ISSN: 2178-7727

DOI: 10.17648/acta.scientiae.8208

The Philosophical-Scientific Ground on Which Young Husserl's Mathematical Education Took Place

Fredy Enrique González [a]

María Aparecida Viggiani Bicudo [b]

[a] Universidade Federal de Ouro Preto – UFOP, Campus Morro do Cruzeiro, Minas Gerais, Brasil

[b] Universidade Estadual Paulista – UNESP, Câmpus de Rio Claro; São Paulo, Brasil

> Almost a century after the publication of Husserl's opus magnum, Logische Untersuchungen and more than half a century after Husserl's death relatively little is known about Husserl's concerns with logic, mathematics and mathematical knowledge. Although Husserl's doctor degree was in mathematics, having been a student of Kronecker, and a student and later assistant to Weierstraß, and, moreover, although his professorship's thesis (venia legendi) and his first major work, Philosophie der Arithmetik, were both devoted to problems related to the nature of mathematics, very few scholars in the Angloamerican analytic circles have been interested in learning what Husserl had to say on problems related to the philosophy and foundations of mathematics.
>
> (Haddock, 2012, p. 91)

ABSTRACT

Context: This research is situated within the field of Philosophy, aiming to highlight the philosophical-mathematical study carried out by the young Husserl. **Objective:** To analyze the philosophical-scientific-mathematical environment in which his initial studies in Mathematics took place. **Justification:** The importance of this study lies in the way Husserl has been recognized as the founding philosopher of Phenomenological Philosophy. Throughout his academic trajectory, he consistently

Corresponding author: Fredy Enrique González. Email: fredygonzalezdem@gmail.com

engaged with Mathematics, dedicating himself to critically reflecting on the production of this science and its significance in the constitution of contemporary European science. **Methodology:** Theoretical study. *Setting and Participants*: Bibliographical study of Husserl, the founder of Phenomenological Philosophy. *Data Collection and Analysis*: Data collection was carried out through bibliographical research. **Results:** The rigor of Husserl's mathematical, scientific, and philosophical education is emphasized, understood through: (a) his participation in the course taught by Weierstrass; (b) his doctoral work, in which he made theoretical contributions to the Calculus of Variations; (c) the work through which he achieved his *Habilitation* as a university professor, where he conducted a psychological analysis of the concept of number. This study constituted the prelude to his Philosophy of Arithmetic, in which he deepened his reflections on number and began his journey through the intricacies of Philosophy and Phenomenology concerning Mathematics and Logic, which he further developed in his Logical Investigations as well as in subsequent studies. This study highlights the absence of references to Edmund Husserl's name in important and well-known texts on the History of Mathematics, despite his significant contributions within the field of Mathematics itself.

Keywords: Mathematical analysis; Arithmetization of mathematics; Crisis of the foundations of mathematics; Karl Weierstrass; George Cantor.

O Solo Filosófico-Científico em que se deu a Formação em Matemática do Jovem Husserl

RESUMO

Contexto: A pesquisa se insere no campo da Filosofia, visando expor o estudo filosófico-matemático realizado pelo jovem Husserl. **Objetivo:** Analisar o ambiente filosófico-científico-matemático em que ocorreram seus estudos iniciais em Matemática. **Justificativa**: A importância desse estudo jaz no modo pelo qual Husserl tem sido conhecido eminentemente como filósofo fundador da Filosofia fenomenológica, embora ao longo de toda sua trajetória acadêmica sempre tenha se ocupado também com Matemática, dedicando-se ao pensar criticamente sobre a produção dessa ciência e de sua importância na constituição da ciência europeia contemporânea. **Metodologia:** Estudo teórico. *Ambiente e Participantes*: Estudo bibliográfico de Husserl, fundador da Filosofia fenomenológica. *Coleta e Análise de Dados:* A coleta de dados se deu pelo estudo bibliográfico. **Resultados**: São enfatizados o rigor da formação matemática, científica e filosófica de Husserl, compreendida mediante: (a) participação no curso ministrado por Weierstrass; (b) o trabalho para seu Doutorado, no qual fez contribuições teóricas ao Cálculo de Variações; (c) o trabalho com o qual conseguiu sua Habilitação como professor de universidade, onde fez uma análise psicológica do conceito de número. Esse estudo constituiu o prelúdio de sua Filosofia da Aritmética, em que aprofundou suas reflexões sobre número e iniciou sua caminhada pelos meandros da Filosofia e da Fenomenologia sobre Matemática e Lógica, que desenvolve nas suas Investigações Lógicas, bem como em estudos posteriores. Nas **Considerações Finais** deste estudo é destacada a ausência da menção do nome de Edmund Husserl em importantes e conhecidos textos de História da Matemática, embora tivesse produzido estudos significativos no âmbito da própria Matemática.

Palavras-chave: Análise Matemática; Aritmetização da Matemática; Crise dos Fundamentos da Matemática; Karl Weierstrass; George Cantor.

La Base Filosófico-Científica sobre la cual se produjo la Educación Matemática del joven Husserl

RESUMEN

Contexto: Esta investigación se sitúa dentro del campo de la filosofía, con el objetivo de destacar el estudio filosófico-matemático realizado por el joven Husserl. **Objetivo**: Analizar el entorno filosófico-científico-matemático en el que se desarrollaron sus estudios iniciales en Matemáticas. **Justificación:** La importancia de este estudio radica en el modo en que Husserl ha sido reconocido como el filósofo fundador de la filosofía fenomenológica. Durante su trayectoria académica, se ha dedicado de forma constante a las matemáticas, dedicándose a reflexionar críticamente sobre la producción de esta ciencia y su importancia en la constitución de la ciencia europea contemporánea. **Metodología**: Estudio teórico. *Diseño y participantes*: Estudio bibliográfico de Husserl, fundador de la filosofía fenomenológica. *Recogida y análisis de datos*: La recogida de datos se llevó a cabo mediante una investigación bibliográfica. **Resultados**: Se destaca el rigor de la educación matemática, científica y filosófica de Husserl, que se comprende a través de: (a) su participación en el curso impartido por Weierstrass; (b) su trabajo de doctorado, en el que hizo contribuciones teóricas al cálculo de las variaciones; (c) el trabajo a través del cual logró su habilitación como profesor universitario, donde realizó un análisis psicológico del concepto de número. Este estudio constituyó el preludio de su Filosofía de la aritmética, en la que profundizó sus reflexiones sobre el número y comenzó su viaje a través de las complejidades de la filosofía y la fenomenología con respecto a las matemáticas y la lógica, que desarrolló en sus Investigaciones lógicas y en estudios posteriores. Este estudio destaca la ausencia de referencias al nombre de Edmund Husserl en textos importantes y conocidos sobre la historia de las matemáticas, a pesar de sus significativas contribuciones dentro del campo de las matemáticas.

Palabras clave*:* Análisis Matemático. Aritmetización de las Matemáticas. Crisis de los Fundamentos de la Matemática. Karl Weierstrass. George Cantor.

INTRODUCTION

Haddock's assertions, as articulated in the above epigraph, continue to hold relevance after over a decade; hence, we undertook this research on the investigations undertaken by Husserl (1859-1938)[1], highlighting his education in Mathematics. He is best known for his work in phenomenological philosophy, as he focuses on topics such as knowledge, logic, language, intersubjectivity, empathy, internal awareness of time, history and historicity and lifeworld (*Lebenswelt*). We believe that Husserl's mathematical education is important and that he had a substantial role in the *Arithmetization of Mathematics*. Moreover, his schooling influences the attributes of his investigative methods, both within Mathematics and Phenomenological Philosophy.

In the education of young Husserl, we will highlight his preoccupation with issues pertaining to the crisis in the foundations of Mathematics which prompted him, from the outset, to contemplate beyond the objects inherent to this discipline, as evidenced by his inquiry into the *genesis of number*. His investigative gaze had already illuminated on the horizon in which the philosopher's work was situated, as he was able to generate a philosophical reflection on this origin (Bicudo, 2020).

The research herein spans from the period in which Husserl began his studies in Leipzig (1876) and Berlin (1878), to the 1900/1901 publication of his work *Logical Investigations (Logische Untersuchungen)* at the University of Halle, following the release of two other seminal works: *On the Concept of Number: Psychological Analysis (Über den Begriff der Zahl. Psychologische Analysen*, Husserl, 1887/1981) and *Philosophy of Arithmetic: Psychological and Logical Studies (Philosophie der Arithmetik. Psychologische und Logische Untersuchungen,* Husserl, 1891/2003). We proceeded to conduct a documentary and bibliographic research, based on translations into English, Portuguese or Spanish of Husserl's original works as well as supplementary materials, such as articles, books, book chapters and encyclopedias, which

[1] Some biographical data of Husserl can be examined at: https://www.igt.psc.br/Aulas/Fenomenologia/EDMUND%20HUSSERL.htm

enabled us to gain a comprehensive understanding of Husserl's mathematical investigations.

