


Mathematical Knowledge for Teaching in the Initial Education of Special Education Teachers

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ABSTRACT

Background: The education of special education teachers is diverse in scope and content. One of the existing approaches corresponds to a five-year university degree. School mathematics is among the various subjects that make up the formative plans for careers. **Objectives:** The goal of this paper is to characterise the knowledge for mathematics teaching offered by the course syllabuses related to school mathematics in the initial education of prospective special education teachers. **Design:** This research is framed within a qualitative approach from a descriptive and interpretative perspective. Specifically, a non-interactive qualitative approach was used. **Setting and Participants:** Thirty-one subject syllabuses from 12 universities of the Council of Deans of Chilean Universities were analysed. **Data collection and analysis:** The syllabuses were analysed through a content analysis that combined concept-driven and data-driven development and was carried out sequentially. **Results:** The results show that the content knowledge focuses on numbers and operations from a procedural and definitional perspective. On the other hand, pedagogical content knowledge focuses on the design of interventions. **Conclusions:** We conclude that knowledge for teaching mathematics in special education needs to include mathematics educators in the discussion about what knowledge special educators should have.

Keywords: Mathematics syllabuses; Content knowledge; Pedagogical content knowledge; Special education teachers; Teachers' education.

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O Conhecimento Matemático para o Ensino na Formação Inicial de Educadores Especiais

RESUMO

Contexto: A formação de professores de educação especial é diversificada em escopo e conteúdo. Uma das abordagens existentes corresponde a um diploma universitário de cinco anos. A matemática escolar está entre as diversas disciplinas que compõem os planos formativos de carreira. **Objetivos:** este trabalho procura caracterizar os conhecimentos para o ensino de matemática oferecidos pelos programas de cursos relacionados à matemática escolar no treinamento inicial de futuros professores de educação especial **Design:** esta pesquisa é enquadrada dentro de uma abordagem qualitativa a partir de uma perspectiva descritiva e interpretativa. Especificamente, foi utilizada uma abordagem qualitativa não-interativa. **Ambiente e participantes:** foram analisados 31 programas de disciplinas de 12 universidades do Conselho de Reitores de Universidades Chilenas. **Coleta e análise de dados:** os programas de estudo foram analisados por meio de uma análise de conteúdo que combinou o desenvolvimento orientado pelo conceito e pelos dados e foi realizada sequencialmente. **Resultados:** os resultados mostram que o conhecimento do conteúdo tem um foco em números e operações e de uma perspectiva processual e de definição. Por outro lado, o conhecimento de conteúdo pedagógico apresenta um foco no desenho de intervenções. **Conclusões:** concluímos que o conhecimento para o ensino da matemática na educação especial precisa incluir educadores matemáticos na discussão sobre o que os educadores especiais devem saber.

Palavras-chave: programas de matemática; conhecimento de conteúdo; conhecimento de conteúdo pedagógico; professores de educação especial; formação de professores.

INTRODUCTION

The education of special education teachers is diverse in scope and content. One of the existing approaches corresponds to a five-year university degree. School mathematics is among the various subjects that make up the formative plans for careers. Each subject has specific purposes and, therefore, offers different learning opportunities to prospective teachers. The notion of learning opportunities takes on different meanings in research (Tatto & Senk, 2011). However, if the syllabus of the formative courses and the curricular experiences proposed are internally coherent, they tend to impact more on prospective teachers' knowledge (Tatto, 2018). Likewise, the research suggests an influence of the topics studied on education and specialised knowledge used in teaching years after teachers finish their initial education (Hiebert et al., 2019; Morris & Hiebert, 2017; Qian & Youngs, 2016).

Notwithstanding this relevant issue, in Chile, Vega (2018) points out that the formative syllabi for basic education careers are dissimilar and do not include all the necessary knowledge to develop proficient teaching. Particularly, this author points out that “in the formative syllabus assessed, a traditional vision underlies, with an emphasis on the conceptual and with few LOs [Learning opportunities] in professional and practical skills” (Vega, 2019, p. 118). Therefore, the syllabuses for other teaching careers are expected to have similar characteristics because all of them are governed by analogous guidelines (e.g., Standards for the teaching career).

