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Reflections on different theories of Mathematics Education: comparisons, similarities and differences

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ABSTRACT

Background: Mathematics Education has evolved as a discipline in recent decades, with several theories emerging to explain the teaching and learning processes. This paper provides a critical reflection on the development of these theories, their similarities and differences. Objectives: To analyze and compare different theories of Mathematics Education, highlighting their historical context, key principles and relationships. Design: This study presents a historical-critical analysis of the main theories in Mathematics Education, focusing on their evolution, similarities and differences. **Setting and participants:** The research examines primary sources and key works associated with the emergence of several theories in Mathematics Education, including the Theory of Didactic Situations (TSD), the Anthropological Theory of Didactics (ADT), the Ontosemiotic Approach (AOS) and the Theory of Objectification (OT). **Data collection and analysis:** The study involves a comprehensive review of original works and critical analyses of each theory, examining their principles, methodologies and applications in Mathematics Education. **Findings:** The analysis reveals that although theories in Mathematics Education have distinct characteristics, they often share more common elements than differences. The study highlights the importance of considering historical theories alongside more recent ones. **Conclusions:** The research emphasizes the value of critically examining both historical and contemporary theories in Mathematics Education, promoting a more comprehensive understanding of the field and encouraging researchers to consider points of convergence between seemingly disparate theories.

Keywords: Mathematics Education; Didactic Theories; Theory of Didactic Situations; Anthropological Theory of Didactics; Ontosemiotic Approach; Theory of Objectification.

Reflexiones sobre diferentes teorías de la Educación Matemática: comparaciones, semejanzas y diferencias

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RESUMEN

Contexto: La Educación Matemática ha evolucionado como disciplina en las últimas décadas, surgiendo varias teorías para explicar los procesos de enseñanza y aprendizaje. Este artículo ofrece una reflexión crítica sobre el desarrollo de estas teorías, sus similitudes y diferencias. **Objetivos:** Analizar y comparar diferentes teorías de la Educación Matemática, destacando su contexto histórico, principios clave y relaciones. **Diseño:** Este estudio presenta un análisis histórico-crítico de las principales teorías en Educación Matemática, centrándose en su evolución, similitudes y diferencias. **Entorno y participantes:** La investigación examina fuentes primarias y trabajos clave asociados con el surgimiento de varias teorías en Educación Matemática, incluida la Teoría de Situaciones Didácticas (TSD), la Teoría Antropológica de la Didáctica (TAD), el Enfoque Ontosemiótico (AOS) y la Teoría de la Objetificación (TO). **Recopilación y análisis de datos:** El estudio implica una revisión exhaustiva de trabajos originales y análisis críticos de cada teoría, examinando sus principios, metodologías y aplicaciones en la Educación Matemática. Resultados: El análisis revela que, si bien las teorías en Educación Matemática tienen características distintas, muchas veces comparten más elementos comunes que diferencias. El estudio destaca la importancia de considerar las teorías históricas junto con las más recientes. **Conclusiones:** La investigación enfatiza el valor de examinar críticamente las teorías históricas y contemporáneas en Educación Matemática, promoviendo una comprensión más integral del campo y alienta a los investigadores a considerar puntos de convergencia entre teorías aparentemente dispares.

Palabras clave: Educación Matemática; Teorías Didácticas; Teoría de las Situaciones Didácticas; Teoría Antropológica de la Didáctica; Enfoque Ontosemiótico; Teoría de la objetivación.

Reflexões sobre diferentes teorias da Educação Matemática: comparações, semelhanças e diferenças

RESUMO

Contexto: A Educação Matemática evoluiu como disciplina nas últimas décadas, com várias teorias surgindo para explicar os processos de ensino e aprendizagem. Este artigo fornece uma reflexão crítica sobre o desenvolvimento dessas teorias, suas semelhanças e diferenças. **Objetivos:** Analisar e comparar diferentes teorias da Educação Matemática, destacando seu contexto histórico, princípios-chave e relações. **Design:** Este estudo apresenta uma análise histórico-crítica das principais teorias em Educação Matemática, focando em sua evolução, semelhanças e diferenças. **Ambiente e participantes:** A pesquisa examina fontes primárias e obras-chave associadas ao surgimento de várias teorias em Educação Matemática, incluindo a Teoria das Situações Didáticas (TSD), a Teoria Antropológica do Didático (TAD), a Abordagem Ontossemiótica (AOS) e a Teoria da Objetivação (TO). **Coleta e análise de dados:** O estudo envolve uma revisão abrangente de obras originais e análises críticas de cada teoria, examinando seus princípios, metodologias e aplicações na Educação Matemática. **Resultados:** A análise revela que, embora as teorias em Educação Matemática tenham características distintas, frequentemente compartilham mais elementos comuns do que diferenças. O estudo destaca a importância de considerar teorias históricas junto com as mais recentes. **Conclusões:** A pesquisa enfatiza o valor de examinar criticamente tanto as teorias históricas quanto as contemporâneas em Educação Matemática, promovendo uma compreensão mais abrangente do campo e encoraja os pesquisadores a considerar pontos de convergência entre teorias aparentemente díspares.

