Capturing Strategies and Difficulties in Solving Negative Integers: A Case Study of Instrumental Understanding

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ABSTRACT

Background: Until recently, many elementary school students made mistakes in solving negative integer problems, especially for students who had instrumental understanding. Studies that focus on strategies and difficulties faced by students can be a solution to finding student weaknesses so that students achieve a better level of understanding. **Objectives:** The objective of this study was to classify cases of 5th grade elementary school students who have instrumental understanding in solving negative integers tasks with a focus on strategies and difficulties faced with number line models. **Design:** This study uses a case study research design. **Setting and Participants:** The study involved 5th grade elementary school students in Sukodono, Indonesia who had studied the material on negative integer operations. Participants who were selected to be explored further were those who had instrumental understanding and had a more varied representation of strategies. **Data collection and analysis:** Qualitative data collection is done by providing a number line model task and interview instructions. Data analysis was performed by means of comparative analysis, namely by comparing all data collected, including the transcribed audio and video recordings. **Results:** Researchers found various types of strategies that experienced difficulties to cognitive completion by students with instrumental understanding, along with difficulties in solving number problems. **Conclusion:** The implications of this study are very useful for further research and lifelong learning practices, especially in the process of how to deal with elementary school students who have difficulty in constructing knowledge and the concept of negative integer arithmetic operations.

Keywords: Strategy and Difficulty; Representation; Instrumental Understanding; Negative Integer; Case study.

Capturando Estratégias e Dificuldades ia Resolução de Números Inteiros Negativos: Um Estudo de Caso de Compreensão Instrumental

RESUMO

Contexto: Até recentemente, muitos alunos do ensino fundamental cometeram erros ao resolver problemas de números inteiros negativos, especialmente para alunos que tinham compreensão instrumental. Estudos que enfocam as estratégias e dificuldades enfrentadas pelos alunos podem ser uma solução para encontrar as fragilidades dos alunos para que eles alcancem um melhor nível de compreensão. **Objetivo:** O objetivo deste estudo foi classificar casos de alunos do 5º ano do ensino fundamental que possuem compreensão instrumental na resolução de tarefas de inteiros negativos com foco nas estratégias e dificuldades enfrentadas com modelos de linha numérica. **Design:** Este estudo usa um projeto de pesquisa de estudo de caso. **Cenário e participantes:** O estudo envolveu alunos do 5º ano do ensino fundamental em Sukodono, Indonésia, que estudaram o material sobre operações de número inteiro negativo. Os participantes selecionados para serem explorados mais detalhadamente eram aqueles que tinham compreensão instrumental e uma representação mais variada das estratégias. **Coleta e Análise de Dados:** A coleta de dados qualitativos é feita fornecendo uma tarefa de modelo de linha numérica e instruções de entrevista. A análise dos dados foi realizada por meio de análise comparativa, nomeadamente comparando todos os dados recolhidos, incluindo as gravações de áudio e vídeo transcritas. **Resultados:** Os pesquisadores encontraram vários tipos de estratégias que enfrentaram obstáculos para a conclusão cognitiva por alunos com compreensão instrumental, juntamente com dificuldades na resolução de problemas numéricos. **Conclusão:** As implicações deste estudo são muito úteis para futuras pesquisas e práticas de aprendizagem ao longo da vida, especialmente no processo de como lidar com alunos do ensino fundamental que têm dificuldade em construir conhecimento e o conceito de operações aritméticas inteiras negativas.

Palavras-chave: Estratégia e dificuldade; Representação; Compreensão instrumental; Número inteiro negativo; Estudo de caso.

INTRODUCTION

Elementary school students understand the concept of negative integers is not as easy as understanding positive integers. According to Bofferding (2014), students' difficulties in solving integer problems are due to their lack of prior knowledge of negative number knowledge. When the concept of the first negative integer is given it will seem abstract to students, as a result it will conflict with previous knowledge of integer arithmetic operations (Cengiz et al., 2018). Relates to solving problems involving negative integers, Bofferding et al. (2017) stated that students who are not accustomed to using negative numbers or students who rely on the concept of positive integers will often ignore the use of negative signs. Likewise, solving problems that involve number operations often overgeneralizes based on previous students' experiences in operating positive integers. This results in a misconception, where students think that addition is always bigger, while subtraction is always smaller (Whitacre et al., 2012).

Rabin et al. (2013) said that student misconceptions continued, students would face many challenges when facing new mathematical ideas that came from previous schemes. Therefore, several studies have tried to bridge students to make it easier to understand the concept of negative integers, including through mental models (Bofferding, 2014), through the problem sequence (Bishop et al., 2014), through problems have different cases (Aqazade et al. al. 2017), and through the opposite model (Cetin, 2019). However, this research has not emphasized the difficulties faced by students in solving the representation-based problems used. Tambychik and Meerah (2010) explains the understanding of educators or researchers about students 'difficulties in solving problems which can be seen from the representation of answers which is very important before developing students' thinking skills.