This fact is especially worrying if we think that teachers’ knowledge of mathematics is key to students’ learning (Hill et al., 2005) and even to the quality of the classes that teachers might dictate (Hill et al., 2008). This suggests that special education teachers’ knowledge of mathematics will impact their students. However, the issue of what mathematical knowledge special education teachers should be trained in is an open line in the literature (Griffin et al., 2014). Specifically, the literature reviews models of professional knowledge that have failed to connect special education teachers’ mathematics skills. In this sense, the review by Griffin et al. (2014) points out that, like in the Chilean case, recommendations and standards on what a special education teacher should know about mathematics are vague. This is because the papers offer minimal guidance in deciding what mathematical content special educators should know and be able to teach (e.g., addition, multiplication, concepts of rational numbers, algebra, statistics, and probability, among others).

The formation of special education teachers to teach mathematics has been a disciplinary area with the least attention and where fewer opportunities to learn mathematics are observed (Greer and Meyen, 2009), perhaps because when special education scholars discuss what teachers need to know, they tend to focus on learning evidence-based practices, teaching experiments, monitoring progress in teaching decisions, and effective collaboration with other professionals and parents (Park et al., 2019). From this perspective, we infer that the role of mathematics knowledge in special education formative syllabuses is not often discussed compared to special education’s specific skills. This issue, despite the teachers’ mathematical knowledge of teaching, indicates the quality of students’ learning, since the activities they perform in the classroom depend largely on it (Brownell et al., 2014; Hill et al., 2008).

From our perspective, special education teachers’ knowledge of mathematics will enable the system to offer and favour access to the

mathematics curriculum to all students, particularly students with educational needs. However, special education teachers perceive that their disciplinary training is weak to adequately implement inclusive legislation decree (Calle, 2020; Inostroza, 2019; San Martín et al., 2017). Moreover, in Chile, the latest National Diagnostic Evaluation (Evaluación Nacional Diagnóstica), performed at the end of any educational degree, shows that mathematics education is the only one in which prospective special education teachers who feel ill-prepared exceed those who feel very well prepared (Centro de Estudios Mineduc, 2020). Therefore, this apparently means that the opportunities they can provide for students with special educational needs are limited (Rojas et al. 2021).

In this context, the research question that guided this research arises: what knowledge for the teaching of mathematics do syllabi related to school mathematics offer to prospective special education teachers? To answer it, we analysed the learning opportunities intended in the curriculum related to school mathematics in the initial education of special education teachers.

THEORETICAL BACKGROUND

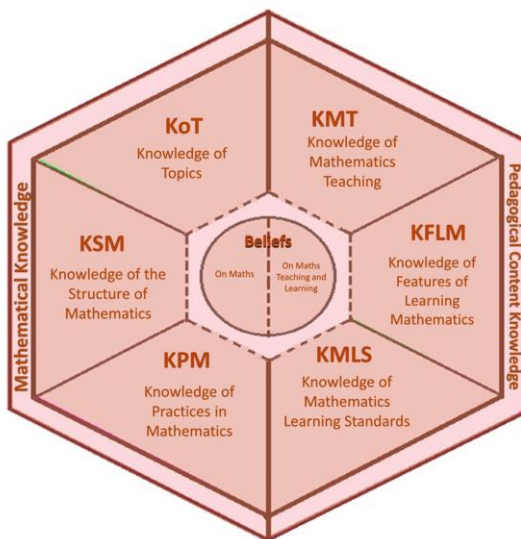
The various conceptualisations of professional competence highlight the teacher's knowledge as a fundamental element. (e.g., Baumert & Kunter, 2013; Niss, 2006; Schoenfeld & Kilpatrick, 2008). While there is no unanimous agreement on how to describe teacher knowledge, it is recognised to be knowledge consisting of several dimensions. Among them, the content and the pedagogical knowledge are seen as suitable for reflecting on the knowledge of prospective teachers (Ponte & Chapman, 2016). These dimensions are often discussed in the light of Shulman's work (e.g., 1986). This author exposes the teachers' own knowledge, a particular type of knowledge that allows teachers to carry out their work: pedagogical content knowledge. However, the literature has noted criticisms that have caused researchers to reinterpret the model (e.g. Depaepe et al., 2013).

Based on Ball and collaborators' (e.g., 2008) work, Carrillo and team (2018) consider that specialised knowledge is not only in the mathematical domain but also relates to aspects of knowledge of the teaching and learning of mathematics. Therefore, this specialisation must refer to professional knowledge as a whole. This stance prompted a redefinition of the knowledge subdomains and the development of the mathematics teacher's specialised knowledge (MTSK) model. Carrillo et al. (2018) point out that "this model features a reconfiguration of mathematical knowledge, a reinterpretation of

pedagogical content knowledge and a new way of conceptualising the notion of specialisation” (p. 240), considering two areas of knowledge: mathematical knowledge and pedagogical content knowledge and a dimension on beliefs. Each of these can be seen in Figure 1.

