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Mediation in Mathematics Teaching and Learning in Deaf Education: Algebraic Thinking

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ABSTRACT

Background: The proposal of inclusive education in the education of deaf children does not limit the issues of access to communication. Bilingual education is necessary for their learning and cognitive development, also considering their way of interacting and organising life. **Objectives**: To investigate how mediation is done through signs and instruments in appropriating algebraic knowledge by deaf students attending non-special schools. **Design**: The design experiment based on educational design research (EDeR) has been chosen as the methodological path. **Setting and participants**: The teaching episodes were developed with two deaf students from two state public schools in a municipality in Paraná, Brazil. **Data collection and analysis**: Field diaries and video recordings of teaching episodes were used for data collection. The collected data were analysed through the use of content analysis. **Results**: Mediation through signs and instruments is the key point for high-quality teaching, as it establishes connections between the student's previous experiences and the mathematical concepts in consolidation. **Conclusions**: Visuality is reaffirmed as essential for the development of mathematical contents, especially in the development of algebraic thinking in the education of deaf students. It also points out the need to expand investments in universal education of quality to guarantee human and material resources for public schools.

Keywords: Mediation; Deaf; Mathematics; Algebraic thinking; Learning.

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A mediação no ensino e aprendizagem de matemática na educação de surdos: O pensamento algébrico

RESUMO

Contexto: A proposta de educação inclusiva na educação de crianças surdas não se limita às questões de acesso à comunicação. A educação bilíngue é necessária para sua aprendizagem e desenvolvimento cognitivo, considerando também sua forma de interagir e organizar a vida. **Objetivos**: Investigar como a mediação é efetivada por meio de signos e instrumentos na apropriação de conhecimentos algébricos por alunos surdos inseridos na escola regular. **Design**: Como percurso metodológico escolheu-se o experimento de *design*, a partir da metodologia *Educational Design Research* (EDR). **Ambiente e participantes**: Os episódios de ensino foram desenvolvidos com dois estudantes surdos de duas escolas públicas estaduais de um município do estado do Paraná. **Coleta e análise de dados**: Para a coleta de dados, foram utilizados diário de campo e gravações em vídeo dos episódios de ensino. Os dados foram analisados por meio da análise de conteúdo. **Resultados**: A mediação efetivada por meio dos signos e dos instrumentos é o ponto central para um ensino de qualidade, de modo que sejam estabelecidas conexões entre as experiências que os estudantes já possuem e os conceitos de matemática que estão em amadurecimento. **Conclusões**: Reafirma-se a visualidade como importante no desenvolvimento dos conteúdos de matemática, em especial no desenvolvimento do pensamento algébrico na educação de surdos. Igualmente aponta-se a necessidade de ampliação dos investimentos na educação, para garantia de recursos humanos e materiais em direção ao ensino público e de qualidade para todos.

Palavras-chave: Mediação; Surdos; Matemática; Pensamento algébrico; Aprendizagem.

INTRODUCTION

The proposal of inclusive education in the education of deaf children does not limit the issues of access to communication. Bilingual education is necessary for their learning and cognitive development, also considering their way of interacting and organising life (Felipe, 2018). In this sense, Prado and Costa (2016) explain that the identities of deaf people are constituted by their relations with the environment. Deaf culture is formed by sharing language, ways of living, and societal organisation.

According to Prado and Costa (2016), it is necessary to reflect on the role of deaf culture in school. The implementation of Brazilian Sign Language, known as Libras, is already in place. However, education also needs to consider the cultural aspects of the deaf community: "[...] consider cultural processes as needed to be structured and made possible from the perspective of deaf individuals" (Prado & Costa, 2016, p. 174).

Similarly, Lacerda et al. (2013) state that, compared to listeners students, it is common for deaf students to arrive at school with little understanding of society's events. The authors attribute the fact to the lack of access to media and debates with interlocutors in their language. Social contact is necessary for the child's development, and when addressing deaf children, they must have contact, from an early age, with people fluent in sign language (Sacks, 2010).

In this sense, the cultural-historical theory is an essential aspect of understanding the education of deaf people, as claimed in the defectology studies defended by Vygotsky (1997). In these articles, the author explains that deaf children are not less developed than others but rather that their development occurs in another way, from stimuli created for the elaboration of compensation. Deaf children take different paths, going through distinct routes, and so the central point is for the school to be understood as part of social life, as a way of assisting the organisation of children in the context they are inserted (Vygotsky, 1997).

According to Vygotsky (1997), when deaf children work with hearing students, there is space for the development of better social inclusion based on the use of higher forms of collaboration, which is essential for deaf education placed as an issue of social education. Thus, the relationships established inside and outside the classroom must be considered while searching for strategies that enhance the forms of knowledge appropriation. In this scenario, mediation through signs and instruments is possible.

Vygotsky (2000) makes an analogy between the use of signs and tools, considering their use for behavioural adaptations based on a psychological point of view. The sign as an instrument of human activity, figuratively speaking, is regarded as a tool when it acts as a mediator to some object or activity (Vygotsky, 2000).

