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Professional Learning Opportunities for Teachers of the Early Years in Algebra Teaching: A Study on the Practice of a Teacher Educator

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ABSTRACT

Background: Teacher education processes directly influence the quality of teaching and, indirectly, students' learning, which leads teacher educators to assume a central role in the development and leading of these processes, justifying in-depth studies about their actions. **Objective:** To understand how the actions and practices developed by a teacher educator during the orchestration of whole-group discussions contribute to creating professional learning opportunities about mathematical knowledge and educational practice regarding working with algebraic thinking aimed at elementary school teachers. **Design:** This is a qualitative-interpretive study along the lines of intervention research. **Settings and Participants**: The study included the teacher educator and 14 teachers who taught in grades 3-5 in a teaching network in a municipality in the state of São Paulo, Brazil. **Data collection:** The data were collected through video recording during the whole-group discussions with two cameras, one directed at the teacher educator and the other at the teachers. **Results:** The results indicate that the teacher educator used a set of actions and practices to conduct the whole-group discussions. **Conclusions:** The teacher educator's practices that led teachers to reflect on their practice are the ones that most provide professional learning opportunities, even though other practices are essential in creating an environment that promotes whole-group discussions.

Keywords: Teacher educator; Teacher professional learning; Teaching of algebra; Early years of elementary school; Whole-group discussion.

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Oportunidades de aprendizagem profissional de professores dos anos iniciais no ensino de álgebra: um estudo sobre as práticas de um formador

RESUMO

Contexto: Os processos formativos influenciam diretamente a qualidade do ensino e, de forma indireta, a aprendizagem dos alunos, o que leva o formador de professores a assumir um papel preponderante na elaboração e condução destes processos, o que justifica estudos aprofundados sobre suas ações. **Objetivo:** Compreender de que forma as ações e práticas desenvolvidas por um formador de professores, durante a orquestração de discussões coletivas, contribuem para a criação de oportunidades de aprendizagem profissional acerca do conhecimento matemático e da prática educativa, no que se refere ao trabalho com o pensamento algébrico voltado para professores dos anos iniciais do Ensino Fundamental **Design:** Trata-se de um estudo qualitativo-interpretativo nos moldes de uma pesquisa de intervenção**. Cenário e participantes:** Participaram do estudo um formador e 14 professores que lecionavam nos 3ºs, 4ºs e 5ºs anos em uma mesma rede de ensino em um município do estado de São Paulo, Brasil. **Coleta e análise dos dados:** Os dados foram coletados nos momentos de discussões coletivas e registrados por meio de duas câmeras, uma voltada ao formador e outra aos professores. **Resultados:** Os resultados indicam que o formador de professores utilizou um conjunto de ações e práticas de forma a conduzir as discussões coletivas. **Conclusões:** As práticas do formador que levaram os professores a refletir sobre sua própria prática são as que mais propiciam oportunidades de aprendizagem profissional ainda que outras práticas sejam imprescindíveis na criação de um ambiente promotor das discussões coletivas.

Palavras-chaves: Formador de Professores; Aprendizagem Profissional do Professor; Ensino de Álgebra; Anos Iniciais; Discussão Coletiva

INTRODUCTION

The need for a high-quality education has driven a series of studies that address, among other aspects, teachers' knowledge, skills, and beliefs, because "teachers' quality is the most important school variable to influence students' performance and improve school quality" (Sancar, Atal, & Deryakulu, 2021, p. 2). By aiming to lead teachers to advance their knowledge and adopt pedagogical routines that provide students with rich learning opportunities (Gibbons & Cobb, 2017), the teacher education processes directly influence teachers' quality and, indirectly, students' learning (Ping, Schellings, & Beijaard, 2018). Despite much work already done, there is a need for additional research on effective teacher education practices (Borko, Jacobs, Seago, & Mangram, 2014).

Addressing the teacher educators' role specifically, we cannot deny its importance in the elaboration and conduction of educational processes (Borko, Jacobs, Seago, & Mangram, 2014), given that "[the] way teacher educators facilitate those professional learning environments is critical to successfully supporting teacher learning" (Gibbons, Lewis, Nieman, & Resnick, 2021, p. 1).

In this article, we focus on a teacher educator's practice, constituted in "something that is done, not just known" (Krainer, Even, Rogers, & Berry, 2021, p. 11), as an alternative to many studies that focus on teacher educators' knowledge (Escudero-Ávila, Monte, & Contreras, 2021; Masingila, Olanoff, & Kimani, 2018; Zopf, 2010). In this regard, the object of the present study lies in the practice of orchestrating whole-group discussions in a teacher education course for teachers in the early years of elementary school, aiming at working with algebraic thinking. The importance of researching the moment of whole-group discussions is justified insofar as studies have shown that at this stage of the teacher education process, "the peak of meaning negotiations takes place" (Ferreira, Ponte, & Ribeiro, in press) and where most professional learning opportunities occur (Elliott et al., 2009). On the other hand, "little is known about how to facilitate such a discussion" (Zhang, Lundeberg, & Eberhardt, 2011, p. 343).

Based on a previous work by Ferreira, Ponte, and Ribeiro (in press), which presented a conceptual framework associating specific characteristics of the teacher educator's actions in orchestrating whole-group discussions with general practices, our study sought to deepen this investigation by aiming to *understand how the actions and practices developed by a teacher educator during the orchestration of whole-group discussions contribute to creating professional learning opportunities regarding mathematical knowledge and educational practice concerning the work with algebraic thinking aimed at teachers who teach in the early years of elementary school.* To this end, we seek to answer the following questions: How do we characterise the actions and practices of a teacher educator when orchestrating whole-group discussions? How do the actions and practices of a teacher educator contribute to creating professional learning opportunities regarding mathematical knowledge and educational practice? Therefore, this work intends to expand the constitution of a body of knowledge necessary for the teacher educator to guide the design and conduction of teacher education processes.

LITERATURE REVIEW

Mathematical knowledge and knowledge of the educational practice

Seeking to understand the knowledge necessary for teaching mathematics, Ponte (2012) considers four dimensions: (i) mathematics knowledge for its teaching, involving mathematical concepts and procedures, its specificities and internal and external connections, (ii) knowledge of the student and learning to understand how students learn mathematics, (iii) knowledge of educational practice, constituted as the core of teacher knowledge, including planning, elaboration of mathematical tasks, evaluation, and regulation of learning, and (iv) knowledge of the curriculum, including its provision throughout schooling and the materials needed to develop student learning.

The focus of this article lies on mathematical knowledge and educational practice, in which the mathematical knowledge necessary for teaching algebra in the early years comes from those contents that students need to learn generalised arithmetic or relational thinking (Molina, 2009), functional thinking, and different interpretations of the equal sign (Hohensee, 2015). Considering the importance of algebra for this stage of schooling, we argue that its development meets a reformulation of the existing teaching practice, adding to the work carried out with arithmetic opportunities for building patterns, generalisations, and mathematical justifications (Mestre & Oliveira, 2011; Molina, 2009). This leads to a perspective of integrating algebraic thinking into lesson planning, beyond working with algorithms (Blanton, 2008), in which generalisation assumes a significant role because it constitutes an essential characteristic of algebra (Kieran, Pang, Schifter, & Ng, 2016).

