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Contributions of a Pedagogical Intervention for the Study of Perspective Representation in Elementary Education Based on Design-Based Research

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ABSTRACT

Context: Given the difficulties presented by Elementary School Mathematics teachers in teaching basic concepts of plane geometry involving the topic of perspective representation, we resorted to a strategy that allowed us to obtain essential information to understand the context and needs of the participants, in a collaborative construction, to enable contributions to teaching and learning, based on Design-Based Research (DBR). **Objective**: to investigate the contributions that a pedagogical intervention, based on DBR, using an educational product consisting of photographic images, can provide to the teaching and learning of notions of perspective for Mathematics teachers in continuing education in Elementary School. **Design**: qualitative and quantitative, supported by the DBR research methodology. **Environments and participants**: three Mathematics teachers, volunteers in the empirical stage, and experts contributed to improving the educational product. **Collection and analysis**: data collection occurred through interviews, participants' activity notebooks, and during each intervention, audio and video recordings, photographs, observations, and field notes by the researcher. The analysis was performed during all phases of the DBR using data triangulation. **Research results**: the results show a significant emphasis, as well as relevant contributions to the teaching and learning of notions of perspective of the participants, who were able to act as agents of their training, reconstructing, improving and contributing to their practice, with the collaboration of all those involved and in line with their daily reality.

Keywords: photography; visualization; learning; collaboration, mathematics teaching.

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INTRODUCTION

In this article, we describe part of the results of a study conducted with a group of teachers working at a Basic Education school, which dealt with the teaching of Plane Geometry in Elementary Education, involving the theme of initial concepts of perspective representation and which had Design-Based Research (DBR) as its methodological approach. This methodology has its origins in different research works that emerged in the 1990s and, according to Rodrigues and Ponte (2020), researchers Brown (1992) and Collins (1992) were the pioneers in works related to Education. The emergence of this methodology was motivated by developing investigations with characteristics distinct from the existing paradigm that took into account the research environment, activities, and participants in collaborative work. In other words, design research focuses on student learning and establishing connections between theory and practice. Thus, DBR is a methodology that has a strongly experimental character, but without separating itself from theory.

This methodology was chosen for our research because DBR is an appropriate approach to address complex problems in educational practice, such as Geometry in Elementary Education. According to Kneubil and Pietrocola (2017), DBR refers to a new, interventionist methodology that aims to combine theoretical and practical aspects of educational research.

According to the Design-Based Research Collective - DBRC (2003), DBR has the following basic principles: (a) focus on complex and relevant problems in educational practice; (b) union between educational theories, design principles, and whenever possible, technological strategies; (c) research in real contexts to refine the intervention; (d) long-term involvement with the object of study in research and development cycles; (e) collaborative nature among participants; and, (f) problem-solving and theoretical construction.

The research began with a meeting with elementary school teachers, who were asked about a problem that was troubling them in their classroom work. They unanimously answered that it was “plane geometry content”, as well as the topic of initial concepts of perspective representation to represent three-dimensional spatial relationships on a flat surface. Thus, we felt challenged to seek alternatives and meet the demand of these teachers for new ways to practice. The challenge was to create something that would be truly interesting for these teachers and that they would be protagonists in proposing collaborative activities. Furthermore, it would come from the real context in which they lived.

In this sense, the idea was to develop an educational product with characteristics of an educational curricular teaching material, that is, a didactic sequence of activities that would enhance the learning of these Mathematics teachers and, at the same time, contribute to the classroom work with their students.

According to Matta, Silva and Boaventura (2014), an educational product can be:

> a) educational products such as teaching materials of all kinds and support; b) pedagogical processes such as, for example, recommendations for teaching attitude, and new teaching proposals; c) educational programs such as curricula, courses, organization of themes and teaching methods, as well as professional development for teachers; or d) educational policies such as teacher or student evaluation protocols, procedures and recommendations for investment, acquisition, options for the relationship between the school and the community (Matta, Silva, & Boaventura, 2014, p. 26).

This article aims to investigate the contributions that a pedagogical intervention, based on DBR, using an educational product consisting of photographic images, can provide to the teaching and learning of initial concepts of perspective representation for Mathematics teachers in continuing education in Elementary Education. We present some reflections on the study carried out in an intervention cycle. For this, we considered the use of photographic images obtained from the realization of a Mathematical Trail, with a previously defined route through the streets and monuments of the city to work on the concepts of geometry in everyday situations.

