



# School Mathematics Tasks Designed by Secondary Education Teachers: An Analysis from the Functional Vision of Mathematics

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## ABSTRACT

**Background:** After mathematics curricular reform in 2012 in Costa Rica, which accentuate the functional vision of mathematics, it is worth inquiring if teachers designed tasks respond to established curricular guidelines and what mathematical and didactic characteristics that these tasks exhibit. **Objectives:** The aim of the study was to describe and analyze the mathematical and didactic elements, linked to the curricular proposal for Secondary Education in Costa Rica, used by in-service mathematics teachers for designing or selecting school tasks oriented to the development of mathematics skills on the subject of functions. **Design:** It corresponds to a descriptive-qualitative research, based on an intrinsic study of cases. **Setting and Participants:** Participants were five mathematics teachers from different educational institutions in Costa Rica during the year 2022 were studied, all teachers participated in a training program to promote teacher reflection competence regarding the design and analysis of school mathematics. **Data collection and analysis:** Data was collected from school mathematical tasks designed by the participants. The analysis of the information was carried out by defining categories and units of analysis based on the curricular provisions established for the teaching and learning of mathematics in Costa Rica, and the components of didactic analysis, as a methodology for the design of school mathematics tasks. **Results:** The analyzed tasks exhibit strengths in the curricular component, as well as in the didactic component. However, they lack elements that allow understanding how and when to put the designed tasks into practice, among other aspects. **Conclusions:** The training of in-service teachers takes a significant enhancement to strengthen knowledge and capacities oriented to the design, selection, and analysis of school mathematics tasks, associated with the functional vision of mathematics that articulates the curricular plan regarding student learning in Secondary Education in Costa Rica.

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## **Tareas matemáticas escolares diseñadas por docentes de educación secundaria: un análisis desde la visión funcional de las matemáticas**

### **RESUMEN**

**Contexto:** Luego de la reforma curricular de matemáticas de 2012 en Costa Rica, que acentúa la visión funcional de las matemáticas, vale la pena indagar si las tareas diseñadas por los docentes en servicio responden a los lineamientos curriculares establecidos y qué características matemáticas y didácticas presentan estas tareas.

**Objetivo:** El objetivo del estudio fue describir y analizar los elementos matemáticos y didácticos, vinculados con la propuesta curricular para la Educación Secundaria en Costa Rica, que utilizan docentes de matemática en servicio para el diseño o la selección de tareas escolares orientadas al desarrollo de habilidades matemáticas sobre el tema de funciones. **Diseño:** Corresponde a una investigación cualitativa descriptiva, basada en un estudio intrínseco de casos. **Escenario y participantes:** Los informantes fueron cinco docentes de matemáticas provenientes de distintas instituciones educativas en Costa Rica, que participaron en un programa de formación para fomentar la competencia reflexión docente desde el diseño y análisis de tareas matemáticas escolares, durante el año 2022. **Recolección y análisis de datos:** La información fue recolectada a partir de tareas matemáticas escolares diseñadas por los participantes. El análisis de la información se realizó mediante la definición de categorías y unidades de análisis basadas en las disposiciones curriculares establecidas para la enseñanza y el aprendizaje de las matemáticas en Costa Rica, y los componentes del análisis didáctico, como metodología para el diseño de tareas matemáticas escolares. **Resultados:** Las tareas analizadas muestran fortalezas tanto en la componente curricular como en el componente didáctico. No obstante, carecen de elementos que permitan comprender el cómo y cuándo poner en práctica las tareas diseñadas, entre otros aspectos. **Conclusión:** significativo para fortalecer conocimientos y capacidades orientadas al diseño y análisis de tareas matemáticas escolares, asociadas a la visión funcional de las matemáticas que articula el plan curricular sobre el aprendizaje en el estudiantado de la Educación Secundaria en Costa Rica.

**Palavras-chave:** diseño y análisis de tareas; docentes en servicio; plan curricular; tareas matemáticas escolares; visión funcional de las matemáticas.

### **INTRODUCTION**

From the framework of learning opportunities, school mathematics tasks have the purpose of mobilizing knowledge and fostering the development of school mathematics competence in the student body (Watson and Mason,

2007). It is characterized by having the intentions of achieving objectives and the development of mathematical abilities or skills through the implementation of “intelligent behaviors that test their intellectual capacity [of the student body]” (Ruiz and Rico, 2016, p. 216). The nature of the tasks and the way they are presented affect what students learn (National Council of Teachers of Mathematics [NCTM], 1991).

Within this order of ideas, school mathematics tasks are an important mechanism that promotes student learning—based on the application of procedures and mathematical reasoning—and the strengthening of capacities such as argumentation, justification, and explanation. In this sense, the need for the mathematics teacher to seek an adequate and justified design of the mathematics tasks that he/she elaborates or selects, as part of the teaching planning process, to ensure the achievement of learning objectives, is highlighted (Henningesen and Stein, 1997; Schwan and Stein, 1998; Sullivan, Clarke, Clarke, & Oshea, 2010). As stated by Chamoso and Cáceres (2018), the mathematics teacher must have the ability to propose tasks.

For Costa Rican Mathematics Education, a change has been promoted in the way in which teachers should plan teaching, with the firm purpose of strengthening the learning of mathematics in Primary and Secondary Education students. The curricular reform in mathematics (Ministerio de Educación Pública [MEP], 2012) has proposed learning based on problem solving and the active contextualization of school mathematics tasks; that is, students will learn mathematics from the analysis and solution of problems of everyday situations (authentic tasks), which will lead to the understanding of mathematical concepts and procedures and the development of specific mathematical skills, according to the curriculum.