Consequently, in the literal sense, signs and tools are different and have specific traits in each activity. The sign does not provoke changes in the object. It acts as an internal element that controls man's behaviour, and its tool is an external activity leading to changes in the object. In doing so, humans may gain dominance over nature (Vygotsky, 1978). Thus, Vygotsky (2007) states that the relationship between sign and tool is mediated action.

Deaf students still experience practices directed at hearing students in school inclusion. Even in an environment mediated by sign language, actions guaranteeing a bilingual education are not developed (Coutinho & Carvalho, 2016). Therefore, in the relationships established in class, in addition to the presence of the Brazilian Sign Language translators/interpreters (BSLIs) (In Portuguese, Tradutor e Intérprete de Libras – TILS), it is necessary to expand the pedagogical-didactic resources, as well as the understanding of learning as a mediated activity in the education of the deaf children.

In the school inclusion of deaf students, changes are necessary so that the school community may acquire an inclusive look, with respect and guarantee of addressing students' particularities (Muniz et al., 2020). From this perspective, diversified pedagogical didactic resources in mathematics teaching and learning enable the appropriation of concepts and mediation of schooling, additionally to the promotion by the teacher of a motivating relationship with his students (Manrique, 2016).

Given this context, the research is directed towards the appropriation of school knowledge by deaf students in the classroom, specifically concerning their mathematics teaching and learning process. The research problem is as follows: How is mediation through signs and instruments carried out in the appropriation of algebraic knowledge by deaf students attending non-special schools?

THEORETICAL FRAMEWORK

Mathematics teaching based on the cultural-historical theory brings possibilities to the classroom by considering culture and social relations as influences in this process. Radford et al. (2018) affirm that knowledge is cultural and historical. Thus, it is essential to consider this to understand the mathematics teaching and learning process.

Vygotsky (2007) explains that the learning process begins before the children reach the school phase. When studying arithmetic in school, they bring previous experiences using operations with quantities, sum, division, and comparison between objects that need to be considered in the classroom. When addressing the development of arithmetic operations, Vygotsky (2000) explains that children move from direct to mediated arithmetic, and this involves culture when it uses instruments (such as toothpicks) to assist in the solution to the problem. From this point, the child starts to operate quantities using signs.

During this process, the child goes through conflicting moments. There is a rupture when they transit from the perception of quantity to abstract operations with signs. The child faces ways of operating that differ from those they elaborated on in a given concrete situation. At this moment, the child begins learning through signs at school (Vygotsky, 2000).

Mathematics teaching in children's early education cannot be reduced to the four fundamental operations. Such operations must be present as problems encompassing different contents and structures (Solovieva et al., 2016). The authors state that language is fundamental for developing intellectual actions, reflection, and generalising the proposed situation. It means that understanding the problem with all the words that compose it is essential in this process.

In this context, Solovieva et al. (2016) state that based on their studies, the logical-mathematical knowledge, and the skills necessary to solve the problem depend on planning an organised teaching program of collective use with specific guidance to students.

The work from the cultural-historical perspective favours the student's learning and highlights the mediation in different manners (language, object, drawing) of consolidating the concepts (Moysés, 1997). In this sense, Vygotsky (2000) explains that, at a given moment of his development, the child perceives the limitation of his arithmetic and begins the process of transition to mediated arithmetic: "School arithmetic is a moment of change" (Vygotsky, 2000, p. 211, our translation), adding that even if there is conflict, the school cannot approach teaching in a purely mechanical way.

The knowledge of mathematics developed in school needs to have meaning and significance to students so that they can read, interpret and explain situations that surround everyday life (Moysés, 1997). The author criticises the work with mathematics seeking to develop formulas, resolution of equations, and symbolic representations, in which the contents addressed are not always presented in a context in which the student can perceive their applications.

Mathematics teaching and learning involve developing concepts, which are to be built along with the students through the interaction and use of objects that assist in this process (Talizina, 2001). The author indicates that students reproduce the concept correctly but cannot apply it while dealing with real-life problems since memorising a concept's definition is not the same as learning it.

During the development and assimilation of concepts, Talizina (2001) explains that the definition of a concept is a starting point, followed by its inclusion in actions that students perform with the aid of objects. In this process, the actions occur through the use of objects as instruments for the construction of a concept. That is a product of the student's actions and requires a logical system of activities and knowledge (Talizina, 2001).

Exploring activities in the zone of proximal development (ZPD) consists of incorporating new knowledge that was not previously accessible to children, enabling discoveries based on the effectiveness of a program that contemplates the collective activity in which students feel capable of understanding, solving, and creating new problems (Solovieva et al., 2016). Accordingly, Radford et al. (2018) report that success in exploring ZPD is due to the teacher's ability to discover a common, effective, and conceptual basis in the progress of students' meanings.

