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Conceptual Maps: A Tool for Assessing the Meaningful Learning of Engineering Students in a Pre-Calculus Course

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ABSTRACT

Context: Students of Differential and Integral Calculus in Engineering programs show difficulties when dealing with the concept of function, one of the fundamental concepts of higher Mathematics, with countless applications in courses of the programs mentioned above. **Objectives:** To evaluate the contribution of a potentially meaningful teaching unit to promote learning of first-degree polynomial, exponential, and logarithmic functions, and to evaluate Engineering students' learning through conceptual maps. **Design:** A potentially meaningful teaching unit of concepts related to first-degree polynomial, exponential and logarithmic functions was applied. The research is based on the Meaningful Learning theory. **Scenario and participants:** Five Pre-calculus students from Engineering programs at a community university in Rio Grande do Sul. **Data collection and analysis:** Data collection took place through initial and final assessment instruments, in addition to questionnaires and conceptual maps, during the students' participation in an extracurricular activity, with analysis of the conceptual maps based on an adaptation of Cañas and co-workers' Topological Taxonomy. **Results:** The results showed that the adopted methodology has the potential to promote meaningful learning in the context of Engineering Education and that conceptual maps are reliable tools for assessing learning. **Conclusion:** it is possible to claim that the first-degree polynomial, exponential and logarithmic functions, once worked in a contextualised way, help in the understanding of different concepts and contribute to the occurrence of meaningful learning, whenever there are learning conditions for such, which requires considering the students' prior knowledge, in addition to their interest in learning.

Keywords: potentially meaningful teaching unit; meaningful learning; teaching functions; concept maps; engineering education.

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INTRODUCTION

It is well known that a considerable fraction of higher education students in Brazil and other countries around the world have many difficulties in programs in which Mathematics is an important course (Bigotte de Almeida, Queiruga-Dios & Cáceres, 2021; Flegg, Mallet & Lupton, 2012; Nasser, Sousa & Torraza, 2012; Nortvedt & Siqveland, 2019; Souza, da Silva & Gessinger, 2016; Van Dyken & Benson, 2019). With the expansion of the number of Engineering programs in Brazil, many students go to university without having developed the necessary cognitive structures at the level of basic education in Mathematics, which leads to high failure rates, mainly in Calculus courses, in addition to dropout rates (de Moraes & Valente, 2016; Madeira et al., 2018; Oliveira et al., 2020). Most of these students are workers looking for professional qualifications and acquisition of scientific knowledge, aiming to become able to compete successfully in the job market (Boff et al., 2014; Reis, Cunha & Spritzer, 2012).

Allied with this, we are still faced with the teaching of Mathematics full of formulas, and algebraic expressions presented ready-made, without building relationships and meanings, promoting, therefore, discouraging classes, lacking in challenges, unattractive and that end up encouraging the memorisation of formulas, that soon after the exams are forgotten (Harris et al., 2015). Thus, as a cause of difficulties in Mathematics, the teaching practice is especially questioned (Peres, 2017). In this context, the new National Curriculum Guidelines for Engineering programs (Brasil, 2019) proclaim that it is necessary to design curricula that consider the adoption of more modern pedagogical strategies and methods that are more adequate to the new global reality, that is, strategies and methods that allow the student to be the leading actor of his learning process. Students, for their part, also need to develop the critical skill of learning how to learn, and, for that, the teacher also needs to be competent to help them learn how to self-regulate learning.

Based on reflections and findings, such as those presented, a didactic sequence called Potentially Meaningful Teaching Unit (PMTU) was built, experimented and analysed, using problem-situations from the area of Science, Technology, Engineering and Mathematics (STEM). The PMTU objective was to help new students in an Engineering program learn first-degree polynomial, exponential and logarithmic functions. Conceptual maps were used to look for evidence of students' meaningful learning.

This article is organised in such a way that, in the next sections, the theoretical framework, methodology, and the analysis of the conceptual maps produced by the students when experiencing the methodology are presented. Finally, some final considerations are presented.

THEORETICAL FRAMEWORK

Studies by Education researchers such as Fry, Ketteridge and Marshall (2015), Demo (2014), Carvalho (2011), Moreira (2011), Biggs and Tang (2011), Zabala and Arnau (2010), Barr and Tagg (1995) have contributed with the discussion of ways to reflect on important questions about the role of education and proven manners of conducting the teaching and learning processes. These same researchers emphasise the need to transform learning environments into privileged spaces for articulation between theory and practice and for a multiplicity of experiences. According to the new National Curriculum Guidelines for Engineering programs, "... the Pedagogical Project of the Program (PPP) must provide for reception and levelling systems, aiming to reduce dropout and course repetition" (Brasil, 2019). Adopting holistic approaches, strategies and methods that contribute to training based on competencies can undoubtedly favour the reception and permanence of students in Engineering programs.

An ethical, reflective and humanised professional is the product of training in which he produced senses and meanings about his learning. For Masini (2011), David Ausubel's Meaningful Learning is a cognitive theory that considers the process of knowledge construction as a process of understanding, reflection and attribution of meanings by the subject, in interaction with the social environment, by forming culture and being formed by it. In line with this position, a Potentially Significant Teaching Unit (PMTU) was proposed in this research work as a teaching methodology based on Ausubel's theory of Meaningful Learning.

A summary of David Ausubel's Meaningful Learning theory is presented in the present theoretical framework. Then, the PMTU methodology, by Marco Antonio Moreira (2011), is discussed and the steps for its development. To conclude this theoretical framework, the theoretical foundation of conceptual maps, one of the tools for evaluating students' learning, is presented.

Meaningful Learning

Many authors agree that students need to be encouraged to study and learn how to learn (Holbrook & Rannikmae, 2007; Novak & Gowin, 1999;

Pozo & Font, 1999; Villas-Boas et al., 2011). Students need to understand that disciplines in Exact Sciences, such as Mathematics, Physics, Chemistry and Biology, help to explain the world in which they live, to know and discover existing technologies and serve as a basis for future technologies.

In this context, the teaching unit in this research work was inspired and theoretically based on David Ausubel's Theory of Meaningful Learning (TML) to offer an alternative to teaching that traditionally promotes rote learning. One of the most essential principles of TML, according to Ausubel and co-workers, can be summarised in the following proposition:

> If I had to reduce all educational psychology to a single principle, I would say this: the single most important factor influencing learning is what the learner already knows. Find this out and teach him accordingly. (Ausubel et al., 1978)

When Ausubel refers to what the student already knows, he refers to the cognitive structure already built, the organisation of ideas and the prior knowledge that the individual brings. It also refers to aspects of the cognitive structure that support the learning of new knowledge. Investigating what the student already knows is not simple, as it implies discovering which concepts, ideas, and knowledge are available in the individual's cognitive structure and their interrelationships and organisations.

This prior knowledge of the individual is called by Ausubel (2003) “subsumer”. The subsumer is the cognitive content that the student builds throughout his life, capable of playing the role of anchoring new knowledge so that it has meaning for him. Through the interaction of the most relevant concepts of the cognitive structure, which are the subsumers, with the new information, meaningful learning occurs.