In relation to the previous idea, teachers have had to face the challenge of adapting teaching to the established purposes, despite the fact that their initial training was based on curricular provisions for the teaching and learning of mathematics based on mathematical content, which precede the current political and educational regulations that promote student education based on the development and strengthening of mathematical skills. It should be noted that the introduction of these curricular approaches was accompanied by training activities for mathematics teachers conducted by specialized personnel in mathematics from the Ministry of Public Education (known locally as MEP) and the staff in charge of educational reform.

After a decade of development of the mathematics curricular reform—which was implemented in 2012 for new students and by way of a transition

plan for students of other grades—, it is worth inquiring about the mathematical and didactic characteristics exhibited by the school tasks designed by some teachers, as opportunities to mobilize knowledge and foster mathematical skills, and if these tasks respond to established curricular guidelines, which accentuate the functional vision of mathematics. The objective of the present investigation is to study five school tasks designed by in-service mathematics teachers for the learning of various concepts and procedures, and the development of mathematical skills in the student body on the subject of functions, by identifying mathematical and didactic elements related to the curricular proposal implemented for Secondary Education in Costa Rica.

## **THEORETICAL BACKGROUND**

A school mathematics task is any demand structured for cognitive performance that is proposed to the student body, which requires reflection on the use of mathematics, and that the teacher intentionally presents as a means for learning or as an evaluation tool (Caraballo, 2014). Proposals that do not contemplate learning expectations, explicit or implicit, and whose result the teacher cannot use to assess student achievement, are excluded from this consideration (Caraballo, Rico and Lupiáñez, 2011).

School tasks play a determining role in the teaching and learning of mathematics. Within this framework, Sullivan, Clarke, and Clarke (2009) highlight that the nature of student learning is determined by the type of tasks that are posed and the way in which they are applied. Those tasks that lead students to become significantly involved with mathematics provide the intellectual context for the development of their mathematical competence (Chapman, 2013; Sanni, 2012). In this sense, Sullivan, Clarke, and Clarke (2013) state “the ways that teachers choose and use tasks are connected to what they know (p. 15).

### **Components of a school mathematics task**

Moreno and Ramírez (2016) describe school mathematics tasks based on three main components, linked to the functional vision of mathematics: mathematical content, situations, and complexity. These components are described below in line with the curricular proposal for the teaching and learning of mathematics in Costa Rica.

*Mathematical content* is presented from the curricular organization of school mathematics. From the perspective of the Ministry of Public Education (MEP, 2012) mathematics is organized into five thematic areas: Numbers, Measurements, Geometry, Relations and Algebra, Statistics and Probability. For the present study, the area of Relations and Algebra stands out, and corresponds to the mathematical area in which the tasks designed by the participating teachers are encompassed.

Particularly, the presence and relationship between mathematical concepts and procedures stands out, that is, tasks promote a link between the concept under study and other mathematical concepts previously addressed, as well as the implementation of algorithms and properties that make them operational. In addition, the use of representation systems that involve various notations, symbols and graphics is included, which accentuate specific aspects and properties of a given mathematical concept that enhances its use and understanding for certain purposes (Lupiáñez, 2016). For the concept of function, four representations are usually recognized: symbolic (analytical), graphic, numerical (value tables) and verbal. Conversions between representation systems “are translations of a certain expression made in one system, to the expression of that same notion in a different system” (Lupiáñez, 2016, p. 122). These are useful in tasks that support various solving strategies.

*Situations* provide meaning to mathematical concept. The situation is the part of the student’s world where the tasks that have been posed are located. In this way the student endows the concept with meaning. Situations show modes of use of the concept, they are indicators of meaning and help to delve into the concept (Caraballo, Rico, and Lupiáñez, 2013). The role of the situation presented in the task statement is an integral part of problem solving, since it allows students to connect mathematics with the real world. Following Moreno and Ramírez (2016), “the importance of studying this variable [situations] lies in the fact that the representations and mathematical strategies chosen to solve a task depend on its meaning, which is linked to the situation from which it arises” (p. 248).

Since the PISA 2012 study, Ruíz-Hidalgo (2016) recognizes four types of situations: (1) personal situations, about the student’s daily activities, (2) educational, occupational, or work situations, identified at school or the workplace, (3) public situations, visible in the community and in the media, and (4) scientific situations, associated with the interpretation of scientific problems. Within the framework of the Costa Rican curriculum, Ruiz (2017) highlights

those situations immersed solely in mathematics, considering them alien to those of scientific nature.

Following the previous idea, the diversity in the presentation of situations that frame the tasks leads to an authentic and relevant use of mathematics in problem solving. The authenticity of a task refers to the possibility of reproducing or simulating in a reasonably realistic way the exposed situation; that during the resolution of the task by the students, conditions similar to those they would face in a real situation arise (Loría, 2021; Palm, 2008). A task is considered to be authentic based on the proximity of the event posed, the adequacy of the question asked, the consistency of the information provided, the clear presence of the purpose pursued, and the specificity of the data of the situation proposed to the responder (Chamoso and Cáceres, 2018). Regarding relevance, it is conceived as the degree to which the issues addressed in the task acquire significance or importance for the student body that solves them.

Finally, Maaß (2010) classifies school mathematics tasks according to their authenticity and relevance. For the purpose of the present study, the tasks in which active contextualization takes place are considered, that is, those that are developed upon the manipulation of information from the surrounding reality through the use and construction of mathematical models. For example, tasks of the types: *problem in real situation and relevant question*, or *problem in situation and authentic question*.