These readings made it possible to highlight his contributions to the clarification of the concept of number, an essential aspect of the *Arithmetization of Mathematics*, a movement advocating that Mathematics should pivot from geometric notions to the concept of number. It was a transition "from the age of intuition to the age of precise demonstrations based on arithmetic" (Magossi, 2020, p. 49) and in which Klein acknowledges Weierstrass' preeminent leadership[2] (Klein, 1895/1896). Thus, by choosing the phrase *Arithmetization of Mathematics*, Felix Klein highlights Weierstrass' thought that "is present in his demonstrations via epsilons (ε′s) and deltas (δ′s). This is opposed to those demonstrations characterized by geometric intuition [...]" (Magossi, 2020, p. 51).

Establishing *Founding Mathematics* on arithmetic bases, as posited by Weierstrass, means the culmination of a longstanding process, as illustrated by Miller (1925), when developing mathematical arguments through propositions articulated in a mathematical language solely comprising natural numbers and the properties of operations applicable to these numbers. In this context, Husserl, in his writings from 1887/1981 and 1891/2003, articulated his concepts regarding number specifically and Mathematics broadly, drawing upon the insights of his mathematical mentors and philosophers, as reflected in the meticulous and lucid methodologies he employed throughout his life.

Young Husserl's academic life seems to have been a true pilgrimage as Vargas (2018) puts it:

---

[2] ***Karl Wilhelm Theodor Weierstrass***, (1815-1897) German mathematician, is considered to be the father of modern analysis. His life was marked by the consequences of the conflict resulting from his father's intentions, who wanted him to follow a career in civil service, which clashed with his passion for mathematics. A teacher at a secondary school, far from the centers of mathematics, it was only at the age of 40 that he began a university career in recognition of the extraordinary quality of the works he had published in the meantime. Endowed with an extremely rigorous thought, he dedicated fundamentally to the theory of functions. His discovery of a continuous function without derivative at any point, which contradicted the intuition of the time, as well as the demonstration that complex numbers are the only commutative algebraic extension of the reals, a result that Gauss had unsuccessfully proposed. Haunted since 1850 by serious health problems, Weierstrass published very little, and many of the works attributed to him were the results of his students' annotations. https://www.infopedia.pt/artigos/$karl-weierstrass

> During his academic peregrination, Husserl attended the universities of Leipzig (WS 1876–WS 1877), Berlin (SS 1878–WS 1880–1), and Vienna (SS 1881–WS 1881–2). Even though most of Husserl's studies were dedicated to mathematics and sciences (inter alia with Karl Weierstraß and Leopold Kronecker, the demigods of contemporaneous mathematics), he also attended philosophy courses given by Wilhem Wundt in Leipzig, as well as by Johann Eduard Erdmann, Moritz Lazarus, and Friedrich Paulsen in Berlin. It is worth highlighting a strange pattern in his studies that seems to suggest he was struggling to find his true vocation: during the last two semesters, Husserl turned to the study of philosophy, dedicating the whole of the second to last semester exclusively to philosophy and registering for a number of philosophy courses with Paulsen in his final semester. 2 This, however, did not immediately result in a change of Husserl's main interest, because the reason for his move to Vienna was to obtain a doctoral degree in mathematics, which he completed between November 1882 and January 1883. His highly technical unpublished doctoral dissertation, submitted in June 1882, is entitled *Beiträge zur Theorie der Variationsrechnung (Contributions to the Calculus of Variations).* Husserl provided simplified proofs for theorems on the extrema of integral functions, which, however, soon became obsolete due to more encompassing results. The young doctor spent the next semester in the mathematical circles of Berlin (contrary to a popular misconception, Husserl did not serve as a formal assistant of Weierstraß) and subsequently volunteered in the Austro-Hungarian army service corps. (VARGA, 2018, p. 144-145).

As Vargas (op. cit.) explains, before dedicating fully to Philosophy, Husserl's academic interest was focused on Mathematics, a science that he studied under the guidance of great mathematicians such as Karl Weierstrass and Leopold Kronecker.

In fact, hoping that an Austrian dissertation would better serve him to secure a position in Austria afterwards, Husserl pursued his doctorate at Vienna under the guidance of Leo Königsberger, a former student of Weierstrass, culminating in his thesis titled Contributions to the Calculus of Variations. This theme was included in one of the disciplines he studied in the courses taught by Weierstrass. This mathematician exerted a notable influence on Husserl; this mathematician exerted a remarkable influence on Husserl; who, as soon as he got approval in his doctoral work, returned to Berlin "to assist Weierstrass in the course on Abelian Functions in 1883-1884" (Hartimo, 2006, p. 322) and thus with him continued to study the Theory of these functions.

Husserl's mathematical education took place in the context of the so-called "Crisis of the Foundations of Mathematics", in which his mentor Weierstrass played a prominent role. Thus, in his study *On the Concept of Number*, he examined the psychological underpinnings of the concept of number. This work served as a foundation for a deeper work entitled *Philosophy of Arithmetic (FdaA)*. In this work, delving into the epistemological bases of Mathematics, presenting an interpretation of the nature of numbers and their relations. Furthermore, Bello (2022) asserts that in *FdaA*, Husserl expanded upon his earlier concepts of number articulated in *On the Concept of Number*, positing that "the genesis of number is to be found in a collective connection" (Bello, 2022, p. 31). Additionally, in alignment with this author, Husserl's discourse in *FdaA* against Frege's critiques reveals the influences of Weierstrass and Cantor on his thought.

Consequently, it can be inferred that Husserl's mathematical education is quite robust, as he not only studied under Kronecker and Weierstrass but also collaborated with and befriended Cantor during his tenure at the University of Halle. Furthermore, he was a member of the Hilbert Circle at the University of Göttingen, where he served as a professor for fifteen years. During this period, Zermelo shared with Husserl his discovery of the so-called *Zermelo-Russell Paradox* (Fonseca and Silva, 2019), as evidenced in Haddock (2012) and corroborated in Ierna (2016). Haddock articulated his thoughts as follows:

> [...] that does not mean that he was unaware of the paradoxes of set theory. Already on April 16, 1902, Zermelo communicated Husserl the so-called Russell Paradox - which Zermelo had discovered before Russell - and they remained very near during their stay in Göttingen. (Haddock, 2012, p. 241).

At that time, he also became acquainted with the works of important mathematicians, particularly Bernard Bolzano[3]. His approach to Bolzano's ideas occurred in three distinct ways: (a) Weierstrass ’classes; (b) Brentano's discussions of *The Paradoxes of the Infinite* (Bolzano, 1991); and (c) his conversations with Cantor. It is conceivable that Husserl encountered Bolzano's ideas through alternative avenues, including his examination of Stolz's paper (1881), which discusses the contributions of this mathematician to the history of Infinitesimal Calculus. In this paper, Stolz mostly addressed technical topics.

From a purely mathematical standpoint, Husserl came in contact with Bolzano's ideas during his studies under Weierstrass, addressed the *convergence of infinite numerical sequences* in his lectures. Bolzano demonstrated that every bounded sequence of real numbers contains a convergent subsequence. This outcome was subsequently independently validated by Weierstrass, and it is now referred to as the Bolzano-Weierstrass theorem. Another route for Husserl's engagement with Bolzano was facilitated by the seminars on *Descriptive Psychology*, conducted by Franz Brentano[4], which Husserl attended from 1884 to 1886, subsequent to completing his

[3] ***Berndard Bolzano*** (1781–1848) was a Czech mathematician, logician, and philosopher, regarded as "one of the most important figures in modern Western thought" (Wang; Wang, 2022, p. 85). His intellectual interests were very broad, including Theology, Philosophy and Mathematics, notably standing out in each of these areas, producing important theological, philosophical and mathematical works; thus, for example: in Theology, he published Science of Religion (1834); in Philosophy, he wrote Theory of Science (1837), a work composed of five volumes and a manual in which he developed new theoretical and conceptual bases for Logic; in Mathematics, he produced his Theory of Quantities, a major work that he unfortunately failed to complete. Bolzano's immense philosophical contribution is composed of the following five volumes: Theory of Foundations, Theory of Elements, Theory of Knowledge, The Art of Discovery (Heuristics), and Theory of Science.

[4] ***Franz Clemens Brentano*** (1838-1917) is known primarily for his work in the philosophy of psychology, especially for having introduced the notion of intentionality to contemporary philosophy. He made major contributions to many fields of philosophy, especially the philosophy of mind, metaphysics and ontology, ethics, logic, the history of philosophy, and philosophical theology. Brentano was heavily influenced by Aristotle and the Scholastics, as well as by the empiricist and positivist movements of the early nineteenth century. Due to his introspective approach to describing consciousness from a first-person point of view, on the one hand, and his rigorous style, as well as his claim that philosophy should be done with exact methods like the natural sciences, Brentano is often considered a precursor of both the phenomenological movement and the tradition of analytic philosophy. A charismatic teacher, Brentano exerted a strong influence on the work of Edmund Husserl, Alexius Meinong, Christian von Ehrenfels, Kasimir Twardowski, Carl Stumpf, and Anton Marty, among others, and thus played a central role in the philosophical development of Central Europe in the early twentieth century. https://plato.stanford.edu/entries/brentano/)

university education. At that time, Husserl exhibited minimal interest in philosophy. He sought out Brentano at the suggestion of his friend Masaryk (Schuhmann, 1988), who had been a student of Brentano (cf. Farber, 1940, p. 238). This attitude originated from mere curiosity and subsequently evolved into genuine excitement. In one of the lectures attended by Husserl, Brentano examined Bolzano's work entitled *The Paradoxes of the Infinite*.