Indeed,

> the central concept of Ausubel's theory is that of meaningful learning, the process through which new information acquires meaning by interaction (not association) with pre-existing relevant aspects in the cognitive structure, which, in turn, are also modified during this process. (Moreira, 1999, p. 26)

In other words, it is the relationship of the new content (concept, idea, proposition) with the subsumers that the student has that can enable the construction of new concepts.

However, for meaningful learning to occur, two conditions are necessary. The first is to have potentially meaningful instructional material, that is, that the instructional material is relatable to the student's cognitive structure. The condition for this material to be potentially meaningful includes two factors: the nature of the material and the nature of the student's cognitive structure (Moreira, 2009).

The second condition, perhaps more difficult to satisfy than the first, is that the student manifests a willingness to learn and thus has an interest in relating the new material to his cognitive structure. According to Novak and Gowin’s vision (1999), the importance of the student's predisposition for learning is intertwined with the integration of thoughts, feelings and actions. It is not exactly about liking the subject or being motivated; the student must be predisposed to interactively relate the new knowledge to what he already has in his cognitive structure, modifying, enriching and giving meaning to this knowledge. Therefore, he must be aware that, without understanding, he will not have good results in learning.

One way of organising concepts is the potentially meaningful teaching units (PMTU), which transpose theoretical assumptions and teaching practice. Therefore, a PMTU can be understood as a “didactic sequence based on learning theories, particularly that of meaningful learning” (Moreira, 2011, p. 1).

Potentially Meaningful Teaching Unit (PMTU)

There is a growing recognition of the importance of studying Mathematics in a structured way, relating contents and interacting in situations that allow learning based on concepts that are meaningful to students. It is possible to understand that an explanation of these conditions, linked to the PMTU methodology, as mentioned above, according to Moreira (2011), is in the understanding that teaching is a means while meaningful learning is the end, and that teaching materials that seek this learning must be potentially meaningful.

Problem-situations are indicated as learning enhancers in teaching and learning activities to be part of didactic sequences (DS). Firme, Ribeiro and Barbosa (2008) highlight the importance of these situations, as they evoke contextualised teaching, with problematic situations related to real contexts, both in social and environmental spheres. Problem-situations allow contextualised questioning with different dimensions such as teaching objectives, varied resources, real problems, experimental activities, technological, environmental, sociocultural issues, among others. Such actions are strategic to develop new skills, to make the teaching process an instrument of human construction committed to the needs of society, as recommended by the National Curriculum Guidelines for Engineering programs (Brasil, 2019) and INOVA Engineering (CNI, 2006).

Learning strategies using DS, understood as sets of planned, experienced and analysed activities, can constitute favourable means for acquiring meanings. Zabala (1998) points out that to understand the educational value of a didactic sequence and the reasons that justify it, it is necessary to identify its stages, defining activities and the relationships established in this construction space. According to the same author, a DS is a set of ordered, structured and articulated activities for achieving educational objectives, which have a beginning and an end known by teachers and students. To contribute to this new educational scenario, the proposal for the construction of PMTU arises, which are theoretically based DS, directed towards meaningful learning, and thus, for both reasons, have a greater potential for success in the occurrence of meaningful learning. (Moreira, 2011).

These DS have guiding principles: identification of previous or subsuming knowledge, advance organisers, problem-situations, progressive differentiation, integrative reconciliation and consolidation.

Being a DS, some sequential steps must be observed in the construction of a PMTU, which Moreira presents as:

1. Definition of the specific topic.
2. Creating and proposing situations in which the student can express prior knowledge.
3. Proposition of problem-situations at an introductory level, preparing the introduction of the knowledge that is intended to be taught.
4. Presentation of general aspects of the knowledge to be taught, considering the progressive differentiation, starting with more general aspects, with an overview of the whole, of what is most important in the teaching unit, for example an oral exposition, followed by a collaborative activity in small groups and complemented with a presentation activity.
5. Resumption of the most general and structuring aspects in a new presentation at a higher level of complexity.

6. To conclude the unit, resume the most relevant characteristics of the content in question from an integrative perspective at higher levels of complexity in relation to previous situations, seeking integrative reconciliation. This consists of relating concepts and pointing out relevant similarities and differences, enabling the description of a new perceptible reality.

7. Assessment of student learning.

8. PMTU evaluation.

Based on the theoretical framework and guiding principles described above, this research sought to elaborate a PMTU with the central theme "Mathematical functions in Engineering Education", addressing the contents of first-degree polynomial function, exponential function, and logarithmic function.

According to Moreira (1999), learning is associated with a particular *corpus* of knowledge, and the actions of teaching and learning are characterised by relations of different representations of the same knowledge: that of the teacher, that of the student and that of the teaching material. This aspect highlights the complex and dynamic character of teaching and points to the importance of assessment in its different stages: planning, teaching itself and final evaluation.

Conceptual Maps

Within the scope of a PMTU, assessment is understood as the search for evidence of meaningful learning, considering that assessment is a progressive process and does not occur only at the end of its application (Moreira, 2011). For this, the teacher's role is to mediate this assessment, focused on capturing meanings, always aiming at meaningful learning, and, consequently, having the possibility of success in the occurrence of meaningful learning (Moreira, 2011).

Joseph Novak drew on David Ausubel's ideas on meaningful learning to lay the groundwork for using concept maps in the mid-1970s (Novak, 2000). According to Moreira (2013), the potential of conceptual maps is evident as a strategy that favours meaningful learning in a formal teaching situation, and as a tool for assessing learning and analysing curricular content.

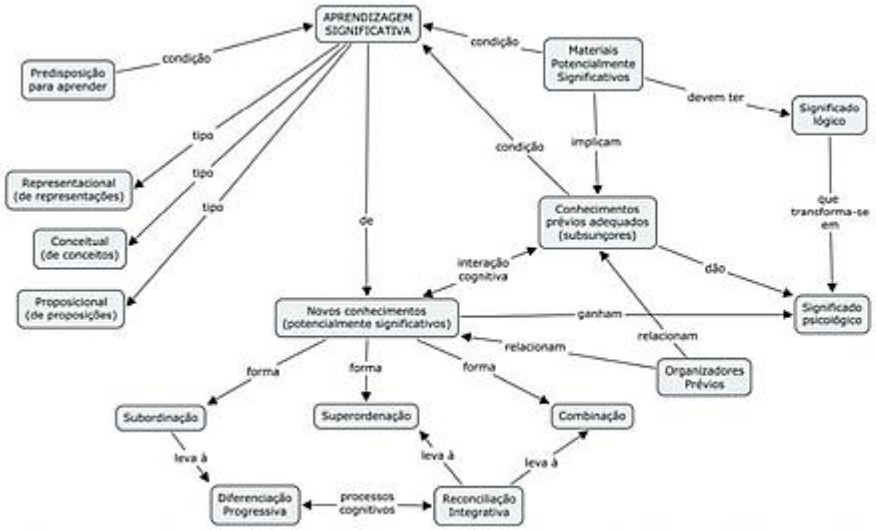
Concepts are essential for human understanding, and a concept map is an organiser of concepts. The conceptual maps proposed by Novak and Gowin

(1999) are tools to represent meaningful relationships in the form of propositions and, therefore, can serve as instructional materials and as assessment tools, as they are external representations that, in some way, reflect internal representations (mental representations) of who made the map (Moreira, 2011). Thus, when preparing a map, the student will be elucidating and explaining his knowledge; that is, in addition to expressing what he knows and how he knows it, he learns with the intellectual effort of making this representation (Trindade & Hartwing, 2012).

