ACTA SCIENTIAE

ISSN: 2178-7727
DOI: 10.17648/acta.scientiae.7160

Movement of Understanding of Mathematical Literacy from Montessori's Perspective: An Approach to the Teaching Processes of Geometry

Luiza Destefani Alves [a]
Luciane Ferreira Mocrosky [a]
Josiel de Oliveira Batista [b]
José Sávio Bicho [c]

[a] UTFPR, Programa de Pós-Graduação em Formação Científica, Educacional e Tecnológica, Curitiba/Paraná/Brasil
[b] UFPR, Programa de Pós-Graduação em Educação em Ciências e Matemática e UNIFESSPA, Faculdade de Ciências Agrárias de Marabá, Marabá/Pará/Brasil
[c] UNESP/Ilha Solteira, Faculdade de Engenharia e UNIFESSPA, Programa de Pós-Graduação em Educação em Ciências e Matemática, Marabá/Pará/Brasil.

Received for publication 30 Apr. 2022. Accepted after review 16 Dec. 2022
Designated editor: Thiago Pedro Pinto

ABSTRACT

Background: Mathematical literacy has been thematised in our studies by the importance of the initial ideas of mathematics in school paths. In this investigative path, the phenomenon of mathematics-literacy-from-Montessori-perspective stood out. **Objectives**: To answer the question: What is this, mathematical literacy from Montessori's perspective? **Design:** Research of a theoretical nature, with a hermeneutic study of three works by Montessori closest to the phenomenon: *The Discovery of the Child*, *Psychoarithmetic* and *Psychogeometry*. After successive readings, we highlighted in each work excerpts that approached the question, calling them Units of Meaning (UM). Each UM was interpreted in dialogue with the work itself and other relevant authors. In all, 84 UMs were evidenced, which converged to 15 core ideas (CI). Placing all these CI side by side and asking what they said in the light of the guiding question, they enabled new convergence movements, evidencing basic characteristics of the phenomenon and geometry revealed itself as one of the guiding threads of mathematics teaching that aims at students' learning. **Setting and Participants:** The theoretical study analysed the three works mentioned above. **Data collection and analysis:** The works that were closest to the highlighted phenomenon were selected and analysed hermeneutically. **Results:** The movement of understanding geometry, emerging from a larger study, clarifies Montessori's understanding of the arithmetic-algebra-geometry triad, with a strong appeal to sensations and perception, emphasising the use of manipulative material, and with sequences that privilege

Corresponding author: Luiza Destefani Alves. Email: luizadestefani@alunos.utfpr.edu.br

abstractions. **Conclusion:** Knowing Montessori's pedagogical proposal favours expanding teaching knowledge about mathematical literacy, woven in the close connection between methodologies, manipulative materials, articulation of mathematics, and teaching posture.

Keywords: Mathematical literacy; Geometry; Montessori; Teacher education.

Movimento de compreensão da alfabetização matemática na perspectiva de Montessori: uma abordagem para os processos de ensino da geometria

RESUMO

Contexto: a alfabetização Matemática tem sido tematizada em nossos estudos pela importância das ideias iniciais da matemática nos trajetos escolares. Nesse fluxo investigativo o fenômeno *alfabetização-matemática-na-perspectiva-montessoriana* se destacou. **Objetivos**: compreender "O que é isso, a alfabetização matemática na perspectiva montessoriana?" **Design**: investigação de cunho teórico, com estudo hermenêutico de três obras Montessorianas mais próximas do fenômeno: A descoberta da criança, Psicoaritmética e Psicogeometría. Após sucessivas leituras, destacamos em cada obra trechos que se aproximavam da pergunta, denominando-os de Unidades de Significado (US). Cada US foi interpretada, no diálogo com a própria obra e outros autores pertinentes. Ao todo foram evidenciadas 84 US, as quais convergiram 15 ideias nucleares (IN). Colocando todas essas IN lado a lado e perguntando pelo que elas diziam à luz da pergunta orientadora, estas possibilitaram novos movimentos de convergência, evidenciando características básicas do fenômeno e a geometria se revelou como um dos fios condutores do ensino da Matemática que visa a aprendizagem dos alunos. **Ambiente e Participantes:** o estudo teórico analisou as três referidas obras mencionadas. **Coleta e análise de dados:** foram selecionadas as obras que se aproximavam mais do fenômeno destacado, analisados hermeneuticamente. **Resultados**: o movimento de compreensão da Geometria, emergente do estudo maior, esclarece entendimento montessoriano da tríade Aritmética-Álgebra-Geometria, com forte apelo às sensações e percepção, enfatizando o uso de material manipulável e com sequências que privilegiam abstrações. **Conclusão**: conhecer a proposta pedagógica montessoriana favorece ampliar o conhecimento docente sobre alfabetização matemática, tecida na estreita ligação entre metodologias, materiais manipuláveis, articulação da matemática e postura docente.