In educational practice, Ponte (2005) distinguishes two approaches: direct teaching and exploratory teaching. These differ in how information is presented and in the nature of the tasks and activities that result from students' involvement in executing them. The type of teaching in which the teacher gives the tools to solve specific problems and then presents the students with analogous situations is called by Skovsmose (2001) traditional teaching of mathematics, designated by Ponte (2005) as teaching based on a direct approach.

In an exploratory approach, a solid component of discussion, of mathematical argumentation, is highlighted, in which the work of discovery and construction of mathematical knowledge is stressed, presenting a clear cut

with a traditional way of teaching mathematics. Furthermore, exploratory teaching presupposes moments of whole-class discussion, favouring students' communication supported by argumentative discourse (Boavida, Gomes, & Machado, 2002). One of the central organising aspects of this approach foresees that the mathematics class should be developed into phases: introduction of the mathematical task, execution, and whole-class discussion (Stein, Engle, Smith, & Hughes, 2008). Ponte, Quaresma, Mata-Pereira, and Baptista (2015, 2016) say that, from this approach, the teacher plays a role in "selecting tasks, anticipating students' possible strategies and difficulties, organising class work, and conducting its execution" (p. 114). To this end, they believe teachers must have specific knowledge that enables them to conduct exploratory teaching in their classrooms. However, they state that traditional or direct teaching is still the most adopted style in schools, assuming "a first moment when the teacher explains and asks questions, followed by students' individual work in paper and pencil exercises" (Ponte, Quaresma, & Branco, 2012, p. 68-69).

Professional Learning Opportunities

To discuss teachers' professional learning, we turn to Hernandez (1998), who claims that learning (whatever it may be) occurs when an individual manages to transfer to a new situation something that has been assimilated with experiences throughout life in learning situations (formal or otherwise) and also in interaction with others. In this sense, the question is: How a teacher education process, more specifically, the actions filed by the teacher educator, can contribute to the teacher's learning?

When analysing different studies that point out which aspects influence teachers' professional learning, we perceived similar aspects: the importance of teachers' previous experiences and knowledge; reflection on practice; the role of professional practice; interaction between teachers; the importance of cognitive conflicts.

Based on the theoretical perspective of complexity, Opfer and Pedder (2011) understand that teachers' professional learning takes place from their previous experiences and knowledge. On the other hand, Webster-Wright (2009) believe that experience alone is not enough for learning, delegating to reflection the transforming role of the learning experience, which is corroborated by other authors (Bransford, Brown, & Cocking, 1999; Ponte, Mata-Pereira, Quaresma, & Velez, 2017). In other words, both adults and children learn through reflection based on practical activity (Ponte, 2005).

Schön (1983) argues that reflection on real-practice situations should be part of the (future) professionals' preparation so that they can be equipped with tools to face the ever-new and different situations they will meet in real life and make the appropriate decisions in the shady areas that characterise professional practice.

Associated with reflection is the role of practice as an important component of the teacher's professional learning. Webster-Wright (2009) considers that professionals learn from practical experience and reflection and that such learning is contextually mediated. Ball and Cohen (1999) also emphasise the role of practice and the importance of interactions and unpredictable situations that arise in the classroom as elements of investigation and reflection. Based on them, the teacher analyses their own teaching.

Another influential aspect of teacher learning is the interaction with other teachers (Ball & Cohen, 1999; Bransford, Brown, & Cocking, 1999) in which teachers learn from each other, thus breaking with the traditional isolation of their work, expanding their opportunities to learn collectively (White, Jaworski, Agudelo-Valderrama, & Gooya, 2013).

Besides these elements, several authors refer to the role that cognitive conflict plays in teachers' professional learning. For example, using the term "imbalance", Ball, Ben-Peretz, and Cohen (2014) emphasise that this element leads to learning and can manifest itself through the teachers' surprise when they face an unexpected situation and that the ability to be startled is the key to being able to learn through experience.

Combining all the aspects presented above, van Es, Tunney, Goldsmith, and Seago (2014) suggest that "teacher learning is more likely to happen when groups of teachers engage in productive imbalance through self-reflection, collegial dialogue, and continuous analysis of practice of teaching and student learning" (p. 343). Finally, to Hiebert et al. (1997), understanding is the result of connections and relationships the individual establishes with social practices through their participation (Lave & Wenger, 1991) being portrayed in changes in the actions, registers, and teachers' speeches (Watson & Mason, 2007).

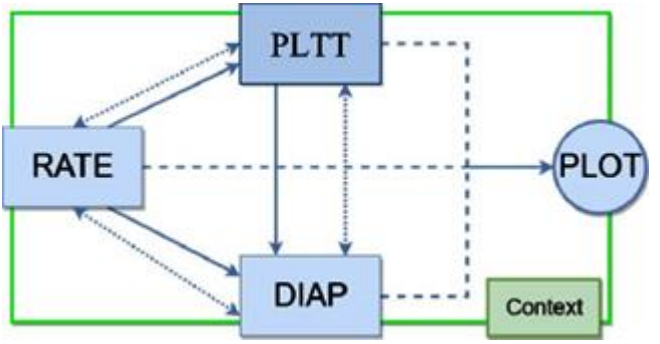
Considering the aspects that influence teacher learning and the characteristics of high-quality teacher education processes (Desimone, 2009), several studies have proposed models that can contribute to designing, conducting, and evaluating teacher education processes. An example is

provided by Ribeiro and Ponte (2020) through the model "professional learning opportunities for teachers" (PLOT) (Fig. 1).

This model presents three domains, observed from an organically complex whole: Role and Actions of Teacher Educator (RATE), Professional Learning Tasks for Teachers (PLTT) and Discursive Interactions Among Participants (DIAP). This model is based on the principle that teachers' learning occurs in their daily practice, in the peers' exchange, and from tasks explicitly prepared for them.

Figure 1

Model of the professional learning opportunities for teachers. (Ribeiro & Ponte, 2020)



Practices and actions of teacher educators

In the same way that, from exploratory mathematics teaching, the teacher plays a fundamental role in conducting whole-group discussions with basic education students, the teacher education exerts a similar influence when orchestrating whole-group discussions to promote professional learning opportunities. Thus, some studies have been pointing out actions the teacher educator can carry out to contribute for the teachers to establish relationships between their previous knowledge and the new challenges (Elliott et al., 2009; van Es, Tunney, Goldsmith, & Seago, 2014; Zhang, Lundeberg, & Eberhardt, 2011).

With regard specifically to the orchestration of whole-group discussions, Elliott et al. (2009) distinguish some of the teacher educator's

actions: directing the discussion to the established objective; supporting teachers' involvement with tasks, encouraging teachers to question each other; providing moments in which teachers explain and justify their ideas; and moderating the discussion to engage the group in mathematical ideas.

By analysing classroom videos, van Es, Tunney, Goldsmith, and Seago (2014) provided a conceptual framework for participants in a teacher education process to develop a critical discourse about students' thinking and mathematically important ideas, based on four practices: Guiding the group towards the video analysis task; Sustaining an investigative stance; Keeping the focus on video and math; Supporting group collaboration. Similar to this research are the studies by Zhang, Lundeberg, and Eberhardt (2011), who pointed out experienced teacher educators' actions by promoting discussions among science teachers: questioning, expressing, making connections, clarifying, reformulating, summarising, dramatising, meta-talking, and modelling.

