ISSN: 2178-7727
DOI: 10.17648/acta.scientiae.7465

Correct-Incorrect Proportional Reasoning Strategies on the Proportional Problems and SOLO Taxonomy

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Received for publication 17 Nov. 2022. Accepted after review 21 Jul. 2023
Designated editor: Claudia Lisete Oliveira Groenwald

ABSTRACT

Background: Proportional reasoning strategies are needed in solving comparison problems. Previous studies are still being researched on proportional problems in general. **Objectives**: This study aims to describe students' forms of proportional reasoning strategies when solving proportional problems and their SOLO taxonomy level. **Design**: A descriptive approach with a qualitative type. **Setting and Participants**: 103 students at10th grade who can communicate their answers clearly and fluently during interviews. **Data collection and analysis**: Data is collected through comparative problem tests and interviews. The data were analysed based on the proportional reasoning strategy and the SOLO taxonomy level. **Results**: The correct strategy used is the proportional reasoning strategy for the inverse proportion problem. Cross multiplication strategy and systematic strategy for direct proportion problem. The incorrect strategies include non-proportional and cross-multiplication (intuitive and additive) strategies. Students' response levels based on the SOLO taxonomy: Students who can answer all the problems correctly are at the relational level, students who only partially answer correctly are at the pre-structural and relational levels, and students who cannot answer all the problems given are at the uni-structural level. **Conclusions**: The results of this study describe the forms of students' proportional reasoning strategies followed by the SOLO taxonomy level so that in the future, these results can be followed up by other researchers to be able to provide solutions related to the incorrect strategies used by students.

Keywords: Proportional Reasoning; Reasoning Strategy; Solo Taxonomy; Proportional Problem.

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Estratégia de Raciocínio Proporcional Verdadeiro-Falso em Problemas Comparativos e Taxonomia SOLO

RESUMO

Contexto: Estratégias de raciocínio proporcional são necessárias na resolução de problemas de comparação. A pesquisa anterior ainda está pesquisando sobre questões proporcionais em geral. **Objetivos**: Este estudo tem como objetivo descrever as formas de estratégias de raciocínio proporcional dos alunos na resolução de problemas proporcionais e seu nível de taxonomia SOLO. **Design**: Abordagem descritiva de tipo qualitativo. **Ambiente e participantes**: 103 alunos do 10º ano que foram capazes de comunicar suas respostas de forma clara e fluente durante a entrevista. **Coleta e análise de dados**: Os dados foram coletados por meio de testes comparativos de problemas e entrevistas. Os dados foram analisados com base na estratégia de raciocínio proporcional e no nível de taxonomia SOLO. **Resultados**: A estratégia correta utilizada é uma estratégia de raciocínio proporcional para problemas de comparação de valores inversos. Estratégia de multiplicação cruzada e estratégia sistemática para problemas de comparação de valores. As estratégias incorretas incluem: estratégias não proporcionais e multiplicação cruzada (intuitiva e aditiva). Níveis de resposta do aluno com base na taxonomia SOLO: os alunos que conseguem responder todas as perguntas corretamente estão no nível relacional, os alunos que respondem apenas parcialmente corretamente estão nos níveis pré-estrutural e relacional e os alunos que não conseguem responder a todas as perguntas dadas estão no nível universitário. nível estrutural. **Conclusões**: Os resultados deste estudo ilustram a forma de estratégias de raciocínio proporcional dos alunos seguidas pelo nível de taxonomia SOLO para que no futuro esses resultados possam ser acompanhados por outros pesquisadores para poder fornecer soluções relacionadas às estratégias erradas usadas pelos alunos.

Palavras-chave: raciocínio proporcional; estratégia de raciocínio; taxonomia SOLO; problema de comparação.

INTRODUCTION

Reasoning that plays an important role in learning mathematics and its application in everyday life is proportional reasoning (Beckmann & Izsák, 2015; Cebola & Brocardo, 2021; Hilton et al., 2016; Im & Jitendra, 2020; Kontogianni & Tatsis, 2019; Lundberg & Kilhamn, 2018; Vanluydt et al., 2021; Weiland et al., 2021). Proportional reasoning is needed in science (physics, chemistry) (Castillo & Fernandez, 2022) and other mathematics at all levels of compulsory education (Kontogianni & Tatsis, 2019; Pişkin Tunç & Çakıroğlu, 2022). Proportional reasoning refers to the ability to use ratios in situations that involve comparisons of quantities (Doyle et al., 2016; Hilton et al., 2016) and is the basis for an understanding of algebra and the transition from informal to formal mathematical thinking (Doyle et al., 2016). Then, proportional reasoning can also be interpreted as a person's ability to understand, construct, and use multiplicative relationships between two quantities from the same or different categories (Dooren et al., 2009). This proportional reasoning is often used by someone to see the relationship between quantities. For example, if we want to estimate the amount of gasoline needed when traveling. The relationship between the amount of gasoline is associated with the distance traveled. In the field of mathematics education, proportional reasoning is developed through the concept of comparison.

The concept of comparison (ratio and proportion) is important in learning at school (Andini & Jupri, 2017; Artut & Pelen, 2015; Buforn et al., 2022; Dougherty et al., 2016; Perumal & Zamri, 2022). This's because comparison topic is the foundation for studying other mathematical material (Dougherty et al., 2016; Vanluydt et al., 2021; Weiland et al., 2021). Other mathematical materials that require comparative concepts are algebra, geometry, statistics, and so on (Beckmann & Izsák, 2015; Vanluydt et al., 2021). Apart from being important in learning mathematics, this comparative concept is also useful in everyday life (Perumal & Zamri, 2022; Phuong & Loc, 2020). Therefore, the concept of comparison is a concern for teachers to be able to convey this material well to students at school.

The concept of comparison is known in schools is direct proportion and inverse proportion. Arican & Kiymaz (2022) dan Johar et al., (2018) state that there are two types of the proportional relationship between two quantities, namely direct proportion and inverse proportion. This topics have been given since elementary school (Andini & Jupri, 2017; Irfan et al., 2020) and continued in junior high school (Bintara & Suhendra, 2021; Castillo & Fernandez, 2022; Irfan et al., 2020). Because of direct proportion and inverse proportion have a proportional relationship, students need to use proportional reasoning strategies to understand and to solve direct proportion and inverse proportion problems.

Given the importance of proportional reasoning and the concept of comparison, it is necessary to study how students' proportional reasoning strategies solve comparison problems. Several researchers have conducted research related to proportional reasoning (Ahl, 2019; Arican, 2019; Arican & Kiymaz, 2022; Buforn et al., 2022; Burgos & Godino, 2022; Cabero-Fayos et al., 2020; Castillo & Fernandez, 2022; Glassmeyer et al., 2021; Karli & Yildiz, 2022; Kontogianni & Tatsis, 2019; Perumal & Zamri, 2022; Pişkin Tunç & Çakıroğlu, 2022; Vanluydt et al., 2021; Weiland et al., 2021). Ahl (2019) designed a research-based detection test to see students' initial understanding of proportional reasoning. The design of this assignment was also carried out by Burgos & Godino (2022) where the focus of this research was on how prospective elementary education teacher students design assignments that involve proportional and algebraic reasoning. Arican (2019) and Arican & Kiymaz (2022) focus on examining the understanding of prospective teacher students and their ability to distinguish proportional relationships from non-proportional relationships. Meanwhile, research conducted by Buforn et al., (2022) aims to characterize how teachers recognize student reasoning. Apart from teachers, there is also research related to student characterization as conducted by Castillo (2022), namely characterizing how high school students solve ratio problems with proportional reasoning. Then, Cabero-Fayos et al., (2020) examines understanding and the strategies used by prospective teacher students in solving the problem of comparison of values with proportional reasoning. Karli & Yildiz (2022) examined the incorrect strategies developed by high school students in solving proportional reasoning problems.

