




Beliefs of Elementary School Teachers and Secondary School Mathematics Teachers During the Teaching Process

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Received for publication 15 Mar. 2022. Accepted after review 29 Aug. 2022
Designated editor: Claudia Lisete Oliveira Groenwald

ABSTRACT

Background: Teacher practices when teaching mathematics are supported by their beliefs on how this subject should be approached. This has an impact on the learning of students who generally perceive mathematics as an incomprehensible mechanical activity in which a series of algorithms must be followed to obtain a solution. Therefore, the following questions arise: What are the beliefs of mathematics teachers regarding the teaching process? Which methodologies do they use in the classroom? **Objective:** The objective was to analyse the beliefs of elementary school teachers and secondary school mathematics teachers during the teaching process. **Design:** The study used a descriptive quantitative approach. **Setting and participants:** The sample consisted of 100 elementary school teachers and secondary school mathematics teachers from three regions in Chile. **Data collection and analysis:** Due to the Covid-19 health crisis, Google Drive was used to send a Likert questionnaire to each of the study participants. The information was processed and quantitatively analysed with the SPSS 25.0 statistical software. **Results:** Statistically significant differences were found based on specialisation, place of work, school administration, and gender of the participants. **Conclusions:** Teachers considered that knowing, learning, and applying mathematical knowledge was possible through rote and routine activities. In addition, teachers with 5 to 8 years of experience believed that mathematics develops problem-solving skills.

Keywords: beliefs; mathematics; teachers; teaching.

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Crenças de Professores do Ensino Geral Básico e Secundário em Matemática no Processo de Ensino

RESUMO

Antecedentes: As práticas dos professores durante o ensino de matemática são sustentadas pelas crenças que eles têm sobre como esse assunto deve ser abordado, aspecto que impacta na aprendizagem dos alunos, que geralmente a percebem como uma atividade mecânica incompreensível. uma série de algoritmos deve ser seguida para se chegar a um resultado. A partir do exposto, surge a pergunta: Quais são as crenças que os professores de matemática têm em relação ao processo de ensino? E a busca pelas metodologias utilizadas em sala de aula. **Objetivo:** Analisar as crenças de professores de Educação Geral Básica e Secundária em matemática durante o processo de ensino. **Desenho:** A pesquisa foi desenvolvida sob uma abordagem quantitativa de natureza descritiva. **Cenário e participantes:** A amostra é composta por 100 professores da Educação Básica e Secundária em matemática de três regiões do Chile. **Coleta e análise de dados:** Devido à atual crise de saúde, um questionário Likert foi enviado a cada um dos participantes do estudo por meio do Google Drive. Posteriormente, as informações foram processadas e analisadas quantitativamente com o software estatístico SPSS 25.0. **Resultados:** São evidenciadas diferenças estatisticamente significativas de acordo com a especialidade, local de trabalho, administração e sexo dos participantes. **Conclusões:** Os professores concebem que conhecer, aprender e aplicar o conhecimento em matemática é possível por meio de atividades de memorização e rotina. Além disso, para professores com 5 a 8 anos de prática, o ensino de matemática permite desenvolver habilidades de resolução de problemas.

Palavras-chave: crenças, matemática, professores, ensino.

INTRODUCTION

The literature reviewed in this study suggests that the beliefs of each stakeholder involved in the teaching-learning process originate from personal experiences, the direct observation of reality, and information (McLeod, 1992). Specifically, the predominant belief in the context of Chilean schools is that learning mathematics is mechanical (Gómez-Chacón, 2008).

The Chilean Ministry of Education (MINEDUC) is concerned about changing the paradigm for teaching mathematics. Teaching is based on the new curricular guidelines, which propose forming a student who perceives mathematics in his/her environment, uses the acquired knowledge to discover and analyse the world, and is able to effectively function in it (MINEDUC, 2015). Therefore, its objective is not only to modify and update the school curriculum but to ensure that, through the teaching process, students develop skills that promote mathematical thinking and its basic concepts and procedures to understand and produce information expressed in mathematical terms (MINEDUC, 2015).

Some of the skills established by the new curricular guidelines are: a) reasoning and communicating, b) modeling, c) visualization, and d) problem solving. However, this challenge involves many problems because there are constant complaints about teaching mathematics and the difficulty of the topics at each level in different social and educational contexts such as transactions, work environment, and school (Erazo & Aldana, 2015); mathematics are conceived as complex, boring, and out-of-touch with reality (Martínez-Padrón, 2008a; 2008b).