Palavras-chave: Educação Matemática; Teorias Didáticas; Teoria das Situações Didáticas; Teoria Antropológica do Didático; Abordagem Ontossemiótica; Teoria da Objetivação.

INTRODUCTION

Both the prehistory and the history of the discipline now called *Mathematics Education* can be explained and analysed by starting from the different interpretative theories that have been proposed throughout the decades since its emergence under its current name: Mathematics Education was baptised as such in the early 1980s, making it just under 50 years old. For different scientific and personal reasons, we found it both pleasing and convenient to focus on 1986, the year of publication of a study that is still considered by many (including ourselves) to be the starting point for our discipline (Brousseau, 1986). (See also: Brousseau, 1988, 1989).

Attention is focused on decades of cases located exclusively in the field of teaching, with naïve positions; this has led us to refer to this phase as *Didactics A* ("A" is not only the first letter of the alphabet and therefore stands for a beginning, a starting point, but also denotes the *Art of teaching*). It was only after the revolutionary scientific work of Guy Brousseau that a return to the problem of learning as the determining scientific factor in investigation prevailed, leading us to define *Didactics B* ("B" as the process that follows "A") (D'Amore, 1999, 2006).

The beginning of the 1980s can also be identified as the birth date of a first theory of actual Mathematics Education, the "theory of didactical situations," whose undisputed creator was Guy Brousseau (Artigue, Gras, Laborde, Tavignot, & Balacheff, 1994).

However, it is not certain (as is often inaccurately stated) that there were no earlier theories. There were theories that dealt with school mathematics, but these had very little to do with the elements of the discipline that we recognise as important today.

Above all, however, various theories have emerged with the emergence of modern Mathematics Education. In order to clarify this attitude, we need to clarify what we mean by *theory*. A detailed study/historical account can be found in D'Amore (2007), but for the sake of brevity and up to date concepts we prefer to quote Radford (2008a, b) to provide a simple and meaningful idea of the content in which the meaning and structure of a theory is expressed.

According to Radford, a theory necessarily comprises principles or a system of conceptually organised principles (P), models of research questions (Q) and a methodology (M). The system of principles (P) comprises some key constructs on which these principles are based. The methodology (M) comprises the techniques for collecting, analysing and interpreting data, events or empirical evidence that support the answers to the research questions (Q). The three components (P, Q, M) of a theory (T) are in a dialectical relationship with each other and therefore significantly alter the results that the theory produces. In other words, no theory is static, all theories evolve over time. This explains how old theories can evolve, precisely thanks to this type of structural analysis and, we believe, thanks to comparison with other theories. It also explains how the various theories and their analyses continue to contribute to the provision of explanations and thus remain interesting, even if the underlying original ideas are retained. In our opinion, this is the case of the theory of didactical situations (TDS), the best known and most fundamental (from a historical point of view) among the theories of Mathematics Education, as shown by Fandiño Pinilla (2020).

In our opinion, none of the theories that preceded Brousseau's TDS can be considered scientifically sound. Nevertheless, we believe that a brief presentation of the most widespread theories in the period we call "prehistory" can be important and useful for people approaching Mathematics Education. It is considered necessary, useful and culturally appropriate to know the historical context and critical development of the discipline.

"PREHISTORY": ZOLTAN DIENES AND GEORGES PAPY

As far as trends dealing with the problem of teaching mathematics are concerned, the decade of the 1960s was certainly dominated by the work of the Belgian couple Georges Papy and Fréderique Lenger Papy and, independently of them, by the Hungarian Zoltan Dienes.

In the 1950s, Georges Papy was one of the strongest proponents of the international proposal known as *New Mathematics* or *Modern Mathematics*, which spread in France, Belgium and the United States, where it was slowly planned in (almost) every detail. According to Papy, the basis of this proposal corresponded to the views of the Bourbakists and therefore consisted of the study (as a starting point and including teaching in kindergarten) of set theory and algebraic structures (whether or not they were formally defined as such). Surrounded by students who uncritically supported him, Papy introduced representative dogmas and forced drawing on students, with special colours for relational words representing algebraic structures and coloured circles in "potato shape" to represent sets ('papygrams') (Papy, 1963, 1969a). In our opinion, however, none of this was really relevant to learning.