The *complexity* component is associated with the ordering and sequencing of tasks according to the learning rhythms shown by students. Here three degrees of task complexity are distinguished based on the difficulty of the resolution process and other aspects (Lupiáñez, 2009; Moreno and Ramírez, 2016).

- *Reproduction*. It corresponds to basic tasks in familiar contexts that imply the reiteration of knowledge or practiced processes. For example, application of standard algorithms, performance of simple operations and use of elemental formulas.
- *Connection*. It is associated with tasks having greater cognitive demand, in less familiar contexts, which imply the management and relationship of different representation systems or different aspects for their resolution. For example, explanation and interpretation, and the selection and use of non-routine problem-solving strategies.

- *Reflection*. It refers to tasks that link a greater number of elements, the resolution of which requires the use of more complex skills. For example, creativity, exemplification, generalization and justification of results.

### **Analysis of school mathematics tasks**

Given the great amount of sources to which teachers have access to select tasks, or to base their task design and preparation, it is convenient that these—the tasks—be subjected to a process of analysis and evaluation with the purpose of establishing their curricular relevance. This assessment would make it possible to provide elements to select or incorporate the task in the teaching and learning processes, in the right way or at the right time, or provide inputs to advance in the design of the task (Ruiz, 2017).

Studying the tasks in a methodical way, through the analysis of its components—the elements that describe and characterize them—, leads to better understand them and decide on their adjustment to the established learning expectations and to the characteristics of students (Moreno and Ramírez, 2016). In this sense, the study of the tasks requires the definition of categories of analysis, which can be based on the provisions of the educational curricula or different theoretical proposals.

For Ruiz (2017), the design and analysis of school mathematics tasks involves different curricular elements such as the intended skills and mathematical knowledge according to the curricular organization, mathematical processes and their degree of intervention in the task, and the situations and contexts that frame the task to make sense of mathematics.

For their part, Rico and Fernández-Cano (2013) and Rico and Moreno (2016) emphasize didactic analysis as a methodology for the design of didactic units—which are made up of school mathematics tasks—. There are five partial analyzes that support this methodology: conceptual analysis, content analysis, cognitive analysis, instructional analysis, and evaluation analysis. Particularly, for the present study, attention is paid to content, cognitive, and instructional analyses, which are mostly associated with curricular planning for the teaching and learning of mathematics in Costa Rica. From content analysis one rescues the study of concepts and procedures, representations and situations; cognitive analysis is linked to *why* and *how far to learn* certain topics, in detail it refers to expectations, limitations and learning opportunities; instructional analysis responds to *how* and *when* to carry out the training, that is, the functions, types

and sequencing of tasks, materials and resources, organization and management of work in the classroom.

The elements described should lead to decision-making on the suitability of the tasks for the promotion and development of school mathematical competence, as opposed to the use of tasks whose solution pathway is specified or obvious, or that require the replication of procedures learned as algorithms. A modeling process is suggested that includes the manipulation of multiple representations, the mathematization of aspects of the situation and the demathematization of the solution, that is, the interpretation of the mathematical result according to the situation presented in the task. Furthermore, through the task, the teacher must seek in the classroom the communication of ideas and conclusions of the students, thus cultivating the explanation and argumentation skills that promote mathematical thinking and reasoning.

## **METHODOLOGY**

The study carried out is of qualitative and descriptive nature based on case studies. The qualitative nature of a study leads to exploring, describing and understanding a particular phenomenon (Ricoy-Lorenzo, 2006). The case study is intrinsic because it provides “a better understanding of this case [...] in all its particularity and ordinary nature. The researcher subordinates, at least temporarily, other curiosities, so that the stories of those who ‘live the case’ are highlighted (Stake, 2013, p. 158).

On this occasion, we attempt to analyze the structure shown by some school mathematics tasks, designed by a group of in-service mathematics teachers, for the learning of different contents and the development of mathematical skills on the subject of functions in Secondary Education, and how their designs respond to the theoretical-curricular approaches proposed by MEP.

### **Information Collection**

The participants were five mathematics teachers from different educational institutions in Costa Rica and who participated in a training program to promote teacher reflection competence regarding the design and analysis of school mathematics tasks during 2022. For their selection, the following criteria was considered: be a teacher in service, have received



training on the curricular reform, and have a minimum of five years of experience in Secondary Education.

The training course sought to achieve five specific objectives aimed at identifying and describing the elements of a mathematical task that are relevant, based on the curricular foundations for the teaching of mathematics in Costa Rica (MEP, 2012) and didactic analysis (Rico and Fernández-Cano, 2013), as theoretical references for the promotion of school mathematical competence; to the justification of the arguments on the analysis of school tasks considered in Secondary Education; to the design of tasks that promote school mathematics competence in Secondary Education; and, finally, to the assessment of practices, own and those of other colleagues, on the analysis and design of tasks for the promotion of school mathematics competence, all these purposes based on the adopted theoretical approach.

The tasks analyzed for the present study were requested from teachers as part of one of the diagnostic activities of the training course in which they participated. Specifically, they had to look in their curricular planning for a task (a problem) that had been developed and used to teach functions. It should be noted that, prior to the selection of these tasks, the teachers had not received any type of information about the training course; the design of the facilitated tasks was based on the training received in the first years of the curricular reform and on their experience as teachers.

### **Information analysis**

The ways in which mathematics tasks can be submitted to analysis is diverse. For the analysis of the tasks, various proposals were taken into consideration, particularizing the curricular provisions established by MEP (2012) for the teaching and learning of mathematics in Costa Rica, and the components of the didactic analysis, as a methodology for the design of school mathematics tasks (Rico, Marín, Lupiáñez, and Gómez, 2008).