Another obstacle to the objective proposed by the new curricular guidelines is related to the beliefs of teachers to develop and work on problem solving. When there are changes in curricula and study programs, it is difficult to modify a pedagogical practice that has been applied for a long time. Fullan and Stiegelbauers (1991) indicated that a high failure rate of educational innovations is due to the inflexible beliefs of teachers. This phenomenon was also analysed by Hanbal and Herrington (2003), who established that teachers can be the opportunity or obstacle of curricular change because they tend to cling to their beliefs and do not accept new pedagogical trends. Similarly, teachers usually distance themselves from innovative challenges when their beliefs about mathematics and its teaching and learning do not coincide with the beliefs implicit in the new educational scenario (Donoso et al., 2016).

THEORETICAL BACKGROUND

Beliefs about mathematics teaching

García (2005) determined that both beliefs and practices build a structure that is complex to change, which shapes pedagogical work and the way of behaving and thinking. According to Felmer et al. (2015), the predominance of traditionalist beliefs would cause an active teacher to develop few tasks that would enable his/her students to solve mathematical problems. Overall, students and trainee teachers have a rather traditionalist belief in mathematics because this subject is only associated with assimilating formulae, procedures, and symbols (MINEDUC, 2015).

This situation was also described by Martínez-Padrón (2014), who stated that such a belief develops because teachers have continued to work with the concept-example-exercise model. This occurs because mathematics teachers still conceive their specialisation as pre-existing knowledge provided with a logical structure to be discovered and focus their attention on the manipulation of rules and procedures (Farfán & Sosa, 2007). Martínez-Padrón (2013) also considered that dimensioning and understanding mathematics through this belief possibly explains the origin of its unpopularity, which is why it continues to be appreciated by a small

group of students, given that it tends to be abhorred or hated by those who do not understand it and causes an almost collective anguish and aversion.

The development of this belief regarding mathematics was established by Gómez-Chacón (2008), who expressed that mathematical beliefs are one of the components of the implicit subjective knowledge that the individual has about mathematics and its teaching and learning. This knowledge is based on experiences that are mainly transmitted and internalised in the educational context in which students learn (Erazo & Aldana, 2015). It is frequently reported that mathematics are a difficult subject to study, understand, explain, and learn; this has given it an unchanging character that has materialised in unfavourable attitudes towards its teaching or learning (Martínez-Padrón, 2013).

These positive or negative events shape and consolidate our beliefs about mathematics; it can be beneficial and decisive in favouring its understanding if it is positive (Fernández, 2010). However, mathematics is usually associated with a mechanical process in which the student is a passive subject who is limited to applying repetitive procedures in each of the proposed classroom activities. The student only adheres to receiving information from the teacher or textbooks and only needs to repetitively assimilate, mechanise algorithms, memorise, and use concepts in which answers are either correct or incorrect (Vesga-Bravo & Losada, 2017).

Although mathematics is increasingly recognised as useful and necessary, its applications are still perceived as a science that is detached from reality. Students perceive it as a useful subject because of its importance but difficult, mechanical, and learned by drill exercises (Erazo & Aldana, 2015). This statement reflects the educational context prevailing in Chile in which mathematics classes are mainly focused on repeating exercises of varying levels of difficulty. This does not generate solid learning in students because contents are only memorised to pass the subject and are decontextualised from their reality (Cantoral, 2009). Not all those who pass mathematics possess the necessary and sufficient competencies to perform the required content and procedures (Martínez-Padrón, 2014).

Consolidation of beliefs

Schoenfeld (1992) encountered a diversity of beliefs regarding mathematical problem solving and decided to list the mathematical beliefs frequently held by students. The list included: a) Mathematical problems have only one correct answer. b) There is only one correct way to solve any problem, which is usually the rule the teacher has just demonstrated in class. c) Ordinary students cannot expect to understand mathematics; they can only expect to memorise and apply it when they

have learned it mechanically and without understanding it. d) Mathematics is a solitary activity performed by individuals in isolation. e) Students who have understood mathematics will be able to solve any problem in 5 minutes or less. f) Mathematics learnt at school has little to do with the real world. g) Formal testing is irrelevant in the process of discovery and invention.

Likewise, Martínez-Padrón (2014) identified the predominant mathematical beliefs in students, highlighting that the mathematics teacher should always provide a model exercise because mathematics is not easy, never has been easy, and it is only important to learn the concepts for the test. The author also recognised the mathematics beliefs held by teachers when teaching and planning their classes. These included: a) Mathematics is easy, but students find it hard to learn. This is why teachers should not complicate matters for students when teaching or evaluating them. b) Teachers need to teach simple things because students will not be able to make progress with the content. c) Mathematics should be taught to be applied but not using very complex activities. d) Mathematics has to be taught in context and students have to solve problems related to daily life.

There is no doubt that the mathematical beliefs mentioned by Schoenfeld (1992) and Martínez-Padrón (2014) are those that predominate in the school population. Leo (2012) recognised the importance of mathematics and its applications in the lives of people; despite its relevance, there is a belief that has led to prejudices and prevented achieving a good academic performance in the subject.