Figure 1

Mathematics Teacher’s Specialised Knowledge – MTSK. (Carrillo et al., 2018)



In this model, mathematical knowledge is understood:

(...) as a network of systemic knowledge structured according to its own rules. Having a good understanding of this network – the nodes and connections between them – the rules and features pertaining to the process of creating mathematical knowledge enables the teacher to teach content in a connected fashion and to validate their own and their students’ mathematical conjectures. (Carrillo et al., 2018, p. 241)

This causes it to be divided into three subdomains: the very mathematical content (knowledge of topics - KoT); the interconnection systems that unite concepts (knowledge of the structure of mathematics – KSM); and how one advances in mathematics (knowledge of practices in mathematics –

KPM). Regarding the pedagogical content knowledge, Carrillo et al. (2018) point out that:

More than being about the intersection between mathematical and general pedagogical knowledge, it is a specific type of knowledge of pedagogy which derives chiefly from mathematics. (Carrillo et al., 2018, p. 246)

This has led them to identify three subdomains that have been labelled Knowledge of Mathematics Teaching (KMT), Knowledge of the Characteristics of Learning Mathematics (KFLM), and Knowledge of Mathematical Learning Standards (KMLS).

METHODOLOGY

This research is framed within a qualitative approach from a descriptive and interpretative perspective. Specifically, a non-interactive qualitative approach was used. According to McMillan and Schumacher (2005), this type of study is characterised by describing and gathering interpretations about the selected sources. This option is chosen because the objective is to understand, discover, and interpret the opportunities for special education teachers to learn mathematical knowledge present in the initial education syllabus.

Sample and units of analysis

For this work, the Universities of the Council of Rectors of Chile (Universidad del Consejo de Rectores de Chile - CRUCH) that impart the special education career were invited to participate. Of the 30 universities that comprise this network, 15 offer this career in different specialisations. Through the University Institutions for Teacher Training in Special Education of CRUCH (Instituciones Universitarias Formadoras de Profesores de Educación Especial del CRUCH), we requested the Heads of Careers to hand us the curricula of subjects related to school mathematics that were included in their formative trajectory. Of the 15 invited universities, 12 replied to the request by sending the documents.

When comparing the information of the syllabuses sent with the curricular meshes published on the official websites of the institutions, we found that only nine had sent all the syllabuses related to school mathematics. We could also observe the diversity in the number of subjects related to school

mathematics. Specifically, the average of subjects is 2.5, ranging from 0 to 6. This process allowed the analysis of 31 syllabuses.

The selection procedure of the units of analysis was carried out with a focus on identifying dimensions of knowledge for teaching mathematics. To fulfil this task, two types of units were used that provided greater reliability to the study: syntactic and thematic. Krippendorff (2004) points to the former as natural syntactic elements, loaded with reliability due to their small size. Regarding the second unit, the author emphasises its correspondence with a particular structural definition of content. As an important aspect, he highlights the need to establish a numbering rule to guide the analysis. In our case, we used the rule of presence (Bardin, 1996) as the primary form of interpretation because our objective is to describe a specific type of knowledge; therefore, the presence or absence is significant. We have also opted for this rule instead of a numbering rule because, as Bardín (1996) points out, using the latter implies that all mentions have the same weight in the discourse. Another reason is related to the characteristics of the syllabus (i.e. there are much shorter syllabi than others). Since this issue could be connected with diverse factors, among which we can find the curriculum model adopted in each university, we opted for the presence rule.

Analysis and Categories

To achieve our goal, we will be guided by the content analysis technique. In this sense, we have carried out a knowledge-oriented analysis in which we are interested, which is the subject of this work. Therefore, data analysis combined a concept-driven and data-driven development and was carried out sequentially (Kuckartz, 2019). First, in the deductive phase (concept-driven), the categories of analysis were established, and we used the knowledge areas and subdomains of the MTSK proposed by Carrillo et al. (2018). In particular, the dimensions of mathematical knowledge and pedagogical content knowledge and the knowledge they consider. The content topics proposed by the school curriculum were also taken into account. This process was used for the first organisation of the units of analysis. Second, this initial analysis was completed with an inductive (data-driven) analysis within each pattern recognition category. It should be noted that the same unit of analysis may contain different ideas, which can refer to different categories and subcategories.