According to Radford et al. (2018), it is necessary to involve students in classroom practices, understanding them as social beings in training. It implies that developing group activities is essential to awaken cooperation and responsibility. Similarly, the authors explain that within the theory of objectification derived from the cultural-historical theory, mathematics education has been seen as a political, social, cultural, and historical enterprise in such a way that it is built upon the construction of reflective and ethical individuals able to make a critical analysis of the situations presented (Radford, 2016).

The production of knowledge is a mediated process, created and recreated through cultural-historical activities, and thus, the choice of the activities to be applied in the classroom is a crucial factor in the effectiveness of teaching and learning (Radford, 2016). The authors emphasise that collaboration between teachers and students, even with different roles, must be related and directed towards the production of mathematical knowledge and complement: "The classroom emerges as a public space for debates in which students are encouraged to show openness towards the others, as well as responsibility, solidarity, care, and critical awareness" (Radford, 2016, p. 201, our translation).

Thought is understood through the relationship between individual reasoning and the different cultural forms of thinking. It happens through language, gestures, artefacts, and general semiotic means (Radford, 2012). In this sense, it is necessary to define algebraic thinking as the object of this study. For this purpose, we sought the research by Radford (2010a, 2010b, 2012) on the development of how the student's algebraic thinking issues are organised in a cultural-historical approach: "Thinking is linked to the context and culture in which it occurs" (Radford, 2010b, p. 4, our translation).

According to Radford (2010a), the objectification process is mediated by a multi-semiotic activity, i.e., understanding knowledge as a theoretical construction of how students engage and perceive something and give it meaning. Thus, classrooms are regarded as interactive zones of mediated activities in which students relate to objects of knowledge and other students through social interaction (Radford, 2010a).

According to Radford (2012), algebraic thinking is a historically elaborated process, going through conceptualisations and reconceptualisations over the years and thus defined by the author: "[...] initial algebraic thinking is based on the possibilities of the student to understand patterns in forms of culturally evolved co-variation and use them to deal with issues of remote and unspecified terms" (Radford, 2012, p. 130, our translation).

Algebraic thinking is defined by three characteristics: the first is the indeterminacy of algebraic objects, i.e., the possibility of replacing unknown variables and objects; the second is that indeterminate objects are studied analytically; and the third is the denotation, the symbolic way of designating their objects (Radford, 2010a). In addition, according to Radford (2010a), it becomes necessary to understand the difference between induction and generalisation. Induction occurs when divination rules are used to resolve the proposed situation. The author calls it naive induction. The algebraic generalisation is based on patterns, on the perception of a shared identity between the terms so that it can be generalised to every sequence. In the author's words:

> Generalising a pattern algebraically rests on the ability to perceive a similarity between some elements of a sequence S, being aware that this similarity applies to all its terms, and being able to use it to provide a direct expression of any element of S (Radford, 2010a, p. 42, our translation).

From a semiotic perspective, Radford (2010b) explains that signs and algebraic formulas, such as the use of letters, are already regarded as algebraic signs. However, he points out that words and gestures can also be considered algebraic signs, as they express algebra in various ways, even if they are not directly using alphanumeric symbols. The author points out that this moment deserves attention, since there is space in it for the emergence of a zone in which the students start thinking algebraically. This "area" is called the emergence zone of algebraic thinking.

The algebraic generalisation is approached by Radford (2010a) in three layers of signification: the factual, which occurs in the elementary layer, expressed in concrete actions through words, gestures, and symbols; the contextual generalisation, referring to contextual objects, in the form of expression, through signs (words); and the symbolic or standard generalisation objects and operations, expressed by an algebraic semiotic system.

Given this, the elaboration and conduction of didactic activities are not enough to lead students toward the forms of generalisation. It is necessary to consider social interactions so that students also learn from discussions with classmates through the available cultural artefacts (Radford, 2010a).

Referring to the Brazilian reality, Moysés (1997) shows that implementing the cultural-historical theory in the educational environment demands teacher education, pedagogical work that favours learning, a clear pedagogical proposal created collaboratively, and an available diversity of material resources.

In the design experiments carried out by Fernandes and Healy (2013, 2016), the authors obtained evidence in the generalisations developed and expressed in the activities carried out with deaf students mediated by sign language: "The students were involved in an emergence zone of algebraic thinking, in which the process of objectification begins, and cultural objects gain personal and subjective meanings" (Fernandes & Healy, 2013, p. 365).

Given this scenario, mathematics teaching in deaf education through the cultural-historical approach shows encouraging perspectives for the process to contemplate, in the educational environment, the issues inherent to this interface, that is, considering the student as a social being, valuing deaf culture and, from that, organise the knowledge to be developed in the classroom.

Pedagogical didactic resources in mathematics classes emerge as a possibility to motivate students in the search for solutions creatively and dynamically, as they act as mediating instruments that assist in teaching and learning by enabling the visualisation of content through manipulative and digital materials. In the education of deaf people, it is essential to seek strategies to motivate and optimise mathematics learning concerning students' specificities, such as sign language and using visual resources in the classroom.