According to Novak and Cañas (2008), conceptual maps present concepts in a hierarchical way, secondarily connecting to other concepts, and establishing meaning between them. In this context, concept maps are restricted to the use of concepts.

Figure 1

Example of Conceptual Map (Moreira, 2013)



Although concept maps usually have a hierarchical structure and include arrows, they should not be confused with organisation charts or flow diagrams, as they do not imply sequence, temporality, or organisational or power hierarchies. According to Moreira (2012, p.1), “conceptual maps are diagrams of meanings, meaningful relations, conceptual hierarchies, if applicable. Concept maps do not seek to classify concepts, but to relate and hierarchise them.” Figure 1shows an example of a conceptual map whose subject is Meaningful Learning.

The propositions, the fundamental units of conceptual maps, are made up of the elements indicated in the equation in Figure 2.

Figure 2

Structure of a proposition

old concept + linking word/phrase + new concept → proposition

The mandatory insertion of a linking word or a linking phrase, which clearly shows the relationship between two concepts, gives the conceptual map its fundamental characteristic of the search for meaning and differentiates it from other visual knowledge organisers (Cañas et al., 2006).

Concept maps are used as one of the primary assessment instruments in this research work. According to Moreira,

> As a learning assessment tool, conceptual maps can be used to obtain a view of the conceptual organisation that the learner attributes to a given piece of knowledge. It is basically a non-traditional assessment technique that seeks information about the meanings and meaningful relationships between key concepts of the teaching subject from the student's point of view. It is most appropriate for a qualitative, formative assessment of learning. (Moreira, 2012, p.5)

As meaningful learning necessarily implies the attribution of idiosyncratic meanings, conceptual maps, represented by teachers and students, reflect the meanings attributed by themselves to their ideas. Both maps used by teachers as a didactic resource, and constructed by students, as a learning or assessment activity have idiosyncratic components. This means that there is no such thing as a “correct” concept map. Therefore, the teacher should not expect the student to present the “correct” conceptual map of a specific content. What the student presents is his map; what is important is not whether it is right or not but whether it gives evidence that meaningful learning has taken place.

From all of this, it can be deduced that conceptual maps are assessment tools with very specific characteristics. It makes no sense to want to assess them as one assesses a multiple-choice test or a numerical problem. The analysis of concept maps is essentially qualitative. Instead of worrying about assigning a score to the map drawn by the student, the teacher should be concerned with interpreting the information given by the student on the map to obtain evidence of the occurrence of meaningful learning.

METHODOLOGY

Based on a diagnostic assessment, a PMTU was constructed tp propose a differentiated teaching and learning environment, which would provide more engagement on the part of students and the occurrence of meaningful learning. To this end, contextualised situations were planned so that students could understand the relationship and applicability of Mathematics in routine engineering situations.

The research was carried out with a group from the Pre-Calculus course, from the first semester of 2016, with students from Environmental Engineering, Automotive Engineering, Civil Engineering, Computer Engineering, Control and Automation Engineering, Mechanical Engineering, Production Engineering and Chemical Engineering, from a university in Rio Grande do Sul.

The research began with a group of students from the Pre-Calculus course. These students participated in a diagnostic evaluation and were invited to participate in an activity outside the course hours, where a PMTU would be developed on the first-degree polynomial, exponential and logarithmic functions. Of these, five signed up to participate. The PMTU development began three weeks after the start of Pre-Calculus classes.

Based on Moreira (2011) and as described in the Theoretical Framework section, the PMTU was organised into eight steps. Steps 1 through 3 are the planning steps. In step 2, the diagnostic evaluation was applied. The other steps, 4 to 8, were performed twice and are briefly described in Table 1.

Aiming to seek evidence of the occurrence of meaningful learning by the students, different assessment instruments were used throughout the PMTU development.

Table 1

Steps 4 to 8 of PMTU and activities carried out in each step

Meeting	Activities carried out
1	**Step 4** - First-degree polynomial function - Construction of an "initial" concept map on first-degree polynomial functions; First problem-situation: experiment on Ohm's Law; Contextualized problems about first-degree polynomial function.
2	**Step 5 –** First-degree polynomial function - Second problem-situation: experiment on the solubility of urea in water; Contextualized problems on first-degree polynomial function.
3	**Steps 6, 7 and 8 -** First-degree polynomial function - Third problem-situation and evaluation: experiment on the uniformly accelerated rectilinear motion; Contextualized problems on first-degree polynomial functions; Construction of a "final" concept map on first-degree polynomial functions.
4	**Steps 4 -** Step 4 - Exponential and Logarithmic Functions - Construction of an "initial" concept map on exponential and logarithmic functions; First problem-situation: magnitude of an earthquake and magnitude of a celestial body; Contextualized problems about exponential and logarithmic functions.
5	**Step 5 -** Exponential and Logarithmic Functions - Second problem situation: experiment on charge and discharge of a capacitor; Contextualized problems about exponential and logarithmic functions.
6	**Steps 6, 7 and 8 -** Exponential and Logarithmic Functions - Third problem-situation and evaluation - Experiment on the hydrogenic potential of some substances; Contextualized problems about exponential and logarithmic functions; Construction of a "final" conceptual map on exponential and logarithmic functions.

In the first meeting dedicated to the study of each function, the students were asked to elaborate on a conceptual map about that function individually. The elaboration of the initial maps aimed to advance in the identification of the subsumers, starting with the analysis of the answers of the diagnostic evaluation to, in the end, use them as a comparative in the process of searching for evidence of constructed knowledge by the students after experiencing the PMTU activities. In addition, as highlighted in the theoretical framework of this research work, the elaboration of the conceptual map favours meaningful learning by promoting the expression of existing relationships between the concepts present in the students' cognitive structure.

In the last meeting dedicated to the study of each function, the students were asked again, individually, to elaborate a conceptual map of the theme. For this, they received the same orientation they had for the preparation of the initial conceptual maps, adding that they should make associations of the theme with the various subjects addressed during the PMTU development, such as experiments, problem situations discussed, exercises worked on, and explanations promoted by the researcher. The objective of preparing this conceptual map was to verify a possible conceptual change, if there was an increase in meanings in relation to the subject studied and to look for evidence of the occurrence of meaningful learning.

Besides the conceptual maps, at the beginning and the end of the study of the first-degree polynomial, exponential and logarithmic functions, records were made of the students' manifestations in relation to the understanding or difficulties encountered during the resolution of problem-situations experienced at the PMTU. Besides these records, an individual final assessment was planned with contextualised open-ended questions related to the studied phenomena. This evaluation process permeated all steps of the PMTU development.

It is important to emphasise the attention given, in this research, to the theoretical framework that underlies it, with regard to the activities planned and carried out during the meetings, according to the steps of development of the PMTU (Moreira, 2011), in particular, concerning advance organisers, the principles of progressive differentiation and integrative reconciliation, and problem-situations. For Moreira, "in the progressive differentiation, more general and inclusive ideas, concepts, propositions of the content must be presented and progressively differentiated, throughout the process." It is understood that this is enhanced by activities that consider subsumers, such as support for constructing new knowledge.