The teacher can influence this perception in the development and consolidation of our mathematical beliefs because it is possible to point out the usefulness of the knowledge being learnt during the teaching process of the content proposed in the programmes at each level (MINEDUC, 2015). This could modify the beliefs of students over time (Agudelo & Valderrama, 2006; Cross, 2009; Handal & Herrington, 2003).

For Martínez-Padrón (2013), the beliefs of a teacher are factors that have a significant impact on his/her work. They provide the basis for decisions in the classroom and influence both the content mobilised in the classroom and the objectives pursued in the selection of learning content and activities. Mathematical beliefs can become a factor that favours the development of the skills established by the MINEDUC in 2015.

Erazo and Aldana (2015) indicated that the beliefs of students are as important as the content; it is therefore essential to encourage them to conduct research in a context that demystifies mathematics. However, reality is far from what is desirable. The teaching of mathematics is rather traditionalist in nature and

students simply replicate a process that was taught; therefore, they fail to visualise the applications and uses of the content, which ultimately produces unfavourable attitudes towards mathematics and problem solving (Bahamonde & Vicuña, 2011). Therefore, together with the acquired negative experiences, students consolidate a belief about mathematics that can be considered an obstacle during the teaching and learning process (Martínez-Padrón, 2013). As a result, Rizo and Capistrós (1999) determined that the emergence of certain beliefs in students about problems can be barriers that are very difficult to overcome and can seriously hinder their behaviour towards this activity.

This is a complex situation because the new curricular guidelines propose a curriculum focused on skill development, but this objective is still far from the classroom reality. These arguments not only enable us to reflect on and discuss the analysed topic but also to ask questions about how we approach the teaching of problem solving. For example, what strategies are implemented in the teaching and learning process when faced with mathematical problems? and how do we prepare future teachers in this area? Furthermore, if we are aware of the poor results in this area, why do we continue to apply the same teaching methodologies? In response to the latter question, Bahamonde and Vicuña (2011) stated that this phenomenon occurs because teachers are extremely mechanical and repetitive and do not progress in non-traditional procedures. Aguilar (2003), Benítez (2013), Contreras (2009), Flores (1998), Gamboa (2014), Gil (1999), Gil and Rico (2003), and Moreno and Azcárate (2003) have indicated that this situation is a reflection of how mathematics teachers understand the teaching and learning process of their students.

However, this statement raises yet more questions. Why do teachers not dare modify or innovate their pedagogical practices? Are the necessary tools for teaching mathematics provided during teacher training? How do mathematical beliefs affect the initial training of mathematics teachers? Vesga-Bravo and Losada (2017) proposed similar questions. For example, what elements need to be considered to significantly change mathematics teacher education programmes so that teachers develop more productive and coherent beliefs and attitudes towards mathematics and its teaching and learning?

Ernest (1989) indicated that it is possible to envisage the positive and negative impact that can be generated by the beliefs held in the area of mathematics during the teaching and learning process. The author stated that these beliefs can have a great impact on the selection of mathematical content, on its teaching, and on the ways of learning it.

METHODOLOGY

Research design and approach

The present study was conducted using a quantitative approach with a quasi-experimental descriptive survey design. It sought to understand and describe the connections between each of the established variables and the study sample (Ghauri & Gronhaug, 2010).

Study sample and selection criteria

A non-probability purposive sample was used to select the participants in the study sample as proposed by McMillan and Schumacher (2005). Based on previously established criteria, that is, elementary school teachers and secondary school mathematics teachers, the aim was to select data that were relevant to the proposed objectives and investigate questions on the basis of the respective analyses (Hernández et al., 2010).

The study sample (N) consisted of 100 male and female elementary school teachers (70) and secondary school mathematics teachers (30) aged between 21 and 65 years with work experience in the mathematics specialisation ranging from 1 to 21 years or more.

Some important characteristics of the study sample included that 78 were women and 22 were men. Of the total, 62 taught mathematics in municipal schools and 38 in private subsidised schools. Educational institutions were located in the following regions: Ñuble (47), Biobío (24), and Maule (29) with 72 in urban and 28 in rural areas.

Data collection instrument

A questionnaire entitled “Beliefs of elementary school teachers and secondary school mathematics teachers during the teaching process” was used to collect data that was relevant to the objectives of the present study. The instrument first collected demographic data for each of the respondents, including academic degree, years of work experience, age, gender, place of work, and type of school administration. The second section of the questionnaire included 26 items, which made it possible to collect data on the dimension of beliefs about mathematics. It was divided into four subdimensions (Table 1).