Using the images captured by the research participants, problem situations were created to study perception and notions of perspective. The proposal was based on the needs presented by the participating Mathematics teachers, so that it could contribute to their ongoing training, and also indirectly contribute to their students, in the understanding and possible construction of mathematical concepts, leading them to “learn to see” (Flores, 2007, p. 27). In this way, we sought to understand how the design principles established for this training experience contribute to the development of teachers' didactic knowledge to teach notions of perspective and three-dimensional spatial representations of the plane for the Elementary School. In this sense, DBR seemed appropriate for this purpose, since it is possible to “invite” people to “eventually participate in this group” (Mckenney & Reeves, 2019, p. 91).

To achieve the proposed objective, we initially tried to understand the importance of this problem. Then, we investigated some factors that allowed us to see it as a systemic problem, which goes far beyond being just an isolated problem of a single teacher or small group. We initially sought information in the documents of the Program for International Student Assessment (PISA), as it is called in Brazil, and also the information contained in the reports provided by the National Institute of Studies and Educational Research Anísio Teixeira (INEP) on the proficiency levels of students. Based on these elements, we researched the results of the average performance in Mathematics and the proficiency levels of the participating students from Brazil and the Organization for Economic Cooperation and Development (OCDE), as well as the performance of Brazilian students in the content, space, and form category of the Mathematics subject.

This research highlights that of the 78 countries participating in PISA in 2018, Brazil appears in one of the last positions, occupying the 70th position. In this assessment, students are distributed into 6 (six) proficiency levels. The percentage of students in each country who reach good levels, for example, proficiency levels 6, 5, or 4, indicates how well these countries can promote sublimity in their educational systems. Brazil, in this case, is classified at level 1 of proficiency on the scale in Mathematics, with 68.1% of students evaluated at levels 1 and below 1, which reveals, according to OCDE parameters, that these students do not have a basic level of knowledge in Mathematics.

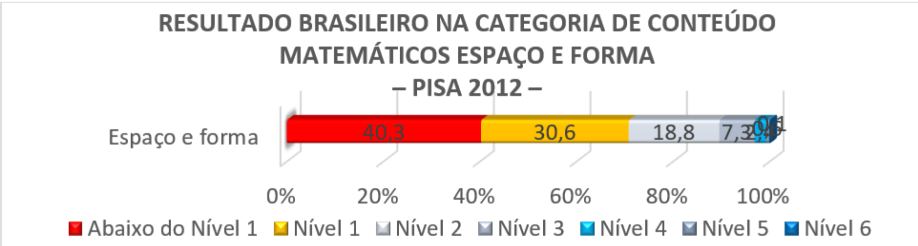
In the 2012 PISA edition, the main focus was Mathematics. Among the four categories of mathematical content that were evaluated in this edition, the area with the most critical performance presented was Space and Shape, which involves Geometry, which shows that 70.9% of Brazilian students evaluated are at level 1 and below 1 in this category, as we can see in the graph in figure 1.

This result is worrying because this category includes the notion of perspective, as well as the execution and reading of maps. Furthermore, it involves the transformation of shapes with and without the use of technology, as well as “the interpretation of views of three-dimensional scenes from different perspectives, and the construction of representations of shapes and, in this case, this result shows that the Brazilian students evaluated were unable to develop such skills”. This category, Space and Form, encompasses a wide range of phenomena that can be found in different environments, both in physical and visual space, such as:

> [...] patterns; properties of objects; position and orientation; representation of objects; encoding and decoding of visual information; dynamic interaction with real shapes as well as their representations. Geometry can be considered a foundation for Space and Form, but this category goes beyond the traditional content of geometry, using resources from other areas of mathematics, such as spatial visualization, measurement, and algebra. (Brasil, 2012, p. 33).

Figure 1

Brazilian results in the content category, Space and Form, in the Mathematics discipline by level in PISA 2012. (Adapted from Inep, 2012)



According to the Ministry of Education (MEC), the Space and Form category is considered to have the lowest score in the PISA performance index, and the graph in Figure 1, described above, shows the low performance of Brazilian students. Brazil had approximately 18.8% of participating students who managed to reach level 2, and only 10.4% of these obtained a score of level 3 or above in the Space and Form mathematical content category. However, this situation is worrying, mainly because it shows that 40.3% of the Brazilian students evaluated scored below level 1, which is considered by the OCDE to be students who do not have a basic level of knowledge of Mathematics.