As he advanced in his study of mathematics and associated philosophical themes, Husserl grappled with the dilemma of continuing as a mathematician or pursuing philosophy. While studying with Brentano, he realized that the reasoning and argumentation of that professor were articulated with great rigor, akin to the methods he esteemed in Weierstrass' procedures. He then understood that Philosophy could be regarded as a rigorous science, deciding on Philosophy, as stated by Farber (1968):

> Husserl related that he did not long resist the power of his personality, despite all prejudices. It was from these lectures that he gained the conviction that philosophy is a field for earnest work which can be treated in the spirit of the most rigorous science, and this led him to choose philosophy as a lifework. (Farber, 1968, p. 9)

Acquainted with Bolzano's contributions through Weierstrass and Brentano, Husserl conducted a more meticulous examination of this author from 1894 to 1896, during which he explored the *Theory of Science* (Bolzano, 1972/1837). Lapointe (2012, p. 5) asserts that Husserl's interpretation of Bolzano's work informs both his critiques of psychologism and his understanding of the function of Logic within the theory of knowledge, as evidenced in his *Logical Investigations*. In this work, Husserl developed six studies characterized as *descriptive-psychological* and *epistemological* whose subjects are: (I) expression and meaning, (II) universals, (III) the formal ontology of the parts and the whole, or mereology[5], (Contarato, 2022; Nunes, 2020a, 2020b) included in Logical Investigations III, entitled, precisely, *"Zur Lehre von den Ganzen und Teilen" (On the Theory of All and Parts)*, (IV) the

[5] Mereology refers to the most fundamental epistemological distinction of Husserl's Phenomenology: that between the real and the ideal: this notion is developed by Husserl, in *the Prolegomena to Pure Logic*.

syntax and mereological structure of meaning, (V) the nature and structure of intentionality, and (VI) the interrelation of truth, intuition, and cognition. Departing from psychologism, Husserl adopted a variation of Platonism[6], influenced by his interpretations of the ideas of Bolzano and Lotze (1817 - 1881), as Husserl elucidated in the following excerpt, inserted in the preface he wrote in 1913 for a second edition of *Logical Investigations*:

> For the fully conscious and radical turn and for the accompanying "Platonism," I must credit the study of Lotze's logic. Little as Lotze himself had gone beyond [pointing out] absurd inconsistencies and beyond psychologism, still his brilliant interpretation of Plato's doctrine of Ideas gave me my first big insight and was a determining factor in all further studies. Lotze spoke already of truths in themselves, and so the idea suggested itself to transfer all of the mathematical and a major part of the traditionally logical [world] into the realm of the ideal. (Husserl, 1913, p. 36)

In the initial volume of his *Logical Investigations*, (Husserl, 2014), acknowledging his indebtedness to Bolzano, asserted:

> The comparison of the present Logical Investigations with the work of Bolzano will teach that these are not at all mere comments or critical improvement exhibitions of the

[6] It is recurrent that scholars of Husserl, who refer to him in the context of his mathematical education and as a mathematician, mention him as a Platonist. They claim that both Plato and Husserl believe in the objectivity and universality of mathematical truths. They argue that, for both, mathematical truths are not merely human conventions or constructs, but rather discoveries about the fundamental structure of reality. However, a deeper analysis of these statements is required, entering into the ways in which both philosophers refer to the reality of mathematical objectualities. Plato understands that mathematical truths are objective and universal. They speak of the fundamental structure of reality, beyond what is shown in the dimension of the sensible world. Husserl takes mathematical objectualities as constituted (in the dimension of meaning, perceived, apprehended and articulated by consciousness within the scope of Leib) and produced (in the dimension of intersubjectivity, of the historicity of Lebenswelt and of the objectification made possible by language), as universal structures that are objectified in idealities. These are not Platonic ideas, but idealities structured according to properties, which are also outlined in this thinking, and which constitutively form these objects. Therefore, they are not entities generated by human convention. Nor are they discovered, as for Plato, the fundamental structure of reality. They are constituted and produced by men, existing in the historicity of Lebenswelt.

> constructions of thought of Bolzano, despite having received decisive influences from him (Husserl, 2014, p. 169).

This statement suggests that Husserl's engagement with Bolzano's work, particularly the Theory of Sciences, significantly impacted his intellectual development. Wang (2022) notes that in Volume I of Logical Investigations, Husserl outlines three purposes for his work while also emphasizing the impact of Bolzano's ideas, since

> Husserl's mission and task of giving pure logic is the same as Bolzano's view of logic as "the rules of thought" or "the rules of truth in itself", that is, pure logic is different from "technology" or "the art of discovery" as scientific practice. The former belongs to "general logic" in a narrow sense and the latter is regarded as "special logic" in a broad sense; it is about the applied logic of specific Science," (Wang; 2022, p. 86)

Despite Husserl's commitment to Philosophy as his primary endeavor, he maintained a connection to Mathematical issues throughout his life. According to Haddock (2012), there are historiographical, or content distortions related to this subject. Moreover,

> [...] some of the few scholars in analytic circles that have dared to mention Husserl when discussing problems in one way or another related to the foundations of mathematics have made either historiographical distortions - e. g., of the relation between Husserl's views and those of Frege - or contextual distortions. (Haddock, 2012, p. 91).

So much so that Centrone (2017) asserts that to gain a comprehensive understanding of Husserl's perspective on Mathematics, it is essential to study not only his primary texts, such as *Philosophy of Arithmetic* and *Logical Investigations*, but also to engage deeply with his numerous other writings that reference the works of various mathematicians. Centrone examines the connections between Husserl's views and those of Leibniz, Grassman, Boole, Schröder, Brentano, Cantor, Frege, Riemann, Hilbert, Brower, Weyl, Kronecker, Carnap, Gödel, and Klein in his essays on the *Logic and Philosophy of Mathematics*. He recognizes that studying the historical-phenomenological perspective of Husserl's works requires consideration of his interactions and influences from other scholars.

Therefore, we agree with Haddock when he states that:

> The fact, however, is not only that Husserl was a mathematician turned philosopher, and - as Frege and Whitehead – one especially concerned with foundational problems, but also one that was very conscious - surely much more than Frege - of the development of mathematics in the nineteenth century, particularly in its second half, and that frequently refers to the research of some of that century's pioneers of contemporary mathematics, like Riemann, Helmholtz, Grassmann, Lie, Klein and Cantor. (Haddock, 2012, p. 92)

It must be pointed out that Husserl's interest in mathematics remained significant, even as his primary focus shifted to philosophy, epistemology, and phenomenology. This is evident in his works after this period, such as *Formal and Transcendental Logic* (1962) and *Experience and Judgement* (1980), published posthumously. Haddock asserts that, although these texts were not explicitly dedicated to Mathematics, "they do contain many insights of special relevance to the foundations of Mathematics" (Haddock, 2012, p. 93).

Claire Ortiz Hill is another author who emphasizes the significance of acknowledging Husserl's mathematical background. She argues that a comprehensive understanding of Husserlian philosophy requires an examination of the significance of his mathematical background in shaping his ideas, as well as a thorough investigation of the mathematical education he received during the early phase of his intellectual development (Hill, 2002). t is noteworthy that Husserl graduated as a mathematician during a period of significant intellectual vibrancy among the foremost mathematicians of the nineteenth and early twentieth centuries. His mathematical concepts provide answers to difficult problems regarding seemingly mysterious portions of his worldview. According to Hill,

> Understanding the evolution of Husserl's views on mathematics is therefore essential to establishing Husserl's proper place in 20th century philosophy of logic and mathematics, a field with deep roots in Austro German ideas

> about mathematics, logic and philosophy, which flowered in English-speaking countries in the twentieth century [..] (Hill, 2002, p. 78)

Hill (2002) explains that Husserl's contributions to the foundations of Mathematics and the Theory of Knowledge have been misunderstood, distorted or simply neglected, resulting in his ideas being inadequately acknowledged and referenced by other scholars who have contributed to this foundation. This assertion is substantiated by Haddock (2012), the previously cited author.

The subsequent sections elucidate pertinent facets of Husserl's education in mathematics and the significance of his contributions.

HUSSERL'S EDUCATION IN MATHEMATICS

The seventeenth century saw a significant milestone in the history of Mathematics: the invention of Calculus. It began almost simultaneously with Newton and Leibniz, figures who are frequently referenced regarded as the most illustrious. However, they were not the only ones who contributed to such a feat. Other important mathematicians, such as the Bernoulli brothers, also conducted essential work.

Calculus rapidly demonstrated its efficacy as a powerful and effective tool, enabling the resolution of longstanding unsolved problems, including those pertaining to curves and physical systems, as well as challenges involving the concept of *infinity*. During the eighteenth and nineteenth centuries, Calculus underwent significant advancement, especially concerning the concept of function and associated notions, like continuity, differentiability, and convergence. All these concepts, whether tacitly or explicitly, pertain to concerns related to the concept of *infinity*.