Based on previous research related to this proportional reasoning strategy conducted by Cabero-Fayos et al., (2020) it was only limited to the problem of turning around values. In addition, recent research by Castillo (2022) characterizes how junior and senior high school students solve comparison problems, especially the problem of comparing ratios with three quantities. However, the problem of comparison of values and comparisons of value returns has not been studied by other researchers. Meanwhile, in learning at school, students are often confused about the difference between the problem of comparison of values or turning values. Such as the results of Bintara & Suhendra (2021) which states that students experience problems when solving problems of comparing values and turning values.

In Karli & Yildiz's research (2022) it only focuses on wrong strategies developed by high school students in solving proportional reasoning problems. The correct strategy of proportional reasoning done by students has not been studied. Following up on some of the previous research that has been mentioned, it is important to examine how students' proportional reasoning strategies are right and wrong through the problem of direct proportion and invers proportion.

The results of this student work will also be identified with the SOLO taxonomy framework. Based on the existing literature, there are several other taxonomies such as Bloom, Anderson, Fink, and Dettmer (Ari, 2013; Koçyiğit & Moralı, 2020). However, the SOLO taxonomy is considered the most appropriate in determining the level of student completion and its categorization.

This research has a high urgency because through this level of proportional reasoning ability it will provide a clear picture of what strategies students use in solving comparison problems. Therefore, teachers can provide instructions and interventions in learning to minimize mistakes and potential failures faced by students. This failure becomes a benchmark for the extent to which students understand a particular material.

Based on the problems above, it is important to examine students' proportional reasoning strategies in solving problems of comparison of values and comparisons of values through the SOLO taxonomy leveling. Therefore, the formulation of the problem of this study is "What is the proportional reasoning strategy used by students in solving the problem of comparison of values and comparison of values based on the SOLO taxonomy?".

PROPORTIONAL REASONING STRATEGIES

Proportional reasoning refers to the ability to use ratios in situations involving quantity comparisons (Doyle et al., 2016). In mathematical terms, this proportional reasoning includes a comparison of ratios between quantities based on the formula a/b=c/d (Thurn et al., 2022; Tjoe & de la Torre, 2014). This proportional reasoning can be identified through the provision of comparison problems.

When solving proportion problems, students use various proportional reasoning strategies. This is in accordance with the statement of Karli & Yildiz (2022) that students develop right and wrong strategies when solving problems that involve proportional reasoning. Several researchers wrote various kinds of proportional reasoning strategies. Avcu & Doğan (2014) wrote that most of the solving strategies that are often used by students are product algorithms. The results of this study are in line with research conducted by Arican (2019), Artut & Pelen (2015), and Ayan & Isiksal-Bostan (2019) which state that the majority of students use the cross-multiplication strategy in solving proportion problems. However, in Ayan & Isiksal-Bostan (2019) study, two new strategies are provided, namely reasonable proportion and questionable proportion strategies. With this reasonable proportion strategy, students are able to see the relationship or relationship between one information and another in the problem. While the questionable proportion strategy is when students directly use the cross-multiplication method.

The reasonable proportion strategy in the results of Ayan & Isiksal-Bostan (2019) research is in accordance with the strategy carried out by Cabero-

Fayos et al., (2020), namely the proportion formula strategy (correct strategy category). In this proportion formula strategy students are able to make equations by stating the correct relationship between problem quantities. Furthermore, Cabero-Fayos et al., (2020) classifies several incorrect proportional reasoning strategies and correct reasoning strategies. The wrong reasoning strategy consists of no answer, intuitive, additive, proportion of attempts, and other errors. Meanwhile, the correct reasoning strategy consists of proportion formulas, proportional reasoning, algebra, and correct other.

Cabero-Fayos et al., (2020) research focuses on student teacher strategies. On the other hand, Karli & Yildiz (2022) focuses on wrong strategies developed by high school students in solving proportional reasoning problems, including: additive relationships, ignoring data, using numbers instead of content, giving emotional responses, and failing to identify non- proportional. This additive strategy is also the strategy most used by students (Artut & Pelen, 2015).

Artut & Pelen's (2015) research yielded seven strategies used by students in solving proportional problems. These seven strategies are adapted to the type of problem. For the missing value problem, the strategies used are factor of change, unit rate, build-up, cross multiplication, and evidence of proportional reasoning. As for the numerical comparison problem, all strategies are used on the missing value problem except cross multiplication is replaced by a common factor or multiple strategy.

SOLO TAXONOMY

The SOLO taxonomy was first introduced by John B. Biggs (1982) which aims to classify students' academic levels (Adeniji et al., 2022; Dole et al., 2015; Karli & Yildiz, 2022; Li et al., 2022). When viewed from the results of student work on a given problem, the SOLO taxonomy classifies students' understanding levels into 5 levels from lowest to highest, namely: pre-structural level, uni-structural level, multi-structural level, relational level, and extended abstract level (Biggs & Collis, 1982).

The first three levels (lower-order learning) in the SOLO taxonomy are referred to as the pre-structural level, uni-structural level, and multi-structural level, where students process quantitatively (Adeniji et al., 2022; Özdemir & Yildiz*, 2015). Whereas for the next two levels, namely the relational level, and the extended abstract level, these enter a higher level where students understand the questions qualitatively and have deeper learning (Adeniji et al., 2022; Özdemir & Yildiz*, 2015).

This level in the SOLO taxonomy provides an understanding of hierarchical complexity in the structure of student responses (Adeniji, Baker, & Schmude, 2022). However, the SOLO taxonomy is not always linear. This is meant by that cycles that occur in stages can occur repeatedly and spiral (Adeniji et al., 2022). The SOLO level of students in each topic can vary. There are some topics (material) that are easy to understand while others are not.

Several studies in mathematics examine from various perspectives related to the SOLO taxonomy and experts agree that this SOLO taxonomy is a basic, practical, and operational theory, which can be used to assess problem solving and student knowledge (Li et al., 2022). Departing from this statement, this study uses the SOLO taxonomy to identify student solutions that require proportional reasoning and categorize student levels based on students' proportional reasoning strategies.