The data studied allowed us to understand the importance of the proposed problem, which is a systemic problem that deserves attention, as it is not just a problem mentioned by the teachers participating in this study, but rather a problem that affects the reality of geometry teaching in Brazil and in several other countries around the world.

THEORETICAL BASES

The theoretical framework consisted of works on the contributions of geometry to teaching, focusing on the National Common Curricular Base (BNCC, 2017), as well as studies carried out by Santos and Nacarato (2014) and Flores (2007), which demonstrate that images and drawings of geometric figures are essential for the development of geometric concepts. To conceptualize visual perception and visualization for teaching and learning mathematics, we investigated works and research by Flores (2007, 2010, 2012), Buratto (2012), Cifuentes (2005), Cifuentes and Santos (2019) and Aumont (2002), which bring a discussion on visual perception in the teaching of Mathematics.

We researched works by authors who investigate and discuss issues regarding connections with images linked to the teaching of Mathematics, reinforced in the studies of Flores (2007), Feldman-Bianco and Leite (1998), Martins and Tourinho (2013), Santaella (2012) and Manguel (2006). These works are the results of research that refers to the study of the image as an object in the academic field, which enables visual perception and thinking.

In the methodological approach of this study, the literature on the history of DBR, proposed by McKenney and Reeves (2019) and Amiel and Reeves (2008), which deals with Design-Based Research, was reviewed. According to these authors, DBR "is a genre of research in which the iterative development of practical solutions to complex educational problems also provides the context for empirical investigations that generate theoretical understandings that can inform the work of others" (Mckenney & Reeves, 2019, p.6). The authors clarify that as the iterative development of practical solutions to complex educational problems occurs, theoretical understandings emerge, and that these two results (development of practical solutions and theoretical understandings) are considered one of the pillars of this genre of research.

Design studies are considered to arise from problems that teachers or students face in real educational contexts. In this sense, according to Reeves (2006), an attempt is made to solve these problems by defining prior and appropriate design principles, which will be evaluated, reviewed, and validated during the process of building and applying the educational product.

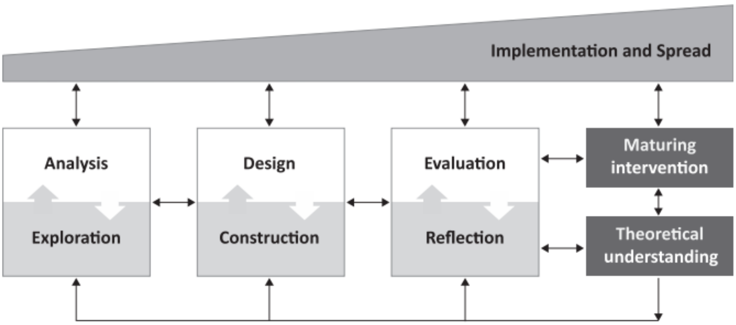
Design research protocols require long-term collaboration involving researchers and professionals. It integrates the development of solutions to practical problems in learning environments with the identification of reusable design principles (Reeves, 2006, p. 52). Therefore, it provides an improvement in educational practice and the creation of a new theory based on the product, since, by contributing to theory, DBR presents a practical product.

In this study, the problem presented by elementary school teachers is related to teaching geometry, involving the topic of perspective. Thus, to help solve the problem presented by the participants in this research, differentiated practical activities were developed using design principles, addressed from theory and practice with collaboration, acting in collaborative work between all actors.

Among the existing models for DBR, we adopted the generic model created by McKenney and Reeves (2019), Figure 2, which presents a flexible process with three main phases: (phase 1) analysis and exploration; (phase 2) design and construction; and, (phase 3) evaluation and reflection, which guide the understanding and development of the intervention.

Figure 2

A generic model for conducting design research in education suggested by McKenney and Reeves (2020, p. 86)



Each of these three phases "involves interaction with practice and contributes, directly or indirectly, to the production of theoretical knowledge and the development of an intervention, which matures over time" (Mckenney & Reeves, 2019, p. 83-84). McKenney and Reeves (2019, p. 83) explain that, in Figure 2, the arrows indicate that the process is iterative and flexible. The process is iterative because the results of some elements repeatedly feed into others, and it is flexible because, although some general flow is indicated, many paths can be taken. The bidirectional arrows indicate that what occurs in practice influences both the central processes underway and the outcomes (contextually responsive) and vice versa. The rectangles highlight the dual focus on theory and practice, which represent the scientific and practical outputs, respectively. And, above, in the form of a trapezoid, implementation, and dissemination, it shows that interaction with the practice is present from the beginning, and increases over time.