In the seventeenth and eighteenth centuries, Calculus procedures were successfully implemented to the resolution of various problems, both internal and external to Mathematics. Calculus exhibited a paradoxical situation: on the one hand, it demonstrated vitality and strength to the extent that its techniques and procedures were able to solve problems that had not been solved before; on the other hand, it mitigates concerns with clarification of its rationale. This indicated that, despite the remarkable success of Newton's and Leibniz's creation in addressing mechanical and physical issues, their justifications for the employed methodologies elicited much scepticism and significant criticism, particularly from Bishop Berkeley. Numerous mathematicians, including Taylor and Maclaurin, endeavoured to account for such criticisms, albeit unsuccessfully.

Consequently, the need to establish more robust foundations for Calculus specifically, and for Mathematics in general, persisted. It was essential to elucidate the conceptual frameworks underpinning the rationale for the procedures and techniques employed in Calculus, as well as to comprehend the principles and concepts that affirm the validity and appropriateness of these techniques for resolving mathematical problems. In this context, Husserl pursued his education as a mathematician from 1877 until 1881.

Husserl began his mathematical studies at the University of Berlin, a city regarded as the global hub for mathematics' education. Kummer, Kronecker, and Weierstrass were eminent figures, the latter possessing an authority universally acknowledged; these three distinguished mathematicians established the Mathematics Seminar in the latter half of the nineteenth century, as noted by Calinger (1996), had "[...] a prominence in mathematics comparable to that of France's École Polytechnique earlier in the century" (p. 153).

The *Seminar* remained active for 20 years (1860-1880). A maximum of 12 students were admitted each year after submission of a paper or the completion of an examination. Students were required to attend the complete cycle of the Conferences that included the following topics: Theory of Analytic Functions; Theory of Elliptic Functions; Calculation of Variations; Applications of Elliptic Functions in the solution of Geometry or Mechanics problems; Theory of Abelian Functions and Advances in the Theory of Analytic Functions (Cf Silva, 2021).

To ensure that students read the original works of the reference authors, there was a library that provided access to the works of Abel, Cauchy, Euler, Monge, Poisson, and many other first-level mathematicians. In addition to these works, the Seminar was connected to the most significant periodicals of the time: *Journal de Mathématiques Pures et Appliquées* (known as Journal de Liouville in honor of its founder, Joseph Liouville, in 1836); *Archiv der*

Mathematik und Physik (The Archive of Mathematics and Physics, also known as *Grunert's Archive*, a scientific journal founded by Johann August Grunert in 1841, and operating until 1920); and *Journal für die reine und angewandte Mathematik (Journal for Pure and Applied Mathematics*, founded by August Leopold Crelle in Berlin in 1826, known there simply as *Crelle's Journal*. After the death of its founder, one of the directors of this periodical was Leopold Kronecker. A prize was established to encourage students, thereby promoting the advancement of their research projects.

The *Seminar* was directed by Weierstrass, who regarded it as the appropriate space for the development of his Theory of Analytic Functions (Knopp, 1945). This research prompted him to explore the foundational principles of Arithmetic, recognizing the significance of establishing Mathematics as a science on a basis distinct from that provided by Geometry. Weierstrass recognized the need to reinforce the foundational concepts of Mathematics, which originated in the seventeenth and eighteenth centuries. He spearheaded a significant movement to establish the fundamental principles of Mathematical Analysis and to derive all of Mathematics from these principles in a rigorously systematic manner.

As for the legacy left by Weierstrass, Massa Esteve (2016) points out that

> One question raised is: how would it be possible to preserve the immense legacy left by Weierstrass, given that due to his insurmountable differences with Kronecker, editor of *Crelle's Journal*, his work might remain unknown? [...] to avoid this problem, Weierstrass decided to edit his complete works himself. In 1894 he edited the first volume, the second in 1895 and after his death his disciples Johannes Knoblauch (1855-1915) and Georg Hettner (1854-1914) edited up to seven volumes, the last in 1927. Finally, all were reprinted in 1967. Weierstrass' interest in ensuring that everything published resulting from his research was true and rigorous must be noted. For him, the most important thing was not the authorship of the publication or the fact that it was cited, but rather that scientific knowledge could be truly advanced. This generous approach to research was not at all common at the time. (Massa Esteve, 2016, p. 33).

The movement led by Weierstrass and others was known as the *Arithmetization of Analysis*. Its core was the creation of a solid theory of real numbers, which is why the clarification of the concept of number becomes a crucial theme in the foundations of Mathematics, according to Guamanga (2021). Young Husserl became interested in this topic and studied it in depth.

On several occasions, Husserl commended Weierstrass' methodical and rigorous methodology, applauding his contributions to establishing a solid foundation for ideas on analytic functions, while also noting his influence on Husserl's intellectual approach. ‘So much so that he came to affirm that he "intended to do with Philosophy what Weierstrass had done with Mathematics" (Hill, 2002, p. 78). Weierstrass’ effort to introduce rigor into Mathematical Analysis can be seen in the ways of working that Husserl preserved throughout his career

> [...] but also in Husserl's struggles with psychologism, his lifelong search for radical foundations for knowledge, his striving to lay bare the original roots, the most primitive concepts and principles of knowledge, to uncover the fundamental building blocks on the basis of which his whole system of philosophy might rest, his ideas about phenomenology as a strict science, [...] (Hill, 2002, p. 79).

Thus, while admiring the working methods of his mentor, Weierstrass, Husserl expressed interest in searching for the radical foundations for Mathematics, which led him to perform deep research on the principles of Arithmetic. According to Hill (2002):

> Weierstrass' thoroughgoing, systematic treatment, ab initio, of the theory of analytic functions had led him to profound investigations into the principles of arithmetic. His scrupulous manner of submitting the foundations of analytic functions to close scrutiny awoke in Husserl an interest in seeking radical foundations for mathematics. (Hill, 2002, p. 78)

This is evident in his early writings, previously referenced, *On the Concept of Number; Philosophy of Arithmetic and Logical Investigations*, in which he conducted an in-depth analysis of fundamental elements of twentieth-century mathematics. Thus, in *On the concept of number (Über den Begriff der*

Zahl), which was his Habilitation thesis (*Habilitationsschrift*), it is possible to discern the convergence of his ideas with those of his contemporaneous mathematicians, including Frege, author of *Basic Notions of Arithmetic: A Mathematical Logical Study Of The Concept Of Number* (1960); Kronecker, author of *On the Concept of Number in Mathematics* (1891); von Helmholtz who wrote *Counting and Measurement from an Epistemological View* (1887); and Dedekind, author of *What Are Numbers and What They Are Useful for* (1887). The treatise in his work on Habilitation suggests that Husserl engaged the debate on the rationale of the idea of number.

It is worth noting that among the sources that fed Husserl's thought is Mathematics, but not only Mathematics, but also Psychology and Philosophy; these three disciplines appear in his work *Philosophy of Arithmetic*, where he synthesizes the mathematical influences derived from Weierstrass' classes, and the philosophical and psychological ideas deriving from his classes with Brentano and other studies.

THE CORE OF WIEIERSTRASS' THEORY OF ARITHMETIZATION AND HUSSERL'S WORK

As previously stated, Weierstrass' fundamental objective in establishing a rigorous foundation for Calculus was to develop a robust framework for the system of real numbers and to derive the essential notions of analysis from this framework.

One of the core issues addressed by Weierstrass pertains to the association of the concept of number with the so-called "number line" (Sinkevich, 2015). He sought to overcome all appeal to geometrical intuitions and:

> [...] build up the system of real numbers in a purely formal manner. Beginning with a rigorous development of the system of positive whole numbers, he proceeded to generate the system of integers in general, the system of rational numbers and, finally, the system of real numbers itself, through a series of precisely formulated steps. (Miller, 1982, p. 2).

Weierstrass' work was part of a broader process, historically known as the "Arithmetization of Analysis"[7], which implied the need to clearly define the notion of real number, a challenge undertaken by several prominent mathematicians of the time, as shown below.

In the second half of the nineteenth century, within the framework of the Arithmetization of Analysis (Klein, 1887), several attempts were made to provide a solid foundation to Mathematics; this procedure underscored the necessity of elucidating the concept of real numbers unequivocally. Thus, the primary objective was to define the real number only on arithmetical bases and without resorting to geometric intuition. According to Lopes (2006), three routes were followed with this intent.

The first route, advocated by Hankel and Frege, supported Analysis on the idea of continuous quantity; the second route, assumed by Dedekind, Weierstrass and Cantor, stated that this notion "[...] should be substituted with a stringent arithmetic formulation of real numbers, namely one grounded on the concept of natural or rational numbers, which was anticipated to be less contentious than the concept of continuous quantity" (Lopes, 2006, p. 3). The third route, endorsed by Heine and Hilbert, asserted "[...] that the fundamental concepts of Analysis could, and should, be constructed simply in a formal way, disregarding philosophical issues as much as possible" (Lopes, op. cit.).

In these three pathways, reaching the real number required beginning with the concept of rational numbers, so underscoring the significance of defining those numbers that are not rational, namely, the irrational numbers. Consequently, this characterization emerged as the initial challenge to be addressed.