Palavras-chave: Alfabetização matemática. Geometria. Montessori.

INTRODUCTION

In the Teacher Education Studies and Research Group (Grupo de Estudos e Pesquisas em Formação de Professores - GEForProf), we have been

dedicated to mathematical literacy, understanding it beyond the intricacies of the mother tongue (Andrade, 2016; Soares, 2012; Smole, Diniz, 2001; Dayrell, 1996; Danyluk, 1991, 1998). For us, mathematical literacy says of the "phenomenon that deals with the understanding, interpretation, and communication of mathematical contents taught in school, considered as initials for the construction of mathematical knowledge" (Danyluk, 1998, p. 20).

In the GEForProf meetings, when we asked about mathematical literacy, the *mathematics-literacy-from-Montessori-perspective*[1] phenomenon stood out in our field of interest. In this path, many relevant questions emerge: How much of mathematical literacy is explicit in the original works of this scientist? How is mathematics present[2] in her work? What does Montessori's pedagogical approach advocate? What possibilities open up with the Montessori method for mathematical literacy? How can the clarification of the Montessori perspective for mathematical literacy join the voices already echoing in the academy under various nicknames and contribute to the education of literacy teachers? First, however, to follow the paths foreseen, we resumed the investigation of the author's work using the hermeneutic study (Gadamer, 1999; Batista, Mocrosky, & Mondini, 2019) to unveil the phenomenon in question (Alves, 2019).

Following the questions, we identified three of the author's works, read and interpreted-comprehended them under the light of an inquiring and investigative enterprise. This research revealed that the Montessori work on mathematical literacy is supported by three pillars, here called open categories: principles for mathematical literacy in the Montessori perspective, movement of understandings of arithmetic, and movement of understandings of geometry (Alves, 2019). Therefore, geometry appeared as one of the guiding threads for the teaching of mathematics by literacy teachers. This article aims to contribute to the initial and continuing education of teachers who teach/will teach mathematics, by explaining aspects of the Montessori perspective and possible articulations with the investigative scenario of that area, intending to open horizons for the teaching of current geometry. To consider the surroundings of the topic, we started the article by explaining

[1]Such reference is seen with hyphen because we start from the relational perspective between the terms, supported by Heidegger's study (2005a, 2005b).

[2]In the Heideggerian sense that it is present.

aspects of Maria Montessori's professional life history, so the interlocutor can have the contextual parameter and the scientist's historical conditions when studying human development and her arrival at education.

A BRIEF BIOGRAPHY OF THE LIFE AND WORKS OF MARIA MONTESSORI

One of the main names of the New School, Maria Tecla Artemísia Montessori was born on August 31, 1870, in Chiaravalle, in the province of Ancona, Italy (Garcia, Mandolini, & Moretti, 2019). She settled in Rome with her family at age twelve, as her father wanted her to enjoy better studying conditions. As she had a clear aptitude for exact subjects, at the age of 17, she completed the engineering course against her parents' will. They wanted her to become a teacher, a profession more open to women of the time (Almeida, 1984). Three years later, she graduated in physics-mathematics but did not work in the area immediately, perhaps to please her parents; she continued with her studies, also graduating in natural sciences at age 22.

However, analysing Montessori's trajectory, we notice that she clearly understood society and the minorities that compose it, which seemed to disturb her. "The social situation engendered by our civilisation highlights the obstacle to the normal development of man. A system of defences analogous to hygiene for the body has not yet been created for the spirit" (Montessori, 2018, p. 22). Touched by the lack of social, infrastructural, and sanitary conditions for marginalised children in Rome, according to Almeida (1984), she enrolled in the third year of the "Medicine and Surgery" course at the University of Rome. The reason she looked for medicine indicates the concept of care in phenomenology, "in the sense of help, of being together with the other, of solicitude" (Bicudo, 2011, p. 91).

According to Almeida (1984), her enrolment in University caused fury, given that the academy was still a scope thought for the male public in the 19th century. Montessori suffered greatly from this issue, but she did not let herself be put down. Her father, very embarrassed by the situation, stopped talking to his daughter for many years. Even so, she remained resilient as an academic in the course she had chosen. Moreover, she worried about women's situation "since they were still in a situation of absolute inferiority before men, in various sectors of society. During her stay at the University of Rome, she gave numerous proofs of this concern" (Almeida, 1984, p. 12).

In 1896, she graduated as a physician, finishing the course with a study on experimental theses in psychiatry: "*Contributo clínico allo stúdio dele alucinazione a contenuto antagonistico*". The following year, she was appointed as an assistant physician at the Psychiatric Clinic of the University of Rome with Giuseppe Montesano and Sante De Sanctis, where she visited psychiatric hospitals and selected cases for the clinic (Almeida, 1984). This experience was the most striking, and why not say decisive, of her career because it triggered the restlessness towards children in the hospital, an environment that was not adapted to receive them.