Ferreira, Ponte, and Ribeiro (in press) present a conceptual framework in which they identify five central practices of a teacher educator in a teacher education process, which, although consistent with previous research (van Es, Tunney, Goldsmith, & Seago, 2014; Zhang, Lundeberg, & Eberhardt, 2011) expand the analysed literature "to the extent that they provide a specific structure for the orchestration of whole-group discussions, based on an exploratory teaching, aimed at teachers of the early years who teach mathematics" (Ferreira, Ponte, & Ribeiro, in press). *Establishing a learning community* encourages teachers to participate in whole-group discussions (Gibbons & Cobb, 2017; Zhang, Lundeberg, & Eberhardt, 2011). To this end, it is up to the teacher educator to encourage the construction of a relational environment in which the teachers feel comfortable and safe to expose their ideas and can also feel encouraged to speak from their standpoint. *Interpreting interactions with teachers and among teachers* involves understanding the teachers' context, speech, and attitudes (Elliott et al., 2009), embodying the teacher educator's interventions during the discussions. Considering that learning is built through connection (Hiebert et al., 1997), *establishing connections* requires the teacher educator to lead teachers to establish such bondings (Escudero-Ávila, Monte & Contreras, 2021) with elements external to the teacher education process, such as teachers' daily practice, and with elements of the teacher education itself, such as previously discussed issues. *Challenging teachers to advance their knowledge* refers to when teachers are confronted with and provoked in their ideas, which results in reflection and search for arguments about them (Gibbons, Lewis, Nieman, & Resnick, 2021).

Finally, *systematising learning* aims to resume the main themes of the discussion, synthesising the knowledge related to the purpose of the meeting (Ferreira, Ponte, & Ribeiro, 2022).

METHODOLOGY

This study follows a qualitative and interpretative approach (Bogdan & Biklen, 1994), as it seeks to study the practice of a teacher educator while orchestrating whole-group discussions in in-service education process, with the participation of 14 teachers who taught in the 3rd, 4th, and 5th grades in the same school network in a city in the state of São Paulo, Brazil. For ethical and confidentiality reasons, the participants' names have been changed[1].

Inspired by the PLOT model (Ribeiro & Ponte, 2020), the in-service education process used carefully planned professional learning tasks (PLTT) with specific objectives and records of practice (Ball, Ben-Peretz, & Cohen, 2014). In this teacher education process, we used the exploratory approach as conduction and organised it in three phases (Stein, Engle, Smith, & Hughes, 2008): (i) Introduction, when we introduced the PLTT. At this stage, we surveyed the teachers' previous knowledge; (ii) Autonomous work, which involved working in small groups in which teachers solved the PLTT; (iii) Whole-class discussion, when teachers presented their solutions to the whole group, leaving it to the teacher educator to manage this moment and systematise the learning process. The objective of the teacher education process was for teachers to understand (i) the need to work with algebraic thinking in the early years of elementary school, considering its recent inclusion in the Brazilian national curriculum (Brasil, 2017); (ii) the mathematical contents necessary for its development – generalised arithmetic, functional thinking and different meanings of the equal sign (Hohensee, 2015); and, (iii) knowledge of educational practice, aiming at its operationalisation in the classroom (Ponte, 2012). It had a workload of 64 hours, distributed over 32 face-to-face hours (eight weekly meetings of four hours) and 32 hours of autonomous work. The teacher educator in this research is the first author of this article. For the PLTT *Equal Sign*, we had the collaboration of facilitator Vitor, a member of the ForMatE research group.

[1] Participants signed the Free and Informed Consent Form (FICF), and the research was approved by the Ethics Committee of the Institute of Education of the University of Lisbon and by the Federal University of ABC (UFABC) under number 3.233.148.

We collected data during the whole-group discussions filmed with two cameras, one aimed at the teacher educator and the other at the teachers, to characterise the dialogic communication process between them.

Data analysis was performed in four phases. First, the teacher education process videos were transcribed, with the production of reports. In the second phase, we identified the teacher educator's actions during the whole-group discussions (the teacher educator's positioning and the consequent dialogic interaction with the teachers) according to the teacher educator's practice categories in Table 1. In this identification, some actions could be framed in different actions and/or practices, which led us to adopt the criterion of defining actions based on the teacher educator's objective for that action.

Table 1

The teacher educator's practices and actions the during the orchestration of whole-group discussions. (Ferreira, Ponte, & Ribeiro, in press)

Educators' practices	Description	Educators' actions
Establishing a learning community	Providing an environment in which teachers feel safe and encouraged to share their ideas and practices	Praising and encouraging Playing Supporting Sharing personal experiences Inviting
Interpreting interactions with teachers and among teachers	Giving meaning and sense to the different interactions	Validating Paraphrasing (*revoicing*) Extending/ Widening Asking for clarification Listening Clarifying/explaining
Establishing connections	Establishing relations elements that are internal and external to the teacher education process	Making connections Resuming

Challenging teachers to advance their knowledge	Asking questions challenging teachers to advance their knowledge	Opposing Questioning
Systematising learnings	Making a synthesis of the discussions and knowledge relating to the teacher education objectives	Summarising the main discussion topics Recovering prior knowledge

In the third phase of analysis, the actions were categorised according to the type of knowledge involved in the discussion: mathematical knowledge for teaching and knowledge of the educational practice. In the fourth phase of analysis, we observed how the teacher educators' actions and practices were established to create opportunities for professional learning, considering the aspects that influence the teachers' professional learning: (i) reflection on practice, (ii) role of professional practice, (iii) cognitive conflicts (iv) teachers' experience and prior knowledge, and (v) interaction between teachers.

Below, we show the episodes representing the teacher educator's different practices and actions when orchestrating whole-group discussions. First, we present PLTT *Equal Sign*, which deals with mathematical knowledge; second, we introduce PLTT *Analysing a Math Class*, focusing on knowledge of the educational practice. Finally, we discuss PLTT *Generalisation*, which addresses both types of knowledge. It is important to note that although the episodes are divided into mathematical knowledge and educational practice in which the data show one or the other knowledge, the objectives of the PLTT cover both types of knowledge since, in a teacher education process, we defend their inseparability (Ribeiro & Ponte, 2020). The teacher educator's practices and actions, categorised according to Table 1, are presented in italics.

RESULTS

Mathematical knowledge

PLTT *Equal Sign* aimed for teachers to understand the meaning of the equal sign from the perspective of equivalence and its importance for the development of algebraic thinking, and the need to work on this content in the early years of elementary school, planning didactical situations. After the

teachers carried out the PLTT in subgroups, in which one of the topics was the survey of the students' possible answers to sentence 8 + 4 = □ + 5, the teacher educator asked the teachers: *From the perspective of equivalence, why is the concept of equality crucial to the development of algebraic reasoning?* After getting imprecise answers, the teacher educator *invited* teachers to solve together the equation $4x + 27 = 87$:

> *Teachers: Put 27 there.*
>
> *Educator: What do I put here?*
>
> *Camila: 4x is equal to 87 minus 27.*
>
> [The teacher educator wrote on the board what Camila had said ($4x = 87 – 27$)].
>
> *Educator: What allows me to do this here: put the 27 here* [on the side of 87] *and -on top of that- negative?*
>
> *Adriana: Oh, the teacher taught that way.*
>
> *Eliana: You had 4x + 27, you took it away and only 4x remained. You took it away from this side and you're going to take it away from the other side too.*
>
> *Educator: Like Eliana did? Explain to me better...*
>
> *Eliana: You had 4x + 27, you took it away and only 4x remained.*
>
> *Educator: How did you take it away?*
>
> *Eliana: You took it away from this side, and you're going to take it away from the other side too.*

In this excerpt, we perceive the combination of four actions of the teacher educator: (i) the *inviting* action, seeking the participation of teachers in solving the equation, (ii) the *asking for clarification* action, leading teachers to reflect and explain better their speeches about the procedure for solving the equation, (iii) the practice of *challenging* through *questioning*, allowing teachers to justify why it was possible to place the –27 on the right side of the equal sign, and, again, (iv) the action of *asking for clarification*, considering that there were clarifications to be made, taking into account that the verb "take away" would need to be better clarified, certifying the common understanding of the idea. Such a combination of actions, and more specifically the actions of *questioning* and *asking for clarification*, challenged teachers to seek

justifications, creating opportunities for reflection on the procedure for solving the equation, which, although internalised in its procedural format, could not be appropriately sedimented in its conceptual understanding.