According to Lopes & Sá (2018), the most important contributions in the search for the resolution of that problem were made by Charles Méray, Karl Weierstrass, Georg Cantor and Richard Dedekind. However, it is essential to acknowledge, Bolzano had contemplated the need for Arithmetization of Mathematics. So great was the contribution of this outstanding scientist that, on account of the principles formulated in his work entitled "*Rein analytischer*

---

[7] Félix Klein (1895) refers to this process with the expression "Arithmetization of Mathematics" (*Über die Arithmetisierung der Mathematik)*

Beweis des Lehrsatzes, dass zwischen zwei Werten, die ein entgegengesetztes Resultat gewaehren, wenigstens eine reelle Wurzel der Gleichung liege..." "(A purely analytical proof of the theorem that between two values with opposite results, there is at least one real root of the equation...)", Klein (1987) considered him as "*one of the fathers of the Arithmetization of Mathematics*." (Klein, 1926, p. 56).

Next, a brief overview of the main theories regarding real numbers will be presented[8]. According to Lopes & Sá (2018), Méray employed successions to define irrational numbers regardless of the notion of limit. Dedekind did not define the real number as a convergent sequence of rational numbers, rather observed that "[...] In any division of the points of the segment into two classes such that each point belongs to one and only one, and such that every point in one class is to the left of every point of the other, there is one and only one point that performs the division" (Lopes & Sá, 2018, p. 88). thus, Dedekind formulated his celebrated strategy of *Dedekind Cuts.*

Another mathematician who made important efforts to define the real number arithmetically was Weierstrass, who

> [...] unlike the other two, he did not limit himself to constructing the real from a presupposed construction of rationals. Weierstrass starts from the more general notion of number and the fundamental operations of Arithmetic; he first introduces the concept of natural number and then that of positive rational number; It will then be from the "aggregates" of these numbers that he will obtain quantities beyond the rational ones. For this reason, in Weierstrass' theory of real numbers, one cannot dissociate the natures of natural, rational, and real numbers. Weierstrass builds his theory in an entirely analytical way, endowing it with a rigor very characteristic of all his mathematical work and elaborating the most complete theory of real numbers of the nineteenth century. (Martins, 2004, p. 1)

8 To examine some of the constructive modalities of the field of real numbers, see Arboleda (2007) and Gray (2015).

Ultimately, Cantor, drawing upon the teachings of his mentor Weierstrass and the exchange of ideas with his associate and friend Dedekind, formulated his definition of real numbers as equivalence classes of sequences of rational numbers. (Costa, 2017)

As a student of Weierstrass, Husserl meticulously documented his lectures, closely engaging with his mentor's efforts to establish Mathematical Analysis on non-geometrically intuitive foundations. Thus, he can be considered as a privileged witness to the inception of Weierstrass' seminal work. In the Introduction to his *Philosophy of Arithmetic*, he asserts that:

> *Weierstrass* usually opened his epoch-making lectures on the theory of analytical functions with the sentences: "Pure arithmetic (or pure analysis) is a science based solely and only upon the concept of number [*Zahl*]. It requires no other presupposition whatsoever, no postulates or premises." (Almost identically the same in the summer semester of 1878 and the winter semester of 1 880/81.) There then followed the analysis of the number concept in the sense of the whole number. (Husserl, 2003, p. 13) (italics, quotation marks and parentheses in the author's original)

Boyer, in his *History of Calculus*, states that:

> To ensure logical accuracy, Weierstrass wanted to base the calculus (and function theory) solely on the concept of number and thus separate it completely from geometry. This required a definition of an irrational number which was totally independent of the idea of limit, since the latter presupposes the former. For that reason, Weierstrass was forced to conduct in-depth research into the principles of arithmetic, particularly with regard to the theory of irrational numbers. In this work, Weierstrass did not enter into the nature of integers themselves, but he began by conceiving the concept of integer as an aggregate of units possessing a characteristic property in common, whereas a complex number is to be considered as a set of units of several species possessing more than one characteristic property. (Boyer, 1959, p. 285)

Husserl considered the mathematical dimensions of number and went beyond. He conducted a psychological analysis, seeking the cognitive origins of the number and, thereafter, engaged in philosophical and logical discussions.

Weierstrass' assertions concerning numbers, fundamentally asserting that they can be traced back to the mental act of counting, were important to Husserl's early research into the philosophy of mathematics. There is no indication that Weierstrass saw the issue of the origin of numbers as particularly challenging or as a matter that should be addressed by philosophers rather than solely by mathematicians. However, for Husserl, this issue was both challenging and significant, necessitating meticulous and thorough research. This was the main issue addressed in the investigation outlined in the *Philosophy of Arithmetic*. His intention was to complete Weierstrass' work and establish robust foundations for Mathematical Analysis. To this end, he recognized that by addressing the issues of the origin of number, with philosophical rigor akin to that employed by his mentor in Mathematics, he would be able to accomplish the intended goal of advancing Weierstrass' program. Thus, Husserl's *Philosophy of Arithmetic* was formulated as an attempt to address the non-mathematical questions to which the program of *Arithmetization of Analysis* inevitably led. According to Miller:

> Weierstrass' assertion that numbers in the most fundamental sense are to be traced back to the mental act of counting served as the starting point for Husserl's own early research in the philosophy of mathematics. There is no evidence that Weierstrass himself saw the problem of the origin of number as a particularly difficult problem or as an issue to be taken up by philosophers, as distinct from mathematicians themselves. His student Husserl did, however, see it as a difficult problem requiring careful and detailed investigations. And when Husserl first began to speak of the 'philosophy of arithmetic,' it was this problem which he had primarily in mind. His intention was to carry through to the completion of Weierstrass' program for securing the foundations of analysis, and to do so by applying to the problem of the origin of number a philosophical rigor comparable to the rigor which his teacher had demanded in mathematics proper. Thus Husserl's philosophy of arithmetic took shape as an attempt to address the non-mathematical

> issues to which the program of arithmetizing analysis inevitably led. (Miller, 1982, p. 4)

In his early studies on the subject of the origin of number, Husserl exhibited a notable proximity to Weierstrass; but as his reflections evolved, he progressively drifted away from his mentor. Husserl points out differences in perspective concerning the *arithmetization of analysis*. While his mentor asserted that it should occur solely within the domain of Mathematics, he admitted that the foundations of Mathematics had to be established in Philosophy and Psychology as well. In fact, he articulated this perspective in his Habilitation work, entitled *On the Concept of Number: Psychological Analyses*. This subtitle underscores the importance he attributed to Psychology in the examination of this essential notion of Mathematics. However, it is in *Philosophy of Arithmetic* that he makes his ideas about the concept of number more explicit:

> [...] in the *Philosophy of Arithmetic*, Husserl divides Weierstrass' view of arithmetized analysis into the system of numerals and blind computations and the system of numbers and conceptual connections. Very sharply, Husserl thus establishes a distinction, which today would be called a distinction between syntax and semantics. The starting point for Husserl's conceptual analysis, i.e., semantics, is to describe the way in which we, in our everyday life, think of numbers. In the first part of Philosophy of Arithmetic, Husserl describes an authentic arithmetic that consists of a descriptive psychological analysis of a concept of number and relations between numbers. Hence, Husserl, already in the Philosophy of Arithmetic differs from Frege, as well as a host of later analytic philosophers, in taking the basis of philosophy to be in analysis of experience rather than in logic. (Hartimo, 2006, p. 334).

Husserl's interest in tracing connections between Mathematics and Philosophy is notable. In the *Philosophy of Arithmetic,* he already indicated that when mathematicians produced their work, they were not able to adequately examine their concepts and methods; hence he recognized the need to conduct both a logical clarification and a more precise analysis of the concepts and methods employed by these professionals. Similar to Weierstrass' plan of not grounding mathematics on empirical experience, Husserl, in his work, had taken internal experience as a foundation to elucidate the origin of the concept of number and, employing methods of Descriptive Psychology. It is important to note that Koenigsberger (1874) asserted that:

> We obtain the concept of whole numbers by becoming aware of a repetition of one and the same mental activity, applied to a given substratum of the phenomenal world, to objects perceived by our senses; a unique mental apprehension [Auffassen] of a single body, e.g. a ball, would not lead us yet to the number, only the perception of a series of objects, that share a common feature [Merkmal], and hence are similar [gleichartig] in a certain respect - similar either in form, or also in color, matter etc. - will imply the repetition of the same mental activity and deliver the concept of the named whole number, the concept of ten balls or of ten red balls or of ten wooden red balls etc. Once we have arrived at various series of named numbers and at various named unities, then the comparison of these series, i.e. the abstraction from the common features of the members of a series leads to the concept of the unnamed number, by mentally considering as common feature of all the observed phenomena just the fact that they can be observed in succession. (Koenigsberger, 1874, apud Ierna, 2006, p. 37-38)

Weierstrass' strategy of conceiving counting as an iterated operation applied to analogous objects was adopted by Koenigsberger (1874) and Husserl. In his work The *Philosophy of Arithmetic*, Husserl regarded abstraction[9] as a fundamental element in the genesis of number. It is worth noting that Koenigsberger made a distinction between *proper numbers and improper numbers*, the former being obtained by abstraction, while the latter obtained through construction. Husserl posits this division, which appears to be derived from Weierstrass' thoughts, who, at that time, had adopted Bolzano's

[9] It must be stressed that the abstract character of a concept comes from the activity of abstracting, which is intentional; in this sense, DeBoer (1978) asserts that "According to Husserl, the general concept is abstracted from that which a number of objects have in common. Abstraction is an act of attention in which we disregard the differences and focus exclusively on similarities. That which a number of things have in common is the fundamentum-in-re of the general concept. (De Boer, 1978 p. 235)".

stance. In his initial work on the genesis of number, Husserl adopted an interpretative approach, influenced by Brentano's Descriptive Psychology.