METHODOLOGY

Research Design

This research was designed using a qualitative approach with a descriptive research type. The choice of approach and type of research is based on researchers' findings regarding the proportional reasoning strategies used by students in solving comparison problems. The researcher gave research instruments in the form of two comparison problems, namely the problem of direct proportion and inverse proportion to 10^{th} grade students in Malang, Indonesia. Then, the results of the student's work were analyzed based on the student's work process which was adjusted to the alternative answers prepared by the researcher. Researchers analyzed various strategies used by students in solving proportional problems, both correct and incorrect strategies. To determine the level of student responses based on SOLO taxonomy, the researcher determined three students with the following criteria: 1) students could answer all correctly, 2) students only answered partially correctly, and 3) students who answered all incorrectly.

Sources of data collected by researchers are not only the results of student work, but also recorded interviews to obtain accurate results. After collecting the data, the researcher then analyzed the findings based on student work and the SOLO taxonomic indicators that had been set by the researcher in Table 2. After doing the analysis, the researcher drew conclusions in the form of correct and incorrect strategies of students' proportional reasoning when solving problems of direct proportion and inverse proportion and its SOLO taxonomy levels.

The research design carried out by researchers is in accordance with the characteristics of qualitative research written by Creswell (2012), namely: 1) Scientific environment (natural setting) where researchers collect data in class with research subjects solving given problems. 2) The researcher as the main instrument where the researcher himself collects data through the results of student work which is documented using a camera along with interviews with research subjects. 3) Various data sources where researchers collect data from various sources such as recordings, observations, and interviews. The data obtained is then studied by giving meaning and processing it into categories. 4) Data analysis. Researchers build categories or topics through analysis of previously collected data. 5) Drawing conclusions. This research process takes place dynamically where all stages may change after the researcher goes into the field and collects data.

Participants

This research involved 103 10^{th} grade students in Malang, namely class X-1, class X-4, and class X-7. The selection of class X students was based on a preliminary study that had been carried out earlier that there were indications that students carried out various correct and incorrect strategies in carrying out proportional reasoning on comparative problems.

The class selection was based on the consideration of the class teacher that the students selected were students with good mathematical abilities. It aims to see how students' strategies are right and wrong in solving comparison problems. Students who participated in this study were students who had studied comparative material.

To categorize the level of student response to this proportional problem, 3 students were selected who represented the condition of student answers, namely 1) students who could answer all questions correctly, 2) students who could only answer some of the problems correctly, 3) students who could not answer all problem properly. These three students were also selected based on their ability to communicate fluently and clearly when conducting interviews in order to obtain accurate data. The three students were named with S1 as the first subject, S2 as the second subject, and S3 as the third subject.

Data Collection and Data Analysis

The researcher gave two proportion problems (inverse and direct) to 10th grade students in Malang. The problems given can be seen in Table 1 below.

Table 1

Proportional Problems

Inverse Proportion	Direct Proportion
Ade, Putra, and Sandi can complete a job alone in 20 days, 30 days, and 60 days respectively. How many days will Ade take to complete the work if Putra and Sandi help him every 3 days? With the order of the first 2 days Ade worked alone.	The price for 14 packets of flour, weighing 750 grams each, is IDR 189,000.00. How much is 25 packets of flour weighing 1kg each?

Based on the results of student work, the researcher will analyze student answers with the rubric of alternative answers that the researcher has prepared. From the results of this analysis will be obtained right and wrong strategies for each problem.

To determine the level of student response, researchers used the SOLO taxonomy rubric as shown in Table 2 below.

Table 2

SOLO Taxonomy Levels

Quantitative Increase and Surface Learning			Qualitative Increase and Deep Learning	
Pre-Structural	**Uni-Structural**	**Multi-Structural**	**Relational**	**Abstract**
Students have not been able to understand or have only a little knowledge	Students understand the problem given but still cannot solve it correctly. Students only focus on one piece of information.	Students can identify more than one piece of information used in the problem but cannot	Students can state the exact relationship between the information used in expressing	Students can reason and make generalizations

related to comparative material. The results of student work do not match the problem.	Students cannot state the relationship between the quantity/information questions that lead to answers.	connect the information. Students are unable to provide logical solutions and fail to complete them.	comparisons. Students cannot make generalizations

Researchers collected data and analyzed student work based on Table 2. From the results of the analysis, the research subjects were determined based on correct, partial, and incorrect answers. Researchers also consider the fluency of student communication and student willingness to be used as research subjects. Researchers conducted semi-structured interviews with students to dig up more information, confirm and deepen students' thinking processes related to their proportional reasoning strategies.

RESULTS

Overall, 37 students were able to answer all the problems correctly, 48 students were only able to answer some of the problems correctly, and 18 students were able to answer all the problems incorrectly. Of the 48 students who were only able to answer part of the problem correctly, there were 5 students who were only correct on the first problem and 43 students who were only correct on the second problem.

For the first problem (inverse proportion), there were 42 students or 40.78% who answered correctly and 61 students or 59.22% who answered incorrectly. Whereas for the second problem (direct proportion) there were 80 students or 77.67% answered correctly and 23 students or 22.33% answered incorrectly.

The results studied from this study are descriptions of students' proportional reasoning strategies in solving the problem of direct and inverse proportion and students' response levels based on the SOLO taxonomy. The research data was obtained from the results of student work during tests and interviews. For data related to the strategies used by students in doing this proportional reasoning based on the overall existing data. In a sense, the data is from the results of identification on 103 10^{th} grade students in Malang. Then, for student response data based on the SOLO taxonomy, it was obtained from three research subjects where the first subject (correct answer in each problem) was called S1, the second subject (one correct answer) was called S2, and the third subject (wrong answer in every problem) was called S3.

The following describes students' proportional reasoning strategies when solving the problem of direct proportion and inverse proportion and levels in the SOLO taxonomy.

Correct Strategies for Inverse Proportion Problem

From the correct answers to problem 1 (inverse proportion), the majority of students use a proportional reasoning strategy that involves calculating fractions. In this proportional reasoning strategy students are able to solve proportional problems by looking at the relationship between the quantities in the problem. With this strategy, students involve calculating fractions as part of a comparative quantity.

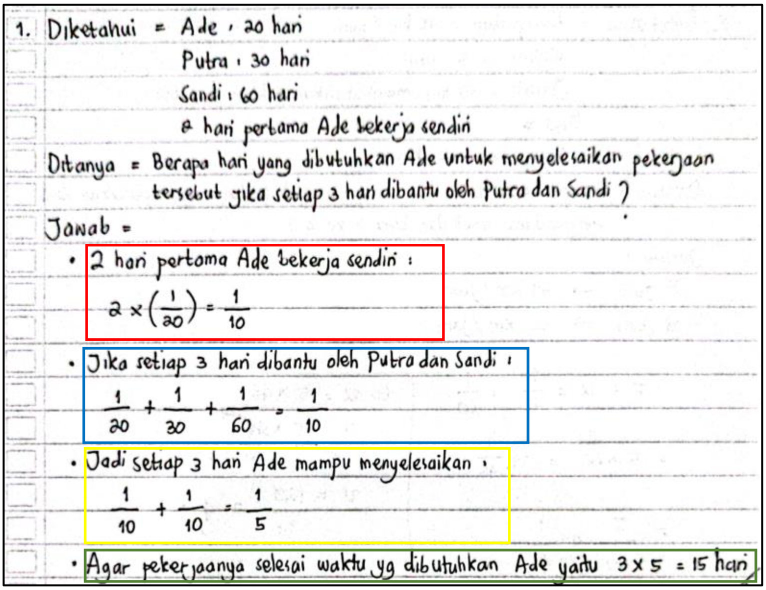
Proportional reasoning strategy

In inverse proportion problem, students are asked to determine the length of a worker's day in completing a job if assisted by a co-worker. To solve this problem, students need to know the relationship between the number of parts of work and the length (time) needed by workers. In this case, the first step taken by students is to determine the number of each part of the work that can be completed in one day. An example of student work that represents a form of proportional reasoning strategy can be seen in Figure 1 below.