He was also famous for his minicomputer, but we will not go into that in this article (Papy, 1969b).

In 1970, Georges Papy founded a study and research group, the GIRP (*Groupe International de Recherche en Pédagogie de la Mathematique*), which was officially based in Walferdange (Luxembourg), very close to the capital, and of which he was president until 1991. The GIRP organised an annual conference for international studies in various European countries, usually in August. In 1996, the conference was dissolved, mainly due to internal cultural disagreements (D'Amore, 2021, pp. 45–47).

Zoltan Dienes was another important international defender of *New Mathematics* or *Modern Mathematics,* and his fame is entirely associated with a collection of elegant and attractive colourful objects in various elementary geometric shapes, the *Dienes Blocks,* which primary school children could play with, and which were supposed to teach them logic automatically. Obviously, this was a simple illusion: in reality, the children simply learned to play with the attractive coloured objects in the box (if and when they learned). At the time, the illusion was created that *cognitive transfer*, the abstract phase of learning, occurs spontaneously under certain circumstances before being transferred to general learning. Today we know that learning does not automatically occur on the basis of a concrete and limited example, because significant conceptual learning requires completely different circumstances and situations.

This dream belongs to that category of illusions that we have defined elsewhere as "panaceas" (D'Amore & Fandiño Pinilla, 2014) and that are still uncritically proposed by people who have no idea about scientific research in Mathematics Education and that are welcome to those (few) acritical teachers who are looking for "educational recipes," rather than information about what it means to learn based on meaningful research studies.

In the early 1980s, Brousseau, with great analytical clarity and singular cruelty (which he later regretted, as he admitted in personal conversations), scientifically examined these proposals by Dienes (and others) through indisputable experiments in classrooms. He then sharply attacked the theories and explicitly named the names of their creators. By referring negatively to the "Dienes effect," we strongly influenced not only those who used logical blocks in the classroom, but also (and especially) those who applied the "Dienes style" in their teaching (D'Amore, Fandiño Pinilla, Marazzani, & Sarrazy, 2020; Dienes 1972, 1975).

In the following sections we give a brief overview of some of the most important, most widely used or best-known theories in the history of Mathematics Education. The best-known and most current of these theories are presented only very briefly because they are so well known that they do not require in-depth analysis here: What interests us on this occasion is merely to mention the various theories because we believe that it is useful and appropriate to avoid unnecessary provincialism.

With appropriate bibliographical references, we will indicate the most specific and detailed texts for each of the theories.

THE BEGINNING OF MATHEMATICS EDUCATION: THEORY OF DIDACTICAL SITUATIONS (TDS)

TDS. The teacher decides to deal with a topic (***t***) that he knows very well from a mathematical-epistemological and historical point of view as well as from a didactic point of view in order to promote the learning of the students in his class. To this end, he selects a suitable, interesting and specific problem that concerns the topic ***t***. He then creates an a-didactic situation in which the goal is for the students to learn and understand ***t***.

After introducing the idea of the topic, it is proposed to the students before they go through the standard steps that constitute the inquiry-based approach: devolution, implication, construction of private knowledge (spontaneous emergence of the individual), validation, socialisation (social construction of ***t***), institutionalisation. We know that the first and last of these phases are the responsibility of the teacher, while the others depend entirely on the personal and group activities of the students. They discuss the concepts among themselves, share their individual insights and make them public, negotiate the concepts and terminology.

The teacher is physically present, collaborates with the students and participates in the joint work – however, his role is not that of a teacher, but rather that of a guide who listens and facilitates the work. During the collaborative work, students also seek private knowledge individually; they must achieve the institutional knowledge expected by society and thus fulfil the teacher’s objectives. Subsequently or simultaneously, the acquired private knowledge is shared and discussed, using a common terminology that lends itself to negotiation.

In this way, students become part of society thanks to their acquired knowledge. At the same time, the teacher also changes, because with this system he has acquired teaching knowledge. It is not necessary to adopt a realist position, certainly not a Platonic one, because it is not necessary that the mathematical object implied in ***t*** is pre-existent; there have been various discussions on this point, but in fact this pre-existence has never been considered necessary throughout TDS. Indeed, a pragmatist position seems more appropriate.