Table 1*Dimensions and subdimensions related to beliefs about mathematics.*

Dimension	Subdimension	Composition
Beliefs in mathematics	What is mathematical knowledge?	Know what mathematical knowledge means for teachers
	Purpose of teaching and learning mathematics in elementary and secondary education	Objective of learning mathematics during initial training
	Activities included in the teaching and learning process in mathematics classes	Objective of activities developed in the classroom during the teaching and learning process
	Strategies for learning mathematics	Know the strategies that teachers more or less consider adequate to learn mathematics

Data collection techniques and analysis

Due to the sanitary crisis caused by the Covid-19 epidemic and the protocols established by the health authorities, data was collected by sending a questionnaire via Google Drive to the email address of each participant. Participants responded on a voluntary basis to the 26 statements included in the Likert questionnaire with a rating between 5 (completely agree) and 1 (completely disagree).

Prior to this process and in accordance with the ethical aspects of research, each of the teachers was informed of the objectives to make them aware of the aim of the study. It was also indicated in the study consent form that their names and responses would remain anonymous and would be used only for research purposes.

The collected data were inputted into a matrix developed with the SPSS 25.0 statistical software for analysis. Descriptive statistics of central tendency (mean), dispersion (standard deviation), frequencies, and percentages were calculated to construct four tables according to the previously established subdimensions. The aim was to determine the beliefs about mathematics held by elementary school teachers and secondary school mathematics teachers during the teaching process.

Finally, an independent samples t-test with $\alpha = 0.05$ was used to establish statistically significant differences according to gender, type of school administration, and years of work experience.

RESULTS

The results were grouped in frequency tables according to the subdimension of beliefs about mathematics. The aim was to show the respective analyses comparing the means according to the type of school administration, specialisation, gender, and location of the study sample by a t-test. Finally, the data was reported according to the years of experience of the teachers by a one-way ANOVA table.

Subdimensions related to beliefs about mathematics

Tables 2, 3, 4, and 5 show the mean, standard deviation, and analysed percentages for each item and subdimensions corresponding to the study sample (N = 100).

Table 2

What is mathematical knowledge?

			CA	A	NAND	DA	CDA
	M	SD	%	%	%	%	%
Know too many definitions	3.04	1.171	13.0	22.0	30.0	26.0	9.0
Know the fundamental theorems of the learning objectives	3.72	1.111	24.0	45.0	16.0	9.0	6.0
Memorise many procedures that can be used to solve exercises	3.08	1.169	10.0	33.0	21.0	27.0	9.0
Decide the importance of a mathematical concept	3.91	0.944	27.0	48.0	16.0	7.0	2.0

Apply creative processes to different situations	4.43	0.856	60.0	29.0	6.0	4.0	1.0
Solve any problem related to the topic being studied	4.22	0.811	41.0	44.0	12.0	2.0	1.0

M: Mean; SD: standard deviation; CA: completely agree; A: agree; NAND: neither agree nor disagree; DA: disagree; CDS: completely disagree.

In the “What is mathematical knowledge?” subdimension, the highest rated beliefs were that knowing mathematics involves applying creative processes in different situations (M = 4.43; SD = 0.856) and solving any problem related to the topic being studied (M = 4.22; SD = 0.811).

As for the items, there is a positive rating of 48% and 45% for deciding the importance of a mathematical concept (M = 3.91; SD = 0.944) and knowing the fundamental theorems of the learning objectives (M = 3.72; SD = 1.111), respectively. In addition, 33% agreed that knowing mathematics implies memorizing many procedures to solve exercises (M = 3.08; SD = 1.169) and 30% neither agreed nor disagreed with the statement that their understanding is due to knowing too many definitions (M = 3.04; SD = 1.171).

Table 3

Purpose of teaching and learning mathematics in elementary and secondary education.

			CA	DA	NAND	DA	CDA
	M	SD	%	%	%	%	%
Develop mathematical skills to address creatively contextualised problem solving	4.54	0.658	62.0	31.0	6.0	1.0	0.0
Provide mathematical knowledge to deal intelligently with	4.49	0.759	60.0	33.0	4.0	2.0	1.0

practical real-life problems							
Enhance skills to deal intelligently with practical real-life problems	4.58	0.606	63.0	33.0	3.0	1.0	0.0
Develop mathematical skills to address creatively non-contextualised problem solving	3.95	0.957	30.0	45.0	18.0	4.0	3.0

M: Mean; SD: standard deviation; CA: completely agree; A: agree; NAND: neither agree nor disagree; DA: disagree; CDS: completely disagree.