The number line model provides visual assistance so that students can check the relationships between integers with each other (Kent, 2000; Whitacre et al., 2011). In addition, Wessman-Enzinger and Bofferding (2014) stated that number lines were introduced to students to facilitate the transition of students' understanding of negative numbers. Therefore, in this study the number line is used as an intermediary for students to solve integer operation problems. Through the number line, the representation of students in adding and subtracting positive or negative numbers will be seen, so that the researcher is able to classify the types of strategies used and identify the difficulties that occur in students in developing strategies

The solution to a problem depends on the student's understanding. Skemp (2006) argues that there are two types of understanding, namely instrumental and relational understanding. The difference between instrumental and relational understanding is the difference in the measure of mathematics and

the reasons for the decision making why the action is taken. Students who are classified as instrumental understanding are students who are able to perform mathematical steps, but students do not know why the measuring instrument is used. Conversely, students who are classified as having a relational understanding are students who are able to take mathematical steps and know the reasons for doing these actions. If the teacher does not find a solution why students who have instrumental understanding or students who have less relational understanding tend to fail in math problems, then this will hamper students' critical attitudes in solving problems (Anderson, 1996).

Several previous studies by Bofferding et al. (2017) explain the strategies students use when facing problems there are differences. Bishop et al. (2014) characterized students' strategies that were classified as relational understanding through a number line model and a series of open sentences. However, previous studies have not characterized the strategies and difficulties used by elementary school students who have instrumental understanding in solving the problem of addition and subtraction of integers through the number line model.

Problems of Study

This study classify the strategies students use with instrumental understanding, along with the difficulties that occur with negative integers to solve problems through number lines and open sentences. This fact leads researchers to eliminate gaps that occur through research questions:

- What are the strategies used by students who have instrumental understanding in solving negative integer arithmetic operations on number lines and open sentences?
- What are the difficulties of students having instrumental understanding in developing problem-solving strategies?

This is so that the results of research findings can contribute to science in determining what strategies are owned by students who are classified as having poor understanding. These findings are expected to be useful for long-term research, namely how students understand development or transition who have instrumental understanding to use relational understanding through settlement strategies.

METHODOLOGY

Research Design

This study tries to explore the strategic cases of students using instrumental understanding in solving the problem of negative integer operations through number lines and open sentences, as well as to identify the difficulties experienced by students in solving these problems, because the case study is

used as the research design (Cresswell, 2012). The types of cases in this study include intrinsic cases of instrumental students who are prone to cognitive problems which are interesting cases to study (van de Walle, 1998).

Participants

A total of 23 students of grade 5 Elementary School in Sukodono, Indonesia were involved in this study. These students have studied the material on negative integer operations. Among 23 students, four students were classified as having instrumental understanding. A student named Yulia (pseudonym) was chosen as the research subject because she was the only student who had good initial knowledge of negative numbers, had poor understanding of number lines, but had a representation strategy that was generally more varied. However, after being confirmed, Yulia could not explain the mathematical arguments in completing the numerical count operation on the number line compared to the other three students. Regarding the involvement of all participants, including Yulia, in this study. Researchers have received ethical clearance from the school committee which the students' guardians or parents also know.

Data Collection and Instruments

To answer the research questions, qualitative data collection was carried out by giving the number line model task instrument and interview instructions. The line model task in this study consisted of negative numbers task, lift task, and open sentence task.

Negative Numbers Task

The negative number task aims to task students' initial knowledge of negative numbers and number lines. The results of this task are used as one of the considerations for finding research subjects, so that it can be ascertained that the subjects obtained really have good initial knowledge. This task was modified by the instrument researcher by (Bofferding, 2014) in terms of identifying students' knowledge of negative numbers. Table 1 below shows the negative numbers task indicators and questions.

Commented [F7]: Editor's Revision Points
1.Needs to include appropriate approval for independent ethical review boards in text papers.

3.If the research does not involve the Ethics Committee, a related explanation in the paragraph or footnote in the Methodology section is required.