Continuing the discussion and given Eliana's pertinent statement about removing the –27 from both sides, the teacher educator began to register the representation of the solution of the equation on the board. When the teacher educator wrote *4x + 27 – 27*, the teachers disagreed with this representation, since the verb "take away" used by Eliana was not necessarily associated with the subtraction operation on both sides of the equal sign:

> *Eliana: No* [it's not like this]. *It must be: 4x is equal to 87 – 27.*
>
> *Educator: How do we write this here, then?*
>
> *Paula: You erase the 27, you remove it.*
>
> *Adriana: You delete it.*

Although the procedure for solving the equation presented by the teachers was correct, they disagreed with the representation of putting –27 on both sides. This fact shows the lack of knowledge about the need to perform the same operation on both sides of the equal sign, but also, and mainly, the lack of understanding that placing –27 on the "other side" is mathematically supported by the justification of the equal sign in the sense of equivalence. During the subsequent debate that the situation provoked, the teachers reflected on and sought arguments and/or justifications to solve the equation. Thus, the teacher educator decided not to speak out, *listening* to the teachers, and giving them opportunities to exchange views with each other, favouring interaction. When the teacher educator realised the justifications did not advance any more, he indicated a possibility of representation:

> *Educator: Couldn't it be like that* [wrote on blackboard $4x + 27 – 27 = 87 – 27$]?
>
> *Camila: You put the –27 on both sides! To give the balance!*
>
> *Adriana: Algebraically.*
>
> *Educator: Can it be like this?*
>
> *Eliana: Yes.*

We noticed that Camila only justified that there had to be a "balance" between the two sides of the equation after the teacher educator represented the mathematical situation by putting –27 on both sides, which *extended/widened*

teachers' knowledge of the understanding of the mathematical phenomenon. Camila's perplexity before an unexpected situation indicates that considering the imbalance posed by the teacher educator's information, learning opportunities could be created. Near the end of the discussion, the teacher educator *clarified/explained* the solution to the equation:

> *Educator: What allows us to do this step here is that you do the same operation on both sides of the equal sign, from the perspective of equivalence* [of equal sign]. *Who of you remembered that?*
>
> *Adriana: I don't think I even learned it.*
>
> *Paula: No* [I did not know it].

To *systematise learning*, facilitator Vitor *summarised* the main topics, *resuming* one of the questions the teacher educator had asked:

> *Vitor: Over there, at 4x + 27, when you put 27 there, it became negative. What's the question, then? Why did the positive number on the left side, + 27, go to the other side and become negative? Many students do it, but they don't know why.*
>
> *Camila: Because the teacher taught how to apply the rule.*
>
> *Vitor: But when you do that* [subtract from one side and the other], *you must write it on both sides. It is not enough to just say in the top line 87 – 27.*
>
> *Adriana: Why is it an equivalence?*
>
> *Vitor: Because it's an equivalence. It's like there at the end. The end is 4x = 60. If we look for answers to this x, we are always thinking about the inverse operation. What is usually done? Take the 4 and divide it by 60: why did a number that was multiplying the x pass dividing to the other side? When students do* [the equation], *they don't know why they do it because they don't understand the equivalence of the equals sign.*

Besides *summarising* the main topics discussed, Vitor *related* what was being discussed with the teacher's practice. This favoured the creation of professional learning opportunities, notably regarding students' knowledge, who often do not understand why the inverse operation is used in solving an equation, which Camila highlighted as a problem in the teaching process that

prioritises procedural knowledge. Finally, the teacher educator *established a connection* between the question initially asked and the solution to the equation:

> *Educator: Look at where the notion of equivalence begins* [point to 8 + 4 = □ + 5]. *Look at the relationship between this* [points to 8 + 4 = □ + 5] *and this* [points to 4*x* + 27 = 87]. *Children need to understand that equality is a relationship that says that two mathematical expressions have the same value.*

In this section, by *resuming* the initial question, the teacher educator showed the need to close the previous question, highlighting the importance of working with the students with the equal sign in the sense of equivalence because it is fundamental for future work with algebra, which translates, among other things, into understanding equations. Although the teachers knew the procedures for solving equations, they did not have it clear that equality is a relationship of equivalence between two numerical sentences, evidenced by Adriana's statement that she had never learned it.

Finally, Rosana witnessed the establishment of a relationship between what was being debated in the discussion and her classroom practice:

> *Rosana: As I'm seeing it for the first time* [the meaning of the equal sign], *what I came to a conclusion was the equivalent [term]. Because when I talk about equivalence here* [on the left side of the equal sign] is *here* [on the right side of the equal sign] *and I get there at x, there I will do the interconnection which is an equivalence, that we teach equivalence only in fractions.*
>
> *Camila: Equivalent fractions, it's true!*
>
> *Rosana: Equivalent fraction, that's where I made this list now. Because I'm going to have to teach equivalent fraction. Here is the equivalence* [in the equation] *and it's in a fraction.*

By providing situations in which teachers can reflect, leading them to *establish relationships*, learning opportunities were created, portrayed in the relationship that Rosana could establish with her practice, with the teaching she was already developing with the students. Equivalent fractions are indicated using the equals sign, not from the perspective of search results, but in the sense of equivalence. It is impossible to define which of the teacher educator's practices and actions contributed to Rosana's possibility to establish a relationship between one of the meanings of the equal sign and her practice of

working with fractions. However, we can say that the actions as a whole, such as *inviting, asking for clarifying, questioning, listening, extending/widening summarising, clarifying/explaining, and relating* that the teacher educator orchestrated contributed to Rosana's relationships with her practice, as she claimed to have seen the meanings of the equal sign for the first time, configuring the creation of opportunities for professional learning.

Considering that one of the fundamental roles of the teacher educator is to make decisions during the whole-group discussion, based on the *interpretation*, she may miss opportunities to delve deeper into the given situation, due to the unpredictability of the practice. For example, when Adriana identified the placement of –27 on both sides of the equation as algebraic, the teacher educator could have asked her to justify the statement, helping to clarify her thinking better and bringing more elements into the discussion.

