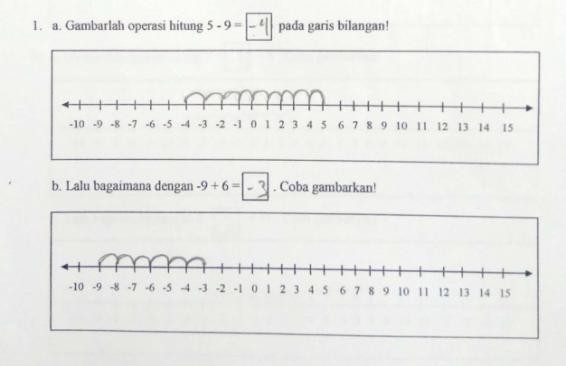
*Yulia's Wrong Strategy and Response*

|  |  |  |  |
| --- | --- | --- | --- |
| **No** | **Problems** | **Strategies** | **Interview responses** |
| 3b | ⎕ - 3 = -5 | Calculating Distance | • "I was the beginning of the numbers." The student points to number three and then moved left to negative number five. "I forgot". Student calculated the distance or jump from three to five negative numbers. The number of jumps is eight. |
|  |  |  | • Student forgot where to start. "Student ranging from negative five plus eight to number three" why added? "I tried" |
| 3c | ⎕ + 3 = -6 | Behind Start | • "Of the negative six plus, three negative results was three" sure? "Not sure, it's hard" |
|  |  |  | • "From the negative six plus three the result was negative three" why him right? "Due to plus". |
| 4c | True or False:  7-2 = 7 - -2 | Count down | Student answered Yes.  • The 7-2 solution, "seven minus two resulted was five". The student points to number seven and steps back two steps then stops at number five.  Solution 7- -2, the student pointed to number seven and moves the pencil to number five, the student steps backed two steps. "Result of five".  • "Correct answer" Were you sure? "I didn't knew, but the result was the same, taken the seconds and I had counted it" |
|  |  |  | Student answered correctly.  • 7-2 settlement, "of seven jumped left twice, a result of five"  • 7- -2 completion "of seven jumped to the left of two, result of five". Why did you answer correctly? Because the result was the same and I have calculated it. |

Yulia suspects that the subtraction or addition is the direction of the shift in the calculation operation steps. Look at the strategy used in solving the problem 5 - 9 = ⎕ and -9 + 6 = ⎕. Student perceive addition as a step forward or a step to the right. Whereas for subtraction student assume that reduction directs their steps to the left. Figure 2 shows the results of Yulia's task.

**Figure 2**

*Yulia's Task Results*



Yulia suspects that the subtraction or addition is the direction of the shift in the calculation operation steps. Look at the strategy used in solving the problem 5 - 9 = ⎕ and -9 + 6 = ⎕. Student perceive addition as a step forward or a step to the right. Whereas for subtraction student assume that reduction directs their steps to the left.

To solve the problem in number 1 "a" and number "b", it can be seen that the student draw the line from numbers -4 to number 5 and numbers -9 to number -3. To clarify the strategy used, the researcher interviewed student about the strategies used in point number 1 "a" and point "b". To solve the problem "a" student step back (move to the left) from number 5 moves to the left as much as 9 steps and ends at -4. Student reason that subtraction moves left or back. So the researchers concluded that the strategy used by student was countdown. In solving the problem of point "b" the student stepped forward (moved to the right) from the number -9 to the right for 6 steps and ended up in the number -3. The student reasons that the addition will move to the right. So that the researchers concluded that the strategy used was calculating forward. Solving problem number 2 "a" ⎕ + 6 = 2, student start the solution step from the result or from behind. The student starts with the number 2 then moves to the left for 6 steps because the sentence the front number is empty, finally stops at -4. The reason for reducing student is that student are experimenting.

In task number 2 points "a" and "b", the student' answers were not correct. Point "b" ⎕- 3 = -5, student use the strategy of calculating the distance. Student start from the middle, which is 3 and move left to the number -5. Student calculate the jump distance between number 3 to number -5. The distance is 8. Point "b" ⎕ + 3 = -6, student starts from behind or results. Student just try to find the answer, student start from -6 then add up 3. The result is -3. Student are not sure of the answer.

The solution to the problem number 3 points "a", "b", and point "c" has been resolved properly. Problem 7 + ⎕ = 3 student start the strategy from the results section. Student starting at number 3 plus 7 result in -4. Student move to the left in 7 steps. Student' understanding of addition will shift to the right as this task changes. In problem 8 - ⎕ = 11 student count backward from number 8 by 11 steps backward. The result is -3. In problem 6 + ⎕ = 4, student starting at 4 plus 6 give -2. Initially the student responded to the way they were added and moved to the left, but in the interview responses the two student stated that the reason was reduced and moving to the left was reduced.