Below, we describe who the participants and collaborators were, and how each of these phases was developed.

METHODOLOGY

DBR used in teaching combines learning-oriented environments with theoretical research, according to Wang and Hannafin (2005). It is based on the planning, implementation, and evaluation of teaching sequences that effectively contribute to the construction of knowledge by combining theory and practice.

In this paper, we present the results of the research carried out in a cycle of the DBR approach which was divided into three phases that consisted of identifying a systemic problem with possible responses to it, defining preliminary design principles and planning, creating and developing an educational product built from the initial principles established, as well as the application and validation of the proposed educational product and the defined principles.

The research had the contribution and assistance of two collaborative groups: one that represents the knowledge of practice, composed of 3 (three) Mathematics teachers who work with Elementary School students and teach in different schools of the municipal education network, in the city of Santa Cruz do Sul - RS; and another that represents academic-scientific knowledge, which we call experts, composed of professors, academics, doctoral students and researchers from the Postgraduate Program in the area of Mathematics and Science Teaching at the Franciscan University (UFN), in the city of Santa Maria - RS, and who contributed to the improvement of the educational product.

Thus, with the meeting of this knowledge (academic-scientific knowledge and practical knowledge), the educational product was produced, applied, refined, and improved. It is worth mentioning that, to preserve their identities, the three participating teachers were identified by the letters 'PA', 'PB', and 'PC'.

With the collaborative groups formed, we describe below the activities developed in each of the three main phases of the process carried out in this study.

Phase 1: Analysis and Exploration. The initial phase 1 "includes the analysis and exploration of the problem in question" (MCKENNEY; REEVES, 2019, p. 85). In this phase, the analysis of needs and context was carried out so that the problem could be solved. The exploration part is more informal and exploratory and was carried out with the first visit to the participants, in which they reported on a problem that was distressing them. The idea was to plan, based on this problem, a sequence of didactic activities that would be related to the students' context and that could contribute to and help these teachers overcome these difficulties. Thus, initially, a dynamic called Mathematical Trail was proposed through the streets of the city of Santa Cruz do Sul, with a previously defined route to observe, search for information, and record through photographs everything that could have a connection with Mathematics, in addition to learning a little more about the local culture. All of this allowed the participants to feel part of what was being built. In addition, it allowed the activities to be developed based on the material captured by them in a real context.

The Mathematical Trail included a sequence of stops highlighted along the route, where questions were proposed. During this route, participants visited the São João Batista Cathedral, which is the largest Gothic-style cathedral in Latin America, located right in the "heart" of the city of Santa Cruz do Sul – RS, with a very rich architecture that can greatly contribute to Mathematics classes, where they were able to collect photographs. These photographs served as a teaching resource for the development of the educational product in phase 2.

Phase 2: Design and Construction. In the second phase, "design and construction," the design was carried out, which "resembles creation" (Mckenney & Reeves, 2019, p. 85), that is, it was planned. The stage of construction of the educational product corresponds to the design of the sequence, which begins with the choice of the previous design principles. These are ideas on which the actions and activities carried out in the sequence are based, which extend to the design. Design principles can be understood as theoretical assumptions that underpin the structure of the design process and the subsequent implementation of the generated product (Kneubil & Pietrocola, 2017). Once the design principles have been defined, the design itself begins.

The educational product proposal was based on the following previous principles: implementation of a Mathematical Trail; proposition of exploratory problem situations based on the dynamics of the Mathematical Trail; use of photographic images captured in a city context; proposition of authentic situations that can be used in elementary school classrooms; and use of technology.

After defining the principles, the initial version was designed with the participation of teachers and the research teacher. After the first version of the product, a micro-evaluation cycle was carried out with the collaboration of specialists, who analyzed and contributed to the improvement of the activities initially proposed. The discussions established with this group were fundamental for us to be able to make corrections and improvements. Only after this first micro-cycle was completed did we move on to our second micro-cycle, in which the activities were applied collaboratively with the teachers participating in the research.