RESULTS

In the 15 universities that belong to the CRUCH and teach the career, we found 24 specialisations (related to intellectual disability, learning difficulties, hearing and language, blind people and cognitive development) and two careers that do not have a speciality. In the 26 careers, three syllabuses from two universities do not have any subject related to school mathematics explicitly in their names in their formative itineraries. In these specialisations, it is possible to observe that the average of subjects related to school mathematics is 2.5, ranging from 0 and 6. However, it is important to note that when receiving the syllabus, we realised that some subjects do deal with school mathematics, but in conjunction with other areas of the school curricula (for example, subjects with names such as “Curriculum in elementary education” (Currículo en enseñanza básica) or similar. Therefore, it is possible that in the initial education programmes of the specialisations we did not have access to (three universities), some subjects effectively deal with school mathematics.

Now, regarding the specialisations of the 12 participating universities (22 specialisations or mentions that sent syllabus) and the number of subjects related to school mathematics, we can say that we find the existence of a) a first group that has one subject (eight specialisations; b) a second group with two subjects (four specialisations; c) a third group with three subjects (four specialisations; d) a fourth group with four subjects (five specialisations; and e) a career with six subjects.

Then, we observed two manners of approaching education in school mathematics. Specifically, we refer to a group of specialisations that include courses in which students are prospective special education teachers and elementary education teachers (three specialisations of two universities). In this first group, we saw that the students have a course or more to focus on the specialisation after the shared courses. Regarding the second group, the courses are only for prospective special education teachers (19 specialisations from ten universities).

On the other hand, we find that the objectives of the subjects have two main focuses: school mathematics or mathematics teaching, which are not exclusive, i.e., a syllabus may have both. In particular, referring to the subjects that have school mathematics as their objective, we find two patterns. The first group of subjects focuses on a topic of school mathematics (numbers and operations, geometry, etc.). A second group aims for school mathematics in a general way, intending to know the mathematics of the curriculum or the mathematics that enables students to develop mathematical thought.

Moreover, the subjects that have the purpose of teaching mathematics can be gathered into three groups: diversification, diagnosis, and curriculum. In the first group are the subjects that aim at teaching strategies to diversify mathematics teaching. The second group corresponds to subjects seeking to develop skills to diagnose learning difficulties in mathematics and implement interventions. Finally, the third and last group has aspects of the mathematics curriculum as its purpose. In these, it is possible to find several that specify some content and others that do not.

Following this general analysis of the syllabus, the results of the in-depth analyses and the full syllabus are presented below.

Content topics

Table 1 shows the different topics in the special education career syllabuses found in the analysis. The topics were identified in mentions both when referring to mathematical knowledge and when referring to pedagogical content knowledge. Thus, the Xs show that said topic is mentioned in both subdomains, while the Xs⁺ were only found in mentions relating to mathematical knowledge. On its side, the Xs* indicate that mentions of the topic were only found when they referred to the pedagogical content knowledge.

Table 1

Content Topics Present in Syllabuses.

U	EoM No.	G	A	DyA	NyO	GyM	H
U1	s/e	X					X*
U2	2	X			X	X ⁺	X*
U3 ^x	1	X			X*		X*
	2	X					X*
	1	X					X*
	1	X			X		
U4	1	X	X	X	X	X	
U5	1	X*			X		X*
U6	2	X*				X	X*
U7	1	X			X	X ⁺	X*
U8	1	X*					X*

U	EoM No.	G	A	DyA	NyO	GyM	H
U9	2	X*	X ⁺	X*	X ⁺		X*
U10	2		X ⁺	X ⁺	X ⁺	X ⁺	
U11	1	X*			X*		X*
U12	s/e	X			X		X

Note. U= university; EoM No.=number of specialisations; G=general; A=algebra; DyA=data and chance; GyM=geometry and measurement; H=ability; S/E=without specialisation; X* =allusions to pedagogical content knowledge; X⁺ =allusions to mathematical knowledge. ^x =The specialisations of U3 have been disaggregated because they had a different number of courses, and they are not common to all of them.

Specifically, the results allow us to identify two groups of mentions; a first group in which it is impossible to identify the content to which they are referring and another group in which the content topic is clearly identifiable. In the latter group, five groups of mentions have been identified: 1) mathematical skills, 2) data and chance, 3) geometry and measurement, 4) algebra and 5) numbers. In them, the topic less addressed in the formative syllabuses corresponds to school algebra and data and chance, which are only seen in the specialisations of three universities. On the other hand, the topic with the greatest presence is numbers and operations in the syllabuses in six universities, followed by geometry, with mentions in the syllabuses of five universities.