Montessori (2019b, p. 119) realised that the performance of the adult in front of the child was also of paramount importance since "the adult, having no notion about the importance of the child's motor activity, limits himself to preventing such activity as if it could be a cause of disturbances". She stated that, in her time, psychology studies pointed out that every human being is imperfect, therefore, the statistics revealed an increasing number of people considered "crazy", and criminals and children considered "difficult", which made her reflect on the "phenomenon of juvenile delinquency" (Montessori, 2018) and the damage that misleads humanity. "The time spent with these children allows her to verify that their needs and desire to play remained intact, which leads her to seek means to educate them" (Röhrs, 2010, p. 13).

It was then necessary to resort to the history of psychiatry so we could realise how children's mental health was understood when Montessori had contact with the children in the clinic. Inoui and Brehm (2017, p. 983) expose an interesting and enlightening historical parameter:

> Historically, psychiatry has taken a long time to distinguish disorders in children, being late the finding for any developmental changes. However, in 1867, Maudsley, a British psychiatrist, included in his studies some research on pathologies of the mind in children, which he called "Insanity at the beginning of life", which was considered a major milestone in the history of child psychiatry. After several studies and research in Italy, in 1906, De Sanctis concluded that some children with mental disabilities could develop psychotic symptoms, while others without neurological mutations and with normal intellectual development could, too. Thus, he understood that there is an association between different clinical conditions under the same diagnosis.

It is necessary to consider the advance of the area of psychiatry in determining each diagnosis, which is today a vast field, thorough and, mainly, careful even with the terminologies used so as not to denote any stigmas and/or stereotypes. For this reason, this research does not determine exactly with which pathologies Montessori had contact. It only expresses primary studies.

To know better the studies already inaugurated in the academy about the patients she attended, Montessori (2017) travelled to Paris, in 1897, to the Bourneville Institute, remaining there for three years. In this institute, she learned about and understood the works of Jean Marc Gaspard Itard (1774 – 1838) and Édouard Séguin[3] (1812 – 1880), both with consolidated and recognised research, at the time, in the area of psychiatry.

Itard was a disciple of Dr Philipe Pinel (1745 – 1826), famous even in contemporaneity for several achievements in his area. One of the studies developed by Itard, to which Maria Montessori (1965) gave special attention, was the case of the "Savage of Aveyron", where he attended a child who today would be considered intellectually disabled. Montessori realised that he was one of the first to observe students. She decided to use this observation in her experiences because she stated that Itard inferred "*a series of exercises capable of modifying the personality*, correcting defects that kept certain individuals in a state of inferiority" (Montessori, 2017, p. 37, emphasis added).

Montessori opened a broad research front to the special method developed by Séguin, called the physiological method, which recommends studying motor activity for conditions such as deafness, paralysis, intellectual disability, and rickets, among others (Montessori, 2017). She also commented that he had not been understood by his contemporaries since he was little mentioned in publications analogous to her own. The method he created, it seems, was left aside because he affirms that regular schools intended to teach children of inclusion using the same method as the so-called "normal" children, a fact that, for a long time, Séguin had shown to be inefficient.

[3] A French physician, Itard's student and collaborator, also considered an educator due to the creation of the physiological method for his patients with intellectual disabilities.

In the author's eyes, the reason seemed evident, as early childhood education teachers levelled up to the children (in the sense of acting like them and using childish language) and used teaching methods that did not allow curricular flexibility and methodological adaptations consistent with each one's conditions. For this reason, in 1898, she participated in the Pedagogical Congress of Turin, defending the thesis of moral education, as she had noticed that the problem that affected the children and adolescents of the clinic "was more *pedagogical* than medical" (Montessori, 2017, p. 37, emphasis added). This concept was new for those times, which made her idea disseminate quickly in the medical and educational areas.

After that, she devoted some years to educating children and adolescents with disabilities and guided the teachers who dealt with them. Montessori realised she was creating a new way of teaching –although we found registers where she commented that she had not created a method (Montessori, 2004; Garcia, Mandolini & Moretti, 2019)– that contained principles of a more rational education than those that had been used in schools, something she wrote in the book *A descoberta da criança* [The Discovery of the Child] (Montessori, 2017, p. 37): "little by little, I became sure that similar methods, applied to normal children, would surprisingly develop their personalities".

Montessori's studies and experiences with children and adolescents in the psychiatric hospital went beyond Itard's and Séguin's to give patients the possibility of literacy in their mother tongue and scientifically and mathematically. She grounded her works in both authors and built a growing understanding of the spontaneous motivation of students for learning.