As for integrative reconciliation, Moreira explains that teaching should explore relationships between ideas, concepts and propositions, relating them, reorganising them and providing the understanding of new meanings. The recombination of existing subsumers in the cognitive structure is also a question here. The problem-situations, in turn, were not proposed as exercises for applying formulas but as situations that aimed to give the opportunity to assign meaning to concepts, at an increasing level of complexity, but never before making sure of a good understanding of what was previously worked on.

Given the above, an assessment of the PMTU potential was carried out in a last meeting. Besides, such potential will be considered based on the comparative analysis of the initial and final conceptual maps of each function, which were produced by the students, in addition to the numerical results of the individual evaluation that the students performed at the end of the study of each function.

With this, it is expected to demonstrate evidence of the occurrence of meaningful learning in the next section.

RESULTS AND DISCUSSION

In this section, we present the analysis of the conceptual maps prepared by the students before and after the study of each type of function, and how these results are presented as evidence that the PMTU favoured the occurrence of meaningful learning in those who participated in its application.

It is important to mention that the diagnostic assessment contained questions that outlined the student's profile and mathematical knowledge questions about the first-degree polynomial, exponential and logarithmic functions. With the results of this diagnostic assessment, some subsumers of the students were diagnosed and considered in the elaboration of the activities proposed in the PMTU and in order to contemplate the advance organisers. In addition, it was also possible to obtain profile data that allowed us to understand better the students with whom we would work.

The initial and final conceptual maps were used, respectively, to identify subsumers, already started with the diagnostic evaluation and to demonstrate knowledge built by the students throughout the application of the PMTU.

When analysing a concept map, the most important thing is not whether it is correct or not; but whether it provides evidence that the student is meaningfully learning the content. This can be observed through the number of links and the quality with which the concepts are presented. That is, a map can present only relevant concepts, interconnected or not, and present them with their respective definitions, which may be correct, partially correct or incorrect. All these possibilities are analysed and classified in different ways.

Besides showing the knowledge constructed by the students, the conceptual maps were used to assess whether there was any change in the organisation of students' thinking and also to assess the effectiveness of the PMTU in the occurrence of meaningful learning.

For the analysis of the maps, in terms of structure, an adaptation of the topological taxonomy proposed and validated by Cañas and co-workers (2006) was used. It is a way of structurally classifying and evaluating the diversity of conceptual maps through common parameters that make it possible to measure advances in the process of building maps.

In the initial concept maps analysis, the map’s structural complexity was considered, as stated by the topological taxonomy, which is valued, even without the correct presentation of the meanings of concepts and propositions. With that in mind, it was possible, in the analysis of the final maps, to evaluate the student's progress during the process, which started with the use of concepts formed by a few words and propositions, which, little by little, revealed the addition of correct definitions, cross-links, interconnections and new concepts.

The topology comprises five criteria known to be good descriptors of concept map structure the size of concept labels, presence of linking phrases, number of branching points, the existence of hierarchical structure, and number of cross-links. Each of these criteria is evaluated on a scale of 0 to 6, where 0 (zero) is the simplest and 6 is the most elaborate. Below is the description of the criteria, as they were adapted for this research:

- Criterion C1 (size of concept labels): the ability to present concepts through summarised texts is a starting point for constructing increasingly complex and sophisticated cognitive structures. Concept maps that do not present concepts can be classified as level 0 (the lowest). About the mathematical functions, which were the subject of this research, algebraic, graphical, numerical and verbal forms were considered. In the latter case of summarised texts or explanatory sentences are also considered.
- Criterion C2 (presence of linking phrases): the observation of the presence or absence of linking phrases is carried out, not exactly the words or phrases used. From this point of view, any symbol intentionally placed on the map to establish a specific relationship between two concepts must be interpreted as a linking phrase.

- Criterion C3 (number of branching points): the degree of branching of a concept map is associated with the number of branching points. A branching point occurs when two or more links branch out from a concept or linking phrase (the exact number does not matter). This criterion refers to the number of concepts with more than one branching point, not the number of links that emerge from a concept.
- Criterion C4 (existence of hierarchical structure): the presence of hierarchical structure was determined by counting the number of links between the root concept and the concept furthest from it. This criterion only makes sense if the map has at least one root concept.
- Criterion C5 (presence of cross-links): a cross-link was considered if two concepts, which belong to two links associated with different branching points of the concept map, are connected. A cross-link is essentially a proposition between concepts, none of which is the root concept, and usually located in different sectors of a concept map in such a way as to form a closed loop.

Based on this, to carry out the structural analysis of the conceptual maps, Table 2 was organised, in which the relationships between the criteria and the levels of this adaptation of the topological taxonomy are presented.

Table 2

Relationship between criteria and levels in the structural analysis of concept maps

Level	Criteria				
	C1 Concepts	**C2 Linking words/ phrases**	**C3 Branching points**	**C4 Hierarchical structure**	**C5 Cross-links**
0	No association with concepts related to the theme	Does not present	Linear (0 or 1 point)	None	None

1	Presence of topics related to functions in one of the forms: algebraic or verbal, less than 50%	Presents less than 50%	Linear (0 or 1 point)	None	None
2	Presence of topics related to functions in at least two forms: algebraic, verbal or graphical, less than 50%	Presents less than 50%	Low degree of branching (2 points)	1 level	None
3	Presence of topics related to functions in one of the forms: algebraic or verbal, equal to 50%	Presents 50%	Medium degree of branching (3 or 4 points)	2 levels	None
4	Presence of topics related to functions, in at least two forms: algebraic, verbal or graphical, equal to 50%	Presents 50%	High degree of branching (5 or 6 points)	3 levels	None

5	Presence of topics related to functions in one of the forms: algebraic or verbal, greater than 50%	Presents more than 50%	High degree of branching (5 or 6 points)	4 levels	1 or 2
6	Presence of topics related to functions, in at least two forms: algebraic, verbal or graphical, greater than 50%	Presents more than 50%	Very high degree of branching (7 or more points)	5 or more levels	More than 2

It is important to emphasise that the levels for each criterion were adapted by the authors for this study, considering the following concepts present in the study of functions: *definition*; *dependent variable* and *independent variable*; *domain* and *image*; *numerical representation*; *algebraic representation*; *graphical representation*; *verbal representation*; *increase/decrease*; *zeros* and *intercepts*. In addition to these, depending on the function, the following specificities were considered: (i) for the first-degree polynomial function: *real coefficients* (customarily called a and b); *linear function with one or two terms* ($y = ax + b$ or $y = ax$); (ii) for the exponential function: *number e*; *conditions for base e*; *asymptotes*; (iii) for the logarithmic function: *base e*, *other bases*; *graphical representation* (Anton, Bivens & Davis, 2014; Hughes-Hallett et al., 2014; Stewart, 2006).

Table 3 shows, in a summarised form, the representative symbols for each criterion and level.