Thus, if one observes the syllabus and the content topic addressed in it, we can identify three groups of syllabuses. The first group does not specify the mathematical content to be taught in the subject, which is present in two universities (two syllabuses). It should be noted that this group does have mentions that allude to some mathematical skill. The second group corresponds to subjects that deal with several subjects of mathematical content in a single subject in eight universities (15 syllabuses). Finally, the third group brings together syllabuses that either focus on a content area (for example, only numbers and operations or only geometry and are found in two universities) or that focus on a content area by adding some mathematical skill such as problem solving.

On the other hand, we found that two careers have subjects that contain mentions on all the content topics, including the skills mentioned in the curriculum. Likewise, the trend of careers seems to focus on the topics of Numbers and Operation and Geometry and Measurement. However, three

careers do not consider mathematical knowledge related to any content topic explicitly in their syllabuses.

Now, if we look in detail at what concepts the syllabuses deal with on each topic, we can find several concepts related to school mathematics. For example, in the three universities with school algebra content in their syllabuses, we find mentions of patterns, equations, inequalities, functions and algebraic language. For example, in programme 1 of U9, a block of content entitled “Patterns and Algebra” (Patrones y Álgebra) appears, and in it, we find concepts such as regularities and patterns, algebraic expressions, notable products, first-degree equations, first-degree function, and linear inequalities.

On data and chance, and in the two universities that have these topics in their syllabuses, we observe that the syllabuses deal both with statistics and with probability. Specifically, in statistics, we find allusions to variables, populations and samples, data representation (tables and graphs), measures of central tendency and some dispersion ones such as range and quartiles and the evaluation of the data representation process. For example, syllabus 5 of U4 notes that,

The first unit begins by establishing the motivation to perform data collection and the criteria that must be taken into account. Emphasis is placed on dealing with the topic at the classroom level. To answer the questions that motivate the research, the topic of analysis of the different types of representation (tables and types of graphs) is discussed. This is followed by data analysis: measures of central tendency, position and dispersion. The way these topics are presented in the classroom –representations and metaphors, and frequent mistakes that students make– is emphasised.

About probability, the syllabuses present concepts such as chance, random experiments, simple and successive events, estimation, calculation and comparison of probabilities. For example, in programme 1 of U9, we find that prospective teachers should “compare probabilities of different events without calculating them.”

In terms of geometric and measurement topics, the syllabus in which they are present shows contents on relative positions and shapes of one, two and three dimensions. However, in these mentions, some syllabuses deal in a general way with phrases such as “Basic components of school geometry”. In contrast, other syllabuses clearly specify what they refer to as syllabus 2 of U6

which contains phrases such as “Measurement of magnitudes (length, area, volume, angles) with both informal and standardised units.”

In the syllabuses of the six universities that include aspects of the Numbers and Operations topic, it is possible to find content related to the concepts of numbers, decimal number system, additive and multiplicative situations, powers and roots, fractions, proportionality, and operation with integers. For example, in the syllabus 1 of U12, we find contents such as “Decimal number system” and “Types of numbering systems”.

Mathematical knowledge

Regarding the mathematical knowledge in the syllabuses, we found that they allude to two subdomains of the MTSK: knowledge of topics and knowledge of mathematical practice. Concerning the first, knowledge of topics, we observed as the most common in the syllabuses analysed, and it is possible to identify four aspects related to a) definitions, properties or foundations; b) procedures; c) representations; and d) phenomenology. As for the first aspect, the syllabuses allude to the fact that the course would deal with some definition or mathematical concept. For example, it is possible to find mentions such as “Define random experiment from an applied problem” or “The second unit begins with the concept of chance and its historical evolution”. On properties of mathematical concepts, the documents mention that mathematical properties would be dealt with in order to deepen the understanding of the concept in question. For example, we found extracts such as “knowing properties of figures and geometric bodies in the plane and space” or “addition and subtraction operations with integers. Justification of the rules of signs”.