To constitute a dynamic and interesting environment for deaf students, the use of manipulative materials is discussed as a manner of facilitating the mediation between the student, the construction of knowledge, and the use of digital materials applied in the classroom.

According to Kiper et al. (2015), the manipulatives for teaching fractions made it possible for deaf students to interact more with the knowledge being taught and allowed integration with other mathematics content without considering a hierarchy between them. The authors highlight the importance of visual activities in deaf people's education, considering that this is how they access information and knowledge.

The exploration of manipulative materials can provide a favourable environment for learning (Colaço, 2018) since the materials act as mediating instruments in deaf students' education and, as stated by Santos (2018), are teachers' allies, favouring the interaction between those involved in the construction of knowledge.

Applying mathematical knowledge in the practical context also constitutes possibilities for the classroom. In their experience using a measuring instrument, the pipette, Jannah and Prahamana (2019) considered that this facilitated the learning of problems. In the study by Husniat et al. (2020), the understanding of geometric concepts was developed using figures built on rectangular cardboard of different sizes. They observed that their students easily understood the concepts being addressed. This was also observed by Santos (2019), who highlights the interaction and collaboration between those involved in developing activities with manipulative materials.

In mathematical activities developed in the classroom for deaf students, it is important to explore visual mathematics (Husniat et al., 2020). As well as other forms of representation suggested by Husniat et al. (2020), it is necessary to investigate verbal deception beyond writing and to know how students interpret and solve problems so that the strategies to be used can be selected.

Exploring manipulatives is not enough to achieve success in learning. It is also necessary to relate the contents to be developed from the expected objectives, and, in this way, the students need to understand the materials being used and their applicability in developing knowledge (Jannah & Prahamana, 2019). Accordingly, Colaço (2018) states that it is not only to adapt the visual materials but to implement them in a context that favours learning.

Thus, the use of pedagogical-didactic resources brings students the possibility of experiencing a reflective practice mediated by tools that allow changes in the construction of mathematical concepts.

According to the National Common Curriculum Base, the teaching and learning of algebra contents contemplate the development of algebraic thinking so that students can establish relationships between quantities in different contexts, thereby learning to solve problem situations and express them through algebraic writing (Brasil, 2017). Regarding algebra, the mathematical skills expected according to the reference document for high school are described in topics that, in short, cover the investigation, interpretation, and resolution of problems involving economic, social, and applied situations in the natural sciences (Brasil, 2017).

Given this scenario, deaf education is part of a context in which the issues that permeate students' teaching and learning processes, such as sign language and visual pedagogy, need to be considered in the planning and execution of strategies in the classroom. According to Dessbesel et al. (2020), empirical studies on teaching algebra for deaf education highlight using visual resources in activities, with manipulative materials and digital materials, to favour the construction of knowledge.

There are many challenges in the development of algebraic thinking since it is necessary to consider that one works with a mathematical language composed of variables, letters, and unknowns. That is, the student is dealing with the indeterminate. According to Donado (2016), dealing with the indeterminate means understanding something that is not present, which, in turn, is not ordinary. Thus, it is necessary to explore the various contexts of application of the contents and the visual resources and tools available to facilitate the appropriation of knowledge.

To Silva (2012), the tool called Matrizmat, which consists of a manipulative material (plastic boxes) used in teaching matrices, enabled the development of students' abstract thinking and fostered collaboration between students in the resolution of activities. It is noteworthy that the tool presented by the author was developed for deaf and blind students and was organised with the objective of: "[...] offering different sensory stimuli to inclusive class learners to facilitate access to the mathematical concept of matrices" (Silva, 2012, p.65).

In teaching mathematics in the education of the deaf, mental representations do not occur directly as in listeners (Frizzarini & Nogueira, 2019). The authors explain that the elements of algebraic representation are first translated into sign language, then into writing, and can be supported by graphic records.

Studying the previous knowledge of deaf students in the teaching of algebraic concepts, Frizzarini and Nogueira (2014) observed that when using sign language, the students could represent the proposed situations, as they complement:

> The students presented a higher level of freedom when translating into Libras, meaning that the students were not bound to a single record, Libras. As it is a visual/motor language, it allows the use of both the visual units of the graphic registers and the symbolic units of the algebraic registers to translate algebraic expressions into sign language (Frizzarini & Nogueira, 2014, p. 386).

Sign language becomes fundamental in deaf students' cognitive development. Regarding its relation to algebraic concepts and their representations, the students used the symbolism associated with typing (Frizzarini, 2014). As a result, as Frizzarini (2014) points out, it is essential to use different registers of content representations for deaf people's education.