Table 3

Structural evaluation of concept maps

Level	C1	C2	C3	C4	C5
0	NC	0	0	0	0
1	$PC_1 < 0{,}5$	< 0,5	0 – 1	0	0
2	$PC_2 < 0{,}5$	< 0,5	2	1	0
3	$PC_1 = 0{,}5$	1	3 – 4	2	0
4	$PC_2 = 0{,}5$	1	5 – 6	3	0
5	$PC_1 > 0{,}5$	1	5 – 6	4	1 – 2
6	$PC_2 > 0{,}5$	1	≥ 7	≥ 5	> 2

Table 4 presents, in descriptive form, the meanings attributed to each acronym.

Table 4

Meaning of acronyms used in criterion C1

Acronym	Meaning
NC	No association with concepts related to the theme.
$PC_1 < 0{,}5$	Presence of topics related to functions in one of the forms: algebraic or verbal, *less than 50%.*
$PC_2 < 0{,}5$	Presence of topics related to functions, in at least two forms: algebraic, verbal or graphical, *less than 50%.*
$PC_1 = 0{,}5$	Presence of topics related to functions in one of the forms: algebraic or verbal, *equal to 50%.*
$PC_2 = 0{,}5$	Presence of topics related to functions, in at least two forms: algebraic, verbal or graphical, *equal to 50%.*
$PC_1 > 0{,}5$	Presence of topics related to functions in one of the forms: algebraic or verbal, *greater than 50%.*
$PC_2 > 0{,}5$	Presence of topics related to functions, in at least two forms: algebraic, verbal or graphical, *greater than 50%.*

As for criterion C2, the absence of linking words/phrases is represented by (0), the presence of half or less of linking words/phrases between concepts by (< 0.5), the presence of more than half by (> 0, 5) and the number (1) represents the presence of linking words/phrases in all concepts presented in the conceptual map.

As for criteria C3, C4 and C5, as shown in Table 2: C3 represents the number of concepts with more than one branching point; C4 represents the number of links between the root concept and the most distant one; and C5 represents the numbers of cross-links, respectively, present in the conceptual map.

Next, we present the analysis of the conceptual maps constructed before experiencing the PMTU and the conceptual maps constructed after completing the PMTU. For each function, the maps of one of the students were selected for analysis.

During the analysis of the materials produced by them, participants are designated as Student 1, Student 2, Student 3, Student 4 and Student 5.

Concept maps for the first-degree polynomial function

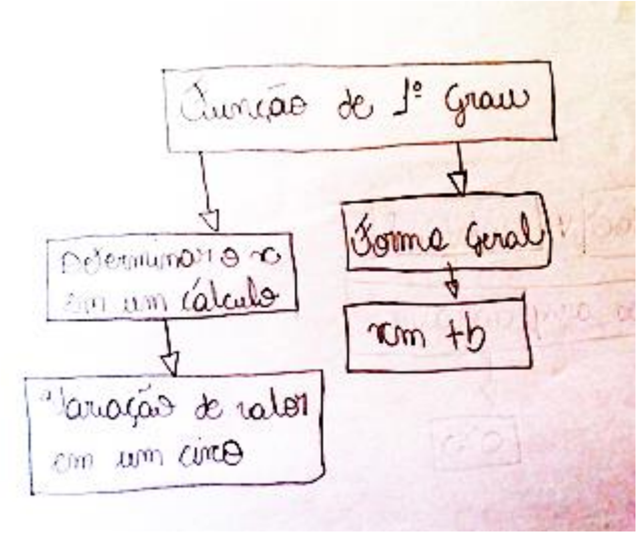
For the study of the first-degree polynomial function, three meetings were held. The first activity proposed in the first meeting was elaborating conceptual maps about the function to be studied. Thus, in Figure 3, the initial conceptual map on the first-degree polynomial function prepared by Student 2 is presented.

In a qualitative analysis of this initial map, produced by Student 2, it is possible to observe the presence of subsumers, such as what he correctly called "*general form*", writing it partially as “*xm* + *b*”. The fact of referring to the "*general form*" and representing it, even if incompletely, is relevant for meaningful learning, as it is important information that can be considered by the teacher when planning activities with the advance organisers.

Indeed, the need for more familiarisation with the mathematical language, evidenced by undergraduate students in Engineering, is one of the difficulties to be overcome. And Student 2, referring to the "*general form*", demonstrates that he has this prior knowledge, which is worth considering as a subsumer in the promoted discussions, which was recorded and, in fact, occurred during the development of the PMTU.

Figure 3

Student 2's initial concept map for the first-degree polynomial function



Other expressions used in mathematical language, such as "*numerical value*", "*intercepts*", "*variable*", "*dependent variable*", "*independent variable*", "*verbal representation*", "*algebraic representation*", "*numerical representation*", "*graphical representation*" or "*geometric representation*", among others, were found in this research in other conceptual maps produced by other students. These are generally used, but they are not mathematical concepts and could be referred to with other words. Therefore, its use already deserves to be considered when promoting meaningful learning since language is fundamental for capturing meanings (Moreira, 2011).

From the point of view of topological taxonomy, the analysis of Student 2's initial conceptual map is presented in Table 5, based on the criteria presented in Tables 2 and 3.

According to Table 5, in criterion C1, level 1 was marked, since Student 2 presents less than half of the concepts considered relevant to the subject. In criterion C2, level 0 was marked, as there are no linking words/phrases in the conceptual map. As for criterion C3, level 1 was marked, as the map only has one branching point. Regarding criterion C4, the marked level was 2, as the map presents two links between the root concept and the concept furthest from it. Finally, in criterion C5, as there are no cross-links, all levels containing 0 were marked.

Table 5

Structural evaluation of Student 2's initial concept map

Level	C1	C2	C3	C4	C5
0	NC	**0**	0	0	**0**
1	**PC_1< 0,5**	< 0,5	**0 – 1**	0	**0**
2	PC_2< 0,5	< 0,5	2	1	**0**
3	PC_1= 0,5	1	3 – 4	**2**	**0**
4	PC_2= 0,5	1	5 – 6	3	**0**
5	PC_1> 0,5	1	5 – 6	4	1 – 2
6	PC_2> 0,5	1	≥ 7	≥ 5	> 2

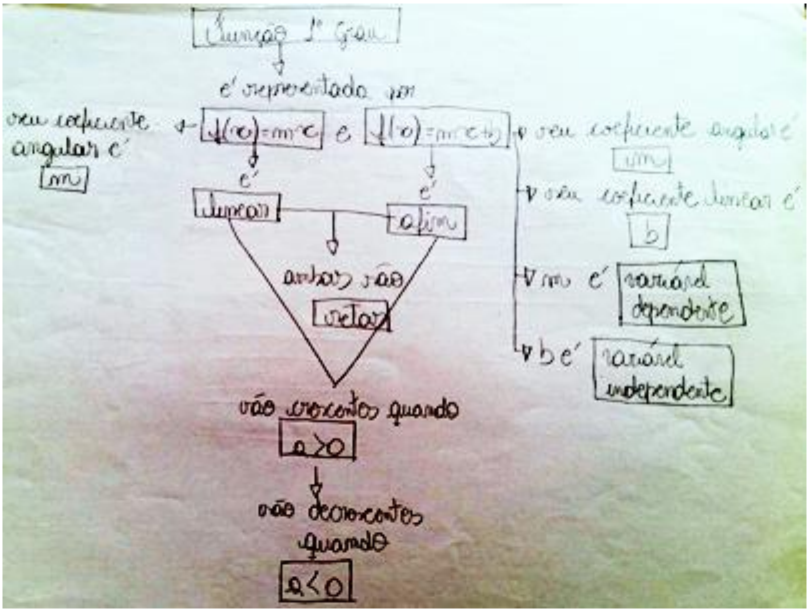
Student 2's conceptual map was classified as level 1 on the topological scale used for its structural evaluation, given that this was the concept obtained in three criteria.