The second group are mentions dealing with procedural elements of mathematics, i.e., they refer to different algorithmic steps. Specifically, mentions such as “conversion between units of measure” or “transform decimal numbers into fractions” were found. Corresponding to the representations, we found mentions that spoke of various registers of representation, concrete, as well as pictorial and symbolic. For example, we found extracts such as “Graphical representations of data: strengths and weaknesses” or “The construction of 2D and 3D figures using a specific material”. On the other hand, phenomenology understood as giving math concepts sense in different contexts and situations, is relegated to a few mentions and appears only when dealing with the arithmetic word problem or the meaning of a specific type of number. For example, we found extracts such as “meanings of the fractions” or “in this

unit, the addition and subtraction operations are treated in depth and simultaneously, understanding them as additive problems; for it, the types of additive problems are determined”.

Regarding the knowledge of mathematical practices, it is possible to observe mentions focused on: a) problem solving; b) argumentation, communication and mathematical language; c) modelling; d) generalisation; and e) visualisation. It is important to note that mentions of mathematical practices have the characteristic of having a greater presence in sections related to the methodology of the subjects or in their evaluation procedures.

Regarding problem solving, phrases such as “it is important to emphasise that problem solving and mathematical discussion (including an appropriate use of mathematical language) are central elements of each class.” However, theoretical elements of this mathematical process were also found in extracts such as “Concept and basic principles of problem solving”. Regarding argumentation and mathematical language, we found extracts such as “arguing the validity of properties, models and procedures with different degrees of mathematical formality, using precise mathematical language to develop in the students the skills of communicating and reasoning, giving meaning and connecting mathematical ideas”. It was also possible to find mentions in which the argumentation and communication were used as methodology or evaluation. In particular, we must point out an evaluation criterion in which the prospective teacher is expected:

Encourage spaces of participation:

-Formulating specific questions that allow the students to account for their reasoning and/or math knowledge to other peers.

-Welcoming students’ interventions and incorporating them into the teaching and learning process.

-Promoting extended and decontextualised discourse in students.

-Providing spaces for mathematical discussion:

-Formulating questions that favour the explanation of the reasoning process of the students to other colleagues.

-Welcoming students’ interventions, incorporating them into the teaching and learning process.

-Promoting extended and decontextualised discourse.

-Enriching the discussion with questions that point to processes and reasoning and consider different levels of complexity.

About modelling, we could identify extracts as “modelling everyday situations using appropriate geometric language”. We must remark that these allusions to modelling are located in one syllabus. On generalisation, the only mention found was “using algebraic language to generalise relationships between numbers.” Finally, on visualisation, mentions such as “developing geometric visualisation skills” were found.

Lastly, there is a group of mentions in four universities that allude to mathematical knowledge, but in them, we cannot infer or identify which specific aspect they refer to. For example, in this group, we found extracts such as “the diagnostic evaluation aims to collect previous information regarding the subject, thus allowing the understanding of terms such as the process of teaching-learning mathematics, alterations, concepts of mathematical skills and knowledge, etc. through developmental questions”. This mention indicates that mathematical knowledge shall be evaluated, but it is not possible to know to which topic it refers.

Table 2 shows a summary of what has been indicated and in which we can observe three trends: two universities that do not present mentions related to mathematical knowledge as conceptualised in this research (empty boxes); a group of two universities that only present general aspects, and the rest of universities that indicate different aspects of mathematical knowledge.

Table 2

Mathematical Knowledge in Syllabuses

U	EoM	P	KoT				KMP			Gral
			D, PyF	R	F	A	G	M	V	
U1	S/E									X
U2	2	X	X			X				X
U3*	1									X
	2									X
	1									X

	1	X	X						
U4	1	X	X	X	X	X			X X
U5	1	X	X	X	X				
U6	2	X	X	X		X	X	X	
U7	1		X					X	X
U8	1								
U9	2	X	X			X			
U10	2	X	X	X			X	X	X
U11	1								
U12	S/E		X		X				X X

Note. U= university; EoM=specialisation; KoT=knowledge of topics; KPM=knowledge of mathematical practices; Gral=general; S/E=no specialisation; P=procedure; D, PyF=definition, property and foundation; R=representations; F=phenomenology; A=argumentation; G=generalization; M=modelling; V=visualisation; RP= problem solving. *=the specialisations of U3 have been disaggregated because they had different number of courses, and they are not common to all of them.