Knowledge of the educational practice

PLTT *Analysing a Math Class* aimed to make teachers analyse in depth the phases of a mathematics class considering the exploratory teaching approach. Regarding the phase of introducing a mathematical task, the teacher educator showed a video in which the teacher (in the video) presented it to her basic education students. Then, divided into subgroups, the teachers discussed the main aspects of introducing a mathematical task. Then, a whole-group discussion in which the teachers were asked to comment on their observations was held:

> *Marina:* [in the task introduction phase] *The teacher needs to ask* [students] *whether they really understood* [the math task]. *Ask the child to explain whether he/she understood what to do in the activity, because we usually overlook it. Because, as they don't read* [what needs to be done], *they take the numbers and start doing the math, so before doing it: I want to know what you must do. Then the behaviour [of the students] changes*.
>
> *Educator:* [addressing the whole-group] *Why are Marina's words important?*

By *interpreting* Marina's positioning, the teacher educator asked a *question* that, although simple, involved several actions that triggered the discussion. The first was the teacher educator's *invitation* to the teachers so that

they could participate and express their ideas based on peer intervention, which may favour the construction of a learning community insofar as the group had the opportunity to support the culture of publicising their ideas. At the same time he *invited* the group to manifest themselves, the teacher educator *validated* Marina's speech, *challenging* participants to reflect and justify the importance of what she had said, explicitly focusing on the content discussed then. Finally, from the teacher educator's *questioning*, the teachers revealed other aspects of introducing a mathematical task, justifying its importance for the teaching and learning process:

> *Debora: If they don't understand the command* [what should be done in the math task], *they won't know where to start from.*
>
> *Amanda: Because, otherwise, they start* [asking]*: What is it that should we do, teacher?*
>
> *Paula: I was reading a part of the text about the importance of asking students to tell us about the task. It is the part that I made a note here, "when students are not interested in the task or they do not understand", when they do not understand they are not interested either and the "success of their learning in mathematics is compromised". So that issue of asking them what they should, what they understood, you call their attention, if they pay attention to what they are reading usually arouses their curiosity.*
>
> *Educator: Do you think this format* [the introduction to the math task] *can contribute to that big complaint that children don't interpret, don't know how to read, don't understand the problem?*

The teacher educator's *questioning* allowed Paula to highlight and share two central characteristics of introducing a mathematical task: getting students interested and performing the task with commitment and understanding of what must be done. After Paula's positioning, the teacher educator posed another *question* that established a relationship with the challenges teachers face daily. It is usual to hear teachers complaining that students, when reading mathematical tasks, do not understand what they read and, therefore, present mistakes in their resolutions. When the teacher educator became aware of this criticism, she created learning opportunities insofar as he sought to establish a direct connection with the teachers' experiences and their daily practice (an external element to the training process), *relating* the

introduction of the mathematical task (focus on teaching) to a student's difficulty (focus on learning), leading teachers to reflect, and contributing to the generation of hypotheses about the basic relationships between teaching and learning.

Continuing the discussion, the teacher educator asked the teachers what else the teacher in the video presented when introducing the mathematical task:

> *Eliana: There is anticipation. Like shaking hands, asking how many* [handshakes] *you think there will be as an answer, I think there will be four handshakes.*
>
> *Debora: There is another aspect which is inviting the student to read.*
>
> *Educator: That is it. Instead of you reading it, because the colleague's voice gives a different intonation. There's something else that teacher Celia did and it's pretty cool.*
>
> *Adriana: Explain in your own words.*
>
> *Educator: This is helping. She did something else, what was it?*

In this section, besides *validating* Debora and Adriana's speech, the teacher educator *paraphrased/voiced* it as she reformulated the teacher's speech in her own words. It is possible to perceive that, when knowing in depth the elements that make up an introduction to a mathematical task, the teacher educator asked the teachers to expose those elements. In this situation, not satisfied with the teachers' answers, the teacher educator stated that he would have other valuable things in the video to highlight, *questioning* the teachers:

> *Adriana: She brought the cube.*
>
> *Educator: Yeah, she brought the cube for visualisation. Because in mathematics, we bring the importance of the different representations. Suppose we go back to the Pythagorean table, if we realise that the child has a lot of difficulties visualising multiplication, the introduction* [of math assignment] *would be a good time* [to ask questions]. *Do you know this table? What do we have to do? Tell me. Do you have to fill it out? So that the child, after you give the task, does not go to your desk all the time asking what they should do, because they did not understand. So, when planning tasks of this nature, we must anticipate the difficulties and try to*

> *overcome those difficulties in the introduction, so that the student can perform the mathematical task.*

This excerpt suggests that Adriana gave him the answer the teacher educator expected. The teacher in the analysed video had brought cubes with self-adhesive stickers as concrete material, a representation that could help students understand the statement of the mathematical task. The teacher educator *established a connection* with elements internal to the teacher education process when he resumed a previously developed PLTT in which he referred to students' difficulties in filling out the Pythagorean table, which can provide teachers with opportunities to *establish relationships*, enhancing the construction of more significant meanings. Another relationship established by the teacher educator, through the actions of *extending/*w*idening*, was to consider the questions that could be asked to the students and consider in the planning the possible doubts or misunderstandings that the students may have in relation to the proposed mathematical task.

Mathematical knowledge and knowledge of the educational practice

PLTT *Generalisation* aimed to discuss the meaning of generalisation, its importance for the development of algebraic thinking, and how to work generalisation with students. Enacting the PLTT, which presented students' registers and justifications for the veracity of some mathematical sentences, required teachers to observe which justifications could reveal that students were generalising. After presenting the subgroups' discussions, the teacher educator returned to the meaning of generalising:

> *Educator: What is generalising in mathematics?*
>
> *Débora: It's when you have a situation... that serves not only for a specific situation, but for more than one, it has a pattern, a regularity...*
>
> *Educator: Who wants to complete or counter-argument? Do you agree with the colleague?*
>
> Silence.
>
> *Educator: Do you agree with Débora?*

At this point, some teachers nodded but did not express their ideas on the question. Situations like those are usual in whole-group discussions, which

may result from the teachers' lack of knowledge about the subject, fear of exposing themselves, but also because they agree with the colleague and do not have more to add. In this section, when *interpreting* the situation, the teacher educator decided to move on by addressing another meeting objective:

> *Educator: And what is the importance of this work* [on generalisation]? *How important is generalisation?*
>
> *Eliana: For the development of algebraic thinking.*
>
> *Débora: There are studies that say that without having this foundation, this algebraic thinking developed in the early years, [students] will have a lot of difficulties in high school or from the 6th year onwards when they start to see the name Algebra.*
>
> *Educator: Well done! So, Eliana mentioned that it is for the development of algebraic thinking and Débora, in a more longitudinal perspective, that it is important to work with generalisation... for the following years.*

One of the objectives of the teacher education process was for teachers to understand the importance of working on algebraic thinking in the early years, considering its recent inclusion in the official Brazilian curriculum and generalisation as a central element in this process. In this section, the teacher educator *praised* and *validated* the position of the teachers, *paraphrasing/revoicing* what was said. By giving voice to Eliana and Débora's ideas, naming them, he gave authenticity to the teacher's role as an active subject of his learning, contributing to the consolidation of the practice of *establishing a learning community*. In addition, the discussion provided an opportunity for Débora to reflect on her practice:

> *Débora: Thinking about my practice, I work with algebraic thinking, but not with that intentionality that I have seen now.*
>
> *Educator: Does it make a difference when Débora asks the question of intentionality? Does it make a difference when you have this knowledge, this intentionality?*
>
> *Eliana: It does.*
>
> *Educator: Why?*