Based on Figure 1, students write that if Ade is able to complete a job within 20 days then in 1 day Ade can complete $\frac{1}{20}$ of a part (red box). Because, for the first two days Ade worked alone, $2 \times \frac{1}{20} = \frac{1}{10}$ of the work was completed. Then, according to the narrative of the problem that Ade will be assisted by his two friends every 3 days, on the third day Ade can complete $\frac{1}{20} + \frac{1}{30} + \frac{1}{60} = \frac{1}{10}$ part of work (blue box). So, every three days Ade can complete $\frac{1}{10} + \frac{1}{10} = \frac{1}{5}$ part of the work (yellow box). To get one complete part (the work can be declared complete), the student performs the multiplication operation, namely $3 \times 5 = 15$ days (green box).

Figure 1

The results of students' work that represent the correct strategy (proportional reasoning) on the inverse proportion problem



1. Diketahui = Ade : 20 hari
Putra : 30 hari
Sandi : 60 hari
2 hari pertama Ade bekerja sendiri
Ditanya = Berapa hari yang dibutuhkan Ade untuk menyelesaikan pekerjaan tersebut jika setiap 3 hari dibantu oleh Putra dan Sandi ?
Jawab =
- 2 hari pertama Ade bekerja sendiri :
$2 \times \left(\frac{1}{20}\right) = \frac{1}{10}$
- Jika setiap 3 hari dibantu oleh Putra dan Sandi :
$\frac{1}{20} + \frac{1}{30} + \frac{1}{60} = \frac{1}{10}$
- Jadi setiap 3 hari Ade mampu menyelesaikan :
$\frac{1}{10} + \frac{1}{10} = \frac{1}{5}$
- Agar pekerjaanya selesai waktu yg dibutuhkan Ade yaitu $3 \times 5 = 15$ hari

From the student's working process on the problem of inverse proportion, in the early stages when students gave statements that Ade could complete $\frac{1}{20}$parts in a day, it showed that students had done proportional reasoning where students were able to see the relationship between the length of time needed and the number of parts of the work. The relationship that occurs in this problem is an inverse proportion. The strategy used by these students is a proportional reasoning strategy. In addition, students are also able to perform fractional calculation operations well. Students are also able to provide opinions on the final answer well that if every three days the work is completed as much as $\frac{1}{5}$ of the part, it takes 15 days to complete the work.

Incorrect Strategies for Direct Proportion Problem

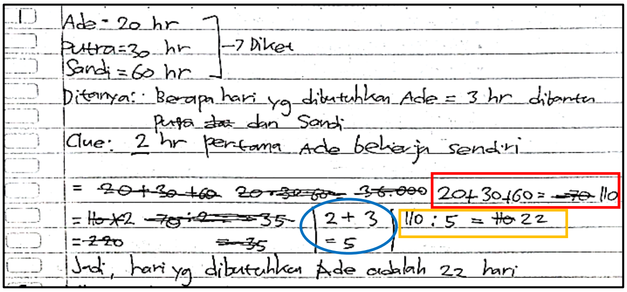
From incorrect answers, students use a counting strategy using all the values/number in the question (without involving the concept of comparison) and can't answer at all question by well.

Non proportional strategy

A non-proportional strategy is a strategy in which students do not use the concept of comparison in solving proportional problems. Students use all the numbers in the problem with addition and division operations. Student work that represents a form of non-proportional strategy can be seen in Figure 2 below.

Figure 2

The results of students' work that represent the incorret strategy (non-proportional) on the inverse proportion problem



Based on Figure 2 above, students add up all the initial information on the questions related to the time the work was completed by Ade, Putra, and Sandi, namely 20 + 30 + 60 = 110 (red box). Then the students added up the numbers 2 + 3 = 5 (blue circle) because the information on the questions stated that Ade worked alone on the first two days and on the third day he was assisted by Putra and Sandi. To get the final answer, students carry out the division operation from the first addition to the second addition, which is 110 : 5 = 22.

From the results of student work it can be seen that students only use all the information about the questions without knowing the meaning of any information provided. In other words, students have not been able to solve proportional problems with the concept of proportion. This is shown by the work of students who carry out the operation of calculating addition and division of the numbers in the problem.

Correct Strategies for Direct Proportion Problem

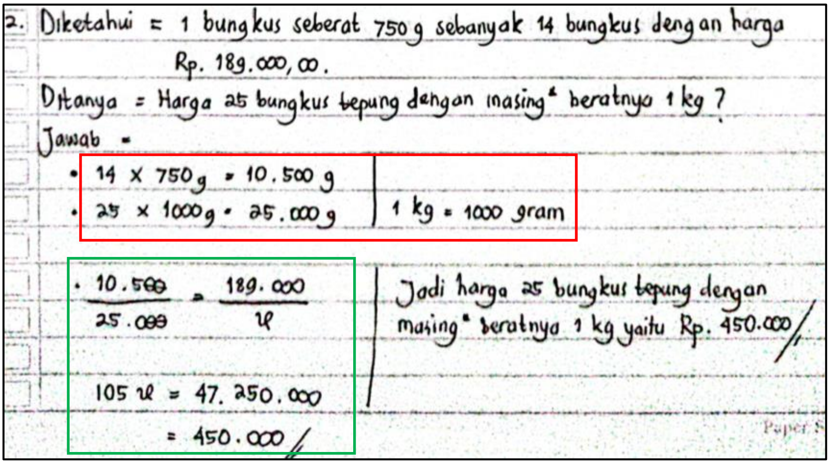
From the correct answers in problem 2 (direct proportion), most students use the cross multiplication strategy. However, there are those who use systematic calculations.

Cross multiplication strategy

In this cross-multiplication strategy students write in the form $\frac{a}{b} = \frac{c}{d}$ such that $bc = ad$. The following is one of the student's work with the cross multiplication strategy can be seen in Figure 3.

Figure 3

The results of students' work that represent the correct strategy (cross multiplication) on the direct proportion problem



2. Diketahui = 1 bungkus seberat 750 g sebanyak 14 bungkus dengan harga Rp. 189.000,00.

Ditanya = Harga 25 bungkus tepung dengan masing² beratnya 1 kg ?