Pedagogical content knowledge

Concerning pedagogical content knowledge, we identified allusions to the three subdomains: knowledge of features of learning mathematics, knowledge of mathematics teaching, and knowledge of mathematics learning standards. About the knowledge of features of learning mathematics, three groups of allusions can be identified: a) a first group that focuses on interpreting and analysing student productions and is expressed in phrases such as: “interpreting school productions to identify common mathematical reasoning”; b) a second group of mentions are those that deal with mistakes and difficulties in learning mathematics. In them, it is possible to identify two clearly identifiable patterns. The first takes a clinical perspective on errors, expressed in phrases such as: “pathologies in mathematical reasoning: etiologies, characteristics, classification”. For its part, the second group treats difficulties and errors from an educational perspective, including even the concept of obstacles, expressed in phrases such as: “difficulties and frequent errors in learning these contents”; and c) finally, a third group related to knowing the different learning trajectories of specific mathematical contents evidenced in phrases such as: “understand the different models for the acquisition and development of the concept of number”.

Regarding the second subdomain, knowledge of mathematics teaching, the results show: a) general aspects of teaching; and other specific aspects such as b) the role of the teacher, c) non-cognitive aspects, and d) the design of interventions. Regarding the general aspects, in these mentions, it is impossible to identify to which teaching aspect it is referred. For example, an excerpt from U4 that was codified under this idea was “Analyse Mathematics Teaching Situations from a Didactic Perspective.” In this sentence we understand that some aspects of teaching shall be analysed.

Within the three specific aspects, the role of the teacher and the non-cognitive aspects were present in 8 syllabuses. The first of these emerged from mentions pointing to teachers’ actions while a student learns mathematics. An example of this was found in U4, and it states that the prospective teacher should “Reflect about and self-evaluate the teaching practice itself and the consequences that his decisions have on the students”. Regarding the non-cognitive aspects, it is possible to observe that the extracts may refer to the prospective teacher or consider them in teaching. For example, on the non-cognitive aspects that the prospective teacher must develop, there are mentions such as “value the importance of contextual variables, fundamentally of educational action, as an instrument for the development of the mathematical skills of young children”. On the other hand, among the examples of the non-cognitive aspects that must be considered in teaching, we find the following extract: “favouring productive effort in the learning of mathematics”.

The aspect we labelled as intervention design is found in all the syllabuses. Specifically, these mentions allude to different aspects such as planning, types of tasks, material resources, evaluation, and collaborative work, but all have the common component of having as their objective that prospective teachers can carry out educational interventions for students who need them. An example of these allusions states the following: “select the methodological strategies and evaluative approaches relevant to the designated socio-educational group”.

In the third subdomain, related to knowledge of learning standards, it was possible to find allusions: a) general, b) to the progression of the curriculum, and c) to its organisation and structure or some specific aspect of these. In the first group, it is not possible to identify any specific content of the mathematics education curriculum, and only general knowledge can be inferred. An example is the following excerpt: “Curricular foundations of mathematics in the first cycle”. The second group, which was labelled as curriculum progression, gathers excerpts that deal with the representation of the

trajectory of a school mathematical content in the Chilean curriculum guideline. For example, an allusion points out that the programme of the subject “considers the curricular progression in the selection of material and didactic activities on the description of positions and 2D and 3D visualisation”. The third and final group deals with knowledge related to the logic that exists and shapes the curriculum. These allusions are made in a general way, pointing out that prospective teachers must have “knowledge and management of the organisation and structure of the Curriculum Guidelines to recognise the know how, know how to be, and know how to perform at different educational levels”. Also, some extracts allude to sections or specific aspects of the document. For example, a subject syllabus indicates that prospective teachers shall develop “fundamental objectives of the school curriculum related to the mathematical contents of the course”.

Table 3 summarises the diversity of pedagogical content knowledge in the formative paths. In addition, it is possible to observe a university syllabus that only mentions the KMT and another university that does not mention the pedagogical content knowledge. The remaining universities present various aspects, with the greatest presence in the syllabuses being the aspects related to designing interventions in the KMT subdomain.

Table 3

Pedagogical Content Knowledge in Syllabuses

U	EoM	KFLS			KMT				KMLS		Gral	
		PE	ED	T	G	RP	NC	DI	G	PC		EO
U1	S/E		X	X			X	X	X			X
U2	2	X	X		X	X	X	X	X		X	
U3*	1							X			X	
	2							X				
	1			X			X	X	X		X	
	1							X	X		X	X
U4	1		X	X	X	X		X	X	X	X	X
U5	1		X	X		X	X	X				
U6	2		X	X				X		X	X	
U7	1			X	X			X		X	X	
U8	1			X				X			X	X
U9	2	X	X		X			X	X			X
U10	2											

U11	1	X		X		X	X	
U12	S/E		X	X		X	X	X

Note. U= university; EoM=specialisation; KFLS =knowledge of learning characteristics; KMT=knowledge of teaching; KMLS=knowledge of teaching standards; Gral=general; S/E=no specialisation; PE= student productions; ED=errors and difficulties; T= learning trajectories; G=general; RP= teacher role; NC= non-cognitive aspects; DI= intervention design; PC= curriculum progression; EO=structure and organisation. *=the specialisations of U3 have been disaggregated because they had different number of courses, and they are not common to all of them.