In the “Purpose of teaching and learning mathematics in elementary and secondary education” subdimension, the study participants positively rated with a high score three items. The first item was to enhance skills to deal intelligently with practical real-life problems ($M = 4.58$, $SD = 0.606$), followed by develop mathematical skills to address creatively contextualised problem solving ($M = 4.54$; $SD = 0.658$) and provide mathematical knowledge to deal intelligently with practical real-life problems ($M = 4.49$; $SD = 0.759$). There was also a positive acceptance, 45% agreed, that the aim of the subject of mathematics is to develop mathematical skills to address creatively non-contextualised problem solving ($M = 3.95$; $SD = 0.957$).

Table 4

Activities in the process of teaching and learning in mathematics classes.

			CA	A	NAND	DA	CDA
	M	SD	%	%	%	%	%
Consolidate the learning objective(s) proposed in class	4.22	0.705	36.0	52.0	10.0	2.0	0.0

Develop logical thinking	4.69	0.506	71.0	27.0	2.0	0.0	0.0
Develop the theorems covered in the subject	3.76	0.933	25.0	41.0	25.0	11.0	0.0
Develop creative thinking	4.62	0.648	69.0	26.0	3.0	2.0	0.0
Develop argumentation skills	4.42	0.806	57.0	32.0	8.0	2.0	1.0
Develop communication skills	4.39	0.737	53.0	34.0	12.0	1.0	0.0
Develop modelling skills	4.45	0.642	52.0	42.0	5.0	1.0	0.0
Develop problem-solving skills	4.77	0.423	77.0	23.0	0.0	0.0	0.0
Develop representation skills	4.49	0.628	55.0	40.0	4.0	1.0	0.0
Establish a relationship with the context of the students	4.48	0.731	59.0	33.0	5.0	3.0	0.0

M: Mean; SD: standard deviation; CA: completely agree; A: agree; NAND: neither agree nor disagree; DA: disagree; CDS: completely disagree.

According to Table 4, there is high acceptance, 62% on average, of participants who completely agree with several statements that emphasise the activities performed in the process of teaching and learning in mathematics classes. These include to enable the development of problem-solving skills ($M = 4.77$; $SD = 0.423$), logical ($M = 4.69$; $SD = 0.506$) and creative ($M = 4.62$; $SD = 0.648$) thinking, and establish a relationship with the context of the students ($M = 4.48$; $SD = 0.731$). In addition, statements related to developing argumentation ($M = 4.42$; $SD = 0.806$), representation ($M = 4.49$; $SD = 0.628$), communication ($M = 4.39$; $SD = 0.737$), and modelling ($M = 4.45$; $SD = 0.642$) skills were also included. Table 4 also shows positive acceptance, 52% agree, for the item associated with consolidating the learning objective(s) proposed in class ($M = 4.22$; $SD = 0.705$). Finally, 41% agreed

that mathematics activities develop the theorems covered in the subject ($M = 3.76$; $SD = 0.933$).

Table 5

Strategies for learning mathematics.

			CA	A	NAND	DA	CDA
	M	SD	%	%	%	%	%
Teamwork	4.15	0.857	41.0	36.0	21.0	1.0	1.0
Reflective situations	4.47	0.658	54.0	41.0	3.0	2.0	0.0
Do a series of exercises	3.65	1.058	24.0	35.0	25.0	14.0	2.0
Constant effort	4.31	0.825	49.0	38.0	8.0	5.0	0.0
Search for complementary information	4.12	0.820	38.0	38.0	22.0	2.0	0.0

M: Mean; SD: standard deviation; CA: completely agree; A: agree; NAND: neither agree nor disagree; DA: disagree; CDS: completely disagree.

In the “Strategies for learning mathematics” subdimension, 54%, 49%, and 41% completely agreed with the items referring to learning through reflective situations ($M = 4.47$; $SD = 0.658$), constant effort ($M = 4.31$; $SD = 0.825$), and teamwork ($M = 4.15$; $SD = 0.857$), respectively. The item associated with complementary information showed a positively high rating ($M = 4.12$; $SD = 0.20$) with 38% of participants stating that they completely agreed. There was also a positive rating, 35% of respondents agreed with the strategy related to doing a series of exercises ($M = 3.65$; $ST = 1.050$).

Student’s t-test

The Student’s t-test for mean equality was used for the dimensions of type of school administration, specialisation, gender, and place of work (Tables 6, 7, 8, 9). Table 6 shows significant statistical differences for type of school administration. Teachers working in municipal schools positively rated the item stating that mathematical knowledge consists of knowing the fundamental theorems of the learning objectives ($M = 3.92$; $SD = 1.076$). Meanwhile, teachers from private

subsidised schools positively rated this item but with a lower score ($M = 3.39$; $SD = 1.104$).

Table 6

Comparison of means according to the type of school administration.

		Group statistics		Student's t-test for mean equality			
		M	SD	t	df	p(bi)	Effect direction
Know the fundamental theorems of the learning objectives	Municipal	3.92	1.076	2.344	98	0.021	M > S
	Private subsidised	3.39	1.104				

M: Mean; SD: standard deviation; t: difference between means of two groups; df: degrees of freedom; p(bi): significance level. Effect direction: M: municipal; S: private subsidised.