Fernandes and Healy's (2016) research investigated algebraic thinking using the microworld of mathematics, supported by digital technology. The authors observed that this made it possible to explore the visual aspects of the proposed content and find solutions to problems: "[...] students reflect a way of thinking algebraically even without resorting to conventional algebraic language" (Fernandes & Healy, 2016, p. 241). In the study made by Conceição (2012), activities using the microworld, exploration of sequences, and generalisation of patterns were also investigated. The author proposes that, differently from the conventional written form, in this environment, the symbolic representation is part of the construction of sequences, thus favouring the appropriation of the concepts involved.

In activities with written language, students presented a little difficulty due to a lack of mastery in the area (Fernandes & Healy, 2013). Conceição (2012) observed that deaf students were not confident in reading the statements of the proposed questions alone and waited for help from the BSLIs.

Sign language has become an essential aspect of the teaching and learning process of algebraic concepts by deaf students (Zanoni, 2016; Silva, P., 2016). In both studies, the authors reported the necessity of mediating the contents in Libras and that using additional tools, such as tables, figures, and drawings, contributed to understanding and developing activities.

In this sense, Fernandes and Healy (2016) state that the creation of signs is present in mathematics classes with deaf students because many of them do not exist or are not known by the community. Silva (2016) points out that due to ignorance or the lack of knowledge of signs, interpretation often occurs by manual spelling (typing), which generates difficulty in understanding the explanation of concepts. The author explains: “[...] the development of the mathematical concept, for the deaf, is a complex associative process between the written and graphic language of mathematics and the Libras practiced by the interpreter” (Silva, 2016, p.111).

With mediation through sign language, generalisations in algebra learning are possible (Silva, 2016). Batista (2016) explains that it is necessary to explore visual-spatial skills combined with Libras, as well as the teacher must have some knowledge of the language to understand how the students elaborate their strategies for solving problems.

Thus, it is up to mathematics teachers to build investigative scenarios that motivate students to explore the objects mediating the available knowledge (Fernandes & Healy, 2016) so that the construction of algebraic knowledge can be carried out in a motivating way and with enthusiasm by teachers and students.

METHODOLOGY

Based on the educational design research (EDR), the design experiment was chosen as the methodological path. This methodology bases itself on the definition established by McKenney and Reeves (2012), according to whom research must be iterative (repetitive), flexible, and focused on theory and practice.

The research was conducted in a public state school in a given municipality in Paraná, Brazil. The sample consisted of thirteen state schools of non-special education having deaf students inserted in their standard classes, as informed by the Núcleo Regional de Educação at the time of application. The choice of schools was defined upon contact with the management team of the respective schools and their availability for the execution of the research. It should be noted that the schools are located in different regions of the city. This essay presents a clipping in which two deaf students from two selected schools compose the analysed sample.

The meetings with the students were called teaching episodes, carried out with a workload of 20 to 30 hours of activities with each deaf student, who willingly volunteered to participate in the extra-classes school support activities. These meetings were individual, held in a classroom, and scheduled according to student availability. They were recorded through audio, video, and annotations on a field diary. Participant observation was chosen as a qualitative research methodology.

The teaching episodes were developed from planning pedagogical actions with mathematics activities in the field of algebra, according to the difficulties the students presented (obtained after interviews with the teachers, BSLIs, and the students themselves), to constitute the iteration cycles. At the end of each meeting, an evaluation was made, as well as the preparation of the materials for the next one. Mediation instruments, such as manipulative and digital materials, were used during the meetings to expand the possibilities and strategies for teaching and learning mathematics.

Content Analysis, characterised by Bardin (2011) as a set of communication analysis techniques, was used for data analysis. This approach involves methodological instruments that can be applied to different sources of discourses.

Therefore, six teaching episodes were carried out with student A1M (deaf student in the 3rd year of high school), and seven were arranged with student A1A (deaf student in the 1st year of high school). Codes according to ethical issues were used to preserve the identity of the participants. The Comitê de Ética em Pesquisa (Ethics in Research Committee) approved the proceedings under protocol number 3443418.0.0000.5547. Once again, it is noteworthy that a clipping is presented (one episode of each student, in detail) in this essay to illustrate and substantiate the research's results.

RESULTS AND ANALYSES

Deaf students enrolled in non-special schools still face several challenges. As explained by Santana et al. (2018), the educational performance of these students does not occur well enough, since classes are still directed to hearing students, with scarce methodologies and directions for the care of the deaf. According to Vygotsky (1997), collectivity is a factor in the development of children. Thus, the exclusion of deaf children from the community due to their language barrier makes social education impossible. The author explains that the collective contact between deaf and hearing children brings the possibility of exploring the various types of language and using higher forms of collaboration, a fundamental condition for their educational progress.

The teaching of sign language from an early age in primary school expands the possibilities for deaf and hearing children through the discovery of a new cultural universe, using resources such as movements and facial and body expressions (Marques et al., 2013, p. 506). Deaf people's education within this context brings the challenge of rethinking the curriculum. According to Kipper et al. (2015), the deaf community wants the school to meet its specificities, respecting its form of communication, to build a curriculum that emphasises the visual perspective.