With greater emphasis, four categories of the mathematics curriculum were considered:

- *Mathematical competence.* The specific area in which the topic of functions (Relations and Algebra) is located and the link of the task with the proposed mathematical skills are identified.
- *Complexity.* The link with the proposed mathematical processes and the degree of intervention of these processes are identified to

determine the degree of complexity of the task (Doyle, 1988; OCDE, 2006; Ruíz, 2017). The mathematical processes are reasoning and argumentation, problem statement and problem solving, communicating, associating, and representing.

- *Problem solving.* It is recognized that the task aims at learning based on the understanding and reflection of a problem situation, and not so much on the direct application of mathematical concepts or results.
- *Active contextualization.* The task is set in some everyday situation familiar to the student; that is, the task is authentic (Verschaffel, Greer, & De Corte, 2000; Palm, 2008). Specifically, a relevant and authentic task is recognized as: problem with real context and relevant question (didactic), problem with real context and relevant question, and problem with real context and authentic question (Maaß, 2010).

From the didactic analysis, the following categories stand out:

- *Content.* The task evidences a relationship among contents (concepts and procedures), the presentation of various representation systems, and the exposition of phenomena about the usefulness of the functions.
- *Cognition.* The corresponding learning expectations are recognized in the task and learning limitations are inferred that may hinder the resolution of the task by the students. That is, the task would make possible the manifestation of errors or learning difficulties in the student body, which can be foreseen and considered by the teacher.
- *Instruction.* The task allows diverse resolutions (more than one learning pathway), allows the use of resources and didactic materials, and indicates a specific organization of the group of students.

In what follows, the most relevant results of this analysis are highlighted for each of the five tasks that were subjected to study.

## **RESULTS AND ANALISES**

The presentation of the results is organized by task; these have been numbered without particular criteria, such as T1, T2, etc. First, the task is described and then the most outstanding mathematical and didactic elements

that characterize it are highlighted, according to the established categories of analysis.

### **T1. Civics assignment.**

To complete a civics task on social phenomena, a fifth-grade student at Liceo Santa Cruz (1500 students) spreads a rumor such that the number of students who know about it is given by  $p(t) = 3^t$ , where “ $t$ ” is given in days.

- a) Determine the number of people who know about the rumor after 4 days.
- b) Determine the number of days that must elapse, after the rumor started, so that the student population that knows about it is 243.
- c) Determine the number of days approximately that must elapse, after the rumor started, for the entire student population to know about it.

#### *Curriculum component*

From the category of mathematical competence, this task shows the exponential function content and is associated with the ability to “identify and apply mathematical models that involve exponential functions” (MEP, 2012, p. 415), proposed for 11th grade in the Relations and Algebra area. Here, the interest is that the student body uses a model exposed in the statement in the treatment of a particular situation.

Regarding complexity, the proposed questions lead to the application of algorithms or the performance of simple operations, associated with the calculation of numerical values and, formulating and solving exponential equations, based on the use of an explicitly formulated mathematical model. In this way, the task has a degree of complexity of reproduction, by mobilizing the mathematical process of statement creation and solving problems through the consideration of simple data and the search for a single solution from the application of the given model.

For its part, the task is part of a situation that could be classified within reality. However, it does not stand out for its authenticity and relevance since the data presented is far from reality, the functionality of the mathematical model is associated more with arithmetic calculation than with the representation and analysis of the situation raised and requested questions do

not resolve a situation of interest to the student body. From this, the consideration that the task moves away from the established foundations for problem solving, as a methodological teaching and learning strategy for the development of mathematical skills, gains strength.

### *Didactic component*

From the content category, the task shows relationships between concepts such as relation, preimage, image, exponential function, and equation; and procedures associated with the calculation of images and preimages, and the resolution of equations. Then, the task statement highlights a symbolic-algebraic representation that models the situation posed; however, the resolution of the task favors the identification of explicit data that does not require the interpretation of the algebraic model or the use of other types of representation, such as tabular or graphic. The task exposes a particular phenomenon of daily life in which the concept of exponential function can be applied.

Regarding cognition, the task is associated with the identification and application of mathematical models on exponential functions. The difficulties or errors that could appear are linked to algebraic procedures, such as substituting  $t$  for the values given in indications b) and c), instead of proposing an equation, product of an incorrect interpretation of the model.

In relation to the instruction, the task requires a single and direct resolution process (learning pathway), evident from the questions raised. The statement does not specify resources or materials that could be implemented, for example, software that allows students to model the function that is addressed in the task. For its part, it is interpreted that the task must be resolved individually, this by the imperative “determine”, established in the three questions. However, depending on the intention of the teacher—despite this indication—the task could be solved by groups of students. Finally, the recognition of aspects about the way in which the task is presented to the group and its organization, typical of classroom management, is not clear.

## **T2. Throwing the ball.**

A ball is thrown upwards with a speed of 64 m/s from the top of a building 80 meters high. It will be at a height of  $A(t) = -16t^2 + 64t + 80$  meters above ground level  $t$  seconds after launch.

- a) Make a drawing of the situation.
- b) How long will it take to hit the ground?
- c) How long will it take for the ball to reach its maximum height and what is that height?

### *Curriculum component*

The mathematical content of the task corresponds to the quadratic function and aims to promote three skills, associated with the area of Relations and Algebra in the 9th grade. The skills are: “graphically and algebraically analyze the quadratic function with the criterion  $f(x) = ax^2 + bx + c, a \neq 0$ ; pose and solve problems in real contexts using the functions studied; and relate the graphic representation with the algebraic one” (MEP, 2012, pp. 411-412). For this educational level, it is of interest that students make use of simple models of phenomena and situations, linked to quadratic functions, complemented by the manipulation of algebraic symbols and their relationships.