> *Eliana: Because, when you have that intention, you can make more meaningful interventions, because if it doesn't go unnoticed, you lose the opportunity.*

At this point, the teacher educator, relying on Débora's speech, considered the word "intentionality" as an object of discussion since, when working on a particular curriculum content, the teacher must be clear about their objective. Although the teacher educator presented a question whose answer would be an affirmation or a denial, in the sequence, he *questioned* by requesting a justification, allowing the teachers to reflect on the need to pursue specific objectives when offering tasks to students. Continuing the discussion, the teacher educator suggested that there could be other arguments for the need to work on the objectives intentionally:

> *Educator: There is another thing....*
>
> *Débora: When you have a goal, a specific goal to achieve, I want to develop that skill, so I need to have that path, follow this path, have a plan on how to help the children.*
>
> *Educator: You must have a goal, like when I put the goals here. We have a goal for our meeting today. At the end of the day, I would like us to have achieved these goals, so, when we enter the classroom, we always have to have a goal. If your goal is never to develop algebraic thinking or work with generalisation, it may even happen, but you won't be able to intervene appropriately or check whether the child has reached your goal.*

The teacher educator sought to establish a direct connection with the practice, emphasising the teacher's work cycle when referring to planning (the need to have a goal to pursue), implementation, and assessment, in the *clarifying/explaining* action. Furthermore, he sought *to list* what is done in the teacher education process (establishing objectives) with classroom practice, creating learning opportunities based on the teachers' experience. By way of *summary*, although the whole-group discussion had not yet ended, the teacher educator presented the main points of the discussion:

> *Educator: You can always work on algebraic thinking and generalisation as long as you first have that objective, and second, that you have eyes to see those particular cases, how they can be transformed into regular cases, this can arise daily in your work.*

Then, the teacher educator asked the teachers to suggest regularities that would be present in their daily lives, establishing a connection with classroom practice and meeting one of the objectives of the teacher education process, that of realising that working with algebraic thinking can happen from deepening arithmetic. After several examples, such as multiplication and division by 10, 100, and 1000, the teacher educator *established a relationship* with the textbook (EMAI) used by teachers:

> *Educator: These situations are present in EMAI.*
>
> *Débora: Yes.*
>
> *Adriana: And sometimes the EMAI does not come with this open proposal to help the teacher because, for example, we will only be able to have this understanding of closed and open* [tasks], *investigation, and exploration when we have the training. If you leave the material with the open questions alone, I think that I, for example, would not be able to find myself in it, I would be more lost. If it weren't for the training, maybe I wouldn't do it the right way.*

In this part, the teacher educator, by alluding to the teachers' textbook, created an opportunity for Adriana to reflect on her relationship with this didactic resource, associating it with the teacher education process, and witnessing its effectiveness. Among the various ways of establishing relationships with the teacher's practice, a textbook is a powerful tool since, used daily, it can lead the teacher to *establish connections* between the teacher education process and what they develop in their daily actions.

DISCUSSION

In previous works about the teacher education process now under analysis, it was possible to register changes in the practice of the participating teachers "both in relation to their understanding of the meaning of algebraic thinking and the way of working in the classroom" (Ferreira, Ponte & Ribeiro, in press). In addition, the teacher educator's work was decomposed during the orchestration of whole-group discussions into actions that, based on their regularities, were categorised into teacher educator's practices (Ferreira, Ponte, & Ribeiro, in press) (Table 1). In this article, we sought to characterise and verify how a teacher educator's actions and practices propelled the creation of

professional learning opportunities for the participating teachers, contributing to the construction of a body of knowledge necessary for the teacher educator.

The results indicate that many of the actions that created opportunities for professional learning, both concerning mathematical knowledge and educational practice, are included in *establishing connections* and *challenging teachers to advance their knowledge*, since they favoured reflection (Webster-Wright, 2009). The actions of *resuming* and *listing* created opportunities for professional learning as they facilitated the establishment of relationships (Hiebert et al., 1997) with the teacher's practice, taking into account their experiences and knowledge. Examples of this direct relationship with practice occurred when the teacher educator referred to the textbook used by the teachers, but also when he problematised the introduction of the mathematical task relating to a common complaint of teachers that students cannot interpret a problem. On the other hand, the *questioning* actions created professional learning opportunities insofar as they provoked reflections, which can be observed when the teacher educator, at PLTT *Equal Sign*, asked questions so that teachers would reconsider their mathematical knowledge about the procedures adopted in solving an equation.

Although those two practices seem to us to be the ones that can most contribute to a change in teacher practice based on reflection, there are others that, although not directly related to the creation of professional learning opportunities – *establishing a learning community* and *systematising learning* – are important to favour or impair learning. The necessary participation and interaction among teachers is not something that results from simply joining desks so that teachers work together. It must be provoked by the teacher educator. *Establishing a learning community* implies that teachers feel safe to participate (Elliott et al., 2009), being a condition for other actions to be filed. This condition was verified when the teacher educator *invited* teachers to speak out but especially when she *praised* and encouraged them to make their teaching public (Kazemi, Ghousseini, Cunard, & Turrou, 2015) and *validated* their ideas, motivating them to get involved and feel more comfortable sharing their experiences. The closure of ideas, albeit provisional, is also part of the discussion, where the teacher educator *summarises* and *rescues* speeches, concepts, and disagreements that arose during the debates and throughout the PLTT process. The practice of *systematising learning* occurred even before the end of the discussion, ratifying the possibility of this practice occurring during this process (Ponte, Mata-Pereira, & Quaresma, 2013).

Most situations in a whole-group discussion are unpredictable, considering it is impossible to control teachers' ideas, questions, and observations. Faced with the many positions of teachers, the teacher educator must decide which path to take while "seeking to take advantage of teachers' ideas and guide the conversations" (Borko, Jacobs, Seago, & Mangram, 2014, p. 261). Regarding the *interpretation* context, at PLTT *Equal Sign*, when the teachers did not satisfactorily answer his question, the teacher educator, *interpreting* the situation, used the representation of the resolution of an equation so that teachers understood the meaning that the equal sign represented in an equation and could transpose it into their practice. The practice of *interpreting interactions with teachers and among teachers* evidenced the fundamental role of the teacher educator in conducting negotiations in a dialogic discourse process. To Jacobs, Lamb, and Philipp (2010), based on professional observation, teachers must be able to pay attention, interpret, and decide how to respond to students' mathematical understandings (*noticing)*. In an analogy to the teachers' role, the teacher educator must be able to apply these three skills in the orchestration of whole-group discussions, considering that it is "crucial to unveil the current teachers' pedagogical reasoning and support them to meet the new approaches to teaching mathematics through discussions in professional groups" (Kim, Metzger, & Heaton, 2020, p. 1225).

We could infer that the combination of the teacher educator's actions may favour the creation of professional learning opportunities, which is also defended by other authors (van Es, Tunney, Goldsmith, & Seago, 2014; Zhang, Lundeberg, & Eberhardt, 2011). At PLTT *Equal Sign*, Rosana testified that after seeing the different meanings of the equal sign for the first time, she was able to establish a relationship with her practice.