The origin of all these considerations lies in the work of Brousseau (1989). [For more details, see also D'Amore (1999)].

OTHER ALTERNATIVE OR SPECIFIC THEORIES (ATD)

ATD. Shared work is part of the *praxeology*, which always emerges as a predominant factor in ATD (Anthropological Theory of Didactics); a relationship to institutional knowledge that comes into play both in TDS and even more so in ATD and is referred to by the French name *Savoir savant* (wisdom, knowledge in itself; academic, official, historical knowledge, without explicit teaching and a desirable outcome in educational institutions). Much of what has been said about TDS can also be applied to ATD. A realist position is not necessary; in fact, a pragmatist position seems most appropriate in this case as well.

Why do we use the adjective "anthropological"? It is not an exclusivity of the focus created by Yves Chevallard in the 1980s, as he himself says, but an "effect of language" (Chevallard, 1999, p. 222). It distinguishes the theory, identifies and characterises it, but it is not exclusive to this theory.

ATD focuses exclusively on cases in the institutional dimension of mathematical knowledge, as the development of the research programme began with the notion of "fundamental didactics" (Brousseau, 1989; see also: Gascón, 1998).

[For further details see: Chevallard (1992)].

Variants of TDS and ATD. Since these two (closely related) theories originated in France, it is obvious that many French academics contributed to their development, either by simply analysing their components differently or by analysing them specifically. Given the nature and aim of this paper, we have decided not to go into detail, but to limit ourselves to mentioning a few names who contributed to this type of development in the 1980s and 1990s and who today could be attributed to one of the theories mentioned as academics who support their development. We will limit ourselves to mentioning Perrin-Glorian (1994), but there would be many other names worth mentioning.

As for the word "theory", we avoid going beyond the exact direction we have chosen (i.e. referring exclusively to theories of Mathematics Education) and we therefore avoid citing and illustrating theories that have contributed to our discipline, not as general theories of Mathematics Education, but as specific theories on particular topics. We limit ourselves to citing the examples of: Perrenoud (1984), Schoenfeld (1985), Vergnaud (1990), Sfard (1991, 2019), Fischbein (1993), Duval (2017).

In section 5, we will briefly present some theories that should apply to Mathematics Education in general.

There are also theories interpreted through models such as the Van Hiele model, which refers to geometric learning (Van Hiele, 1986) [for more information see: D'Amore (1999), pages 84–89]; or the APOS of Edward (Ed) Dubinsky, a model to explain mathematics learning inserted in the style of neo-Piagetian constructivism, as it refers to the so-called "reflective abstraction in learning". [Dubinsky (1987, 1989, 1991)].

We will not go into these types of theories in detail here.

MOST RECENT THEORIES [(OSA), (TO)]

The theories we mention in this paragraph are so well known and widespread that we will not discuss them in detail, but confine ourselves to a very brief mention. We will, however, provide a consistent bibliography.

EOS. The OSA (Onto-Semiotic Approach) is a very broad theoretical system that brings together different foci and theoretical models for the investigation of Mathematics Education, starting from anthropological and semiotic hypotheses about mathematics and its teaching/learning. It emerged in the early 1990s in the Mathematical Education Theory Research Group at the University of Granada and is currently being developed and applied by various other international (especially Spanish and Latin American) research groups.

The analysis of OSA examines the group formed by students and teachers who spontaneously perform a common task, i.e. a so-called *community of practice*, which is formed not only around a specific mathematical topic ***t***, but also through the working method associated with sharing, exchange and joint work. To analyse these factors, a pragmatist view is required, because it is precisely the shared work that creates and shapes ***t***, that configures it and makes it emerge, shared thanks to the work (we emphasise: shared) between the students themselves, and between students and teacher. The emergence and development of ***t*** implies, ontologically speaking, a clear entry into the adult and historical society that permeates OSA, while the exchange of the constructive elements of shared labour constitutes a semiotic framework.

See: Godino and Batanero (1994), Godino (2002), D'Amore and Godino (2006), Godino, Batanero and Font (2019), D'Amore and Fandiño Pinilla (2017, 2020), D'Amore (2024).

TO. In TO (theory of objectivation), the topic ***t*** of collective work and therefore of learning is proposed directly by the teacher or may arise spontaneously in the context of an activity in which the class or part of it participates (group of students, with or without the specific participation of the teacher). The exchange process is strictly focused on the personal positions of the learners, but within the context of a shared effort. The spontaneous appearance of ***t*** marks the key moment of objectification, of personalization of the engagement from a Marxist point of view, of production and thus of entry into the desired society as a demarcation of the learning that has taken place.