Table 7

Comparison of means according to the specialisation of the participants.

		Group statistics		Student's t-test for mean equality			
		M	SD	t	df	p(bi)	Effect direction
Know too many definitions	Elementary education pedagogy	3.20	1.098	2.123	98	0.036	EEP > MEP
	Mathematics education pedagogy	2.67	1.269				
Memorise many procedures that can be used to solve exercises	Elementary education pedagogy	3.36	1.064	3.867	98	0.000	EEP > MEP
	Mathematics education pedagogy	2.43	1.165				

Develop the theorems covered in the subject	Elementary education pedagogy	3.90	0.935	2.343	98	0.021	EED > MEP
	Mathematics education pedagogy	3.43	0.858				

M: Mean; SD: standard deviation; t: difference between means of two groups; df: degrees of freedom; p(bi): significance level. Effect direction: EEP: Elementary education pedagogy; MEP: Mathematics education pedagogy.

Table 7 indicates significant statistical differences regarding the specialisation of the respondents. Elementary school teachers positively rated that mathematical knowledge consists of knowing too many definitions (M = 3.20; SD = 1.098) and memorising many procedures that can be used to solve exercises (M = 3.36; SD = 1.064). Meanwhile, secondary school mathematics teachers negatively rated the abovementioned items (M = 2.67; SD = 1.269). As for the purpose of teaching mathematics, there were statistically significant differences when considering that its objective is to develop the theorems covered in class. Elementary school teachers positively rated this statement (M = 3.90; SD = 0.935), while secondary school mathematics teachers had an equally positive but lower rating (M = 3.34; SD = 0.858).

Table 8 discloses significant statistical differences according to the gender of the participants, especially when considering that mathematics can be learnt by doing a series of exercises. Male teachers (M = 4.05; SD = 0.722) provided a higher rating than female teachers (M = 3.54; SD = 1.113) for this statement.

Table 8

Comparison of means according to the gender of the participants.

	Group statistics	Student's t-test for mean equality					
		M	SD	t	df	p(bi)	Effect direction
Do a series of exercises	Female	3.54	1.113	-2.016	98	0.047	M > F
	Male	4.05	0.722				

M: Mean; SD: standard deviation; t: difference between means of two groups; df: degrees of freedom; p(bi): significance level. Effect direction: F: Female; M: Male.

Table 9 shows the significant statistical differences for the place of work of the participants. Teachers working in rural areas positively rated that mathematical knowledge consists of knowing too many definitions (M = 3.43; SD = 1.136), while those in urban areas (M = 2.89; SD = 1.157) negatively rated this statement. Both urban and rural groups positively rated the item referring to mathematical knowledge as knowing the fundamental theorems of the learning objectives. However, those working in rural areas (M = 4.18; SD = 0.723) had a higher rating than those from urban areas (M = 3.54; SD = 1.186).

Table 9

Comparison of means according to the place of work of participants.

	Group statistics		Student's t-test for mean equality				
		M	SD	t	df	p(bi)	Effect direction
Know too many definitions	Urban	2.89	1.157	- 2.104	98	0.038	R > U
	Rural	3.43	1.136				
Know the fundamental theorems of the learning objectives	Urban	3.54	1.186	- 2.652	98	0.009	R > U
	Rural	4.18	0.723				

M: Mean; SD: standard deviation; t: difference between means of two groups; df: degrees of freedom; p(bi): significance level. Effect direction: U: Urban; R: Rural.

One-way ANOVA and Tukey's post-hoc test

A one-way ANOVA was performed followed by Tukey's post-hoc test to find statistically significant differences. Results are displayed in Tables 10 and 11, respectively. Table 10 shows that for the "Activities in the process of teaching and learning in mathematics classes" subdimension (F = 2.479; p = 0.037), the "develop problem-solving skills" item exhibits statistically significant differences among teachers who have between 5 to 8 years (M = 4.96) of work experience and those with 9 to 12 years of experience (M = 4.62).

Table 10

One-way ANOVA for work experience.

		Sum of squares	df	Quadratic mean	F	α
Develop problem-solving skills	Between groups	2.063	5	0.413	2.479	0.037
	Within groups	15.647	94	0.166		
	Total	17.710	99			

df: Degrees of freedom; F: ratio of two variances; α : significance level.**Table 11**

Tukey's post-hoc test for work experience.