After experiencing all the activities proposed at the PMTU on first-degree polynomial functions, Student 2 elaborated the conceptual map shown in Figure 4.

Although the final map still presents some wrong information, such as considering the independent variable "*b*" and the dependent variable "*m*" instead of "*x*" and "*y*", respectively, it can be observed that there was the attribution of meanings when able to differentiate the linear function with one or two terms, definitions that were not even mentioned in the initial conceptual map produced by this student. In addition, regarding evidence of the occurrence of meaningful learning, progressive differentiation is observed when correctly writing the general forms of the linear function with one or two terms, distinguishing them correctly, and presenting their main terms, as well as correctly and with the respective meanings. Indeed, the subsumer "*General form → xm + b*", shown by Student 2 in the initial map, may have been the support point for the advance organisers, planned to be part of the study material, to serve as a bridge to the expansion, with meaning, of the concepts related to first-degree polynomial functions. And the recombination of those pre-existing elements in the cognitive structure, demonstrated by correctly writing *first-degree function → represented by → f(x) = mx and f(x) = mx + b*, highlighting the meaning of the terms *m* and *b*, can be understood as integrative reconciliation. Therefore, during the studies, it is up to the teacher to promote the integration of the concepts approached, solve different questions, and consider the knowledge constructed through the proposed activities. Meaningful learning can thus be recognised, because of this organisation, in the assessment activities promoted by the teacher.

Figure 4

Student 2's final concept map for the first-degree polynomial function



In the final conceptual map analysis, it can be observed that Student 2 presented more than half of the concepts relevant to the subject, and thus, in criterion C1, level 5 was marked. Regarding criterion C2, levels containing 1 were marked (levels 3 to 6), as there are more than 50% of linking words/phrases in the concept map. As for criterion C3, level 3 was marked, as it has three branching points. Regarding criterion C4, the marked level was 4 because, as can be seen, there are three links between the root concept, the first-degree polynomial function, and the concept farthest from it. Finally, in criterion C5, as the map has only one cross-link, level 5 was marked.

Therefore, considering the topological taxonomy, the analysis of the final conceptual map is presented in Table 6.

Table 6

Structural evaluation of Student 2's final concept map

Level	C1	C2	C3	C4	C5
0	NC	0	0	0	0
1	$PC_1 < 0,5$	< 0,5	0 – 1	0	0
2	$PC_2 < 0,5$	< 0,5	2	1	0
3	$PC_1 = 0,5$	**1**	**3 – 4**	2	0
4	$PC_2 = 0,5$	**1**	5 – 6	**3**	0
5	**$PC_1 > 0,5$**	**1**	5 – 6	4	**1 – 2**
6	$PC_2 > 0,5$	**1**	≥ 7	≥ 5	> 2

The combination of these results, for this student, resulted in a conceptual map considered at level 5 since this is the most repeated level.

In the comparison between the initial conceptual map produced by Student 2, classified as level 1, and the final map of level 5, it is evident that there was an acquisition of meanings about the subject worked on at the PMTU and, therefore, the occurrence of meaningful learning.

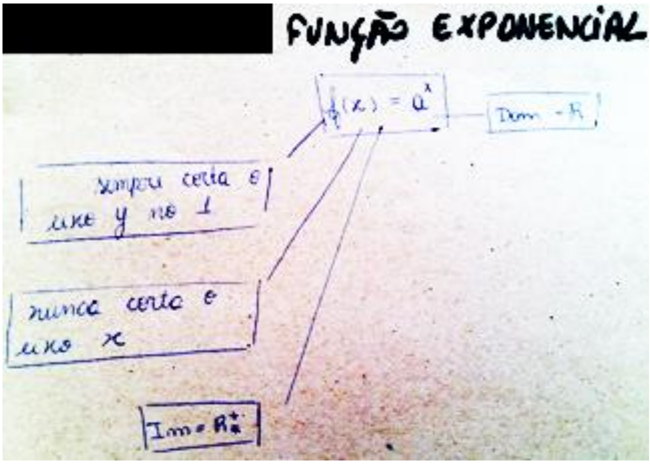
Concept maps for the exponential function

At this point, it is important to point out that the students, when producing the initial conceptual map on the exponential function, had already studied this content in Pre-calculus classes and, most likely, in high school.

Figure 5 shows the initial conceptual map of Student 5 on the exponential function, prepared at the first PMTU meeting that addressed this subject and was selected for this analysis.

Figure 5

Student 5's initial concept map for the exponential function



An analysis of the elements present in the initial conceptual map of Figure 5 reveals that everything that was presented has some relation with the exponential function, except for $f(x) = a^x$, although they seem to be meaningless expressions, at that moment, for Student 5, because of the way they were written. However, if properly considered by the teacher, as subsumers, they may facilitate the recognition and integration of concepts in the student's cognitive structure. This can be justified because: $f(x) = a^x$ is an expression commonly associated with the definition of the exponential function; the proposition *always cuts the y-axis at 1* is associated with the idea that the graph of such a function contains the point (0,1); *never cuts the x-axis* means that the referred function, in the way it was written, does not contain zero and, finally, $Im = IR_*^+$ is related to the image of the function, presented here incorrectly, regarding the notation used. Besides, the restriction was not made for the "*base a*", which, as is known, must be greater than 1. However, these are ideas that Student 5 had as prior knowledge and which, recognised by the teacher, can give meaning to the knowledge to be constructed.

Therefore, the analysis of this conceptual map was carried out according to the same criteria observed in the analysis of the maps on first-degree polynomial function produced by Student 2; that is, it was based on the criteria presented in Tables 2 and 3, resulting in Table 7.

Table 7

Structural evaluation of Student 5's initial concept map

Level	C1	C2	C3	C4	C5
0	NC	**0**	0	0	**0**
1	$PC_1 < 0,5$	< 0,5	**0 – 1**	0	**0**
2	**$PC_2 < 0,5$**	< 0,5	2	**1**	**0**
3	$PC_1 = 0,5$	1	3 – 4	2	**0**
4	$PC_2 = 0,5$	1	5 – 6	3	**0**
5	$PC_1 > 0,5$	1	5 – 6	4	1 – 2
6	$PC_2 > 0,5$	1	≥ 7	≥ 5	> 2

As can be seen in Table 7: in criterion C1, level 2 was marked, since Student 5 presents less than half of the concepts relevant to the subject. In criterion C2, level 0 was marked, as there are no linking words/phrases in the conceptual map. As for criterion C3, level 1 was marked, as the map has only one branching point. In relation to criterion C4, the marked level was 2, because, as can be seen, there is only one link between the root concept, $f(x) = ax$, and the concept farthest from it. Finally, in criterion C5, as the map has no cross-links, levels containing 0 were marked.

Because three criteria fit level 2, this concept map was classified as level 2 on the topological scale used for its structural assessment.