Table 1

Indicators and Questions from the Negative Numbers Task

No	Indicators	Questions
1	• Find positive and negative integers on the number line	• Find the 10 numbers before 5 on the number line below!
	• Determine the number of negative numbers that appear on the number line	• How many negative numbers are there on the number line?
	• Compares the number of integers	• When compared, 0 and - 6 which is greater?
	• Summarize the existence of an integer on the number line	• So what can you deduce from the number line?
2	• Draw arithmetic operations on a number line.	• Draw the addition and subtraction of 0 + 2 + 5 - 3 on a number line!
	• Describe arithmetic operations of positive integers on a number line, starting at zero.	• How many steps to move from number 0 to number 2? In which direction is it moving?
	• State the number of steps for the displacement of each jump in the direction of the line number.	• How many steps to move from number 2 to number 7? In which direction is it moving?
		• How many steps to move from number 7 to number 4? In which direction is it moving?
	• Define other forms of arithmetic operations with results similar to previous arithmetic operations.	• Are there any other forms of addition and subtraction that have the same result as the problem above? If there is, write down and draw the number line below, if not, give a reason!
3	• Draw arithmetic operations for positive integers on a number line that does not start at zero.	• Draw the addition and subtraction of 2 + 5 - 3 = ☐ on the number line!
	• Describe the displacement steps and direction of the number line in the description.	• Describe the steps of displacement and the direction of the number line!
	• Summarize the number line at this number and the previous number.	• So, the conclusion after solving problems number 4 and 5 is …
	• Declare deficiency must draw a line starting at zero.	• Then is it possible to draw the form of addition and subtraction on a number line that is not preceded by the number 0 (zero)?
4	• Draw negative integer arithmetic operations on a number line.	• Draw the addition and subtraction of 3 + 4 - 9 on a number line!
	• Explain understanding after completing arithmetic operations on a number line	• Try to explain what you understand after seeing the form of addition and subtraction on the number line above!

Lift Task

The lift task aims to identify student problem solving from negative number operations related to the elevator context. The aim of this task is to train and cultivate students' instrumental

understanding of number lines in everyday contexts. This task was modified by an instrument researcher by Bofferding (2014). Figure 1 and Table 2 show the description and form of the lift task.

Figure 1

Descriptions of Lift Task

1. Rani was at the mall with her brother. Those on the 3rd floor are buying clothes. After that they went up 4 floors to buy food. After being full, they went home and went down 9 floors to the parking lot.

2. Rudi entered the lift on the 5th floor, in the elevator he met a workmate and he was asked to help deliver a letter to Susi on the 8th floor. When she arrived, Susi was in the parking lot taking something. Finally Rudi went down 9 floors to meet Susi.

Table 2

Indicators and Questions of Lift Task

No	Indicators	Questions
1	• Illustrating the problem of the story in a more contextual situation, namely in an elevator or elevator	• Describe the position of the related problem presented in the lift!
	• Determine the final position according to the problem presented	• What floor is the parking lot located on?
	• The arithmetic operation determines the form of the presented problem process	• Determine the arithmetic operations according to the form of the story!
2	• Illustrating the problem of the story in a more contextual situation, namely in an elevator or elevator	• Describe the position of the subject as the problem presents the story in the elevator or elevator below!
	• Determine the final position of the story subject according to the problems presented	• On what floor do the two people meet?
	• The arithmetic operation determines the form of the presented problem process.	• Determine the arithmetic operations according to the form of the story!

Open Sentence Task

This task aims to classify the strategies used by students who have instrumental understanding in completing a series of open sentences in negative integer arithmetic operations. These tasks are formed in two types, eg open sentences such as ☐ + 6 = 2 and comparison tasks. Table 3 shows the indicators and questions from the Open Sentence Task.

Table 3

Indicators and Questions of Open Sentence Task

No	Indicators	Questions
1	Illustrates negative integer operations in the form of an open sentence at the end.	Illustrate the counting operation 5 - 9 = ☐ Illustrate the counting operation -9 + 6 = ☐
2	Illustrates the operation of a negative integer in the form of the initial open sentence.	Illustrate the calculation operation ☐ + 6 = 2 Illustrate the calculation operation ☐ - 3 = -5 Illustrate the counting operation ☐ + 3 = -6
3	Illustrates a negative integer operation in the form of an open sentence in the middle.	Illustrate the counting operation 7 + ☐ = 3 Illustrate the counting operation 8 - ☐ = 11 Illustrate the counting operation 6 + ☐ = 4
4	Declare true or false statements of negative integer arithmetic operations.	9 + (-7) = 2 True or False of the calculation operation? Try to prove the arithmetic operation by drawing the number line. What is the reason you answered right or wrong? 9 + -7 = 9 - 7 True or False of the calculation operation? Try to prove the arithmetic operation by drawing the number line. What is the reason you answered right or wrong? 7-2 = 7- -2 True or False of the calculation operation? Try to prove the arithmetic operation by drawing a number line. What is the reason you answered right or wrong

Interview Instruction

Interview instructions were used to explore students' thinking about strategies and instrumental student difficulties in solving number line problems. This interview was conducted 2 times at different times, this was also intended as data triangulation to ensure the validity of the data. Students are asked to explain problem-solving steps to see the results of tasks that have been done previously while recording audio and visual of student activities. After completing the entire task and interviewing the researcher screening of the video whether it is clear or not. If there are answers that are different from the results the students wrote, the researcher will clarify this. Questions will be developed according to

the answers expressed by students about the problem-solving process carried out by Polya in making plans and carrying out the analysis of the completion plan.