Mathematics teaching in deaf education needs to value visual mathematics and sign language as a form of communication, not only in translating the content into Libras but also in seeking strategies for realising learning (Kipper et al., 2015). The use of manipulative and digital tools brings to the classroom the possibility of exploring visual-spatial skills, facilitating the appropriation of knowledge and its alignment with the education of deaf people (Arroio et al., 2016).

Knowledge is based on cultural and historically constructed forms, and this is the necessary scenario for investigating the development of thought based on the selection of activities to be explored (Radford, 2012). In this article, the activities carried out with two high school students, Student A1M and Student A1A, are described in more detail as two moments of the teaching episodes, in which the central focus was the development of algebraic thinking.

The teaching episode developed with A1M was divided into six meetings (three hours each, on average), all in the presence of the Libras translator and interpreter. At the first meeting, a playful activity was developed using the Tower of Hanoi and problem-solving activities as a diagnostic assessment. At the second meeting, the activities of the previous meeting were carried out in a commented manner with the student. Also, the topic of the "study of polynomials" was introduced. In the third meeting, the study of the polynomial operations of addition and subtraction was introduced using manipulative material as a mediation tool, as described below.

In the situation (meeting 3, A1M), we used a manipulative material (plates in EVA and using coloured sheets to solve polynomials operations) to solve polynomial operations of addition and subtraction. We found that this material was fundamental to overcoming A1M's difficulties with the content. In the initial activities applied in this episode, we found that the student had not understood the concepts of polynomial operations. As so, mediation aided by the tool enabled the appropriation of knowledge. As suggested by Muniz et al. (2020), it is important to develop activities that prioritise mathematical investigation by manipulating material resources, emphasising visual elements, and establishing bilingual communications.

Figure 1 shows the activity proposed to A1M and then an excerpt of the student's resolution, letters c and d of this activity (Figure 2).

Figure 1

Situation Problem 2 teaching episode, meeting 3

Atividade 2: Dadas as funções polinomiais
$A(x) = 2x^5 + 4x^4 - 3x + 8$ *e* $B(x) = -x^5 + 3x^3 - 2x + 4$ **calcular as subtrações:**

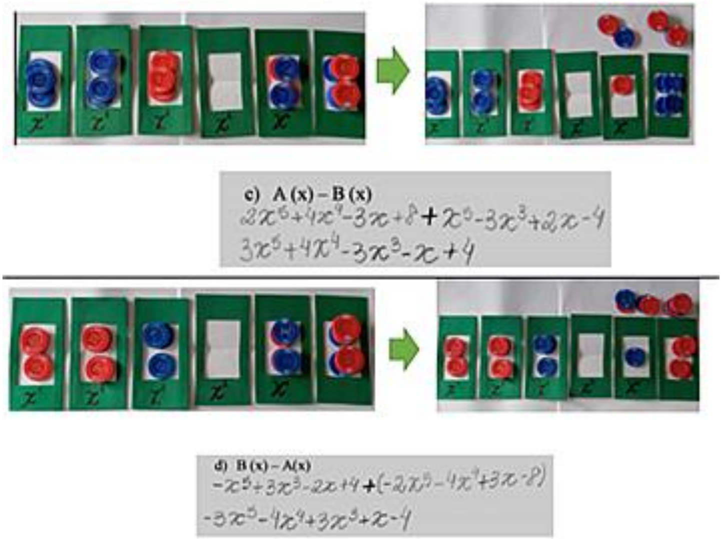
a) A (x) + B (x)
b) B (x) + A (x)
c) A (x) – B (x)
d) B (x) – A(x)

A1M solved it correctly and was enthusiastic while manipulating the objects, showing himself motivated by his success in solving the activity. At the end of this meeting, the student was asked to solve a similar activity without using the material, and he could do it. This fact shows us that there was an appropriation of knowledge, which means that he internalised the concepts and could use them in other situations developed during the episode. To be able to operate with the variables proves that there was a development in algebraic thinking. This activity with A1M shows that the student abstracted the concept after the pedagogical actions.

The multiplication of polynomials was developed in the fourth meeting and the division in the fifth. Finally, in the sixth meeting, activities were carried out that contemplated all the addressed contents, and the student solved them successfully, proving that the process of teaching and learning algebraic topics was internalised through the appropriation of knowledge, as well as with the aid of mediation tools proved to be efficient for understanding the concepts. Afterwards, the student commented that he enjoyed participating in the activities on the night shift. He also stated that he would like to continue participating in more meetings.

Figure 2

A1M student resolution of problem 2



Radford (2012) explains that for development to occur, students must access numerical and spatial structures. Moreover, to structure thinking and seek resolution strategies regarding the proposed situations, it is necessary to employ signs and tools consciously.

Historically constructed knowledge enables reflection and involvement with the situations that surround everyday life. Mathematics Cartesian graphs offer possibilities for thinking, conjecturing, acting, and experiencing situations. Nonetheless, this does not happen naturally, so the learning process is necessary (Radford et al., 2018). Thus, as stated by Vygotsky (2001), education is an organised activity planned to intervene in the natural process of personal's growth.