The complexity of the task is characterized by the presence of questions for the iconic representation of a model and the application of mathematical algorithms, mainly associated with the formulating and resolution of quadratic equations, based on the exposed model. Thus, the degree of complexity of the task is of the *reproduction* type, with some proximity to a degree of connection since the need for an iconic representation enables the relationship between this iconic representation and the symbolic-algebraic representation given in the statement. However, the requested representation is not required for the following two indications since they are solved independently and only with the use of the algebraic model.

The situation exposed in the task approximates a phenomenon that could be considered an everyday situation; however, this “real” situation is incorporated into the task to give it a didactic character. In this way, the task lacks authenticity, since the situation cannot be replicated in the classroom, and relevance, since the data provided is not significant for its resolution, for example, the incidence of the speed of the ball and the height of the building. In addition, the mathematical model allows, through arithmetic calculation, to obtain the required data.

### *Didactic component*

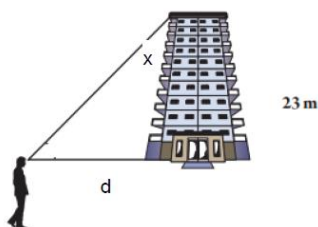
Conceptually, the task involves relationships between basic concepts of the quadratic function, such as: parameters that determine the criterion of a quadratic function ( $a$ ,  $b$ ,  $c$ ), vertex (maximum), concavity, intersection with the abscissa axis, preimage and image, quadratic equation; and with mathematical procedures such as drawing an iconic or graphic representation, calculating preimages of a function, solving a quadratic equation, and numerical substitution. It should be noted that the problem statement does not specify the meaning of the variables  $A$  and  $t$ , essential for the interpretation of the model. On the other hand, the statement of the task exposes a symbolic-algebraic representation that models the situation and the first of the indications promotes the use of an iconic representation that illustrates the situation, as part of the resolution process. Although the task exposes a situation that shows the application of a quadratic function, it is far from being an everyday situation.

The cognitive component is characterized by the presence of learning expectations linked to the iconic representation and the application of mathematical models. The difficulties or errors that could manifest themselves are perceived from the algebraic procedures necessary to determine the requested values, such as the formulation of a quadratic equation and the incorrect interpretation of the solutions, in which a negative value can be assumed as a possible value of the elapsed time, or the values of the coordinates of the maximum point.

The specific indications shown in the task lead to a specific learning pathway, that is, to a guided resolution. Also, the task statement lacks information about the resources or materials that the students could use. As with T1, technological resources that favor visualization and graphic modeling of the situation or verification of results, for example, are not used. Regarding the organization of the group, the imperative of the first request "Make a drawing" suggests that the task must be resolved individually, but the information in the statement is scarce to affirm this.

### **T3. Engineer's calculation.**

An engineer determines that the distance a person is from a 23m high building is modeled by the function  $d(x) = \tan x$ .



Analyze the following statements and determine if they are true or false.

- I. As the person approaches the building the distance decreases and the angle “ $x$ ” increases.
- II. If the person moves away the angle “ $x$ ” increases.
- III. If at a given moment the person is  $40\text{m}$  away, then the angle “ $x$ ” is approximately  $48^\circ$ .

### *Curriculum component*

Even though the task statement falls within the thematic area of Relations and Algebra, the intended skill is associated with the content proposed for the 9th year, on trigonometric ratios, in the area of Geometry, and requires prior knowledge of the measure of angles and the estimation of these—which are developed in the 3rd and 4th grade of primary education—. It is expected that the ability to “apply the basic trigonometric ratios (sine, cosine, tangent) in various contexts” (MEP, 2012, p. 317) will be fostered in the student body, as a means of recognizing the usefulness of the basic trigonometric ratios and their relation to everyday situations. It should be noted that the Costa Rican mathematics curriculum does not address the study of trigonometric functions in Secondary Education. Despite this, we proceeded with the analysis of the task considering the intention of the teacher that it be a means for learning the topic of functions.

For solving the task, three mathematical processes linked to problem solving, reasoning and communication have been identified. Respectively, the processes highlight the resolution of a problem from simple and explicit data and the application of known algorithms or models, which only admits a single solution; the interpretation of results extracted from an applied procedure; and the communication of results obtained through routine procedures. In this way, from the degree of intervention of these mathematical processes, the task is one of reproduction.

Likewise, the task reflects a situation that is reasonably realistic, in the field of civil engineering. However, it is considered that it lacks a relevant question that allows a simulation in the classroom, while being highly significant and motivating for the student body. The task is limited to closed answers, without allowing spaces for explanation and argumentation by the group of students.

### *Didactic component*

From the category of mathematical content, the definition of the tangent function and the relationship between concepts such as angle, triangle, triangle side, trigonometric tangent ratio, estimation, and measurement, stand out. Regarding procedures, the calculation of a numerical value (that of angle  $x$ ) stands out. The representations that emerge are the symbolic-algebraic (the model) and iconic. Of the latter, it should be considered that the indication of the right triangle is omitted as a transcendental aspect for the application of the trigonometric model. Looking at the figure, the triangle that includes the angle “ $x$ ” is not a right triangle, since the edge of the building is inclined. On the other hand, when considering the height of the building, it may induce to misinterpretation since the data of the height of the person is not clarified and, with this, the angle of elevation. Also, the indications omit specifics about the fact that the increase or decrease that could occur are with regards to the measure of the angle and not literally to it as a figure.