DISCUSSION AND CONCLUSIONS

This work focuses on the intended curriculum that the schools of education specify in the formative paths of special education teachers. One of the aspects that this work reveals is related to the focus that the training of special education teachers grants to aspects related to the number and operations from a procedural perspective and prioritising definitions over other aspects of mathematics. On the other hand, the pedagogical content knowledge strongly focuses on actions that help diagnose, plan, and assess the performance of students with special educational needs, i.e., what we have indicated as the design of interventions. In this sense, the results suggest that the prioritisation of aspects of mathematics teaching over elements of the discipline may not have the expected results. This is because knowledge about the subject to be taught is fundamental to building pedagogical content knowledge (Agathangelou & Charalambous, 2021). These results must be addressed with caution because we must consider that the intended curriculum is not synonymous with the implemented curriculum. However, the research suggests that when teachers need to plan mathematics teaching, they draw on the knowledge they were exposed to in their initial education (Morris & Hiebert, 2017) and even answer better questions related to teaching (Hiebert et al., 2019). In this sense, specifying the aspects recognised as essential to teaching mathematics in the syllabuses (e.g. Carrillo et al., 2018) will help clarify what aspects should be taught to the beginning teachers' educators.

Currently, in the Chilean educational system, several professionals who accompany the learning processes of students who need specialised support co-exist. Specifically, among them are teachers of different subjects and special education teachers. In this sense, the results suggest that bringing to the classroom co-teaching actions that generate learning poses a challenge since the knowledge of teachers is a key element to achieving successful co-teaching

experiences (Scruggs et al., 2007). As shown in this work, prospective special education teachers are not getting this kind of education.

On the other hand, the results shown here may explain what Rojas et al. (2021) stated regarding the limited opportunities to learn mathematics provided by special education teachers, as well as the low perception that this group has at the end of its initial education (Centro de Estudios Mineduc, 2020). In this sense, we believe this may also be due to the manuals generated in this regard. In particular, we refer to those that the manuals, both private (e.g., Martínez-Montero, 2010) and public initiatives (e.g., Unidad de Educación Especial y Fundación Down 21-Chile, 2017), focus on numbers and operations. This perspective may have led to a limitation in understanding what mathematics should be taught and what can be learned by students with special educational needs. However, research has shown hopeful signs regarding what this group of students can do in other topics of school mathematics (Gil-Clemente & Cogolludo-Agustín, 2019; López-Mojica & Ojeda, 2015).

Likewise, our results highlight a possible cause of the difficulties in implementing Decree 83 (Ministerio de Educación, 2015) and regulating inclusive education in Chile. This document has tensioned the role and identity of special education teachers since they must no longer only perform specialised work with some students but diversify teaching and teach whole classes, where disciplinary knowledge becomes a critical node (Inostroza, 2019). This fact is in addition to those indicated by Calle (2020) regarding the implementation of Decree 83.

In short, this analysis of the syllabuses of subjects shows the great variability in the education of mathematical knowledge for teaching special education careers, which may owe to the vagueness with which it appears in the documents that rule these aspects. In particular, these results allow us to visualise the different interpretations that Chilean schools of education that form special education teachers have made regarding the fact that prospective teachers must know the curriculum, identifying the concepts and central skills. In this sense, the results presented here can serve as input for teacher educators in mathematics education and special education to discuss what aspects should be included in the education of prospective special education teachers. This fact is relevant because having unclear standards and guidelines on what is expected of prospective teachers regarding mathematics teaching, only collaborative work between both areas of knowledge may contribute to improving initial education.

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AUTHORS’ CONTRIBUTIONS STATEMENTS

Both authors actively participated in the all-research process, reviewing and approving the final version of the text.

DATA AVAILABILITY STATEMENT

The data are kept in the authors’ archives. The corresponding author is in charge of their custody and consultation by the interested parties.

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