The didactic sequence of activities was composed of 3 blocks of content composed of activities supported by the photographic images recorded in the Mathematical Trail dynamics. The content of perspective representation shown in this article is presented in block 3. In this work, we present a selection of some activities that make up this block.

Application of the educational product

In this article, we present activity 10 developed in block 3, which focused on the investigation of initial concepts of perspective representation, to develop notions of depth and distance in space when analyzing images, as well as to learn techniques for representing three-dimensional spatial relationships on a flat surface and representing a three-dimensional image in perspective on a plane. From the analysis of the photographic images and observation, it was possible to make conjectures and identify properties.

In activity 10.1, participants were expected to define what parallel lines are and identify, in the photographic image, the idea of perspective, which is the art of representing objects on a plane as they appear to the eye, according to their position and distance, giving the idea of depth.

Activity 10 - Objective: To explore the concepts of perspective, parallel, perpendicular, and concurrent lines.

Upon entering the Cathedral through the main door, one sees a central corridor that leads to the altar, along with other side corridors in the same direction.

Following the tradition of Gothic architecture, we can see that, on the ceiling, the naves are covered by ribbed star-shaped vaults, supported internally by slender and tall octagonal columns made of brick, with capitals decorated with floral motifs.

Activity 10.1 Look at the photographs below and answer the questions.

Photograph 1

Photograph 2

a) In the columns in photograph 1, draw straight lines along the length of the columns and describe their positions. Are the distances between them equal? Are they the same height? Are they parallel?

b) In photograph 2, draw straight lines along the length of the columns and describe their positions. What do you notice about the distances and heights of these columns? Are they parallel? Describe what you noticed.

c) If in photograph 1 the height of the columns is always the same, why do in photograph 2, at the end of the corridor, these columns seem smaller when we look at them?

d) In photograph 2, how was the width of the corridor measured at the back? And the actual width of the corridor? Explain your answer.

When analyzing the responses, it was found that the participants, in item ‘a’, drew straight lines over the columns in photograph 1 and described that the columns were positioned vertically and parallel to each other, with equal distances and heights.

The researcher asked the participants to explain why the columns are parallel. One of the comments was the following:

> **PA**: *“The columns are parallel because, basically, to be parallel, straight lines that do not meet do not have points in common, they never cross, but they have the same direction”*.

It was possible to analyze, by the participant's comment, that he was able to explain what parallel lines are.

In item 'b', the participants drew lines in photograph 2 along the length of the columns and described that they were at different angles. They also reported that their sizes decreased as they moved away from the altar.

In item 'c', the participants tried to explain the difference in height of the columns in the photographs and it was observed that most of them realized that it was a notion of perspective, except one participant.

In item 'd', the participants were able to identify that the width of the corridor in the image was decreasing. Participant 'PC' cited the example of counting the tiles to prove the real width of the corridor, as we can see in the comments, described below.

> **PA**: *"The width of the corridor is different at the beginning and the end."*
>
> **PC**: *"This image is so cool! I can work on proportions and ratios. This image is symmetrical. Everything I see in it is symmetrical. It's amazing, you can see the tiles. You can count how many tiles there are here and how many there are at the end. Students can imagine the tiling or even the area to prove that, in the end, there are the same number of tiles and the same measurements as in real life."*

It can be seen from the speech of one of the participants that the activity allowed him to make possible connections with other mathematical concepts such as ratios and proportions.

Next, in activity 10.2, the participants were expected to identify the symmetry in the image, be able to identify perpendicular lines, and verify that the image changes as its position is changed.

Activity 10.2
In the photograph below, the lower points E, G, and I and the upper points F, H, and J were initially marked on the columns, highlighting the height. The photograph was then folded along the line r, which divides the image in its center, and then unfolded, opening it again.

Source: Prepared from the photograph by participant PB

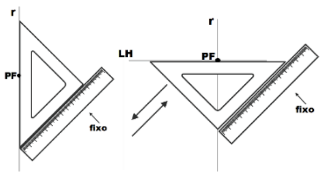
a) When the photograph was folded, did the points on the columns coincide exactly?

b) Are the points E', F', G', H', I' and J', which highlight the height of the columns in the photograph, symmetrical to the points E, F, G, H, I, and J? Justify.

c) In the photograph, is the corridor converging towards the altar? Are the points highlighted on the columns also converging towards the altar? Draw line segments from the points and in the corridor that converge towards the altar to prove this fact.

d) Observe the line segments drawn, called vanishing lines (LF), they intersect at a single point, called the Vanishing Point (PF). Highlight this point (PF). What effects do this point and the line segments have on the image?

e) Draw a horizontal line (LH), perpendicular to this Vanishing Point (PF) and the line r. Find out how. If you prefer, use a ruler and a set square, as shown in the model below.