Data Analysis

All interview data were collected, audio recordings, and videos were transcribed and copies of student work were combined with each transcript. Data analysis procedures involving open, axial, and selective coding processes for continuous and qualitative data involve permanent comparative analysis between each category and new categories emerge (Cresswell, 2012).

Researchers conducted interviews and compiled transcripts of students' answers. To analyze, the interview data and transcripts were reduced to fragments containing explanations of the students' main ideas. Data were coded, sorted, and read repeatedly to answer research questions. Data validity was ensured by determining accurate and complete data collection by managing written tasks and interview transcripts after recording.

RESULTS

The results of the study were shown by students' learning on initial knowledge, namely Yulia's understanding of the context of the situation, strategies, and difficulties in solving problems involving the number line model.

Yulia Understanding of The Context Situation

Sometimes students still don't know the existence of negative numbers that actually exist in real situations, for that in the next task more researchers want to identify students' understanding of negative numbers in real situations through contextual problem elevators. Through contextual problems, they will understand more easily so that they can solve problems properly because they are related to everyday life. After the researcher knows the understanding and knowledge that students have, the students will be introduced to problems related to everyday life, in this case, negative integers that contextualize these problems in the real life of students. Students are introduced to the existence of negative numbers in real life students are the tools that usually make it easier for someone to get to a location quickly in a building or mall which is often called an elevator. Students illustrate problems that make up a story into a picture that resembles an elevator. Students illustrate by marking by shading in the appropriate position on the elevator problem image. Then students add a curved line next to the picture as a sign of displacement.

Yulia responds to the number line problem, that when someone uses the elevator and the process goes up, Yulia assumes it includes addition, while when the elevator goes down, Yulia assumes it includes subtraction. Through Yulia, contextual problems are easier to solve and she can also write

down the form of arithmetic operations that occur according to the problem presented. The focus of this research is how Yulia's strategy in solving problems related to negative integers by solving using a number line. Questions are designed using open sentences. The researcher hopes that he can solve the problem with a strategy that suits the understanding of each student.

The strategy used by Yulia for this problem tends to use signs or lines as the position and direction of the perpetrators of movement in the elevator. Students successfully solve the problem about the story presented and students can write the form of addition and subtraction according to the problem. When students are able to solve the problem, the researcher hopes that students can solve the problem of counting negative integers using number lines.

Yulia Strategies In-Line Numbers Problem

This point will be explained in detail about Yulia as a research subject. Knowledge about negative numbers is categorized as Yulia. Students can determine 10 numbers under the number 5 in sequence and right on the number line, students can also say the number of negative numbers that appear on the number line if there are five numbers -5, -4, -3, -2, -1, students are also able to compare between 0 and -6 which is greater than 0 on the grounds that the number 0 has no negative sign, and students can also conclude that the right of the number that lies on the number line, the greater the farther to the left and vice versa the number that lies on the line numbers getting smaller and smaller.

The next problem is almost identical to the previous problem, it's just that the set of questions before the operation starts with a count of zero while in this problem it doesn't start with zero. After completing this and previous questions, students are expected to conclude. Students have been able to solve the problem well and are able to conclude between this problem and the previous question is the same result, students also conclude that the drawing operation on the number line does not have to start with the number 0. This last problem is an evaluation of the previous questions about how students understand the addition and subtraction of negative integers on the number line. Students can answer questions and draw number lines correctly, but when students are asked to write down what the students understand the problem is, they cannot be concluded.

Yulia Strategies and Difficulties in Solving Numbers Line Problems

In the previous negative numbers task and lift task, the students tended to model operations involving negative integers vertically on the number line. In the open sentence task, students are asked to illustrate horizontal number lines. Yulia already understands the problem, seen from the correct steps to solve it, but when asked to make addition and subtraction forms that are almost the same as before with the same results, she cannot solve them. It appears in the blank on the student answer sheet. This is one of the reasons researchers found that Yulia is classified as instrumental because she can perform mathematical steps but has not been able to connect previous concepts with new concepts. In this case the researcher deliberately started the operation of counting with the number 0 so that the future Yulia

could compare with the next question. When conducting an interview with Yulia, the researcher submitted an task sheet that had been done to the previous students in the form of a series of open sentences. Students are interviewed about how and why students solve problems so that researchers know the strategies used.