Frege's critiques of Husserl's Philosophy of Arithmetic, published in 1891, mostly focused on psychological elements. In this work, Husserl expressed his expertise in mathematics, psychology, and philosophy. Frege scrutinized it, and, according to Mohanty (1977), critiques it for its due to its *underlying psychologism*. However, it is worth noting that they were friends and maintained correspondence discussing topics that dealt with some specific aspects of their theories of meaning, as well as Frege's explicit rejection of any form of psychologism. Between 1891 and 1894, the letters they exchanged concerned the objective nature of logic and mathematics, the philosophy of arithmetic, number theory, the distinction among representation, sense, and meaning (*Repräsentation, Sinn,* and *Bedeutung*), and the theory of concept as a referent of predicates. As stated by Soares (2010):

> The correspondence between the two philosophers shows that a dialogue and a confrontation were established between them on some specific aspects of their theories of meaning as well as the rejection of any form of psychologism. Between the years 1891 and 1894, the letters they exchanged concerned the objective status of logic and mathematics, the philosophy of arithmetic, number theory, the distinction between *Vorstellung*, *Sinn* and *Bedeutung*, and the theory of the concept as a referent of predicates. In a letter dated May 1891, Frege acknowledged receipt of a copy of Husserl's Philosophy of Arithmetic, which was the subject of a review by Frege, in which he attributed to Husserl's work traces of psychologism in the way he understood the theory of number. (Soares, 2010, p. 26-27)

UNDERSTANDINGS ABOUT HUSSERL'S MATHEMATICAL EDUCATION

The aforementioned considerations indicate that Husserl's mathematical education primarily occurred during the 1880s, when he actively engaged in the movement of the Arithmetization of Analysis alongside prominent mathematicians, including Leopold Kronecker and Karl

Weierstrass, who had already established significant and influential work at that time.

As pointed out throughout this text, Husserl's mathematical education was quite rigorous. Alongside his mentorship under mathematicians Kronecker and Weierstrass, he was a colleague and friend of Cantor when he was at the University of Halle; he was a member of the Hilbert Circle at the University of Göttingen, where he served as a professor for fifteen years. As shown in our investigations, during this period, Zermelo shared with Husserl his discovery of the so-called *Zermelo-Russell Paradox*.

His initial studies in mathematics, conducted in Berlin from 1877 to 1881, were marked by his association to great masters, as previously noted. In this environment, the debates centered on the *principles of arithmetic*. With his doctorate, *Contributions to the Calculus of Variations*, in 1883, he began his academic career, demonstrating his distinctive approach from the outset. His sustained engagement in Mathematics was characterized by his habilitation at the University of Halle, culminating with the production of "*On the Concept of Number"*, followed by his *Philosophy of Arithmetic*, wherein he began his exploration of the field of the Philosophy of Mathematics, aiming to clarify the logical nature of its essential principles and concepts, which he further elaborated in his *Logical Investigations*. Thus, it may be inferred that, from their *Contributions...* up to his *Investigations...*, there is an uninterrupted mathematical trajectory that demonstrates Husserl's interest in actively participating in the pursuit of resolving the Mathematical Crisis in which he was immersed at the time.

The comprehension of the fundamental issue addressed by these prominent nineteenth-century mathematicians prompted Husserl to persist in his inquiry. The research he conducted prompted na exploration of radical foundations of Mathematics, centered on the comprehension of the concept of number as being the basis of these foundations. He delved deeply into the concept of the origin of number, articulating his theories alongside those of other contemporary scholars, particularly *Friedrich Ludwig Gottlob Frege, Leopold Kronecker; Hermann von Helmholtz* and *Julius Wilhelm Richard Dedekind.*

As we can understand, Husserl was very knowledgeable about mathematical concepts and their associated issues. He was aware of the advancements regarding numbers up to that moment, when he contributed to the program of the *Arithmetization of Analysis*, proposed by Weierstrass. This work is evident in numerous ideas articulated in his seminal works that were produced during his tenure at the University of Halle. It is noteworthy that, during the period, he was a colleague and friend of Georg Ferdinand Ludwig Philipp Cantor, known simply as Georg Cantor, with whom he shared understandings of the problems that captivated both of them, particularly those regarding the foundation of Mathematics.

FINAL CONSIDERATIONS

In this essay, we explain the context in which Husserl's education as a mathematician was developed. He had the privilege of witnessing the causes of the so-called Crises of Mathematics as a student of Weierstrass and of being a protagonist in the efforts to resolve this crisis, as he participated in the Arithmetization of Analysis and, therefore, of Mathematics. Evidence of this participation were his works: *On the Concept of Number; Philosophy of Arithmetic* and *Logical Investigations*. In this context, Husserl's name appears alongside other mathematicians, philosophers and logicians such as Weierstrass, Kronecker, Cantor, Dedekind, Leibniz and Frege.

Key references to follow the trajectory of Husserl's education in Mathematics are as follows: (a) his participation in the *course taught by Weierstrass*; (b) the work for his Doctorate, in which he made *theoretical contributions to the Calculus of Variations*; (c) the work with which he achieved his habilitation as a university professor, whereby he conducted a *psychological analysis of the concept of number*, which constituted the prelude to his Philosophy of Arithmetic, in which he furthered his reflections on number and embarked on his journey through the complexities of philosophy, which he would further develop in his *Logical Investigations*.

As he conducted his research, Husserl came to understand that his contribution to the program of his mentor Weierstrass on arithmetization of mathematics had to transcend disciplinary boundaries and develop a type of metamathematics. This disposition is evident in the fact that, while investigating the origin of number, he sought its roots, which extended beyond the concept of number and the presentations of the construction of number. At first, in *Philosophy of Arithmetic*, he clarified psychological analyses regarding the origin of numbers. Advancing in his investigations, he penetrated through logic and philosophy. Simultaneously, while researching and exposing understandings of the topics focused, he became concerned with the manner in which he conducted the movement of this search, always concerned with the clarity and rigor of the paths taken in this search, specifically questioning the validity of his methodological approaches. This perpetual inquiry is a hallmark of his phenomenological thought, prompting him to formulate his own phenomenology and corresponding phenomenological methodology.

We emphasize two essential points highlighted in this research. The initial aspect, noted by numerous other scholars, refers to Husserl's enduring fascination with Mathematics, which is manifested in clearer concepts and the stringent demands of his investigative methodologies. The second point, which we believe to be outstanding in the long review conducted to author this essay, refers to the lack of mentions of Edmund Husserl's name in important and well-known texts regarding the History of Mathematics, such as, for example, that of Boyer (1959). We understand that his absence from the list of distinguished mathematicians, despite his substantial mathematical background and involvement in the Arithmetization of Analysis movement prompted by the Crisis of the Foundations of Mathematics, can be attributed to the nature of his research, which seeks to explore Mathematics, its properties, and its methodologies, extending inquiries beyond the discipline itself. Thus, he conducted a meta-analysis of the senses and meanings of concepts, as well as a philosophical examination of this discipline. For him, it was insufficient to merely perform or comprehend mathematics; one must also grasp it in relation to reality, considering the subjectivity of the individual and that of the historicity of the *Lebenswelt (life-world)*, as well as the discourse regarding the world and the perspectives from which its truths may be acknowledged and utilized. His name does not appear, therefore, in demonstrated theorems, but he is regarded as one of the most prominent philosophers of the 20th century.

AUTHORS' CONTRIBUTION STATEMENTS

GF and MAVB, together, developed the research, organizing the theoretical part, the methodological design, the FG carried out the coleta and data analysis. Finally, GF, with the guidance and supervision of MAVB, wrote the article for this research.

DATA AVAILABILITY STATEMENT

Data supporting the results of this study will be made available by FG upon reasonable request.

References

Arboleda, L. C. (2020). MODALIDADES CONSTRUCTIVAS Y OBJETIVACION DEL CUERPO DE LOS REALES. *Revista Brasileira de História da Matemática*, 19. https://doi.org/10.47976/RBHM2007vn19

Bello, A. A. (2022). *Husserl e as Ciências*. (M. A. V. Bicudo; J. C. Bortolete; R. de F. Batistela, Tradutores para português). São Paulo, SP: Livraria da Fisica,

Bicudo, M. A. V. (2020). The origin of number and origin of geometry: issues raised, and conceptions assumed by Edmund Husserl. *Qualitative Research Journal. São Paulo (SP), v.8, n.18, p. 387-418, Special Edition: Philosophy of Mathematics.* DOI: http://dx.doi.org/10.33361/RPQ.2020.v.8.n.18.337

Bolzano, B. (1950, original de 1831). *The Paradoxes of the Infinite*. London: Routledge and Kegan Paul.

Bolzano, B. (1972, original de 1837). *Theory of Science. Attempt at a Detailed and in the main Novel Exposition of LOGIC. With Constant Attention to Earlier Authors*. Edited and translated by Rolf George. Berkeley and Los Angeles: University of California Press.