In addition to the teacher educator's successive and combined actions, we could perceive that focusing on the objective of the PLTT is an essential element in the orchestration of whole-group discussions (Elliott et al., 2009), considering that it is natural that peripheral subjects and also distant from the central objective are part in the discussion. At PLTT *Analysing a Math Class*, the results showed that the discussion was developing in a growing movement of ideas improvement, in which the teacher educator pursued the objective of raising the central characteristics of introducing a mathematical task.

This work also indicates that just like the students benefit from working with different representations of the same mathematical situation (NCTM, 2014), we can infer that in teacher education, the role of different

representations exerts the same influence, since that in working with the equal sign, teachers could benefit from the representation of the equation to understand its sense of equivalence.

CONCLUSION

Given the ever-present need to leverage teachers' knowledge and skills in and for teaching mathematics, notably of teachers in the early years of elementary school who have gaps in their knowledge (Ferreira, Ribeiro & Ribeiro, 2017), the teacher education processes need to be continually improved through research that focuses on the multiple elements that compose their conduction effectively (Desimone, 2009).

The conceptual framework "*Practices and actions of the teacher educator during the orchestration of whole-group discussions*" allowed characterising the teacher educator's actions at the three analysed PLTT. To achieve the proposed objectives, in addition to the PLTT containing practice registers (Ball, Ben-Peretz, & Cohen, 2014), the teacher educator went through the whole-group discussions: interpreting, establishing connections, challenging, systematising, and establishing a learning community. Although in each analysed PLTT the actions did not occur in the same sequence because the dialogical process makes the unfolding of each discussion unique, the actions filed in each discussion were similar, both in terms of the development of mathematical knowledge and educational practice. In this sense, one of the contributions of this work lies in legitimising the five practices of teacher educators since, operationalised together, they provided reflections on practice, a fundamental aspect in expanding teachers' knowledge and skills (Kim, Metzger, & Heaton, 2020).

Regarding creating professional learning opportunities, we found that *establishing connections* and *challenging teachers to advance their knowledge* was central as they led to cognitive imbalances (Ball, Ben-Peretz, & Cohen, 2014) based on reflection, which can foster changes in practice. Thus, when orchestrating whole-group discussions, the teacher educator must have these two practices as a guide, challenging through *relating, resuming, opposing, and questioning* teachers in their ideas. On the other hand, *establishing a learning community* favours the creation of professional learning opportunities to the extent that it encourages teachers to expose themselves. Challenges can be emptied into evasive answers if teachers feel uncomfortable exposing themselves and contributing to discussions. Finally, *interpreting interactions*

with teachers and among teachers seems to us to be the foundation and generator of many other actions for the continuation of the discussions, being the fruit of the teacher educator's knowledge (Ferreira, Ponte & Ribeiro, in press), but also of the learning objective to be pursued.

We conclude by highlighting some of the limitations of the work and suggesting future research. The characterisation of the teacher educator's practices and actions when orchestrating whole-group discussions was observed from a single teacher education process and the conduction of a specific teacher educator and should be studied in other contexts and disciplines, including prospective teacher education. Furthermore, interpreting moments of tension portrayed in the data indicates that research should investigate the teacher educator's role in moments when teachers (i) deviate from the subject; (ii) present conceptual or procedural mistakes; (iii) remain silent before the teacher educator's questions; and (iv) do not answer according to the proposed objective. Faced with these tensions, it is pertinent to ask what the actions and practices the teacher educator can use are, both to take advantage of the situation and to get out of it and continue with the proposed objective, seeking to constitute the role of *noticing* of the teacher educator.

Finally, analysing the teacher educators' practices is a fruitful path as they reveal the complexities specific to the teacher educators' work (Borko et al., 2021). Our work contributes to the literature by pointing out that the frame of reference *"Practices and actions of the teacher educator during the orchestration of whole-group discussions"* provides ideas on effective actions to orchestrate whole-group discussions that can materialise in the teacher educator's planning by including dialogue strategies, reducing the unpredictability of their orchestration, so present in situations of professional practice.

AUTHORSHIP CONTRIBUTION STATEMENTS

MCNF collected the data and analysed and wrote the first version of the article. All authors, MCNF, JPP and AJR discussed all parts of the article, reviewed, and contributed to the final version of the work.

DATA AVAILABILITY STATEMENT

The authors agree that the data supporting the results of this study are available upon reasonable request, at the authors' discretion.

REFERENCES

Ball, D., Ben-Peretz, M. & Cohen, R. B. (2014). Records of Practice and the Development of Collective Professional Knowledge. *British Journal of Educational Studies*, *62*(3), 317-335.

Ball, D. & Cohen, D.K. (1999). Developing practice, developing practitioners. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession* (pp. 3-32). Jossey-Bass.

Blanton, M. L. (2008). *Algebra in elementary classrooms: Transforming thinking, transforming practice.* Heinemann.

Boavida, A. M., Gomes, A. & Machado, S. (2002). Argumentação na aula de matemática: Olhares sobre um projecto de investigação colaborativa. *Educação e Matemática*, *70*, 18-26.

Bogdan, R. & Biklen, S. (1994). *Investigação qualitativa em educação. Uma introdução à teoria e aos métodos.* Porto.

Borko, H., Jacobs, J., Seago, N., & Mangram, C. (2014). Facilitating video-based professional development: Planning and orchestrating productive discussions. In Y. Li, E. A. Silver, & S. Li (Eds.), *Transforming mathematics instruction* (pp. 259-281). Springer.

Borko, H., Carlson, J., Deutscher, R., Boles, K. L., Delaney, V., Fong, A., Jarry-Shore, M., Malamut, J., Million, S., Mozenter, S., & Villa, A. M. (2021). Learning to Lead: An Approach to Mathematics Teacher Leader Development. *International Journal of Science and Mathematics Education*, *19*, 121-143. https://doi.org/10.1007/s10763-021-10157-2

Bransford, J., Brown, A. L., & Cocking, R. (1999). *How people learn: Brain, mind, experience and school*. National Academies Press.

Brasil, Ministério da Educação; Secretaria de Educação Básica (2017). *Base Nacional Comum Curricular*: MEC/SEB.

Desimone, L. M. (2009). Improving impact studies of teachers' professional development: Toward better conceptualizations and measures. *Educational Researcher, 38*(3), 181–199. https://doi.org/10.3102%2F0013189X08331140

Elliott, R., Kazemi, E., Lesseig, K., Mumme, J., Carroll, C., & Kelley-Petersen, M. (2009). Conceptualizing the work of leading mathematical tasks in professional development. *Journal of Teacher Education*, *60*(4), 364-379.