The first two problems raised involved addition and subtraction of negative numbers with open lines as a result of addition or subtraction operations. The first task is task 5-9 = □ and followed by -9 + 6 = □ students must complete the task correctly. Yulia's first response resolves 5-9 = □ by jumping 9 times from number 5 to the left or towards the smaller number value. The researchers assumed the students used a countdown strategy. Students' understanding of this problem assumes the reduction will move to the left. If the task before the student moves to the left then the task -9 + 6 = □, Yulia completes the task by jumping to the right of the number -9 jumping 6 steps, the student assumes that because it is added it will move to the right. Students' strategies regarding the two problems were the same when students were interviewed a second time. Students use the definition of a number line and the notion of addition as a move to the right (forward) on the drawn number line and reduced as a move to the left (backward). Researchers think so because of the plus it will move to the right. Students' strategies regarding the two problems were the same when students were interviewed a second time. Students use the definition of a number line and the meaning of addition as a move to the right (forward) on the drawn number line and reduced as a move to the left (backward). Students think that because it is added, it will shift to the right. Strategy = students regarding the two same problems when students are interviewed for the second time. Students use the number line and the notion of addition as a move to the right (forward) on the number line drawn and reduced as a move to the left (backward).

The second problem lies in the open sentence in the middle of the task such as 7 + □ = 3, 8 - □ = 11 and 6 + □ = 4. Students have completed all three tasks correctly. In the fourth task, students were asked to prove the correctness or error of tasks such as 9 + -7 = 2, 9 + -7 = 9-7, and 7-2 = 7- -2. The first problem students solve it well, the second problem the answers are wrong but the strategy is right, the third problem students answer wrong.

The third task is task □ + 6 = 2, □ - 3 = -5 and □ + 3 = -6. Students must complete the task □ + 6 = 2 correctly. Students start the strategy from the results and go back 6 steps because the open sentence in front of the student's understanding is slightly different from before. When there is addition and students start from the results of the backward movement. On tasks □ - 3 = -5 and □ + 3 = -6 students still find it difficult and finally the students' answers and strategies are still wrong. Students start from the middle to the task □ - 3 = -5, which is from number 3 then left to number -5, to find out the results of students calculating the distance between 3 and -5. Students start from the results then add 3 moves to the right. In this task students forget the concepts that have been worked on in the point □ + 6 = 2. Researchers have differentiated Yulia's answers based on the right and wrong strategies when interviewed (Table 4).

Table 4

Yulia's Strategy and Response

No.	Problems	Strategies	Interview response
Strategy and Correct Answers			
1a	$5-9 = \square$	Count down	"From the number five, if you lower it to the left, counting from steps five to nine, the result is negative four" why to the left? "Because if it is reduced to the left".
			"Five minus nine" refers to student number five then moves left nine steps, the result is negative four "why move left?" Because minus.
1b	$-9 + 6 = \square$	Calculating Forward	"From negative nine plus six, one, two, ..., six. The result is negative three." "Right to maturity plus"
			"From negative nine plus six. The result is negative three "why step right? "Because added" are you sure you answered correctly? "Certain"
2a	$7 + \square = 3$	Behind Start	The student "Three plus seven results in a negative four" points to the number three then shifts to the left seven steps. Then the result is negative four. The student's concept changes at the beginning when the student states that if the addition moves forward, in this case the student actually moves backward.
			"Three minus seven negative answers four" Why did he go left? "Due to minus".
2b	$8 - \square = 11$	Count down	"From the number eight add eleven, one, two, ...eleven. The result is negative three. "The student's concept changed at the beginning of the student stating that if added it moved forward but in this case the student actually moved backwards.
			"Eight minus eleven, becomes negative three" why go left? "Due to minus"
2c	$6 + \square = 4$	Behind Start	The student "four plus six results in negative two" points to the number four then moves left six. The result is negative two. The student's concept changes at the beginning when the student states that if the addition moves forward, in this case the student actually moves backward.
			"Four minus six is negative two." Why go left? "Because it's reduced."
3a	$\square + 6 = 2$	Behind Start	"Because the numbers are almost non-existent, the numbers start behind". Students point to number two then move to the left for six steps and finally stop at negative number four. So the result is negative four.
			"From number two the result is negative four minus six" why is it reduced? "Trying to write".
4a	True or false: $9 + -7 = 2$	Behind Start	Students answered correctly.
			From number two Students begin to draw a line number from the number two rungs to the left of nine rungs and end at a negative number seven. But unable to explain why the student has answered correctly "don't know, grade was calculated beforehand"
			Students answered correctly.
			"From the negative number seven plus nine the result is two" why did you answer correctly? "Because the result is the same as and because I have calculated it"
One Right Answer Strategy			
4b	True or false:	Calculating	Students answered incorrectly.