Jawab =

• 14 × 750 g = 10.500 g

• 25 × 1000 g = 25.000 g

1 kg = 1000 gram

• $\frac{10.500}{25.000} = \frac{189.000}{x}$

105 x = 47.250.000

= 450.000

Jadi harga 25 bungkus tepung dengan masing² beratnya 1 kg yaitu Rp. 450.000

In Figure 3 above, the initial stages of students doing multiplication calculations between the amount of flour and the weight of flour so that the overall weight for 14 packets of flour, each weighing 750 grams, is 10,500 grams (red box). Because what was being asked was the price of 25 packets of flour weighing 1 kg each, the students also did the calculations to determine the total weight of the flour in question, which was 25000 grams (red box). In the next stage, students begin to write a comparison form of value, namely $\frac{10500}{25000} = \frac{189000}{x}$ (green box). From this form, students state that if flour weighing 10500 grams costs Rp. 189000.00 then if 25000 grams the price is x. To determine the value of x, students carry out a cross multiplication strategy from the comparison form that has been written. The result of this multiplication is $105x = 47250000$. Therefore, the value $x = 450000$ (green box).

Based on the completion steps carried out by students using the cross multiplication strategy, students determine the equivalence of the price value of the total weight of flour. In determining the total weight students also convert the weight from grams to kilograms. After obtaining value equality, students begin to write it in the form of a comparison of value. Through interviews students were able to explain that the heavier the flour mass, the more expensive the price. Therefore, students use the concept of direct proportion in solving the problem.

Systematic strategy

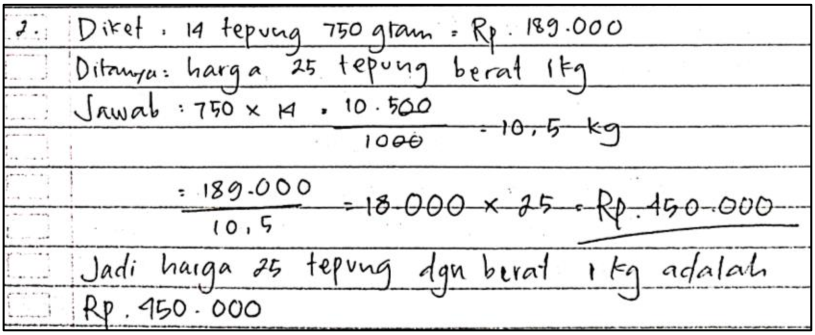
In addition to using the cross-multiplication strategy, students carry out calculations systematically. The following is one student work that represents a systematic calculation in Figure 4.

From Figure 4, the first step is for students to calculate the total weight of flour with a weight of 750 grams each for 14 packets of flour, namely $750 \times 14 = 10500$. The total weight is converted into kilograms so students divide 10500 by 1000 to produce a weight of 10.5 kg. The next step is for students to determine the price per 1 kg of flour from the information that 10.5 kg of flour costs Rp. 189,000. Therefore, students get the price for 1 pack of flour weighing 1 kg, which is Rp. 1 kg each so students multiply 18000 by 25 so that we get 450000.

The process carried out by these students is also a correct strategy because the procedures given are logically acceptable. Therefore, the strategy carried out by these students was categorized as a systematic strategy by determining the unit price of 1 packet of flour weighing 1 kg.

Figure 4

The results of student work that represent the correct strategy (systematic) on the direct proportion problem



Incorrect Strategies for Direct Proportion Problem

In this comparison problem, there were 22.33% of students who answered incorrectly or there were 23 students out of 103 students. For wrong answers, namely not answering at all, using the wrong concept, calculating all existing values without regard to proportional reasoning, other errors, namely wrong calculations.

Non proportional strategy

Non-proportional strategy is a strategy in which students do not use the concept of comparison in solving proportional problems. Students use all the numbers in the problem with addition, multiplication, and division operations. Student work that represents a form of non-proportional strategy can be seen in Figure 5 below.

Based on Figure 5, students calculate all the values in the questions without paying attention to the interrelationship of the information about the questions. At the beginning the students wrote 14+14=28 and 189+189=378. Students also do not understand the purpose of writing. Students then clarify their answers later by writing 189000 : 14=13500. Students state that the price is the price for 1 pack of flour. To determine the price of 25 packets of flour, students multiply 13500 by 25 to obtain 13500 × 25 = 337000. Students assume that the weight of each flour in 1 pack is the same. Even though the initial information stated that for 14 packets of flour at a price of 189000 each weighed 750 grams. While what is asked in the question is the price of 25 packets of flour weighing 1 kg each.

Figure 5

The results of students' work that represent the incorrect strategy (non-proportional) on the direct proportion problem

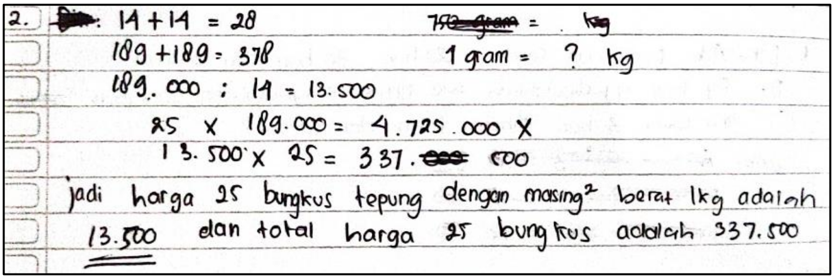
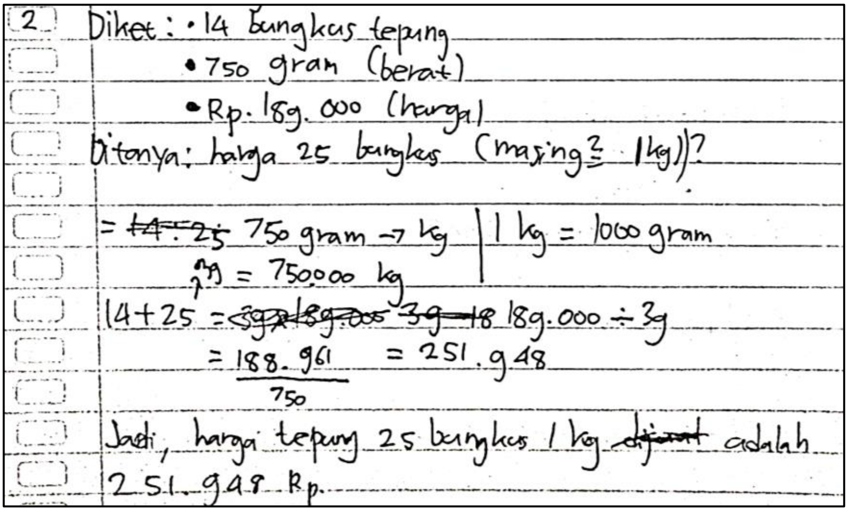

2. 14 + 14 = 28 ~~750 gram~~ = kg
189 + 189 = 378 1 gram = ? kg
189.000 : 14 = 13.500
25 x 189.000 = 4.725.000 X
13.500 x 25 = 337.~~000~~ 500
jadi harga 25 bungkus tepung dengan masing² berat 1kg adalah
13.500 dan total harga 25 bungkus adalah 337.500

Figure 6

The results of students' work that represent other incorrect strategies on the direct proportion problem


2 Diket: • 14 bungkus tepung
• 750 gram (berat)
• Rp. 189.000 (harga)
Ditanya: harga 25 bungkus (masing² 1kg)?
= ~~14 : 25~~ 750 gram → kg | 1 kg = 1000 gram
?g = 750000 kg
14+25 = ~~39 x 189.000~~ ~~39 - 18~~ 189.000 ÷ 39
= 188.961 / 750 = 251.948
Jadi, harga tepung 25 bungkus 1 kg ~~dijawab~~ adalah
251.948 Rp.