Taking into account the elements of the cognitive category, it is conceived that the task promotes the analysis of situations, shown in the three assumptions, through the use of iconic representation. However, mathematical argumentation is left aside to solve or describe the posed situation. The limitations that can be manifested are associated with the task statement and the visualization capacity of the student; particularly, due to wording, the lack of precision in the iconic representation and the calculation of the requested angle measure (error in defining the tangent ratio or in determining the angle measure).

Within this framework, from the statement, it is pointed out that the task refers to a single form of resolution, in which the main action is to analyze; that is, the determination of the true value of three propositions through the interpretation of the iconic representation shown in the statement and the criterion of the given function. Concerning the indication “analyze”, it could be interpreted as a suggestion for an individual approach to the task, however,



there is no precision on this. Regarding resources or materials, the task statement lacks details.

#### T4. The pharmaceutical laboratory.

In a pharmaceutical laboratory, it was determined that if 5mg of a certain drug is injected into a patient, the remaining amount of said medicine, in the bloodstream, after  $t$  hours, is determined by the expression:

$$f(t) = 5 \left(\frac{1}{3}\right)^{\frac{t}{3}}$$

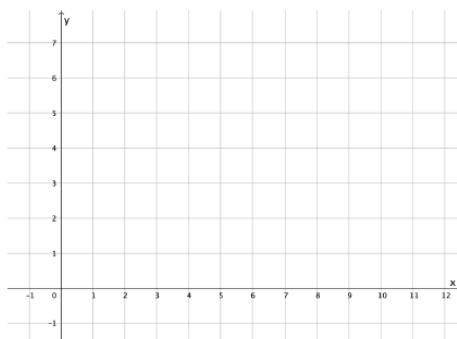
So, after how long will only 1 milligram of the drug remain in the patient's bloodstream?

To solve the above problem, you can guide yourself by answering the following questions:

- a. If  $t$  represents the time in hours, and  $f(t)$  the milligrams of the drug, how many milligrams will there be after 0h, 1h, 3h, ...? Complete the following table.

$t$	0	1	3	4	4,5	5	6	10
$f(t)$								

- b. Based on the information obtained above, what is the graphical representation of  $f(t)$ ? Locate the points of the table, in the following Cartesian plane.



- c. From the above two representations of  $f(t)$ , for what value of  $t$  do you think  $f(t)$  is close to 1?
- d. If  $f(t)$  approximates the number of milligrams of the drug in the bloodstream from the hours that have elapsed, then what does  $f^{-1}(t)$  allow us to determine? Represent in the Cartesian plane above, the graph of the inverse function of  $f(t)$ .
- e. After the analysis performed, what is the best approximation for  $t$ , as requested in the problem? Check your answer by evaluating the function at the respective time.

### *Curriculum component*

The task belongs to the thematic area of Relations and Algebra on the study of exponential and logarithmic functions. The skills promoted with the task, as stipulated in the study program (MEP, 2012) are three: “graphically, tabularly and algebraically analyze exponential functions” (p. 414), “identify and apply mathematical models that involve exponential functions” (p. 415) and “identify the logarithmic function as the inverse of the exponential function” (p. 415).

Regarding the complexity of the task, the activation of the five mathematical processes is recognized. Especially, the establishment of a resolution strategy where sequential actions are executed is identified; the use of argumentation and various representations to answer specific questions; the use of links between concepts and mathematical procedures in solving a real situation; the interpretation of a sequence of reasoning from the application of mathematical concepts and procedures, the description of the actions carried out, the communication of conclusions through natural and mathematical language; and the use of two representations to solve problems and the mobilization between them. In this way, from the degree of intervention of the mathematical processes, it is recognized that the task, according to its complexity, is of the connection type. The task and its execution plan lead to the understanding of a phenomenon through the model that represents it, rather than the direct application of mathematical concepts or procedures.

On the other hand, the situation considered in the task is close to a scientific reality, which gives it a degree of authenticity. Likewise, the question may be relevant for the student body, since the data provided and the

relationship between the concepts involved favor its resolution in the real situation posed; it is a task in *real situation and relevant question*.

### *Didactic component*

The task highlights concepts such as inverse, exponential, and logarithmic function, as well as other implicit concepts necessary for its resolution (preimage, image, numerical value, estimation). The relationships between these concepts are evident from the mathematical procedures that must be applied. For example, to complete the table and plot the graph of the given function, it is required to substitute values corresponding to preimages in the criterion to find their respective images and determine the ordered pairs. The representation systems are diverse; relationships are presented and established between the symbolic-algebraic, tabular, and graphic representations. The task highlights an everyday situation associated with medicine.

Relative to the learning expectations, the design of the task favors the scope or development of the proposed skills. The dynamics of resolution promotes the analysis of different representations of the exponential function and its inverse, as well as the application of an exponential model for the calculation of numerical values that support their answers and strengthen the mathematical argumentation. The difficulties that could manifest themselves are related to errors during the calculation of images—considering that the power includes a fractional base and exponent—, the location of points in the Cartesian plane, the shift between representations and the incorrect interpretation of the information, particularly that obtained from the graph.

The task statement favors a learning pathway; the proposed work scheme suggests a single resolution method. The use of the personal pronoun “You” suggests the individual approach of the task by the students. From the task statement, the inclusion of a grid for drawing the graphs of the functions that model the situation stands out; this implies the suggestion of specific resources for the resolution of the task.

### **T5. Vacation at Zancudo beach.**

Álvaro wants to stay at a hotel located in Playa Zancudo, once all this COVID 19 is over, and the authorities of the Ministry of Health give the go-ahead to vacation. He consults at the hotel reception, and they tell him that the cost is €50,000 per day, which includes the room, as well as breakfast. The right

to use the safe in the room has a cost of €6,000, which is only paid for the entire stay, suppose Álvaro chooses to use the safe.