In task number 4 the "a" strategy the student used was correct. Student answered correctly. Student starting from number 2 move to the left as many as 9 so that they end in -7. However, in the second interview, student started with the number -7 plus 9 and ended at number 2. Student did not know the reason for the solution in the first interview. In solving the problem at 4 points "b" 9 + -7 = 9-7, student use counting forward and backward strategies. Student start the strategy from -7 to the next 9 steps and end at number 2, for the counting operation 9-7 student count backwards from the number 9 backwards by 7 steps and finally end up in number 2. However, the student still answer that the comparison is wrong. Even though the results are stated to be the same. Thus, the researcher concluded that the student' answers were wrong, but the strategy used was correct.

Problem solving at 4 points "c" 7-2 = 7 - -2, student use a countdown strategy. Student start the 7 minus 2 number strategy, student start from 7 then walk backward two steps the result is 5. As for the task in the counting operation 7 - -2 student start from number 7 and go back 2, the result is 5. The completion strategy used by student is lacking right. Each of Yulia's strategy difficulties in answering were analysed (See Table 5).

**Table 5**

*Yulia's Difficulty is Based on the Strategy Used*

|  |  |  |
| --- | --- | --- |
| **Strategies** | **Problems** | **Student difficulties with the strategies used** |
| Count down | 5-9 = ⎕  8 - ⎕ = 11  True or false:  9 + -7 = 9-7  True or false:  7-2 = 7 - -2 | Student do not experience problems on tasks where the open sentence is behind. Student feel confused about the change in their counting operation signs when completing a task where the open sentence is in the middle, resulting in a difference in arguments between the oral statement and the strategy depicted on the number line. For example, the student states 8 + 11, the addition always goes forward, but the student moves the pencil backwards. In the problem of comparison, student have difficulty in counting operations, because the addition symbol is lined with negative symbols. Student ignore one of these symbols. Student also have difficulty explaining the reasons for the completion steps. |
| Count forward | -9 + 6 = ⎕  True or False: 9 + -7 = 9-7 | Student do not have difficulty on tasks where the open number is behind. The strategies and reasons that the student gave were correct. In the problem of comparison, student have difficulty in counting operations, because the addition symbol is lined with negative symbols. Student ignore one of these symbols. Student also have difficulty explaining the reasons for the completion steps. |
| Behind Start | ⎕ + 6 = 2  ⎕ + 3 = -6  7 + ⎕ = 3  6 + ⎕ = 4  True or False: 9 + -7 = 2 | Problems that arise in tasks where the opening sentence is at the beginning, student have difficulty starting the strategy. Student then start the strategy from the results section. In addition, the difficulty also lies in the change in the operation markers due to their changing location. Sometimes student state the wrong statement and influence the strategy to answer results. |
| Calculating distances | ⎕ - 3 = -5 | The problem that arises lies in the use of count operation marks. These inaccuracies can affect the strategy and result in wrong answers. |

The student' difficulties are described in each of the strategies student undertake. In the strategy of counting down or counting forward, difficulties occur when solving comparison problems. Student who complete the task have difficulty subtracting or adding symbols that are close to negative symbols. In the behind the start strategy, the difficulty of student starting to complete the task is because the open sentence is in the middle. Meanwhile, in the strategy of calculating the distance, difficulties occur in the use of operating marks.

**Discussion**

Student' views about the existence of negative numbers or numbers smaller than zero are different. Like the initial task given to research subjects, student are asked to name 10 numbers under the number 5 on the number line based on knowledge. The research subjects can mention regularities well. In contrast to the others there are still student who cannot say. This task raises the number line with the numbers 5, 6, 7, 8, 9, 10. To the left of the numbers 5, 10 are the squares left blank by the researcher. Student are asked to name 10 numbers in the box that are intentionally left blank. Students who do not have good knowledge of negative numbers will name them randomly. Even though the number line should also be a position code. Representation according to (Fischer, 2003) is automatically associated with left space, the coding and representation of the number line can be adjusted according to the numerical context and according to the problem.

When student are asked to compare these numbers, the researcher wants to know the extent to which student understand negative numbers. Student are required to compare numbers 0 and -6 which are bigger, it turns out that there are those who answer -6 which is greater than 0. However, the subject has good knowledge and understanding of negative numbers, it can be seen from the student' conclusions. Thus, with good knowledge and understanding, it will be the basis for further student to solve problems related to negative numbers. Ask students why and use number line representations, help students make connections between problems, come up with students 'answers and strategies, describe students' thinking (Aqazade et al., 2017).