After experiencing all the proposed activities of the PMTU, Student 5 created the final conceptual map, shown in Figure 6.

In the analysis of this final conceptual map, it can be observed that Student 5 presented more than 50% of concepts relevant to the subject, and thus, in criterion C1, level 6 was marked. As for criterion C2, levels containing 1 were marked. (Levels 3 to 6), as there are more than 50% linking words/phrases in this concept map. Regarding criterion C3, level 5 was marked because the map has 6 branching points. Concerning criterion C4, the marked level was level 6 because, as can be seen, there are 6 links between the root concept, *exponential function*, and the most distant concepts, which are *the displacements of the function graph*. Finally, regarding criterion C5, as there are no cross-links, levels containing 0 were marked.

Figure 6

Student 5's final concept map for the exponential function

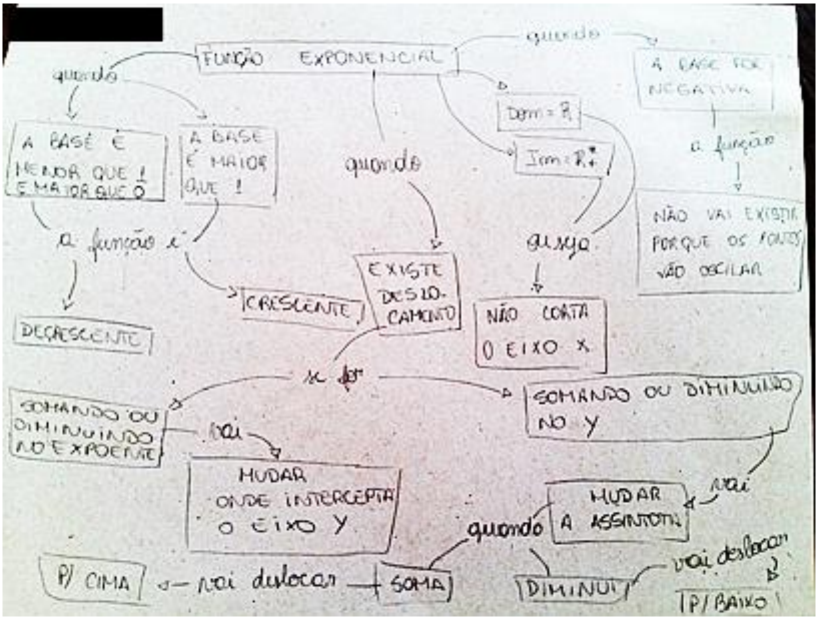


Table 8 presents the evaluation corresponding to this analysis.

Table 8

Structural evaluation of Student 5's second concept map

Level	C1	C2	C3	C4	C5
0	NC	0	0	0	**0**
1	$PC_1 < 0,5$	< 0,5	0 – 1	0	**0**
2	$PC_2 < 0,5$	< 0,5	2	1	**0**
3	$PC_1 = 0,5$	**1**	3 – 4	2	**0**
4	$PC_2 = 0,5$	**1**	5 – 6	3	**0**
5	$PC_1 > 0,5$	**1**	**5 – 6**	4	1 – 2
6	**$PC_2 > 0,5$**	**1**	≥ 7	**≥ 5**	> 2

Considering Table 8, the highest number of marks is at level 6. Therefore, the level defined for the final conceptual map produced by this student was level 6, which demonstrates a great evolution in the meanings on the subject addressed compared to the initial conceptual map.

In elaborating the final conceptual map on exponential function, Student 5 presents an expressive variety of concepts. In this aspect,

> [...] the number of concepts present in relation to the amount intended by the teacher can signal whether learning is closer to memorisation or closer to meaningful. The presence of links between concepts and the appropriate words to indicate the relationship involved are signs that meaningful learning has occurred, demonstrating that the learner has perceived the relationship between the concepts. (Novak, 2000, p. 58)

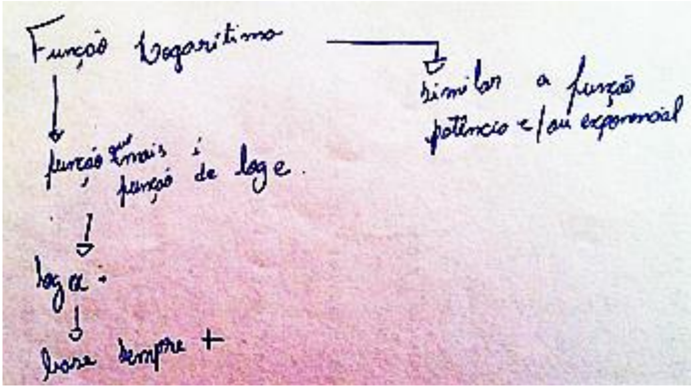
Considering the main concepts of the Meaningful Learning theory and the elaboration of the final concept map (Figure 6), there is no doubt that the principles of progressive differentiation and integrative reconciliation are present in Student 5's cognitive structure.

Concept maps for the logarithmic function

In the first meeting that addressed the logarithmic function, the students elaborated conceptual maps related to it. In Figure 7, the initial conceptual map prepared by Student 3 is presented.

Figure 7

Student 3's initial concept map for the logarithmic function



In the analysis of this map, the following can be considered as subsumers: the symbol commonly used for the logarithmic function, *log x*, the number "*e*", mentioned in the expression *log e*, the expression "*base always* +", which is associated with the condition for the existence of the logarithm of a number, and the expression "*similar to the power and/or exponential function*". Despite the expression "*similar to the power and/or exponential function*" being somewhat confusing and even meaningless, in the way it was written, it should be valued as a subsumer to be used in the planning of activities, so that the student can relate what he understood about the written expression with the new knowledge. What the teacher has at his disposal, in this case, are ideas that, if well explored, through advance organizers consistent with the concepts to be built, could lead to the occurrence of meaningful learning.

Table 9 presents the analysis of Student 3's initial conceptual map for the logarithmic function, based on the structural evaluation criteria found in Tables 2 and 3.

Table 9

Structural evaluation of Student 3's initial concept map

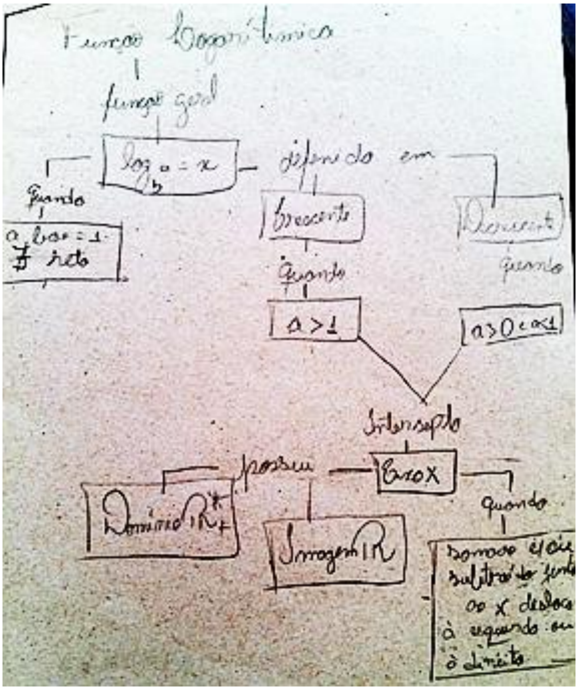
Level	C1	C2	C3	C4	C5
0	NC	**0**	0	**0**	**0**
1	**$PC_1 < 0,5$**	< 0,5	**0 – 1**	**0**	**0**