Dependent variable	(I) Work experience	(J) Work experience	Difference of means (I-J)	SE	α
Develop problem-solving skills	1-4 years	5-8 years	-0.108	0.124	0.951
		9-12 years	0.229	0.119	0.388
		13-16 years	0.350	0.223	0.623
		17-20 years	0.250	0.204	0.823
		≥ 21 years	0.072	0.133	0.994
	5-8 years	1-4 years	0.108	0.124	0.951
		9-12 years	0.338*	0.113	0.039
		13-16 years	0.458	0.220	0.307
		17-20 years	0.358	0.201	0.479
		≥ 21 years	0.181	0.127	0.715
9-12 years	1-4 years	-0.229	0.119	0.388	
	5-8 years	-0.338*	0.113	0.039	
	13-16 years	0.121	0.218	0.994	
	17-20 years	0.021	0.198	1.000	

	≥ 21 years	-0.157	0.122	0.793
13-16 years	1-4 years	-0.350	0.223	0.623
	5-8 years	-0.458	0.220	0.307
	9-12 years	-0.121	0.218	0.994
	17-20 years	-0.100	0.274	0.999
	≥ 21 years	-0.278	0.226	0.820
17-20 years	1-4 years	-0.250	0.204	0.823
	5-8 years	-0.358	0.201	0.479
	9-12 years	-0.021	0.198	1.000
	13-16 years	0.100	0.274	0.999
	≥ 21 years	-0.178	0.206	0.955
≥ 21 years	1-4 years	-0.072	0.133	0.994
	5-8 years	-0.181	0.127	0.715
	9-12 years	0.157	0.122	0.793
	13-16 years	0.278	0.226	0.820
	17-20 years	0.178	0.206	0.955

SE: Standard error; α : significance level.

DISCUSSION

Based on the study results, it can be established that the beliefs of elementary school teachers and secondary school mathematics teachers regarding the teaching of this subject are consistent with the MINEDUC (2015) objectives. These aim to educate a student who perceives mathematics in his/her environment and uses the acquired knowledge to discover and analyse the world to be able to function effectively in it (MINEDUC, 2015).

An analysis of the responses indicated that the purpose of knowing and learning mathematics is to apply creative processes in different situations that develop the ability to solve intelligently real-life problems by reflective learning situations. According to Hanbal and Herrington (2003), the abovementioned shows how the study participants were able to see an opportunity in the curricular objectives proposed by the MINEDUC (2015) in the mathematics curricular guidelines.

According to the analyses and statistically significant differences found by the Student's t-test for independent samples, it was observed that female elementary school teachers working in rural areas under a municipal administration had traditionalist beliefs (Ferlmer et al., 2015). For these participants, knowing, learning, and applying mathematical knowledge was only possible by rote, mechanical, and routine learning activities (Martínez-Padrón, 2014; Vesga-Bravo & Losada, 2017; Cantoral, 2009).

Likewise, the work experience of teachers, specifically those who have been working for 5 to 8 years, was confirmed by Martínez-Padrón (2013), who stated that the beliefs of teachers influence the objectives they wish to attain in the teaching process. For this group in particular, the activities that are carried out in the mathematics class are aimed at developing problem-solving skills.

CONCLUSIONS

It can be established that the beliefs of each of the participants were mainly focused on the fact that teaching mathematics provides a range of knowledge specific to this area of study, which fosters the development of the skills proposed in the new Chilean Ministry of Education (MINEDUC, 2015) curricular guidelines. Students can enhance these skills through reflective learning situations, teamwork, and constant effort.

The beliefs of the participants regarding the question "What is mathematical knowledge?" refers to the application of creative processes in different situations. There was a prevalent belief that the purpose of mathematics teaching and learning in elementary and secondary education is to enhance skills to deal intelligently with practical real-life problems. As for activities in the teaching and learning process in mathematics classes, there is a belief that such tasks will develop problem-solving skills. Finally, regarding the strategies used to learn mathematics, beliefs are expressed that learning is possible to the extent that the mathematics teacher generates reflective situations.

It was determined that statistically significant differences existed for gender in the subdimension on strategies for learning mathematics. Male participants believed that learning could be achieved by a series of exercises that the student has to do.

There were statistically significant differences for the type of school administration. Teachers in municipal schools believe that mathematical knowledge consists in knowing the fundamental theorems of the learning objectives.

There were statistically significant differences between secondary school mathematics teachers and elementary school teachers related to their specialisation. There is a prevalent belief in elementary school teachers that mathematical knowledge is related to knowing too many definitions and fundamental theorems. Elementary school teachers also expressed their belief that the purpose of the activities carried out in the teaching and learning process in mathematics classes is to develop the theorems that are covered in the subject.

There were statistically significant differences related to the place of work of the participants as to what mathematical knowledge means. The beliefs of teachers working in rural areas were associated with a student who understands mathematics when he/she knows too many definitions and the fundamental theorems of the learning objectives.