The first and last phases are the responsibility of the teacher, the rest is participation and guidance. We are of course aware that this act can be interpreted in different ways, even oppositely. We can also see the stage in TO where the teacher stops acting as a teacher, steps back to collaborate with the students without giving feedback or solutions, and instead simply solicits opinions, compares opinions, and gives the floor to those who are on the sidelines. Thus, the action in TO can be interpreted as a regulating action of the processes of adaptation to the learning situation, an activity referred to as collaborative and collective labour.

See in particular Radford (2021); but also D’Amore (2015) and D’Amore and Radford (2017).

RELATIONSHIPS BETWEEN THEORIES

Some notable works explain the reasons for the emergence and specific spread of different theories (including different arguments) with high-level historical-critical analysis (Teppo, 1998; Lerman, 2006; Prediger, Bikner–Ahsbahs, & Arzarello, 2008; Sriraman & English, 2005, 2010; Prediger, Bosch, Kidron, Monaghan, & Sensevy, 2010; Bikner–Ahsbahs & Prediger, 2014).

The study of the relationships between the theories can lead to at least a partial unification, a decisive contrasting, or a partial comparison (Prediger, Bikner–Ahsbahs, & Arzarello, 2008; Radford, 2008a, b). By nature, we generally tend to look for (at least partial) similarities. For this reason, we have participated with conviction in the writing of recent works that compare and contrast (rather than simply contrast) theories such as TDS and TO (Asenova, D’Amore, Fandiño Pinilla, Iori, & Santi, 2020).

Based on the results obtained and our conviction that it is important not to emphasise only the contrasts between theories, we decided to extend this investigation by exploring the commonalities between theories in Mathematics Education, both in historical offerings such as TDS and ATD and in more recent ones such as EOS and TO.

Among the different possible research studies, we only mention Fandiño Pinilla (2020).

For example, by studying the original sources (which is not always easy), we now believe that the origin of some of the phases that characterise TDS: implication, construction of private knowledge, validation and socialisation within a didactic situation, is absolutely not opposed to the collective work of TO [although at that time, in the 70s and early 90s, it did not seem necessary to take a position on the characteristics of this theme (shared labour), but was perhaps taken for granted].

Again, we have devoted a large part of our analysis to presenting this position, which at first glance seems far removed from some perspectives that tend to emphasise the differences between theories rather than the shared elements (however partial they may be) or the analogies (however hidden they may be).

This position always requires a careful examination of the primary sources and key works associated with the emergence of a theory. This includes what actually seems contradictory from the point of view of those (including research centres) who blithely and with ingenuity, ignorance or even malice refer only to the latest theories and neglect or simply dismiss earlier work. For we know that a theory that emerges in 2022 may be superseded in a decade by another theory that, unbeknownst to its authors, repeats earlier viewpoints that are now considered outdated.

A very clear example (without resorting to direct and precise citations) is the recent rediscovery of the idea of the didactic contract (a typical feature of TDS), clearly baptised with a different name and great self-claim (even by a national journal), by non-experts without any theoretical basis historically grounded in relation to the world of Mathematics Education. They seem to be rediscovering mathematical principles and creating new ones. When a French psychologist conducted a classroom test in 1980 and discovered the famous *captain's age* assumption, he publicly complained about it and accused primary school teachers of teaching mathematics badly; the question was published in the most widely read and non-specialised publications. However, no one (neither the psychologist nor the journalists) knew that the question was at an advanced stage of a scientific-critical study by Brousseau, based on the much more general theme of the *didactic contract*.

CONCLUSION

The aim of this brief contribution is simply to return to critical elements in order to reflect on the differences, but also the similarities, between theories that many (in our opinion, sometimes hastily and superficially) consider distant, incompatible and opposed.

Our in-depth analyses, based on original work on each of the theories mentioned above, show that it is ultimately possible to highlight points of convergence or at least similarities. In fact, there are often more shared elements than differences.

In the full study by Fandiño Pinilla (2020), for example, there are numerous examples of this type; they can also be found in Asenova, D'Amore, Fandiño Pinilla, Iori and Santi (2020); in D'Amore and Fandiño Pinilla (2020) and in other works.

This is therefore a (concrete) invitation to researchers in the field to consider theories, indeed all theories worthy of serious study (even if they are only historical), avoiding this absurd but generalised approach of neophytes to "forget" the oldest theories and only promote the study of the most recent ones. This pseudo-science of "modernism at all costs" is of no use to the serious scientist conducting a critical and, above all, competent study.

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