Answer the following questions:

- a) If Álvaro stays for 12 days, what is the amount to pay for the stay?
- b) Finally, Álvaro paid € 406,000, how many days did he stay?
- c) Write a formula (algebraic expression) that allows calculating the amount “ $M$ ”, to be paid by Álvaro according to the days “ $x$ ” that he decides to stay.

### *Curriculum component*

The task belongs to the area of Relations and Algebra, according to the program for 8th grade. The promoted ability is “to identify given situations that can be expressed algebraically in the form  $y = ax + b$ ” (MEP, 2012, p. 331). The purpose is that contextualized problems that imply a linear relationship are presented to the student body, through the relation between variables.

The design of the task, especially its resolution, mobilizes five mathematical processes: problem solving, reasoning, connecting, communicating, and representing. Students face a task with simple data—explicitly presented—and a single solution, they are faced with the need to implement known algorithms and resolution strategies that are executed through sequential actions, and to establish a mathematical model that represents the exposed situation. In addition, direct questions that involve the identification of information and the implementation of mathematical procedures must be answered; relate concepts to respond to the situation in context; communicate through an algebraic representation a result obtained through routine procedures or the interpretation of a sequence of reasoning. The degree of intervention of these processes leads to the classification of the task as a connection task.

Regarding the problem-solving category, it is noteworthy that the task requires more than a direct application of mathematical algorithms; that is, a combination of mathematical elements linked by the understanding of a real situation that is treated as a linear relationship between specific variables. The situation shown in the task is close to reality, it is characterized as a *problem with a real context and relevant question*. It grants the possibility of being replicated in class, maintaining the conditions in which the situation is framed.

On the other hand, the question is relevant because it enhances skills that students would put into practice in similar situations.

### *Didactic component*

The task is oriented to the identification of situations that can be represented by linear algebraic expressions; with the purpose of introducing the study of the linear function. In this case, it is intended to integrate numerical calculation and the resolution of first-degree equations, as well as the symbolic-algebraic representation of a real situation to the study of the linear function. For example, to establish the criteria that represents the amount to be paid for hotel accommodation, the student must first explore the calculation of the amount to be paid for a specific number of days and then determine the number of days based on an amount paid, for this the student must solve a first-degree equation with one unknown. Here, the concepts of dependent and independent variable, constant, numerical value and equation stand out. In this sense, the task highlights a daily situation associated with a personal activity of the student. From a cognitive analysis of the task, it is possible to establish that it promotes the mathematical skill to which it is associated. Regarding learning limitations, errors linked to numerical calculation, identification and wrong clearance of variables can manifest.

Taking into consideration that the task statement does not include the algebraic model of the exposed situation, to solve the task the students could resort to different learning pathways, despite the fact that the answer is unique in each of the questions. That is, for questions (a) and (b) students could use additive or multiplicative strategies. It is ruled out that the task proposes teaching materials or resources for its resolution. Similarly, there is no proposal for the organization of the students at the time of presenting the task, despite the indications “answer and write” that could refer to an individual treatment of the task.

Finally, Table 1 shows a summary of the curricular and didactic elements identified in the statements of the tasks studied.

As can be seen in Table 1, from the curricular perspective, the categories on mathematical competence and complexity stand out. Of the analyzed tasks, the inclusion of elements that promote the development of mathematical skills and the consideration of the contents established in the curriculum stand out. This is reinforced with the identification of conceptual

structures drawn from the relationship between different mathematical concepts.

The tasks show a clear reference to the use of complexity degrees—especially reproduction and connection—from the activation of some of the mathematical processes suggested in the educational guidelines, with a greater presence: state and solving problems (with a preponderance of the second component), reasoning and argumentation (with a preponderance of the first component), representing and communicating.

Problem solving, as a teaching strategy to promote learning, shows different characteristics between tasks; some lead to the direct application of procedures, while others are inclined to bring students closer to comprehension processes.

Another of the strengths identified in the tasks is the use of a particular situation as the frame of reference for the use of the concepts and mathematical procedures involved. In this sense, an intention to show students authentic tasks is recognized, despite the fact that the relevance factor lacks representation in some of these.

**Table 1**

*Characterization of the tasks*

Math. competence		Complexity			Problem solving		Active contextualization			Content		Cognition			Instruction	
Am	Hb	P	I	Cj	Ap	Cp	Rel	Au	Ec	Sr	Fe	Ex	Li	Ca	R	O
		r													m	
Task 1																
✓	✓	✓	✓	✓	✓	-	-	-	✓	-	✓	✓	✓	-	-	-
Task 2																
✓	✓	✓	✓	✓	✓	✓	-	-	✓	✓	✓	✓	✓	-	-	-
Task 3																
✓	✓	✓	✓	✓	✓	-	-	✓	✓	✓	✓	✓	✓	-	-	-
Task 4																

✓	✓	✓	✓	✓	-	✓	✓	✓	✓	✓	✓	✓	✓	✓	-	✓	✓
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Task 5

✓	✓	✓	✓	✓	-	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	-	-
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

*Note:* Am=mathematical area; Hb=Skill; Pr=Mathematical process; I=Intervention; Cj=Complexity; Ap=Application; Cp=Comprehension; Rel=Relevance; Au=Authenticity; Ec=Conceptual structure; Sr=Representation systems; Fe=Phenomenon; Ex=Expectation; Li=Limitations; Ca=Learning pathways; Rm=Resources and materials; O=Classroom organization

As part of the didactic framework for the analysis, a tendency is recognized in the way in which task statements include specific purposes (mostly implicit) for the learning of mathematics in the student body. Similarly, when solving the tasks, it is possible to recognize possible errors and difficulties, which can manifest or appear at the moment the student solves them, and that the teacher could take into consideration as a teaching strategy that promotes adequate learning, from the understanding of the content and the development of the proposed skills.