Other student work that only involves all the values in the questions without regard to quantity or the interrelationship of information between questions can be seen in Figure 6.

In Figure 6 above, students convert weight from 750 grams to kg. Students wrote that 1 kg = 1000 grams, but when students converted from 750 grams to 750000 kg. The results of this conversion are not included in the calculation. However, from the results of these calculations it can be seen that students have not been able to do their reasoning properly. After that, students immediately add up 14 and 25 where 14 is the number of packets of flour known in the question and 25 is the number of packets of flour asked in the problem. Students add up the two quantities with the aim of calculating the total number of packets of flour. Then the students calculated the division of 189000 by 39 which resulted in 188961. The results obtained from this division were wrong and the procedure for solving them did not make sense. Then from these results students divide 188961 by 750 to produce 251948. The working procedure carried out by these students does not make sense and students only perform arbitrary calculations by entering all known number values in the problem.

SOLO Taxonomy Level on S1

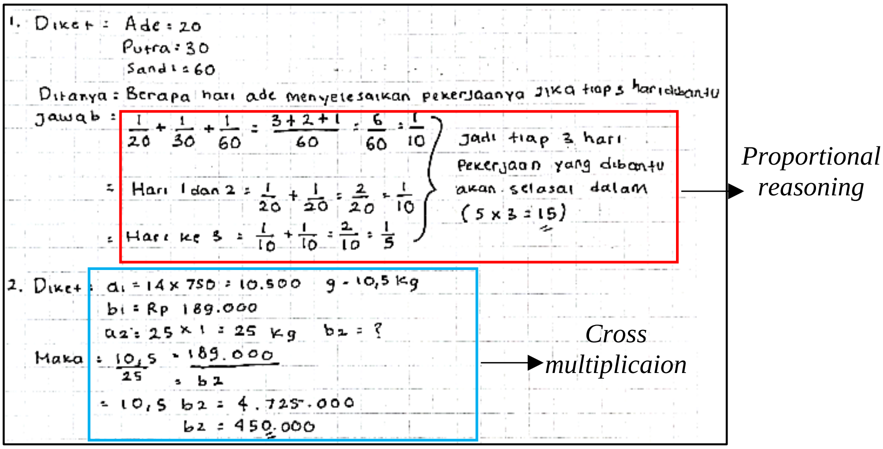
The first subject can solve the problem of direct proportion and inverse proportion correctly. Following are the results of S1's work for numbers 1 and 2 as shown in Figure 7.

In Figure 7, S1 can write the correct solution for each problem. In the process of working on number 1, S1 uses a proportional reasoning strategy. It can be seen that S1 uses the information in the problem in such a way that S1 writes down the number of parts of the work that can be completed if done together using the fraction "$\frac{1}{10}+\frac{1}{10}=\frac{2}{10}=\frac{1}{5}$". When interviewed, S1 can explained that for every 3 days, $\frac{1}{5}$ of the work is completed, so to get the full part it takes 15 days to complete it together with the condition that the first two days are Ade working alone.

In problem number 2, S1 solves it with the concept of equivalent comparisons where S1 solves it using a comparison formula with a cross multiplication strategy. This can be seen when S1 writes the formula "$\frac{10,5}{25}=\frac{189000}{b_2}$". If it is related to the research results from Cabero-Fayos et al., (2020), the proportional reasoning strategy used by S1 is included in the proportion formula category. This proportion formula is when students can use reasoning strategies correctly involving relationships between quantities. S1 can also explain that the intended b_2 is the price of 25 flours weighing 1 kg each.

Figure 7

The result of S1's work



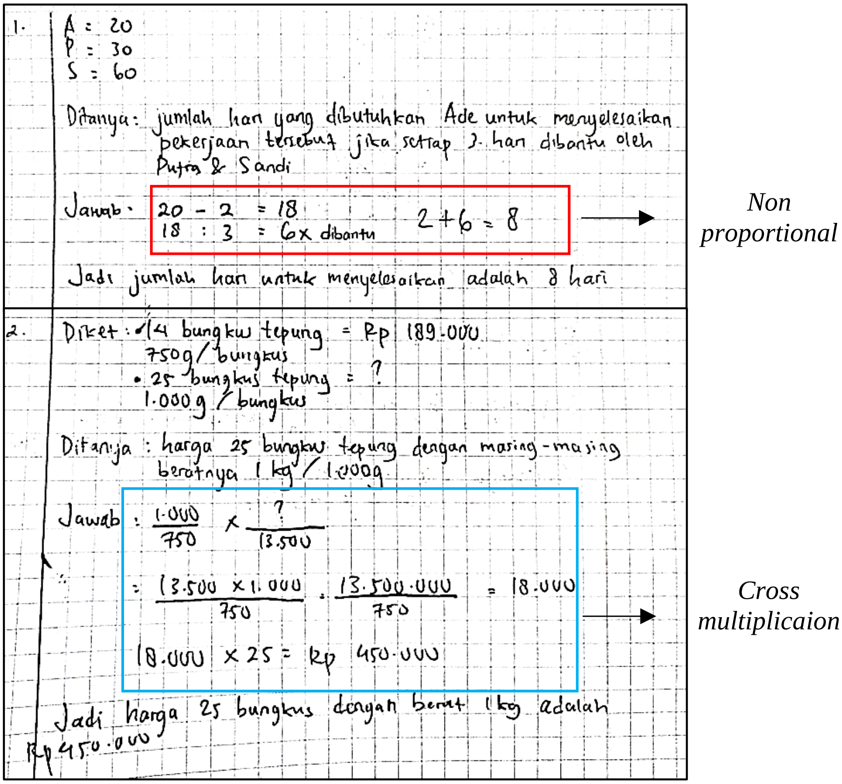
Based on the reasoning strategy used by the S1 in solving this comparison problem and the S1's ability to explain the relationship between information about the questions so that the correct answer is obtained, the SOLO S1 taxonomy level is included in the relational level. This is in accordance with the opinion of McGill (2013) that students can understand the problem and then can provide links between the information provided so that the correct answer is included in the relational level.

SOLO Taxonomy Level on S2

The second subject (S2) was unable to solve the problem of inverse proportion in question number 1 but S2 was able to solve the problem of direct proportion in question number 2 correctly. Following are the results of S2's work for numbers 1 and 2 as shown in Figure 8.

Figure 8

The result of S2's work



In Figure 8 above, S2 cannot write the correct solution for question number 1. In the process of working on number 1, S2 cannot solve the problem with the concept of comparison (red box in Figure 8). Therefore, S2 is said to use a non-proportional strategy. Based on the interview, S2 considered that when Ade was able to complete it on his own in the first 2 days, the total days (20 days) were reduced by the number of days Ade worked alone, which was 2 days, so S2 wrote "20 – 2 = 18". Because S2 thinks that the work will be completed by 3 people (Ade, Putra, and Sandi), the remaining 18 days are divided by 3 people so that "18: 3 = 6". S2 gave the opinion that Ade was assisted 6 times so that the total number of days needed to solve the problem was "2 + 6 = 8".