About the elevator problem as a contextual problem. Many students do not have direct experience, but are interested in the context of the problem (Yilmaz, Akyuz, & Stephan, 2019). One of the characteristics that a good contextual problem of the problem will bring about mathematical interpretation and strategy solutions, an informal strategy that serves as a development into more formal mathematics (Widjaja, 2013). Through an elevator or elevator, in addition to knowing the existence of actual negative numbers students are also introduced to the use of number lines, these numbers are only vertical lines because of the shape of the elevator. The ability of students to relate mathematics to real life is very important so that students know the usefulness of mathematics in real life. The ability of students to make connections between mathematics and real life is very important because it relates to the introduction of mathematics in everyday life, the accuracy of the introduction to develop certain conceptual and mathematics learning (Altay, Yalvaç, & Yeltekin, 2017).

Several implications are related to the researcher's findings. First, how to draw a number line by the research subject provides a different understanding in compiling and implementing a completion plan. The number line that has been made instrumental-only describes the number line without being emphasized by the large number of sequences. The freedom of students to formulate and implement a plan causes differences in the symbols or lines students use when using number lines as a tool to help solve problems. Second, different types of problems affect students' understanding of the concept of the material. Understanding the concept of student material can be seen from the way the students use the solution. Tasks for preliminary studies with tasks to study strategies provide insight into students' conceptual understanding. The task is designed so that students remain directed at their understanding of using number lines, students' understanding of the concept of the material is more striking.

Regarding the number of students who completed the task, they remembered more formulas in finding results so that students did not know the actual concept of the material. Even though with this type of quantity students are given more freedom in using strategies according to their respective understandings and ways of thinking. Students 'freedom of thought to operate number lines provides insight into the development of students' thinking. So that the strategy used will be in accordance with student thinking. Students do not have to start from zero in arithmetic operations to solve negative number tasks using number lines.

In fact, number lines are introduced in the student book, but indirectly students are introduced to a set of rules that must be followed to get the correct answer. Although the research subjects used number lines to solve negative integer problems, it seems that it can resemble the size of a textbook if using number lines, but in the process students develop very different rules. Students are free to start where in using the number line to complete the task being asked, especially in this case the arithmetic operation task is in the form of an open sentence. So if students have to complete the task by giving a point at the beginning as the starting point for moving the step number line and students will find it difficult if they open a sentence to find the number in front or begin to purposely leave it blank. Therefore the number line can be used as an appropriate tool so that students can provide strong reasons for negative numbers (Bishop, 2011). Negative integer count operations with number lines and open sentences can be used as a teacher's solution so that students are better able to understand the concept of negative integers and can explore students in problem-solving strategies that are in accordance with student thinking. Students' differences in interpreting negative numbers will affect the strategies students use in solving problems. The gap that occurs when introducing negative integers is that students interpret negative numbers in various ways according to students 'knowledge of numbers, so that it can affect students' problem solving strategies in solving negative integer problems (Aqazade et al., 2017). Students' differences in strategy use depend on their contrast in analyzing the problem gives them different benefits in interpreting the problem (Aqazade et al., 2017; Bofferding & Wessman-Enzinger, 2017).

Student difficulties occur because students do not understand the difference between the signs of operation and positive or negative integer signs. Students should understand the problem first when completing the elusive negative integer concept task, even though students have no difficulty placing negative numbers on the number line but will have problems comparing values (Cetin, 2019). Giving tasks at the basic education level that are difficult to make students tired and pessimistic so that students cannot solve problems (Stein & Burchartz, 2006). The difficulty of students in solving these problems is possible because of the cognitive barriers, tasks that are too difficult at the beginning can become students' cognitive obstacles. The cognitive barriers to understanding and knowledge that allow students to learn preclude negative integers (Bishop et al., 2014). Sometimes the process of perceptual memory, there is a possibility that students' perceptions of concepts can be misunderstood which can cause cognitive barriers (Fischer, 2003).

**CONCLUSIONS**

Researchers have found various strategies, these strategies look at the instrumental ability of students in linking existing material concepts to new concepts. Students who have instrumental understanding are still unable to connect existing concepts to new concepts. Instrumental students have been able to solve the problem, but when asked for an explanation of the reasons for completing the steps, students still cannot be held accountable. Strategies for students who have an instrumental understanding include counting down, counting forward, calculating distance and starting backwards. Generally, students' difficulties occur because they do not understand the difference between positive or negative operating signs and integer signs and decide the type of strategy used in problem solving.

**AUTHORS’ CONTRIBUTIONS STATEMENTS**

MFA and MDKW conceived the presented idea, research methodology, and writing - review & editing. MF collected the data. HER and NQW conducted data analysis. All authors actively participated in the discussion of the results, reviewed and approved the final version of the work.

**DATA AVAILABILITY STATEMENT**

The data that support the results of this study will be made available by the corresponding author, MFA, upon reasonable request.

**REFERENCES**

Altay, M. K., Yalvaç, B., & Yeltekin, E. (2017). 8th grade student’s skill of connecting mathematics to real life. *Journal of Education and Training Studies*, *5*(10), 158–166.