2	$PC_2 < 0,5$	< 0,5	2	1	**0**
3	$PC_1 = 0,5$	1	3 – 4	2	**0**
4	$PC_2 = 0,5$	1	5 – 6	3	**0**
5	$PC_1 > 0,5$	1	5 – 6	4	1 – 2
6	$PC_2 > 0,5$	1	≥ 7	≥ 5	> 2

According to Table 9, in criterion C1, level 1 was marked, since Student 5 presents less than half of the concepts relevant to the subject and that he presented them only in the verbal way. In criterion C2, level 0 was marked, as there are no linking words/phrases in the conceptual map. As for criterion C3, level 1 was marked, as the map has only one branching point. Regarding criterion C4, the marked level was 0 and 1, because, as can be seen, the conceptual map does not present any hierarchy. Finally, in criterion C5, as there are no cross-links, the levels containing 0 (levels 1 to 4) were marked.

Figure 8

Student 3's final concept map for the logarithmic function



As can be seen in Table 9, level 0 was scored three times and level 1 was scored four times. The measure used to evaluate this conceptual map was the use of the C1 criterion, as the concepts related to the subject are an essential part in the construction of a conceptual map. Thus, the level defined for the final concept map produced by Student 3 was level 1.

After experiencing all the proposed activities of the PMTU, Student 3 elaborated the final conceptual map on the logarithmic function, shown in Figure 8.

In the analysis of this final conceptual map, it can be observed that Student 3 presented more than 50% of concepts relevant to the subject, and thus, level 5 was marked in C1. As for criterion C2, levels containing 1 were marked (levels 3 to 6), as there are more than 50% linking words/phrases in this concept map. Regarding criterion C3, level 3 was marked, because the map has 4 branching points. About criterion C4, the marked level was level 6, because, as can be seen, there are more than five levels between the root concept, logarithmic function, and the most distant concepts on the map. Finally, regarding criterion C5, as the map has only one cross-link, level 5 was marked.

Table 10 presents the result of this analysis.

Table 10

Structural evaluation of Student 3's second concept map

Level	C1	C2	C3	C4	C5
0	NC	0	0	0	0
1	$PC_1 < 0,5$	< 0,5	0 – 1	0	0
2	$PC_2 < 0,5$	< 0,5	2	1	0
3	$PC_1 = 0,5$	**1**	**3 – 4**	2	0
4	$PC_2 = 0,5$	**1**	5 – 6	3	0
5	$\mathbf{PC_1 > 0,5}$	**1**	5 – 6	4	**1 – 2**
6	$PC_2 > 0,5$	**1**	≥ 7	**≥ 5**	> 2

In Table 10, it is observed that level 5 was considered in three of the criteria. So, the level defined for the final conceptual map produced by this student was level 5, which shows a great evolution in the meanings constructed on the subject addressed, when compared to the initial conceptual map.

Finally, it is worth mentioning that the previously mentioned initial and final assessments were also analysed to seek evidence of knowledge built by students throughout the application of the PMTU. The evidence of the occurrence of meaningful learning, already observed in the analysis of the conceptual maps, as previously presented, was also revealed in the comparative analysis of the initial and final evaluations. It was observed that there was conceptual evolution on the part of all participating students. It should be noted, however, that the use of mathematical concepts in everyday situations is part of a process that requires the attention of teachers, at all levels of education. In particular, in Engineering undergraduate programs, in which the applications of Mathematics concepts in other courses of the program are approached and worked on with an emphasis on practice. Therefore, the learning process, with regard to the interpretation and resolution of problem-situations, must be continuous and must deserve attention in other courses of the program.

FINAL REMARKS

The application of the PMTU stimulated the development of critical and reflexive postures on the study of mathematical functions. Students had the opportunity to be more active in their learning processes. They were able to seek solutions to doubts that arose during the development of the PMTU. They had greater autonomy facing the different situations that were proposed. Students clearly, in most cases, developed and perfected their oral and written communication and graphical expression skills.

The approach of conceptual, procedural and attitudinal contents, through the PMTU, enabled the resolution of contextualized problem-situations, thus favouring the construction of knowledge and understanding of the world, as claimed by Quartieri and collaborators (2014).

The elaboration of conceptual maps demonstrated the potential of this resource to help in the search for evidence of the occurrence of a progressive differentiation and, consequently, of meaningful learning, as mentioned in the analysis of the conceptual maps of Students 2 and 5, shown in Figures 4 and 6, respectively. Even though none of the final conceptual maps presented all the expected concepts, in relation to each of the functions studied, we consider the quantity of concepts present in the referred maps as a positive sign, in relation to the quantity found in the initial maps.

According to Novak (2000), the expressive number of concepts and links used in a conceptual map signals the approximation with meaningful learning. In addition, Moreira (2011) emphasizes that if, in explaining the map, the learner goes up and down in the conceptual hierarchies, this is an indication of integrative reconciliation and can be found in the final conceptual maps, on first-degree polynomial function, of Student 2. In the case of exponential and logarithmic functions, the final concept maps of Students 3 and 5 also attest to this.

The teaching and learning processes using a PMTU – with the theoretical basis of Moreira (2011) and the concepts of meaningful learning by Ausubel (2003) – favoured the development of new learning.

The results of this research demonstrate the importance of verifying and considering the students' prior knowledge, as recommended by Moreira and Masini (2006), since, based on their investigation, the subsumers present in the cognitive structure of each student can be highlighted. This is how the teacher manages to plan, organize and propose activities that may make sense to his students, which, in fact, occurred in the development of this PMTU.

In general, the learning of the students participating in the UEPS was considered meaningful, since the students established relationships between the studied concepts and the previously constructed knowledge. Therefore, it was verified that the work and the material produced met the expectations, intentions and purposes of the research.

Therefore, it is possible to claim, that the first-degree polynomial, exponential and logarithmic functions, once worked in a contextualized way, help in the understanding of different concepts and contribute to the occurrence of a meaningful learning, whenever there are learning conditions for such, which requires considering the students' prior knowledge, in addition to their interest in learning.

The use of a PMTU enables the teacher to carry out a formative evaluation throughout the process, not restricting it to just one final test, thus favouring more than one way of approaching each content, in a progressive and integrative way, in addition to being carried out with individual and collective stages between student-student, student-class and student-teacher.

The assessment process represented a paradigm shift by abandoning the traditional attitude of measuring results through tests to use other elements and instruments that enabled students to express their progress in learning. (Hoffmann, 2019).

The theme of this research was not exhausted in this discussion, which motivates the development of new studies, and can be adapted and modified according to the characteristics of the context to be considered for its application.

AUTHORS' CONTRIBUTION STATEMENT

BCB, LZS and VVB conceived the research work. BCB developed the research work under the strict guidance and monitoring of LZS and VVB. BCB, LZS and VVB actively participated in the construction of this text.

DATA AVAILABILITY DECLARATION

The data produced and supporting the results of this study can be provided by the authors upon request.

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