In this sense, Radford (2010a) explains that algebraic thinking presents itself as one out of many forms of thinking mathematically. The activities carried out with A1A are described in a teaching episode that contemplated seven meetings (of three hours each, on average). In the first meeting, we used the Tower of Hanoi as a motivational activity and some problem-solving activities for further diagnostic assessments. In the second meeting, the activities of the previous class were carried out in a commented manner with the student, highlighting the points in which he presented more difficulty, followed by activities in which operational sets were resolved, these contents being those addressed in the regular class. In the third meeting, problems were deepened, with sets based on external evaluation questions and increasing difficulty. We used the Venn diagram on the blackboard as a visual material to solve the activities.

The fourth meeting started with some logical challenges using Tangram and studying sequences, aiming to introduce the concept of functions for problem-solving intuitively. The situation described in the clipping (meeting 4, Student A1A) of activity 3 (Figure 3) stands out.

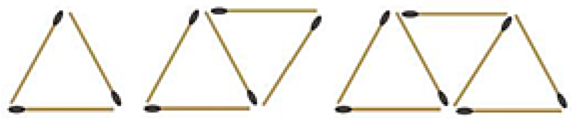
Figure 3

Situation Problem 3 teaching episode, meeting 4 (Adapted from Tinoco, 2009, p. 33)

Com palitos construa um triângulo.

a) Quantos palitos você usou?

Continue a formar figuras outros triângulos como na figura:



b) Ao formar três triângulos, quantos palitos você usou?
c) E se formar cinco?
d) E se formar dez?
e) Se alguém quiser saber quantos palitos serão usados para formar um número n qualquer de triângulos, você saberia escrever uma expressão para ajudá-lo?
f) Verifique se esta expressão dá o número de palitos que você usou para fazer 5 triângulos. O mesmo para três triângulos.
g) Descubra agora quantos palitos são necessários para formar 58 triângulos.

In the dialogue transcribed below, from the resolution of Activity 3 (Meeting 4, A1A), we found that the student's algebraic thinking evolved from factual generalisation, expressed in concrete action and representation with the manipulative material, in which A1A used the sticks to represent the figure, to contextual generalisation, in which he anticipates the formation of the next figures. These happened when the student referred to the number of toothpicks used to construct the fifth figure, as described by Radford (2010a).

> C: (After reading question 3, I handed the student a pack of toothpicks so that he could assemble the situation, which helped him solve it).
>
> PP: *build a triangle with the toothpicks. How many toothpicks did you use?*
>
> A1A: *Three*
>
> PP: *Very well. Continue forming the figures as in the drawing.*
>
> C: (Student mounted the triangle sequence on the desk)
>
> PP: *When forming three triangles, how many toothpicks did you use?*
>
> A1A: *Seven*
>
> PP: *When forming five triangles, how many toothpicks did you use?*
>
> A1A: *Eleven*
>
> PP: *And to form ten triangles?*
>
> A1A: *Twenty-one*
>
> PP: *Right. Tell me: "If someone wants to know how many toothpicks will be used to form a number n of triangles, would you know how to write an expression to help him?"*
>
> A1A: *Multiply by two and add one more*
>
> PP: *Very well. You can write it down on the sheet.*
>
> PP: *And how did you come up with that answer?*
>
> A1A: *It is that here there were three and here seven. For three triangles, I used seven sticks. Then I saw that there were three triangles and figured that each triangle used three sticks. I multiplied it by two and got six, then I added one more.*
>
> PP: *And this solution works for eleven as well?*
>
> A1A: *Yes, because it is five multiplied by two plus one.*
>
> PP: *Very well.*

PP: *The letter f asks: "Make sure this expression gives you the number of toothpicks you used to make five triangles. And the same for three triangles." Did it work?*

A1A: *Yes*

PP: *Okay. Now "find out how many toothpicks does it take to form 58 triangles"?*

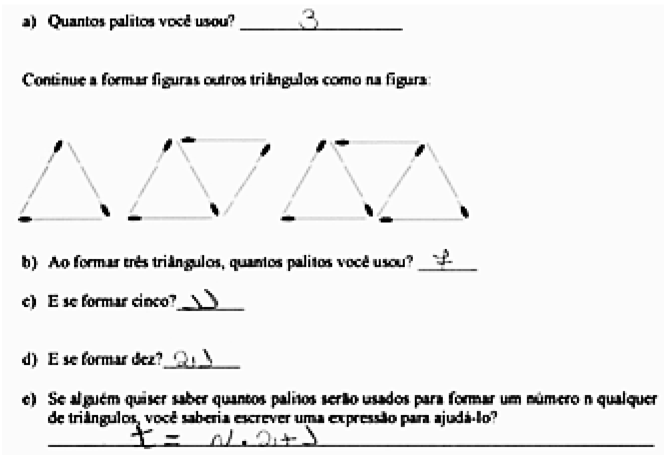
A1A: *One hundred and seventeen*

PP: *Very good!*

The explanation of the formula found by A1A in the transcribed dialogue characterises the internalisation of the concept. The letters n and t used by the student (Figure 4) are semiotic representations of the solution. According to Radford (2010a), the letters used to represent the formula indicate the function of words in the contextual and factual generalisation of the students.