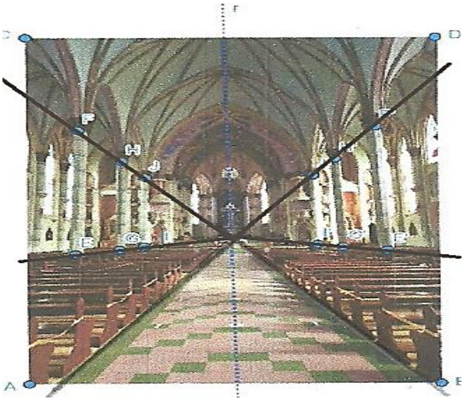
f) What is a perpendicular line?

g) Where was the person who took this photograph positioned? Was the image taken from above, from the front, from the right side, or from the left side? Does the image change if we change the position of the person who took the photograph?

In items 'a' and 'b', participants confirmed that the points in the columns coincided exactly and were symmetrical. Participants stated that the hallway and the highlighted points in the columns were converging toward the altar. They drew straight line segments on the image to show that they were indeed converging, as can be seen in Figure 3.

Figure 3

Line segments drawn by participants in the image



In item ‘d’, participants responded about the effect that this vanishing point and the line segments gave to the image, with the following comments:

> **PC:** *“It gives the effect that it is converging towards the altar”.*
>
> **PA:** *“It gives the effect that it seems to be decreasing”.*
>
> **PC:** *“It is the notion of perspective”.*
>
> **PB:** *“It would be the notion of depth, of distance”.*

In items ‘e’ and ‘f’, the participants tried to draw perpendicular lines and correctly explained what a perpendicular line is, as we can see in the speech of participant PC.

> **PC:** *“Perpendicular lines are those that intersect at a common point and form a 90° angle, a right angle.”*

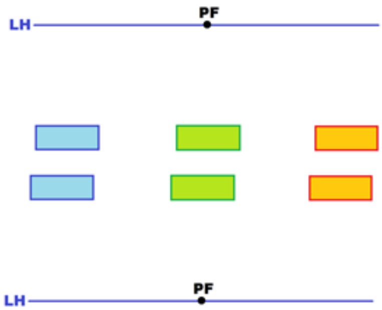
In item ‘g’, the participants observed the image and responded that the person who took the photograph was positioned at the beginning of the hallway, in front of the entrance door, in the center. They reported that if the position of the person taking the photograph changes, the image will also change position.

When analyzing the participants’ responses, it was possible to see that they were able to respond correctly.

In activity 10.3, the participants were expected to identify the different positions of the vanishing lines and realize that the images presented the idea of perspective.

Activity 10.3

In the figures below, we have a Vanishing Point (PF) on a Horizon Line (LH) and rectangles in different positions. Mark the vertices of each rectangle and draw, from these vertices, vanishing lines (LF) that converge towards the vanishing point (PF).



a) Observe the images that were formed. What effects do the Vanishing Lines (LF) give to the image?

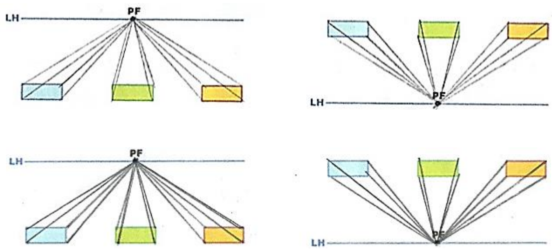
b) What is the difference between the image seen, of the rectangles that are below the horizon line (LH) and the image seen, of the rectangles that are above the horizon line (LH)?

c) Considering the way the vanishing lines (LF) are presented in the images, is it possible to say that they are all in perspective?

The participants initially marked the vertices and drew vanishing lines towards the vanishing point. This can be seen in the image. They also stated that the effect given to the images was depth in different positions.