From the results of this Master's work, if it is identified with the reasoning strategy carried out by Cabero-Fayos et al., (2020), this is included in the intuitive and additive categories. It is categorized as intuitive because S2 uses the wrong information in the problem which causes S2 to be unable to see the relationship/relationship between quantities (additives).

In problem number 2, S2 solves it with the concept of direct proportion where S2 solves it using the comparison formula. This can be seen when S2 writes the formula "$\frac{1000}{750} \times \frac{?}{13500}$". In the previous discussion, the form of strategy carried out by S2 is included in the cross multiplication strategy. However, when seen from the results of Cabero-Fayos et al., (2020), the proportional reasoning strategy used by S2 is included in the proportion formula category. However, S2 is still wrong in writing the proportion formula, namely S2 writes the multiplication of that proportion.

Based on the reasoning strategy carried out by S2 in solving comparison problem number 1, actually S2 is still in the pre-structural level category where S2 has not been able to use his knowledge in solving comparison problems of inverse values. However, when faced with problems of value, S2 is included in the relational level. Seeing conditions like this, researchers assume that the level of students in solving this problem is adjusted to the conditions of the problems faced by students. There are problems that are not in accordance with the structure of students' thinking so that students have not been able to solve them properly.

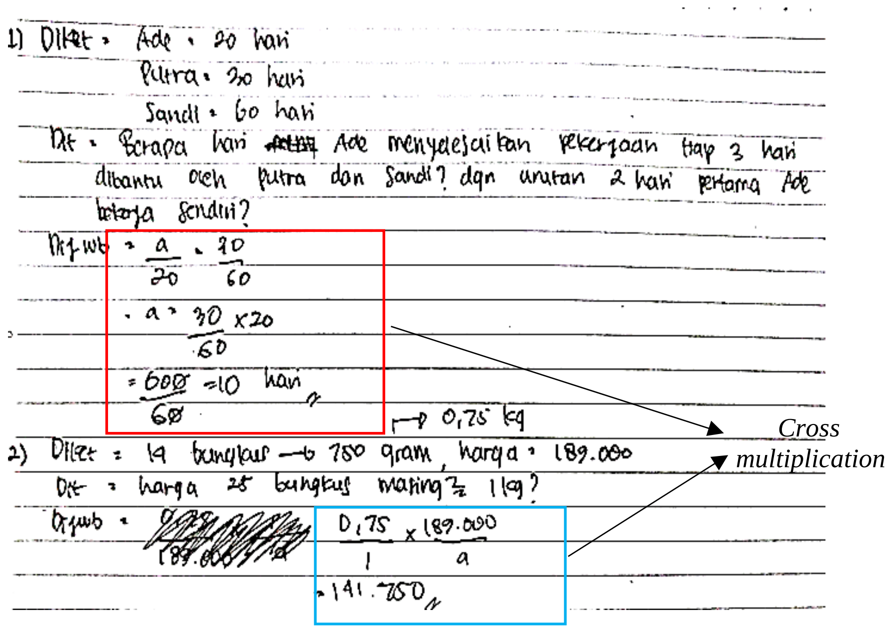
SOLO Taxonomy Level on S3

The third subject (S3) cannot solve the comparison problem in every given problem. Following are the results of S3's work for numbers 1 and 2 as shown in Figure 9.

Based on the results of S3's work in Figure 9, for both question number 1 and number 2, S3 carried out a proportional reasoning strategy with cross multiplication. However, S3 only includes any value regardless of the relationship between the quantities. Therefore, the results of the doctoral work are further identified through the results of Cabero-Fayos' research, (2020) where the results of the doctoral work fall into the following categories: intuitive, additive, and proposition attempt. In the red box, S3 writes "$\frac{a}{20} = \frac{30}{60}$". The a written means the number of days it will take Ade to complete the work if assisted by his two friends. Then S3 wrote down the process of carrying out the comparison in such a way that the result "a = 10 days" was obtained. When interviewed, S3 was able to explain that the more people who help with the work, the faster the work will be completed. Armed with this statement, S3 considers that the answer, which is 10 days, is correct because if Ade works alone, it takes 20 days to complete the work, whereas if he is assisted by his two friends, it will take less time, namely only 10 days. S3's statement is included in the proportion attempt category where S3 can explain the correct quantity relationship but in solving the problem S3 is still wrong. This is affected because S3 is still included in the intuitive and additive category where S3 is still unable to use the question information correctly so that the comparison concept used is wrong.

Figure 9

The result of S3's work



Answer number 2 is also the same as number 1. S3 writes the work process " $\frac{0,75}{1} \times \frac{189000}{a} = 141.750$ " as in the blue box. The same thing as in previous work number 1, S3 writes a as the price of 25 packets of flour weighing 1 kg each. S3 states that if flour weighing 0.75 kg costs 189000 then flour weighing 1 kg costs 141750. From S3's statement, the researcher sees that S3 ignores information that flour weighing 0.75 kg costs 189000 for 14 packs of flour and the problems solved is the price for 25 packets of flour weighing 1 kg each. Therefore, this S3 work falls into the category of intuitive, additive, and proposition attempt.

Based on the reasoning strategy carried out by S3 in solving this comparison problem, the SOLO S3 taxonomy level is included in the Uni-structural level. This is in accordance with the opinion of McGill (2013) that at this uni-structural level students only focus on one piece of information. Students cannot state the relationship between the quantity/information questions that lead to answers.

DISCUSSION

Based on the results of the research that has been described, the correct strategy in solving this comparison problem depends on the problem given. However, most students use the correct cross multiplication strategy in solving comparison problems. This is in accordance with some of the results of previous studies which state that students often use cross multiplication strategies (Avcu & Doğan, 2014; Ayan & Isiksal-Bostan, 2019; Öztürk et al., 2021; Tunç, 2020). For the cross multiplication strategy, students cross the denominator and quantifier multiplication, from the form $\frac{a}{b} = \frac{c}{d}$ such that bc=ad (Çalışıcı, 2018; Im & Jitendra, 2020). Because most of the students finished with this strategy, the students' answers lacked variety (Ayan & Isiksal-Bostan, 2019).

There are several reasons why this cross-multiplication strategy is often used by students. One of them is that students are often taught with cross multiplication strategies in solving comparison problems (Öztürk et al., 2021). This is reinforced by the research results of Andini & Jupri (2017) that students only remember the methods/procedures given by the teacher. In addition, proportional conditions are often related to multiplication (Cebola & Brocardo, 2021).

However, apart from using the cross-multiplication strategy, there is a proportional reasoning strategy. This proportional reasoning strategy is carried out by students when they are able to see the interrelationships between information in the questions. This is in accordance with the definition of

Cabero-Fayos et al., (2020) that the proportional reasoning strategy is the use of the correct reasoning strategy in which students are able to express relationships between quantities. This strategy is almost the same as the strategy expressed by Ayan-Isikal, namely the strategy of reasonable proportions. This reasonable proportion strategy determines the dependent variables and the relationship between these variables. The variable in question is the quantity of information about the problem. Another correct strategy that students do when solving a comparison problem is a systematic strategy. This strategy is when students are able to carry out systematic calculations involving multiplication and division calculation operations.