Bolzano, B. (1991). *Las Paradojas del Infinito*. México, DF: Mathema (UNAM), f. 161.

Boyer, C. (1959). *History of Calculus*. New York: Dover.

Calinger, R. (1996). The Mathematics Seminar at the University of Berlim: Origins, Funding and the Kummer-Weierstrass Years. In: R. Calinger (Ed.). *VITA MATHEMATICA. Historical research and integration with teaching*. Washington, DC: Mathematical Association of America. II. Historical Studies: From the Scientific Revolution to the Present, p. 153-176.

Centrone, S. (Ed.). (2017). *Essays on Husserl's Logic and Philosophy of Mathematics*. Dordrecht, The Netherlands: Springer Verlag. (Synthese Library. Studies in Epistemology, Logic, Methodology, and Philosophy of Science. Volume 384).

Centrone, S.; Silva, J. J. da. (2017). *Husserl and Leibniz: Notes on the Mathesis Universalis*. In: Centrone (2017), Chapter 1, pp 1-23.

Contarato, T. S. R. (2022). *Mereologia, a composição do mundo – reflexões filosóficas sobre as partes que compõem os objetos*. Ponta Grossa - PR: Atena.

Costa, P. C. (2017). *A Construção dos Números Reais e Aplicações no Ensino Médio*, (Dissertação de Mestrado). Programa de Pós-Graduação Mestrado Profissional em Matemática em Rede Nacional, Universidade Federal de Viçosa (Minas Gerais, Brasil). https://locus.ufv.br/server/api/core/bitstreams/55ac7ea3-74b5-4bda-b1ba-dd7dcf0f8c81/content

De Boer, T. (1978). *The Development of Husserl's Thought*. (Translated by Theodore Plantinga). The Hague/Boston/London: Martinus Nijhoff.

Dedekind, R. (1998). *Qué son los números y para qué sirven? y otros escritos sobre los fundamentos de la Matemática*. Edición e Introducción a cargo de José Ferreirós. Madrid: Ediciones Alianza Editorial – Universidad Autónoma de Madrid. 188p (Versão digitalizada transcripta do original publicado em alemão no 1887 disponível aqui: http://www.opera-platonis.de/dedekind/Dedekind_Was_sind_2.pdf.

Farber, M. (2012, jul-dez). Edmund Husserl e os Fundamentos de sua Filosofia. *Revista da Abordagem Gestáltica* – XVIII(2): 235-245. Título original: "Edmund Husserl and the Background of his Philosophy," publicado na revista Philosophy and Phenomenological Research, Vol. 1, Nr.1, p. 1-20 (1940), editada pela International Phenomenological Society

Fink, E. (Ed.). (1975, original de 1913). *Edmund Husserl: Introduction to the Logical Investigations; A Draft of a Preface to the Logical Investigations* (Translated with Introductions by Philip J. Bossert and Curtis H. Peters). The Hague: Martinus Nijhoff.

Fonseca, A. & Silva, D. (2019). Estudo Histórico do Paradoxo de Russell: A fecundidade de uma matemática falível. En Pérez-Vera, Iván Esteban; García, Daysi (Eds.), *Acta Latinoamericana de Matemática Educativa* (pp. 544-553). México, DF: Comité Latinoamericano de Matemática Educativa. https://funes.uniandes.edu.co/wp-content/uploads/tainacan-items/32454/1153759/Fonseca2019Estudo.pdf

Frege, G. (1960, original de 1884). *The foundations of arithmetic: A logico-mathematical enquiry into the concept of number*. Translated by J. L. Austin. xxii + 119. New York: Harper & Brothers.

Gray, J. (2015). *The Real and the Complex: A History of Analysis in the Nineteenth Century*. xvi + 350 pp., figs., illus., apps., bibl., index. Cham, Switzerland: Springer.

Gray, J. (2015). The Construction of the Real Numbers. In: J. Gray. *The Real and the Complex: A History of Analysis in the 19th Century*. Springer Undergraduate Mathematics Series. Springer, Cham. Chapter 25, pp 253-258. https://doi.org/10.1007/978-3-319-23715-2_25

Guamanga, M. H. (2021). Husserl: ¿Fenomenología de la matemática? *Eidos: Revista de Filosofía de la Universidad del Norte*, 36:170-192. http://www.scielo.org.co/pdf/eidos/n36/2011-7477-eidos-36-170.pdf

Haddock, G. E. R. (Ed.) (2016). *Husserl and Analytic Philosophy*. Berlin/Boston: Walter de Gruyter GmbH, pp. viii + 338.

Haddock, G. E. R. (2012). *Against the Current: Selected Philosophical Papers*. Frankfurt: Ontos. ISBN: 978386838148; PP. xii + 456.

Haddock, G. E. R. (2012). *Husserl's Relevance for the Philosophy and Foundations of Mathematics*. In Haddock, (2012). Chapter 3, pp 99-109.

Hartimo, M. (2006). Mathematical roots of phenomenology: Husserl and the concept of number, *History and Philosophy of Logic*, 27:4, 319-337, https://doi.org/10.1080/01445340600619663 https://www.tandfonline.com/doi/abs/10.1080/01445340600619663

Hill, C. O. (2002). On Husserl's Mathematical Apprenticeship and Philosophy of Mathematics. In: Tymieniecka, A. T. (Editor). *Phenomenology World-Wide: Foundations - Expanding Dynamics - Life-engagements, a Guide for Research and Study*. Seção II: LAYING THE FOUNDATIONS OF PHENOMENOLOGY, pp 78-93, 2002.

Hopkins, B.; Crowell, S. & Col. (Eds.). (2005). *The New Yearbook for Phenomenology and Phenomenological Philosophy*, *Vol. V*. New York: ROUTLEDGE-Taylor and Francis Group. f. 414

Hopkins, B.; Crowell, S. & Col. (Eds.). (2006). *The New Yearbook for Phenomenology and Phenomenological Philosophy*, *Vol. VI*. New York: ROUTLEDGE-Taylor and Francis Group. f. 364

Husserl, E. (1975, original de 1913). *Introduction to the Logical Investigations; A Draft of a Preface to the Logical Investigations.* (Eugen Fink, ed. Translated with Introductions by Philip J. Bossert and Curtis H. Peters). The Hague: Martinus Nijhoff.

Husserl, E. (1981, original de 1887). Über den Begriff der Zahl. Psychologische Analysen. Halle: Heyneman. In: P. Mc Cormick and F. Elliston (eds.), *Husserl: Shorter Works*, Notre Dame: University of Notre Dame Press, 1981, 92-120.

Husserl, E. (1980, original de 1948). *Experiencia y Juicio. Investigaciones Acerca de la Genealogía de la Lógica.* (Redacción y edición de Ludwig Landgrebe Con un epílogo de Lothar Ele y Traducción: Jas Reuter. Revisión de Bernabé Navarro) (Título original en alemán: Erfahrung und Urteil. Editada por Claassen. Hamburg: 1948.). México 20, D. F.: Universidad Nacional Autónoma de México, Ciudad Universitaria (Primera edición en español, 1980)

Husserl, E. (2014). *Investigações Lógicas, Primeiro Volume, Prolegômenos à Lógica Pura*. De acordo com o texto de Husserliana XVIII Editado por Elmar Holenstein; Tradução de Diogo Ferrer). Rio de Janeiro: Forense Universitária, 2014, f. 114.

Husserl, E. (1962). *Lógica formal y trascendental.* (Trad. de L. Villoro). México: UNAM. (Original: *Formale und transzendentale Logik. Versuch einer Kritik der logischen Vernunft*. Hua XVII. Hrsg. von P. Janssen. Den Haag: Nijhoff, 1974).

Husserl, E. (2003, original de 1891). *Philosophy of Arithmetic. Psychological and Logical Investigations with Supplementary Texts from 1887–1901.* Collected Works X, D. Willard, trans., Dordrecht: Kluwer. [Original: Philosophie der Arithmetik. Psychologische und Logische Untersuchungen. Halle (Saale): C.E.M. Pfeffer (R. Stricker), 1891].

Ierna, C. (2005). The Beginnings of the Husserl's Philosophy, Part 1: On the Concept of the Number to Philosophy of Arithmetic. In: Hopkins, B.; Crowell, S. & Col., *The New Yearbook for Phenomenology and Phenomenological Philosophy, Vol. V*. Chapter 1, pp 1-56

Ierna, C. (2006). The Beginnings of Husserl's Philosophy, Part 2**:** From Über den Begriff der Zahl to Philosophie der Arithmetik. In: Hopkins, B.; Crowell, S. & Col., *The New Yearbook for Phenomenology and Phenomenological Philosophy, Vol. VI*. Chapter 3, pp 33-82

Ierna, C. (2016). The Reception of Russell's Paradox in Early Phenomenology and the School of Brentano: The Case of Husserl's Manuscript A I 35α. In: G. E. R., Haddock (ed.), *Husserl as Analytic Philosopher*. De Gruyter. pp. 119-142 (2016) /

Imaguire, G. & Cid, R. R. L. (eds.) (2020). *Problemas de Metafísica Analítica / Problems in Analytical Metaphysics*. Pelotas: Editora da UFPel / UFPel Publisher.