	$9 + \text{-}7 = 9\text{-}7$	Forward	Solution 9 + -7 "of negative seven. The result is two" students starting at the negative number seven and then progressing to nine steps and ending at two.
			9-7 completion, "nine" refers to student number nine then takes seven steps back and ends at two. The answer should be correct because the number line represents both. But the student answered incorrectly. "Because I had counted but I slipped"
		Count down	Students answered incorrectly.
			Solution 9 + -7 "of negative seven plus nine, a result of two"
			9-7 solution, of nine minus seven results in two.
			Why are line numbers 9 + -7 going to the right and line numbers 9-7 going to the left? "Because 9 + -7 is added, and 9 -7 is subtracted.
			Then the reason you answered wrong? "Because I have counted"

Strategy and Wrong Answers

|			
3b	$\square - 3 = \text{-}5$	Calculating Distance	"I am the beginning of the numbers." The student points to number three and then moves left to negative number five. "I forgot". Students calculate the distance or jump from three to five negative numbers. The number of jumps is eight.
			Students forget where to start. "Students ranging from negative five plus eight to number three" why added? "I try"
3c	$\square + 3 = \text{-}6$	Behind Start	"Of the negative six plus, three negative results is three" sure? "Not sure, it's hard"
			"From the negative six plus three the result is negative three" why him right? "Due to pluses and trials".
4c	True or False: $7\text{-}2 = 7 - \text{-}2$	Count down	Students answered Yes.
			The 7-2 solution, "seven minus two results is five". The student points to number seven and steps back two steps then stops at number five.
			Solution 7- -2, the student points to number seven and moves the pencil to number five, the student steps back two steps. "Result of five".
			"Correct answer" Are you sure? "I don't know, but the result is the same, take the seconds and I have counted it"
			Students answered correctly.
			7-2 settlement, "of seven jumps left twice, a result of five"
			7- -2 completion "of seven jumps to the left of two, result of five". Why did you answer correctly? Because the result is the same and I have calculated it.

Figure 2

Yulia's Task Results
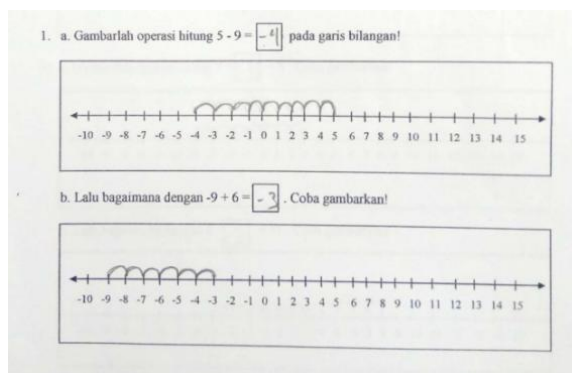
Yulia suspects that the subtraction or addition is the direction of the shift in the calculation operation steps. Look at the strategy used in solving the problem 5 - 9 = ☐ and -9 + 6 = ☐. Students perceive addition as a step forward or a step to the right. Whereas for subtraction students assume that reduction directs their steps to the left. Figure 2 shows the results of Yulia's task.

To solve the problem in number 1 "a" and number "b", it can be seen that the students draw the line from numbers -4 to number 5 and numbers -9 to number -3. To clarify the strategy used, the researcher interviewed students about the strategies used in point number 1 "a" and point "b". To solve the problem "a" students step back (move to the left) from number 5 moves to the left as much as 9 steps and ends at -4. Students reason that subtraction moves left or back. So the researchers concluded that the strategy used by students was countdown. In solving the problem of point "b" the students stepped forward (moved to the right) from the number -9 to the right for 6 steps and ended up in the number -3. The student reasons that the addition will move to the right. So that the researchers concluded that the strategy used was calculating forward. Solving problem number 2 "a" ☐ + 6 = 2, students start the solution step from the result or from behind. The student starts with the number 2 then moves to the left for 6 steps because the sentence the front number is empty, finally stops at -4. The reason for reducing students is that students are experimenting.

In task number 2 points "a" and "b", the students' answers were not correct. Point "b" ☐- 3 = -5, students use the strategy of calculating the distance. Students start from the middle, which is 3 and move left to the number -5. Students calculate the jump distance between number 3 to number -5. The distance is 8. Point "b" ☐ + 3 = -6, student starts from behind or results. Students just try to find the

answer, students start from -6 then add up 3. The result is -3. Students do not know the reason for solving the problem. According to him, this problem is difficult, making students unsure of the answer.

The solution to the problem number 3 points "a", "b", and point "c" has been resolved properly. Problem 7 + □ = 3 students start the strategy from the results section. Students starting at number 3 plus 7 result in -4. Students move to the left in 7 steps. Students' understanding of addition will shift to the right as this task changes. In problem 8 - □ = 11 students count backward from number 8 by 11 steps backward. The result is -3. In problem 6 + □ = 4, students starting at 4 plus 6 give -2. Initially the students responded to the way they were added and moved to the left, but in the interview responses the two students stated that the reason was reduced and moving to the left was reduced.