Anderson, J. A. (1996). Experiments to investigate instrumental and relational understanding among undergraduates. *International Journal of Mathematical Education in Science and Technology*, *27*(6), 813–819.

Aqazade, M., Bofferding, L., & Farmer, S. (2017). Learning integer addition: Is later better? *North American Chapter of the International Group for the Psychology of Mathematics Education, Paper Presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, 219–226. Indianapolis.

Bishop, J. P. (2011). First graders outwit a famous mathematician. *Teaching Children Mathematics*, *17*(6), 350–358.

Bishop, J. P., Lamb, L. L., Philipp, R. A., Whitacre, I., Schappelle, B. P., & Lewis, M. L. (2014). Obstacles and affordances for integer reasoning: An analysis of children’s thinking and the history of mathematics. *Journal for Research in Mathematics Education*, *45*(1), 19–61.

Bofferding, L. (2014). Negative integer understanding: Characterizing first graders’ mental models. *Journal for Research in Mathematics Education*, *45*(2), 194–245. https://doi.org/10.5951/jresematheduc.45.2.0194

Bofferding, L., Aqazade, M., & Farmer, S. (2017). Second graders’ integer addition understanding: Leveraging contrasting cases. *Proceedings of the 39th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, 243–250. Indianapolis: Hoosier Association of Mathematics Teacher Educators.

Bofferding, L., & Wessman-Enzinger, N. (2017). Subtraction involving negative numbers: Connecting to whole number reasoning. *The Mathematics Enthusiast*, *14*(1,2,3), 241 – 262.

Cengiz, C., Aylar, E., & Yildiz, E. (2018). Intuitive development of the concept of integers among primary school students. *International Electronic Journal of Elementary Education*, *11*(2), 191–199. https://doi.org/10.26822/iejee.2019248599

Cetin, H. (2019). Explaining the concept and operations of integer in primary school mathematics teaching: opposite model sample. *Universal Journal of Educational Research*, *7*(2), 365–370.

Cresswell, J. W. (2012). Planning, conducting, and evaluating quantitative and qualitative research. *Educational Research*, *10*, 1–12. https://doi.org/10.4135/9781483349435

Fischer, M. H. (2003). Cognitive representation of negative numbers. *Psychological Science*, *14*(3), 278–282.

Kent, L. B. (2000). Connecting integers to meaningful contexts. *Mathematics Teaching in the Middle School*, *6*(1), 62–66.

Rabin, J. M., Fuller, E., & Harel, G. (2013). Double negative: the necessity principle, commognitive conflict, and negative number operations. *The Journal of Mathematical Behavior*, *32*(3), 649–659. https://doi.org/10.1016/j.jmathb.2013.08.001

Skemp, R. R. (2006). Relational Understanding and Instrumental Understanding. *Mathematics Teaching in the Middle School*, *12*(2), 88–95.

Stein, M., & Burchartz, B. (2006). The invisible wall project: Reasoning and problem solving processes of primary and lower secondary students. *Mathematical Thinking and Learning*, *8*(1), 65–90.

Tambychik, T., & Meerah, T. S. M. (2010). Students’ difficulties in mathematics problem-solving: What do they say? *Procedia-Social and Behavioral Sciences*, *8*, 142–151. https://doi.org/10.1016/j.sbspro.2010.12.020

van de Walle, J. A. (1998). *Elementary and middle school mathematics: Teaching developmentally*. United Kingdom: Addison-Wesley Longman, Inc.

Wessman-Enzinger, N., & Bofferding, L. (2014). Integers: Draw or discard! game. *Teaching Children Mathematics*, *20*(8), 476–480.

Whitacre, I., Bishop, J. P., Lamb, L. L. C., Philipp, R. A., Schappelle, B. P., & Lewis, M. (2011). Integers: History, textbook approaches, and children’s productive mathematical intuitions. *Proceedings of the 33rd Annual Conference of the North American Chapter of the International Group for the Psychology of Mathematics Education*, 913–920.

Whitacre, I., Bishop, J. P., Lamb, L. L. C., Philipp, R. A., Schappelle, B. P., & Lewis, M. L. (2012). Happy and sad thoughts: An exploration of children’s integer reasoning. *The Journal of Mathematical Behavior*, *31*(3), 356–365. https://doi.org/10.1016/j.jmathb.2012.03.001

Widjaja, W. (2013). The use of contextual problems to support mathematical learning. *Indonesian Mathematical Society Journal on Mathematics Education*, *4*(2), 157–168.

Yilmaz, A., Akyuz, D., & Stephan, M. (2019). Middle grade students’ evoked concept images of number line models and their calculation strategies with integers using these models. *International Journal of Education in Mathematics, Science and Technology*, *7*(1), 93–115.