Figure 4

Student A1A resolution of situation problem 3



Mediation through the tool. In this case, the manipulative material (toothpicks) used to construct the figures contributed to achieving the proposed objective. According to Arnoldo Jr. et al. (2014), visual materials can act as mediators and potentials in mathematical activities. Applying algebraic symbols consists of the cultural way of using signs and meanings in symbolic generalisations, that is, to understand how letters are employed in a formula (Radford, 2010a).

The fifth and sixth meetings continued with the resolution of problem situations over the concept of functions, addressing the exploration of sequences using manipulative material (sticks previously described), and, finally, obtaining the law of formation of a function. Finally, the seventh meeting ended with a recapitulation of the concepts developed and the resolution of problem situations proposed by the external evaluations for high school. The student obtained a good result in the resolution, bringing elements that showed us the appropriation of mathematical knowledge, his dedication to studies, and his eagerness to learn and challenge himself more and more.

In this sense, in the teaching episodes presented in this study, it was possible to verify the existence of a point of convergence between the two educational environments regarding communication with the students, specifically, oral mediation and mediation through sign language. There are diverse situations concerning the students' characteristics, as the participating students bring with them particular questions regarding their learning time and fluency in sign language, school trajectories, and relationships with the deaf community, school community, and family.

According to Radford (2016), in the school environment, the teachers and students collaboratively produce knowledge based on the history and culture of those involved in the mediation. Classrooms are interactive places dedicated to mediated activities (Radford, 2010a).

Thus, the mediation carried out through signs and instruments is the central point for quality teaching, thereby establishing connections between the student's previous experiences and the mathematical concepts in consolidation.

CONCLUSION

This study presented the following research problem: How is mediation carried out through signs and instruments in the appropriation of algebraic knowledge by deaf students attending non-special schools? Deaf students are understood as members of a community that acts and establishes social relations based on the artefacts of their culture, using sign language as a form of interaction. Thereby, transformations in school become necessary.

Deaf education is based on social education and forms of mediation established in classroom interactions. W observed that school inclusion still lacks investment in human and material resources, initial and continuing education of teachers and BSLIs, collaborative work among the school community, and implementation of public policies to consolidate the purposes of education for all. Given the data collected, we see that deaf students attending no-special schools face many challenges, such as restricted access to information in the school environment, since communication through sign language is not part of interactions with teachers, classmates, and the pedagogical team. As so, it requires mediation with BSLIs at all times of communication. They also have to deal with the lack of material and digital resources, as well as the recognition of deaf culture.

Therefore, it is noteworthy that the collaborative work between mathematics teachers and BSLIs is fundamental since this combination makes it possible to expand the strategies used in the classroom, deepening the knowledge of the specificities of the deaf students and the mathematical knowledge that is being developed, while respecting each other's role in this process.

The algebraic concepts were the object of study of the teaching episodes, in which pedagogical actions were applied to the deaf students individually. Based on the mediation carried out through signs and instruments, we could act in the students' ZPD and develop strategies that lead them to the appropriation and development of knowledge, thereby expanding the superior functions of these students.

Algebraic thinking is developed throughout the school years, and due to the necessity of understanding abstraction and generalisation, it represents a challenge to deaf and hearing people. However, using visual resources and manipulative and digital materials allows knowledge to be structured from real situations, contributing to the learning process. Tools in solving mathematical problems depend on how they are employed because they do not cause transformations without teachers' interaction. Mathematics applied to everyday life arouses interest in students since it mixes the contents developed in the classroom with students' social practices.

In this sense, we highlight the activities carried out by pedagogical support teams during the day shift, and preferably in the non-special schools attended by the students, like developing tasks to review the contents addressed in the classroom. As presented in this study, this strategy benefits students, giving them opportunities to overcome their difficulties regarding mathematical content through teacher mediation and the use of mediating instruments.

We reaffirm the importance of visuality in developing mathematics content, especially regarding the development of algebraic thinking in deaf children's education. It is also necessary to expand investments in education to guarantee human and material resources for public education and quality for all. The ideal school becomes a challenge, and it is necessary to be careful while defending the best education model for deaf people to avoid accentuating the exclusion.

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AUTHORS' CONTRIBUTIONS STATEMENT

RSD, EMS, and SCRS conceived the idea here presented. RSD developed the theoretical scope, adapted the methodology to the research context, developed the field research with data collection, and analysed the data. All authors actively participated in the discussion regarding the results, and reviewed and approved the final version of the work.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the corresponding author, RSD, upon reasonable request.

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