PC:

PA:



In activity 10.4, participants were expected to see in the images that perspective is related to the idea of depth. They were also expected to be able to identify the different positions of the vanishing lines that converged to a vanishing point in each image.

Activity 10.4

In the three photographs below, are there lines converging to the same point?

Trace Vanishing Lines (LF). Locate and highlight the Vanishing Point (PF) and the horizon line (LH) from it in each image. What effects do the (LH), the (PF), and the (LF) give to the image?

a) Is it possible to construct perspective images by positioning the (PF) in different positions on the (LH)?

The participants analyzed the images and were able to identify the escape lines, as we can see in their dialogues, below.

> **PC:** *"Yes, in the hallway of the Cathedral, towards the door, on the path that leads to the fountain in the square, and on the street. They all have lines converging to the same point."*

PA: *“There are two images that are not centered, but in the background, I can see the vanishing point, where the lines converge.”*

PC: *“The entire square converges to the fountain. How cool!”*

PA: *“I liked that! It worked here! I was able to draw the lines and they all converge to a point. What a cool effect! It really gives the notion of depth. You can also work on angles with the students.”*

By analyzing the participants' dialogue, it was possible to see that they were able to meet the expectations of the activity. They confirmed that it is possible to construct images in perspective, positioning the vanishing point in different positions.

In activity 10.5, participants were expected to use the techniques learned and draw in perspective.

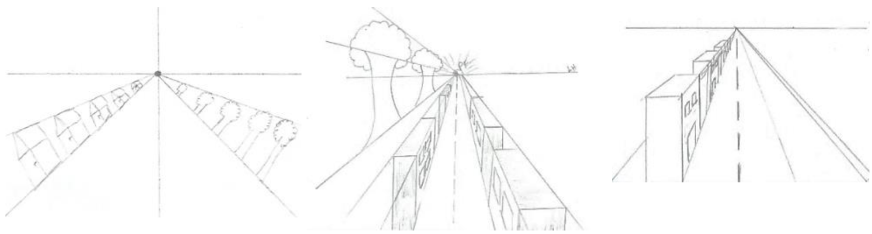
Activity 10.5
Now, draw a horizon line (LH) on a sheet of paper, and highlight a vanishing point (PF) on this horizon line (LH). Draw vanishing lines (LF) from this (PF). Be creative and draw in perspective. Suggestion: draw a highway, road, or corridor as a reference and decorate drawings with houses, trees, buildings, etc.

In this sense, the participants ended up drawing, as we can see in their lines and in Figure 4, below.

Figure 4

Perspective on the plan drawn by the participants

PA **PB** **PC**



PC: *"It doesn't matter if the point is anywhere, right?"*

PA: *"Yes, I'll draw vanishing lines up and down. I'm not very good at drawing (laughs)".*

PB: *"The further away you go, the smaller it gets. It gives the idea of depth".*

PC: *"In high school, we have to work on solids in perspective, from the front, from the side. I'll tell you that I did that here in the 9th grade, they almost cried. In the first few days, I worked on the notions of perspective for two or three weeks, at the end. After you learn and understand, then it becomes clear".*

PA: *"Lately, I haven't worked with 9th graders anymore, so I haven't worked on that yet. I've never been good at drawing".*

It was possible to observe that the participants were able to draw in perspective, using the vanishing point, the horizon line, and the vanishing lines, within their possibilities. Participant 'PA', at several moments, mentioned that he was not very good at drawing, but it was possible to observe, in his comments, that he was able to understand, at the end of the activities, the idea of perspective.

Phase 3: Evaluation and Reflection. After implementing the activities, we moved on to phase 3 of evaluation and reflection. In our study, evaluation and reflection were carried out throughout the research period. At the end of the activities, a general discussion was established in which the participants were able to discuss the activities developed and their suitability for the level of their students and the difficulties of implementing them in the classroom. We noticed that they understood the concepts involved in addition to establishing connections with other content. According to Krasilchik (2004), this fact may have occurred because the concepts were presented in a contextualized manner and, also, the participants were co-authors of the activities constructed. These activities proved to be understandable and motivating for learning, as well as challenging the teachers' imagination and understanding.

This study allowed learning about the initially proposed content to actually occur and mobilized teachers about motivation, interest, and participation in teaching activities.