Escudero-Ávila, D., Monte, M. & Contreras, C. (2021). What do mathematics teacher educators need to know? Reflections emerging from the content of mathematics teacher Education. In M. Goos, K. Beswick (eds.), *The learning and development of mathematics teacher educators*, https://doi.org/10.1007/978-3-030-62408-8_2

Ferreira, M. C. N., Ribeiro, C. M. & Ribeiro, A. J. (2017). Conhecimento matemático para ensinar álgebra nos anos iniciais do ensino fundamental. *Zetetiké, 25*(3), 496-514. https://doi.org/10.20396/zet.v25i3.8648585

Ferreira, M. C. N., Ponte, J. P., & Ribeiro, A. J. (2022). Towards an approach to teachers' professional development: how to work with algebraic thinking in the early years. *PNA*, *16*(2), 167-190. https://doi.org/10.30827/pna.v16i2.22234

Ferreira, M. C. N, Ribeiro, A. J. & Ponte, J. P. (in press). Práticas e ações do formador de professores que ensinam matemática na orquestração de discussões coletivas. *BOLEMA*

Gibbons, L. K. & Cobb, P. (2017). Focusing on teacher learning opportunities to identify potentially productive coaching activities. *Journal of Teacher Education*, *68*(4), 411-425. https://doi.org/10.1177/0022487117702579

Gibbons, L. K., Lewis, R. M., Nieman, H., & Resnick, A. F. (2021). Conceptualizing the work of facilitating practice-embedded teacher learning. *Teaching and Teacher Education*, *101*, 103304. https://doi.org/10.1016/j.tate.2021.103304

Hernández, F. (1998). Formação docente: O desafio da qualificação cotidiana. A importância de saber como os docentes aprendem. *Pátio Revista Pedagógica, 4*.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K.C., Wearne, D., & Murray, H. (1997). *Making sense: Teaching and learning mathematics with understanding.* Heinemann.

Hohensee, C. (2015). Preparing elementary prospective teachers to teach early algebra. *Journal of Mathematics Teacher Education, 20*(3), 231-257. https://doi.org/10.1007/s10857-015-9324-9

Jacobs, V. R., Lamb, L. C., & Philipp, R. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*(2), 169–202.

Kazemi, E., Ghousseini, H., Cunard, A., & Turrou, A. C. (2015). Getting inside rehearsals: Insights from teacher educators to support work on complex practice. *Journal of Teacher Education, 67(1), 18–31*. https://doi:10.1177/0022487115615191

Kieran, C., Pang, J., Schifter, D., & Ng, S. F. (2016). ICME-13. *Early algebra: Research into its nature, its learning, its teaching*. Springer.

Kim, Hj., Metzger, M. & Heaton, R. M. (2020). Teacher planning sessions as professional opportunities to learn: An elementary mathematics teacher's re-conceptualization of instructional triangles. *International Journal of Science and Mathematics Education, 18*, 1207–1227. https://doi.org/10.1007/s10763-019-10019-y

Krainer, K., Even, R., Rogers, M.P., & Berry, A. (2021). Research on learners and teachers of mathematics and science: Forerunners to a focus on teacher educator professional growth. *International Journal of Science and Mathematics Education, 19*, 1–19. https://doi.org/10.1007/s10763-021-10189-8

Lave, J. & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge University Press.

Masingila, J. O., Olanoff, D. E., & Kimani, P. M. (2018). Mathematical knowledge for teaching teachers: Knowledge used and developed by mathematics teacher educators in learning to teach via problem solving. *Journal of Mathematics Teacher Education, 21*, 429–450. https://doi.org/10.1007/s10857-017-9389-8

Mestre, C. & Oliveira, H. (2011). O pensamento algébrico e a capacidade de generalização de alunos do 3.º ano de escolaridade do ensino básico. In: Guimarães, C. & Reis, P. (Orgs.) *Professores e infâncias: Estudos e experiências* (pp. 201-223). Junqueira & Marin.

Molina, M. (2009). Uma propuesta de cambio curricular: integración del pensamento algebraico em educación primaria. *PNA*, *3*(3), 135-156. https://doi.org/10.30827/pna.v3i3.6186

NCTM (2014). *Princípios para a ação: Assegurar a todos o sucesso em Matemática.* APM.

Opfer, V. D., & Pedder, D. (2011). Conceptualizing Teacher Professional Learning. *Review of Educational Research*, *81*(3), 376-407. https://doi.org/10.3102/0034654311413609

Ping, C., Schellings, G. & Beijaard, D. (2018). Teacher educators' professional learning: A literature review. *Teaching and Teacher Education*, (75), 93–104. http://doi:10.1016/j.tate.2018.06.003

Ponte, J. P. (2005). Gestão curricular em matemática. In GTI (Ed.), *O professor e o desenvolvimento curricular* (pp. 11-34). APM.

Ponte, J. P. (2012). Estudando o conhecimento e o desenvolvimento profissional do professor de matemática. In: N. Planas (Ed.), *Educación matematics: Teoría, critica y prática* (pp. 83-98). Graó.

Ponte, J. P., Mata-Pereira, J., & Quaresma, M. (2013). Ações do professor na condução de discussões matemáticas. *Quadrante, 22*(2), 55-81.

Ponte, J. P., Quaresma, M. & Branco, N. (2012). Práticas profissionais dos professores de Matemática. *Avances de Investigación en Educación Matemática, 1*, 65-86.

Ponte, J. P., Quaresma, M., Mata-Pereira, J., & Baptista, M. (2015). Exercícios, problemas e explorações: Perspectivas de professoras num estudo de aula. *Quadrante, 24*(2), pp. 11-134.

Ponte, J. P., Quaresma, M., Mata-Pereira, J., & Baptista, M. (2016). O estudo de aula como processo de desenvolvimento profissional de professores de matemática. *BOLEMA*, *30*(56), 868-891.

Ponte, J. P., Mata-Pereira, J., Quaresma, M., & Velez, I. (2017). Formação de professores dos primeiros anos em articulação com o contexto de prática de ensino de Matemática. *Revista Latinoamericana de Investigación En Matemática Educativa*, *20*(1), 71-94. https://doi.org/10.12802/relime.17.2013

Ribeiro, A. J., & Ponte, J. P. (2020). Um modelo teórico para organizar e compreender as oportunidades de aprendizagem de professores para

ensinar matemática. *Zetetiké*, *28*, 1-20. https://doi.org/10.20396/zet.v28i0.8659072

Sancar, R., Atal, D., & Deryakulu, D. (2021). A new framework for teachers' professional development. *Teaching and Teacher Education, 101*, 103305, 1-12. https://doi.org/10.1016/j.tate.2021.103305

Schön, D. A. (1983). *The reflective practitioner: How professional think in action*. Averbury.

Skovsmose, O. (2001). *Educação Matemática Crítica: a questão da democracia*. Papirus.

Stein, M. K., Engle, R. A., Smith, M., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning, 10*, 313-340. http://dx.doi.org/10.1080/10986060802229675

van Es, E. A., Tunney, J., Goldsmith, L., & Seago, N. (2014). A framework for the facilitation of teachers' analysis of video. *Journal of Teacher Education, 64*(4), 340-356. http://doi:10.1177/0022487114534266

Webster-Wright, A. (2009). Reframing professional development through understanding authentic professional learning. *Review of Educational Research, 79*, 702–739. https://doi.org/10.3102/0034654308330970

White, A. L., Jaworski, B., Agudelo-Valderrama, C. & Gooya, Z. (2013). Teachers learning from teachers. In M. A. (Ken) Clements et al. (Eds.), *Third International Handbook of Mathematics Education* (pp. 393-430). Springer.

Zhang, M., Lundeberg, M., & Eberhardt, J. (2011). Strategic facilitation of problem-based discussion for teacher professional development. *Journal of the Learning Sciences*, *20*(3), 342-394. https://doi.org/10.1080/10508406.2011.553258

Zopf, D. (2010). *Mathematical knowledge for teaching teachers: The mathematical work of and knowledge entailed by teacher education*. Unpublished doctoral dissertation. http://deepblue.lib.umich.edu/bitstream/handle/2027.42/77702/dzopf_1.pdf.