The greatest weakness of the tasks has to do with the instruction category. From this framework, the tasks lack—almost in their entirety—elements that lead to the organization of the students for the resolution of the tasks, the use of didactic materials or resources within the resolution process and the possibility for students to have different options or resolution pathways that allow them to obtain the requested response.

## CONCLUSIONS

The analyzed tasks show a follow-up, by the teachers who designed or selected the tasks, of the curricular approaches on the mathematical content of relations and algebra, the skills linked to this content, the methodological problem-solving strategy, the inclusion of everyday situations, and the activation of mathematical processes.

The design of the tasks responds to at least one of the mathematical skills proposed in the study plan, as one of the means to promote school mathematical competence in Secondary Education students.

Regarding the problem-solving strategy, it is evident that the tasks constitute problem-situations. Their statements bring them closer to the conceptual framework of the functional vision of learning mathematics, which seeks the development of skills through the activation of mathematical

processes to attend to situations of everyday life (Doyle, 1988; Loría and Lupiáñez, 2019). However, for the most part, the tasks become routine exercises, by leading students to or expecting from them the direct identification of the necessary actions for the resolution of the tasks (MEP, 2012). In addition, the relevance and authenticity identified in the tasks lead to a divergence in the manner in which they actively contextualize mathematics; that is, they give meaning to mathematical concepts and procedures according to the situation in which each task is framed. A certain degree of authenticity is recognized in the tasks, without this allowing the student to generate interest for their resolution, due to the little significance that these hold in their reality.

Notoriously, the tasks mobilize one of the five mathematical processes suggested in the curricular guidelines; this is accompanied by precise actions that establish the degree of intervention of these processes, resulting in the degrees of complexity that characterizes them. Most of the tasks promote actions or reproduction processes that distance them from the promotion of school mathematics competence, which is mainly advocated by tasks with a complexity of connection or reflection (Ruíz, 2017).

From the Didactic Analysis framework, specific aspects of mathematical content and learning were recognized. The tasks promote the relationship between the concept under study (functions) with other mathematical concepts, but the implementation of mathematical procedures refers to simple operations or the application of basic properties; for the most part, the dynamics of solving the tasks directs the students along a specific solution pathway that steers them away from actions that favor the understanding, analysis and interpretation of the information, the argumentation and explanation of the result.

The theoretical proposals and the curricular provisions recommend the multiplicity of representations and their manipulation during the development of proposed tasks. On the contrary, the presentation of the studied concept is characterized by the use of particularly algebraic representations; in very few tasks this use of representations is diverse and promotes the relationship among them.

In addition to the above, although the tasks refer to the application of the concept in the attention of daily-life phenomena and promote the mathematization of the exposed situation, the resolution strategy of these hinders the demathematization of the result for its interpretation within the situation, since its establishment—of the result—comes from the repeated and



direct application of known algorithms without giving room for reflection and argumentation of what is obtained.

Despite the fact that the tasks did not explicitly detail the expectations and learning limitations considered in their design, it was possible to determine—in their entirety—that they are oriented to the development of mathematical skills, according to the curricular proposal for the educational level in which they fall, and they are an opportunity for the manifestation of errors and the recognition of learning difficulties in the students that solve them.

From the instruction, the statements of the tasks made it impossible to determine the resources and materials suggested for their resolution, and the classroom management for their presentation to the students. However, most of the tasks are characterized by favoring unique learning pathways, especially established from the statement itself and by the direct application of routine algorithms.

Based on the above, the tasks constitute good initiatives for the students to review the mathematical contents and apply procedures associated with the skills that are expected to be promoted in the subject of functions, this from the framework of complexity of reproduction that characterizes them. As Chamoso and Cáceres (2018) conclude when they analyzed tasks designed by teachers in initial training, sometimes the statements shown in the tasks seem forced, in the sense that the situation is a “set” for the mathematical model in which each one is focused.

On the other hand, for the tasks to favor the development of mathematical competence, within the framework of the functional vision of the discipline, important modifications are required in their design to make them a means with which the students construct mathematical learning (Hiebert and Grouws, 2007). Namely, a close link between the mathematical concept, the skills that are expected to be developed, the mathematical processes that are activated in the task, and the situation that frames it. In addition, the use of various representation systems, a clear definition of the resources or materials that support the resolution of the task, and a justified choice of aspects of group organization that contribute to student learning.

Finally, the training of in-service teachers takes on a significant enhancement for the design, selection, and analysis of school mathematical tasks, which correspond to the functional vision of mathematics and respond, in this way, to the curricular provisions on the learning of mathematics in Secondary Education in Costa Rica.

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## **CONFLICT OF INTERESTS**

The authors declare that they have no conflict of interest.

## **INFORMED CONSENT**

The authors declare that the participants were informed about the research, agreed to participate and signed a voluntary consent.

## **AUTHORS' CONTRIBUTIONS STATEMENTS**

All the authors affirm that the final version of this article was read and approved. The total percentage of contribution for the conceptualization, preparation and correction of this article was the following: M.P.A. 50% and J.R.L.F. 50 %.

## **DATA AVAILABILITY STATEMENT**

Data supporting the results of this study will be made available by the corresponding author [J.R.L.F.], upon reasonable request.

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