The previously defined design principles were evaluated and validated throughout the process of implementing the activities. In addition to the principles initially defined, it was possible to define new principles that emerged throughout the training process. We highlight:

a) the construction of a collective and cooperative learning process, prioritizing activities of interaction and discussion with the participants;

b) the use of photographs provided teaching material that facilitated the understanding of the content covered;

c) the contextualization of the subjects covered made it possible to establish connections with other content, giving meaning to Mathematics;

d) the co-participation in the planning and construction of the educational product awakened motivation for the work carried out;

e) the connection with the local reality made it possible to create original material that can be used in the classroom at the elementary level;

f) the enthusiasm of the participants when faced with the study of geometry concepts in a playful way based on images of the place where they live;

g) the development of the ability to perceive mathematical objects in the environment in which we live;

h) the development of the ability to create and solve problems.

It was possible to perceive, in general, that the teachers had knowledge of the concepts of geometry at the end of the intervention, but were unable to relate them in a didactic way to their daily lives, giving voice to difficulties in working with their students.

We observed that the proposed educational product, based on images, mobilized the interest, motivation, and enthusiasm of the teachers for teaching geometry because throughout the process they showed concern for applying it to their students, always trying to adapt the work to the elementary level. The contributions and feedback from the experts, which were taken during the first evaluation microcycle, also helped in validating the pre-established principles.

Activity 10, described here, did not require the use of technological resources for its completion. Therefore, this principle, initially defined, could not be validated, since the participants preferred to use only pencil and paper.

FINAL CONSIDERATIONS

The present work aimed to present the partial results of a DBR cycle, which included the definition of a systemic problem related to plane geometry in elementary education, specifically related to the content of perspective, the definition of preliminary design principles, the design of an educational product, application, and validation, in addition to the establishment of new principles, based on observations during the development of the training process.

The present study demonstrates success in terms of the validation of part of a sequence of activities on the concepts of perspective from material collected in a Mathematical Trail carried out through the streets of the city of Santa Cruz do Sul. The success observed is due to the fact that the activities described here mobilized the participants in terms of motivation, interest, and participation in the didactic activities. The co-participation in the construction of the activities, and the discussions established between the participants, with the research teacher and the group of researchers from the university, transformed the performance of the teachers and made possible the creation of an environment rich in interactions.

However, some considerations should be made to improve the educational product. We noticed that even after the development of the activities, some misconceptions persisted in the participants' responses related to geometric concepts and that these need to be revisited in another DBR cycle. Here it is necessary to resume the survey of the difficulties that persist, the discussion and systematization, and propose a redesign of the activities so that learning actually occurs. This fact configures a new design principle that emerged from the results obtained from the application.

The contributions and feedback from the group of experts helped to improve the educational product and validate the results obtained. The ongoing meetings with this group, even online, made it possible to monitor the implementation of the activities and possible corrections that were made throughout the training course.

The use of photographs taken by the research participants was crucial to obtaining the results described, as it allowed them to include them in the construction of the activities, and, at the same time, they were able to play a leading role as teachers.

The idea of the concept of perspective was understood by the participants and the photographs helped in their understanding. They realized and recorded in their reports that the issue of depth, in which an object can appear to be further away, closer, larger, or smaller than its real size, was central and the material captured inside the cathedral contributed to their clear perception.

We believe that the approach, with the use of photographic images, collaborative activities for knowledge construction, and contextualization of the participants' daily lives, enabled more meaningful learning. The environment created provided spaces for creation, exploration, discovery, and knowledge construction. This is in line with the ideas of Vergnaud (2009, p. 156) when he states that it is the situations that give meaning to the concept and that, in the teaching and learning process, the same "[...] cannot be reduced to its definition [...]. It is through the situations to be resolved that a concept acquires meaning", and must be constructed through contextualization, through different activities. In this way, the participating teachers were able to give meaning to what they were studying and would later be able to work with their students.

The report presented is a fragment of a broader research and the next steps are related to the new cycles of DBR, that is, its implementation with elementary school students, data collection, analysis, validation, and redesign.

AUTHOR CONTRIBUTIONS

DSFSF and VB conceived the idea and mobilized the theoretical-methodological framework. DSFSF adapted the methodology to this context, created models, carried out practical activities, collected data, and analyzed the documentary data. VB guided the research and revised the theoretical framework, supervised and guided the research. All authors actively participated in the discussion of the results and reviewed and approved the final version of the work.

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