In task number 4 the "a" strategy the students used was correct. Students answered correctly. Students starting from number 2 move to the left as many as 9 so that they end in -7. However, in the second interview, students started with the number -7 plus 9 and ended at number 2. Students did not know the reason for the solution in the first interview. In solving the problem at 4 points "b" 9 + -7 = 9-7, students use counting forward and backward strategies. Students start the strategy from -7 to the next 9 steps and end at number 2, for the counting operation 9-7 students count backwards from the number 9 backwards by 7 steps and finally end up in number 2. However, the students still answer that the comparison is wrong. Even though the results are stated to be the same. Thus, the researcher concluded that the students' answers were wrong, but the strategy used was correct.

Problem solving at 4 points "c" 7-2 = 7 - -2, students use a countdown strategy. Students start the 7 minus 2 number strategy, students start from 7 then walk backward two steps the result is 5. As for the task in the counting operation 7 - -2 students start from number 7 and go back 2, the result is 5. The completion strategy used by students is lacking right. Each of Yulia's strategy difficulties in answering were analyzed, Yulia's difficulties were described in Table 5 as follows.

Table 5

Yulia's Difficulty is Based on the Strategy Used

Strategies	Problems	Student difficulties with the strategies used
Count down	5-9 = □ 8 - □ = 11 True or false: 9 + -7 = 9-7 True or false: 7-2 = 7 - -2	Students do not experience problems on tasks where the open sentence is behind. The strategy and reasons given are correct. Students feel confused about the change in their counting operation signs when completing a task where the open sentence is in the middle, resulting in a difference in arguments between the oral statement and the strategy depicted on the number line. For example, the student states 8 + 11, the addition always goes forward, but the student moves the pencil backwards. In the problem of comparison, students have difficulty in counting operations, because the addition symbol is lined with negative symbols. Students ignore one of these

		symbols. Students also have difficulty explaining the reasons for the completion steps.
Count forward	-9 + 6 = ☐ True or False: 9 + -7 = 9-7	Students do not have difficulty on tasks where the open number is behind. The strategies and reasons that the students gave were correct. In the problem of comparison, students have difficulty in counting operations, because the addition symbol is lined with negative symbols. Students ignore one of these symbols. Students also have difficulty explaining the reasons for the completion steps.
Behind Start	☐ + 6 = 2 ☐ + 3 = -6 7 + ☐ = 3 6 + ☐ = 4 True or False: 9 + -7 = 2	Problems that arise in tasks where the opening sentence is at the beginning, students have difficulty starting the strategy. Students then start the strategy from the results section. In addition, the difficulty also lies in the change in the operation markers due to their changing location. So that students sometimes state the wrong statement and can influence the strategy and answer results. In comparison problems students have difficulty stating the reasons for the solution.
Calculating distances	☐ - 3 = -5	The problem that arises lies in the use of count operation marks. These inaccuracies can affect the strategy and result in wrong answers.

The students' difficulties are described in each of the strategies students undertake. In the strategy of counting down or counting forward, difficulties occur when solving comparison problems. Students who complete the task have difficulty subtracting or adding symbols that are close to negative symbols. In the behind the start strategy, the difficulty of students starting to complete the task is because the open sentence is in the middle. Meanwhile, in the strategy of calculating the distance, difficulties occur in the use of operating marks.

DISCUSSION

Students' views about the existence of negative numbers or numbers smaller than zero are different. Like the initial task given to research subjects, students are asked to name 10 numbers under the number 5 on the number line based on knowledge. The research subjects can mention regularities well. In contrast to the others there are still students who cannot say. This task raises the number line with the numbers 5, 6, 7, 8, 9, 10. To the left of the numbers 5, 10 are the squares left blank by the researcher. Students are asked to name 10 numbers in the box that are intentionally left blank. Students who do not have good knowledge of negative numbers will name them randomly. Even though the number line should also be a position code. Representation according to Fischer (2003) is automatically associated with left space, the coding and representation of the number line can be adjusted according to the numerical context and according to the problem.

When students are asked to compare these numbers, the researcher wants to know the extent to which students understand negative numbers. Students are required to compare numbers 0 and -6 which are bigger, it turns out that there are those who answer -6 which is greater than 0. However, the subject has good knowledge and understanding of negative numbers, it can be seen from the students' conclusions. Thus, with good knowledge and understanding, it will be the basis for further students to solve problems related to negative numbers. Ask students why and use number line representations, help students make connections between problems, come up with students 'answers and strategies, describe students' thinking (Aqazade et al., 2017).