The incorrect strategy that is most often carried out by students is the non-proportional strategy. This is in accordance with the results of Tunç (2020)'s research that students often make mistakes in implementing reasoning strategies where proportional problems are carried out in a non-proportional way. This can be caused because students have difficulty distinguishing between proportional problems and non-proportional problems (Arican, 2019; Artut & Pelen, 2015; Karli & Yildiz, 2022; Tunç, 2020). There are several things that cause this student difficulty in distinguishing proportion problems and non-proportion problems. For example, students misunderstand the meaning of the problem and there are misconceptions that interfere with their ability to understand ratios and proportions (Dougherty et al., 2016). When students begin to understand the intent of the problems given, students' mathematical language ability is also a factor in failing to distinguish proportion problems. This is because mathematical language skills affect proportional reasoning abilities (Vanluydt et al., 2021).

When students carry out non-proportional strategies, students often use any number to carry out the calculation process. In this case students no longer pay attention to the quantity or the relevance of information in the problem. This is the same as the results of research conducted by Karli & Yildiz (2022) and Öztürk et al., (2021) that students perform illogical calculations using any number. In this case students try to show the results of the work without knowing the truth of the answer (Ayan & Isiksal-Bostan, 2019). Another wrong strategy is a cross multiplication strategy. In this study, students attempted to solve comparison problems using a cross multiplication strategy. However, students only use numbers in their calculations.

When carrying out non-proportional and cross multiplication wrong strategies, student work can be identified through the Cabero-Fayos's wrong strategy category. This wrong strategy is both intuitive and additive. It is referred to as intuitive when students incorrectly use information about questions that cause students to be unable to connect or link between quantities (additives). The wrong strategy experienced by these students can also be caused by students ignoring some of the information. This is referred to as “data neglect” (Castillo & Fernandez, 2022; Kahraman et al., 2019; Karli & Yildiz, 2022; Öztürk et al., 2021). Other wrong strategies carried out by these students did not solve the problem (no solution), unclear work results, and inaccurate calculations (Tunç, 2020).

The students' SOLO taxonomy level was analyzed based on the students' correct answers. First, students who can answer all questions correctly are at the relational level. This is in accordance with the research results of Karli & Yildiz (2022) and Özdemir & Yildiz* (2015) that students with high abilities are at the relational level. The high ability in question is a student who can answer math problems well.

Second, students who can only answer one problem correctly and one problem with an incorrect answer are at the pre-structural level for problem number 1 and are at the relational level for problem number 2. Students are at this pre-structural level when students solve inverse proportion problem, students cannot determine the relationship between quantities so students use non-proportional strategies in solving comparison problems. However, when students solve direct proportion problem, students are at the relational level because students are able to correctly determine the relationship between quantities. So from the second subject it can be seen that the level of student response based on the SOLO taxonomy is non-linear. Students can be at the lowest level when dealing with problems they are not good at. However, at the same time when students are faced with problems that are in accordance with the schema they have, students can be at the relational level. This is in accordance with the statement of Adeniji et al., (2022) that the SOLO taxonomy is not always linear. This means that the cycles that occur in stages can occur repeatedly and in a spiral.

Third, students who cannot answer all the problems given are at the Uni-structural level. At this uni-structural stage students understand the problem given but students only focus on some of the information. Students cannot state the relationship between the quantity/information questions that lead to answers. This is in accordance with the research results of Karli & Yildiz (2022) and Özdemir & Yildiz* (2015) that students with low reasoning abilities are at the uni-structural level.

CONCLUSIONS

Based on the results and discussion of this study, the first condition related to the forms of students' proportional reasoning strategies is right-wrong in solving this comparison problem including: (a) the correct strategy used by students for the comparison problem is in the form of a proportional reasoning strategy that involves calculations fractions. In this proportional reasoning strategy, students are able to see the interrelationships between information in the questions well. For comparison problems, the correct strategy used by students is a cross multiplication strategy and a systematic strategy. The strategy with cross multiplication is that students make the form a/b = c/d such that bc = ad. In making this form of comparison with cross multiplication, students have carried out a proportional reasoning process by paying attention to each part of the quantity that has a relationship with one another. The cross-multiplication strategy can also be referred to as the proportion formula strategy. This is because in the process of working with the cross-multiplication strategy, students write formulas from comparative forms. For this systematic strategy, students do not provide a formula for the process, but students carry out a logical calculation process in accordance with the proportional reasoning process. (b) the wrong strategy used by students when solving problems, both the value-reversal comparison problem and the value-comparison problem, is a non-proportional strategy and does not answer at all. In this non-proportional strategy students do not use the concept of comparison in solving proportional problems. Students use all the numbers in the problem with the operations of addition, multiplication, and division without paying attention to the relationship between the information in the problem. Then in the case of subjects who do all wrong on each given problem this shows a solution with a cross multiplication strategy. However, the given cross multiplication strategy does not properly involve proportional relationships. If further identification of the wrong strategies used by these students can be categorized into intuitive and additive forms from the results of Cabero-Fayos, et al., (2020). It is said to be intuitive when students use the wrong information to state their proportional relationship. This additive is when students are not precise in giving proportional relationships.

The level of student response based on the SOLO taxonomy is divided into three conditions. First, for students with all correct answers in each given problem where these students are able to use proportional reasoning strategies and cross multiplication/proportion formulas. Students who are able to answer all the answers correctly are at the relational level based on the SOLO taxonomy. At this relational level, students can state the exact relationship between information used in making comparisons. This can be seen when students are able to solve comparison problems with good value and value. Second, for students who can only answer one problem correctly and one problem with an incorrect answer, this is at the pre-structural level for problem number 1 and is at the relational level for problem number 2. Students are at this pre-structural level when students are faced with with the inverse comparison problem, students cannot determine the relationship between quantities so students use non-proportional strategies in solving comparison problems. However, when students are faced with the problem of equivalent comparisons, students are at the relational level because students are able to determine the relationship between quantities correctly. So from the second subject it can be seen that the level of student response based on the SOLO taxonomy is non-linear. Students can be at the lowest level when dealing with problems they are not good at. However, at the same time when students are faced with problems that are in accordance with the schema they have, students can be at the relational level. Third, students who cannot answer all the problems given are at the Uni-structural level. At this uni-structural stage students understand the problem given but students only focus on some of the information. Students cannot state the relationship between the quantity/information questions that lead to answers.

AUTHORS' CONTRIBUTIONS STATEMENTS

P.P., P.W., S.D., and S.S., conceptualised research ideas, research theories, methodology, and instruments. P.P., was responsible of collecting data, analyzing data and writing article. P.W., coordinated research activities and reviewed data completeness. S.D., validated the instrument and reviewed the writing. S.S., checked the completeness of the writing and the references used All authors actively discussed results, reviewed, and approved the final revision of the work.

DATA AVAILABILITY STATEMENT

Data supporting the results of this study will be made available by the corresponding author, [S.D.], upon reasonable request.

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