There were no statistically significant differences between groups according to age; however, differences were significant for work experience related to the purpose of activities carried out in the teaching and learning process in mathematics classes. Specifically, teachers with 5 to 8 years of teaching experience were highlighted because they believed that each mathematics activity could develop problem-solving skills.

ACKNOWLEDGEMENTS

We are grateful for the support of the National Commission for Scientific Research and Technology (Comisión Nacional de Investigación Científica y Tecnología, CONICYT) through the Fondecyt project N°1180993 entitled “Estudio sobre las configuraciones del conocimiento matemático que desarrollan profesores en comunidades educativas en contextos de ruralidad, migración y comunidades mapuche” (Study of mathematical knowledge configurations developed by teachers in educational communities in rural, migrant, and Mapuche community contexts). We acknowledge the doctoral scholarships of the National Research and Development Agency (Agencia Nacional de Investigación y Desarrollo, ANID) and the Education and Interculturality Research Group 195023 GI/VC of the Universidad del Bío-Bío, Chile.

AUTHORSHIP CONTRIBUTION STATEMENT

Each of the authors contributed to the tabulation, analysis, interpretation and conclusion of the data analysed in each of the tables presented.

DATA AVAILABILITY STATEMENT

The documents used in the present study are available at:

<https://www.dropbox.com/s/fara91kqqmcc5e1/Disponibilidad%20de%20datos.sav?dl=0>

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ANNEXES

QUESTIONNAIRE

Beliefs of elementary school teachers and secondary school mathematics teachers during the teaching process

Dear Teacher:

As part of a doctoral study, a group of researchers is interested in studying the beliefs about mathematics held by elementary school teachers and secondary school mathematics teachers.

For this purpose, you are requested to complete the information in this questionnaire as thoroughly as possible. The collected information will only be used for academic and research purposes and kept confidential.

I) Personal data:

Observation: Complete each of the following.

1) Teacher education

a) Elementary school teacher

b) Secondary school mathematics teacher

2) Work experience year(s)

3) Age

.....

4) Gender:

 M F

For questions 5 to 7, consider the location where you work the most hours if you work in more than one school.

5) Where is the school in which you work?

a) Urban Name of institution.....

b) Rural Name of institution.....

c) Other Name of institution.....

6) What is the type of administration of the school where you work?

a) Municipal

- b) Private subsidised
- c) Private

7) In which region is the school where you work?

- a) Ñuble
- b) Biobío
- c) Maule
- d) Other

Which one?

II. Questionnaire

Observation: Below is a series of statements that you must answer in the box by circling the option that most closely reflects what you think or believe.

Indicator	Acronym	Score
Completely agree	CA	5
Agree	A	4
Neither agree nor disagree	NAND	3
Disagree	DA	2
Completely disagree	CDS	1

Mathematical knowledge is to:	CA	A	NAND	DA	CDS
1) Know too many definitions.	5	4	3	2	1
2) Know the fundamental theorems of the learning objectives.	5	4	3	2	1
3) Memorise many procedures that can be used to solve exercises.	5	4	3	2	1
4) Decide the importance of a mathematical concept.	5	4	3	2	1
5) Apply creative processes to different situations.	5	4	3	2	1
6) Solve any problem related to the topic being studied.	5	4	3	2	1
The most important purpose of teaching mathematics is to:	CDA	A	NAND	DA	CDS
7) Develop mathematical skills to address creatively contextualised problem solving.	5	4	3	2	1

8) Provide mathematical knowledge to deal intelligently with practical real-life problems.	5	4	3	2	1
9) Enhance skills to deal intelligently with practical real-life problems.	5	4	3	2	1
10) Develop mathematical skills to address creatively non-contextualised problem solving.	5	4	3	2	1
The objective of activities included in the teaching and learning process in mathematics classes is to:	CDA	A	NAND	DA	CDS
11) Consolidate the learning objective(s) proposed in class.	5	4	3	2	1
12) Develop logical thinking.	5	4	3	2	1
13) Develop the theorems covered in the subject.	5	4	3	2	1
14) Develop creative thinking.	5	4	3	2	1
15) Develop argumentation skills.	5	4	3	2	1
16) Develop communication skills.	5	4	3	2	1
17) Develop modelling skills.	5	4	3	2	1
18) Develop problem-solving skills.	5	4	3	2	1
19) Develop representation skills.	5	4	3	2	1
20) Establish a relationship with the context of the students.	5	4	3	2	1
Mathematics can be learnt through:	CDA	A	NAND	DA	CDS
21) Teamwork.	5	4	3	2	1
22) Reflective situations.	5	4	3	2	1
23) Doing a series of exercises.	5	4	3	2	1
24) Constant effort.	5	4	3	2	1
25) Searching for complementary information.	5	4	3	2	1

Thank you for your cooperation!