Klein, F. (1895, nov. 2). Über die Arithmetisierung der Mathematik. Nacharichten von der Königl. *Gesellschaft der wissenschafen zu Göttingen*, v.2, p. 82-91.

Klein, F. (1926). Vorlesungen ¨uber die Entwicklung der Mathematik im 19. *Jahrhundert.* Vol. 1. Berlin: Verlag Von Julius Springer. p. 611.

Klein, F. (1996, original de 1887). The arithmetizing of mathematics. In: Ewald, W. B. *From Kant to Hilbert: a source book in the foundations of mathematics*. Oxford: Oxford University Press, v. 1, pp 965-971.

Klein, F. (1895). *Ueber Arithmetisirung der Mathematik*. Göttinger Nachrichten (Geschäftliche Mittheilungen), 1895, p.82. (The arithmetizing of mathematics. Miss Maddison's translation in the Bulletin, 2d series, vol. 2, p.241, 1896).

Knopp, K. (1945). *Theory of Functions. Part I. Elements of the General Theory of Analytic Functions*. (Translated by Frederick Bagemihl, M.A.). New York: Dover Publications; 8 figures. vii + 146pp.

Knopp, K. (1945). *Analytic Continuation and Complete Definition of Analytic Functions*. In: Knopp, K. (1945), Ch. 8 pp 92-111.

Koenigsberger, L. (1874). *Vorlesungen über die Theorie der elliptischen Functionen nebst einer Einleitung in die allgemeine Functionenlehre* (Palestras sobre a teoria das funções elípticas e uma introdução à teoria geral das funções), Volume 1 (Leipzig: Teubner, 1874), 1.

Kronecker, L. (2001, original de 1891) UBER DEN BEGRIFF DER ZAHL IN DER MATHEMATIK: Offentliche Vorlesung des Herrn Prof. Dr. L. Kronecker, gehalten an der Friedrich Wilhelms Universit¨at zu Berlim im Sommer Semester 1891. Nach stenographischen Aufzeichnungen. (Sobre o conceito de número em matemática. Palestra aberta do professor Kronecker, realizada na Universidade Friedrich Wilhelms em Berlim, no verão de 1891. De acordo com registros estenográficos) In: Boniface, J.; Schappacher, N. 'Sur le concept de nombre en mathématique' Cours inédit de Leopold Kronecker à Berlim (1891). *Revue d'histoire des mathématiques*, Tome 7 (2001) no. 2, pp. 207-275. http://www.numdam.org/item/?id=RHM_2001__7_2_207_0

Lapointe, S. (2012). *Bolzano's Theoretical Philosophy, An Introduction.* Houndmills, Basingstoke, Hampshire. Palgrave Macmillan; Series: History of Analytic Philosophy.

Lopes, A. C. M., & Sá, P. F. de. (2018). NÚMEROS REAIS: ASPECTOS HISTÓRICOS. *Boletim Cearense de Educação e História da Matemática*, *3*(9), 79–90. https://revistas.uece.br/index.php/BOCEHM/article/view/56

Lopes, P. C. R. (2006). *Construções dos Números Reais.* (Dissertação de Mestrado em Matemática), Universidade de Madeira; Funchal, Portugal; Departamento de Matemática e Engenharias) p. 163. https://digituma.uma.pt/entities/publication/1ea14606-7fcb-468a-bdd7-67c8609c24ae (https://digituma.uma.pt/bitstreams/bd7362c5-56bf-4460-b05f-373d1fb79d82/download)

Lotze, R. H. (1884). *Logic in Three Book of Thought, of Investigation, and of Knowledge*. Oxford: Clarendon Press.

Magossi, J. C. (2020). O sonho de Lagrange. *Professor de Matemátca On Line (PMO), Revista Eletrônica da Sociedade Brasileira de Matemática,* v.8, n.1, https://doi.org/10.21711/2319023x2020/pmo8

Martins, A. P. A construção do Sistema dos Números Reais por Weierstrass. *In*: Encontro do Seminário Nacional de História da Matemática, 17, 2004, Lisboa. *Anais eletrônicos* [...] Lisboa: Museu de Ciência da Universidade de Lisboa, 2004, jun. 25 e 26. p. 1. https://repositorio.ipv.pt/handle/10400.19/1403.

Massa Esteve, M. R. (2016). Karl Weierstrass (1815-1897). El padre del análisis matemático. *Métode*, Nro. 88, p. 28-34. Disponível em: https://metode.es/revistas-metode/article-revistes/karl-weierstrass-1815-1897.html

Miller, G. A. (1925). Arithmetization in the History of Mathematics. *Proceedings of the National Academy of Sciences of the United States of America*, Vol. 11, No. 9 (Sep. 15, 1925), pp. 546-548 (3 pages). https://www.jstor.org/stable/84857

Miller, J. P. (1982). *Numbers in Presence and Absence: A Study of Husserl's Philosophy of Mathematics*. The Hague/Boston/London: Martinus Nijhoff Publishers.

Mohanty, J.N. (1977). Husserl and Frege: A New Look at their Relationship. In: Mohanty, J.N. (eds) *Readings on Edmund Husserl's Logical Investigations*. Springer, Dordrecht, 1977. https://link.springer.com/chapter/10.1007/978-94-010-1055-9_3

Novák, J. (ed.). (1988). *On Masaryk: texts in English and German*. Amsterdam: Rodopi.

Nunes, R. de O. (2020a). *Totalidades e estrutura mereológica: Um estudo sobre a natureza dos objetos compostos*. (Tese de Doutorado). Universidade Federal do Rio de Janeiro (UFRJ); Doutorado em Lógica e Metafísica. https://sucupira.capes.gov.br/sucupira/public/consultas/coleta/trabalhoConclusao/viewTrabalhoConclusao.jsf?popup=true&id_trabalho=9310076#

Nunes, R. de O. (2020b). *Mereologia e o Problema da Composição*. Em: Imaguire & (eds.) (2020); Capítulo 4, pp 109-156.

Schuhmann, K. (1988). *Husserl and Masaryk*. In: Novák (1988), pp. 129-156.

Silva, C. M. S. da. (2021). As Notas de Aula de Karl Weierstrass em 1878. *Revista Brasileira de História da Matemática*, *21*(42), 294–328. https://doi.org/10.47976/RBHM2021v21n42294-328

Sinkevich, G. I. (2015). On the History of Number Line. *Antiquitates Mathematicae*, Vol. 9(1), p. 83–92. https://arxiv.org/pdf/1503.03117

Couto Soares, M. (2010). Notas sobre referência e intencionalidade: Frege e Husserl. *Phainomenon*, (20-21), 25-42. https://phainomenon-journal.pt/index.php/phainomenon/article/view/262

Stolz, O. (1881). "B. Bolzano's Bedeutung in der Geschichte der Infinitesimalrechnung" (A importância de B. Bolzano na história do cálculo). *Mathematische Annalen* 18, pp 255-279. Disponível In: https://gdz.sub.uni-goettingen.de/download/pdf/PPN235181684_0018/PPN235181684_0018.pdf

Tymieniecka, A. T. (Editor). (2002) *Phenomenology World-Wide: Foundations - Expanding Dynamics - Life-engagements, a Guide for Research and Study*. Kluwer Academic Publishers.

Varga, P. A. (2018). Husserl's Early Period: Juvenilia and the Logical Investigations. In: Zahavi, Dan. (ed.). *The Oxford Handbook of the History of Phenomenology*. Chapter 6, pp 144–177.

Vilela, D. S. (1993). A caracterização dos números reais por Georg Cantor. *Revista da Sociedade Brasileira de História da Ciência*, *[S. l.]*, n. 10, p. 85–94, https://rbhciencia.emnuvens.com.br/rsbhc/article/view/444.

Von Helmholtz, H. (1977, original de 1887). Numbering and Measuring from an Epistemological Viewpoint From: Philosophische Aufsätze Eduard Zeller zu seinem fünfzig-jährigen Doktorjubiläum gewidmet ["Philosophical essays dedicated to Eduard Zeller on the occasion of the fiftieth anniversary of his doctorate"], Leipzig, Fues' Verlag, 1887, pp. 17-52. Reprinted in Wissenschaftliche Abhandlungen, vol. III, pp. 356-391. In: Von Helmholtz, Hermann. *Epistemological Writings*. The Paul Hertz/Moritz Schlick Centenary Edition of 1921, with notes and commentary by the editors; newly translated by Malcolm F. Lowe; edited, with an introduction and bibliography, by Robert S. Cohen and Yehuda Elkana. Dordrecht-Holland /Boston-U.S.A: D. Reidel Publishing Company, 1977. Chapter III, p. 72-114.

Wang, Y., Wang, C. Bolzano and Phenomenology. *Cultural and Religious Studies*, February 2022, Vol. 10, No. 2, 85-90.

https://www.davidpublisher.com/Public/uploads/Contribute/6232f93f3419e.pdf

Zahavi, D. (Ed.). (2018). *The Oxford Handbook of the History of Phenomenology*. Oxford: Oxford University Press, Chapter 6, pp 107–134. https://doi.org/10.1093/oxfordhb/9780198755340.001.0001.