About the elevator problem as a contextual problem. Many students do not have direct experience, but are interested in the context of the problem (Yilmaz et al., 2019). One of the characteristics that a good contextual problem of the problem will bring about mathematical interpretation and strategy solutions, an informal strategy that serves as a development into more formal mathematics (Widjsaja, 2013). Through an elevator or elevator, in addition to knowing the existence of actual negative numbers students are also introduced to the use of number lines, these numbers are only vertical lines because of the shape of the elevator. The ability of students to relate mathematics to real life is very important so that students know the usefulness of mathematics in real life. The ability of students to make connections between mathematics and real life is very important because it relates to the introduction of mathematics in everyday life, the accuracy of the introduction to develop certain conceptual and mathematics learning (Altay et al., 2017).

Several implications are related to the researcher's findings. First, how to draw a number line by the research subject provides a different understanding in compiling and implementing a completion plan. The number line that has been made instrumental-only describes the number line without being emphasized by the large number of sequences. The freedom of students to formulate and implement a plan causes differences in the symbols or lines students use when using number lines as a tool to help solve problems. Second, different types of problems affect students' understanding of the concept of the material. Understanding the concept of student material can be seen from the way the students use the solution. Tasks for preliminary studies with tasks to study strategies provide insight into students' conceptual understanding. The task is designed so that students remain directed at their understanding of using number lines, students' understanding of the concept of the material is more striking.

Regarding the number of students who completed the task, they remembered more formulas in finding results so that students did not know the actual concept of the material. Even though with this type of quantity students are given more freedom in using strategies according to their respective understandings and ways of thinking. Students 'freedom of thought to operate number lines provides insight into the development of students' thinking. So that the strategy used will be in accordance with student thinking. Students do not have to start from zero in arithmetic operations to solve negative number tasks using number lines.

In fact, number lines are introduced in the student book, but indirectly students are introduced to a set of rules that must be followed to get the correct answer. Although the research subjects used number lines to solve negative integer problems, it seems that it can resemble the size of a textbook if using number lines, but in the process students develop very different rules. Students are free to start where in using the number line to complete the task being asked, especially in this case the arithmetic operation task is in the form of an open sentence. So if students have to complete the task by giving a point at the beginning as the starting point for moving the step number line and students will find it difficult if they open a sentence to find the number in front or begin to purposely leave it blank. Therefore the number line can be used as an appropriate tool so that students can provide strong reasons for negative numbers (Bishop, 2011). Negative integer count operations with number lines and open sentences can be used as a teacher's solution so that students are better able to understand the concept of negative integers and can explore students in problem-solving strategies that are in accordance with student thinking. Students' differences in interpreting negative numbers will affect the strategies students use in solving problems. The gap that occurs when introducing negative integers is that students interpret negative numbers in various ways according to students 'knowledge of numbers, so that it can affect students' problem solving strategies in solving negative integer problems (Aqazade et al., 2017). Students' differences in strategy use depend on their contrast, analyzing the problem gives them different benefits in interpreting the problem (Aqazade et al., 2017; Bofferding & Wessman-Enzinger, 2017).

Student difficulties occur because students do not understand the difference between the signs of operation and positive or negative integer signs. Students should understand the problem first when completing the elusive negative integer concept task, even though students have no difficulty placing negative numbers on the number line but will have problems comparing values (Cetin, 2019). Giving tasks at the basic education level that are difficult to make students tired and pessimistic so that students cannot solve problems (Stein & Burchartz, 2006). The difficulty of students in solving these problems is possible because of the cognitive barriers, tasks that are too difficult at the beginning can become students' cognitive obstacles. The cognitive barriers to understanding and knowledge that allow students to learn preclude negative integers (Bishop et al., 2014). Sometimes the process of perceptual memory, there is a possibility that students' perceptions of concepts can be misunderstood which can cause cognitive barriers (Fischer, 2003).

CONCLUSIONS

Researchers have found various strategies, these strategies look at the instrumental ability of students in linking existing material concepts to new concepts. Students who have instrumental understanding are still unable to connect existing concepts to new concepts. Instrumental students have

been able to solve the problem, but when asked for an explanation of the reasons for completing the steps, students still cannot be held accountable. Strategies for students who have an instrumental understanding include counting down, counting forward, calculating distance and starting backwards. Generally, students' difficulties occur because they do not understand the difference between positive or negative operating signs and integer signs and decide the type of strategy used in problem solving.

AUTHORS' CONTRIBUTIONS STATEMENTS

MFA and MDKW conceived the presented idea, research methodology, and writing - review & editing. MF collected the data. HER and NQW conducted data analysis. All authors actively participated in the discussion of the results, reviewed and approved the final version of the work.

DATA AVAILABILITY STATEMENT

The data that support the results of this study will be made available by the corresponding author, MFA, upon reasonable request.

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