We found in her writings a direct approximation to the constructs of Pestalozzi (1746 – 1827) and Froebel (1782 – 1852), always in a critical way, showing new forms of educational work with children. Regarding Froebel, she could study the manipulative materials he had created, as he dedicated himself to hand work. On the other hand, based on Pestalozzi's writings, Montessori delved into studies of the intimate issues of being, which denotes that she sought readings that tended to a more accurate look at the autonomy and humanisation of educational processes, seeking to consider each student as a unique being and capable of evolving through their efforts (Montessori, 1939).

By observing the number of congresses she was invited to attend and the honours earned throughout her life, we assume that her achievements were recognised in the academic sphere. According to Almeida (1984), from her participation in the orthophrenic congresses of London and Paris, she was

invited to direct the Orthophrenic School, where she joined her schoolmates in research on children who were considered abnormal at the time. The result of this investigation is the "Method of Classification of the Disabled" based on the works of Itard, Séguin, and Bourneville from the analysis of the methods used for the education of the physical senses (touch, smell, taste, sight, and hearing).

When she attended to the patients of the clinic in a more pedagogical way, she could see that by giving patients freedom and spontaneity, they calmed and got organised naturally to understand themselves as a collective and in an environment where they were encouraged to understand how to act functionally and healthily.

> Today, most of the children we meet are unstable, lazy, disorganised, violent, stubborn, disobedient, etc.: they are *functionally ill*. However, they can heal themselves by submitting to a kind of mental hygiene. They can *normalise*. In this case, they become disciplined children capable of presenting many happy surprises. In this normalisation, children do not become "obedient to a teacher who would instruct and correct them" but find their ways in the laws of nature: they begin to function normally, and thus can reveal this kind of *psychology* that, as in the case of the body, is located inside, in a complicated labyrinth of psychic organs (Montessori, 2018, p. 47, emphasis added).

The term "normalisation" has become very common in the Montessori method but not always understood in its essence. In a decade when people/children could be referred to as "abnormal," Montessori gave evidence that she could educate them to achieve as close as possible to the "normality" of others. Normalisation was gradually considered as "active discipline" and also as "spontaneous discipline" since it suggests to the individual a self-belonging and a responsibility with whom and with what surrounds him (Montessori, 2004, 2003, 2015, 2017, 2018, 2019b, 2021).

We consider it pertinent to comment that, according to Almeida (1984), in the clinic of the psychiatric hospital, the children and adolescents received were considered incapable of learning, however, they received education through special methods. As Montessori saw them in constant cognitive and social progression, she began to study pedagogy and applied new experiences in this group, who, besides patients, also became students.

Montessori used something more original that would allow them to advance in school content, a fact their predecessors had not yet done.

Around 1904, when Montessori obtained the title of "distinguished associate professor in anthropology", she began teaching at the Faculty of Mathematical, Physical, and Natural Sciences of the University of Rome and maintained a private practice and work in public clinics and hospitals. Notably, despite being quite busy, "her great concern is still the educational problem and, until 1906, deepens her pedagogical research that now also includes normal children" (Almeida, 1984, p. 16).

As Montessori's work was in evidence, she received the opportunity to have a space of her own in a poor neighbourhood of Rome, San Lorenzo, taking care of the education of the children of the community, initially between three and seven years old. She founded the "Casa dei Bambini" in January 1907; she would soon realise the social and pedagogical importance of her institution (Montessori, 2017). It was her first school and a significant milestone for the method, because there she could show society that regardless of the physical, psychic, economic, or social condition, children can learn if they have opportunities and stimuli, showing immense respect to the children and their pace of learning, considered by her also as "inner life", as it deals with a very intimate factor that emerges through necessity.

The first measure she took when receiving the possibility of working with so-called "normal" children at the time was to study the environment that the other schools proposed, which, in this case, was complete immobility (since the school desks were fixed on the pavement and students were not freedom to "come and go") so that learning could occur, a fact that seems to refer to how the inpatients of the psychiatric hospital stayed. What did Montessori do about it? Exactly the opposite! The Casa dei Bambini was all designed for children, with small and light furniture, bookcases, and shelves accessible to their eyes, materials that stimulated sensorimotor education – everything planned and commissioned by her, consistent with the audience she would receive – in a spacious environment, advocating all other principles that she conceived as fundamental in education. All this was prepared according to the way Montessori (2018) conceived the environment.

This is one of the pillars of the Montessori approach, configured as a basis for its correct application, a theme she addressed in several works (Montessori, 1939; 2004; 2017; 2018; 2019b; 2021). In contemporaneity, we noticed that the environment with furniture adapted to the age group was well accepted and even incorporated by most educational institutions that serve

children; however, the origin of this idea is not always recognised nor mentioned.

That school environment was carefully designed for children to learn to move, acquire skills, and control their movements with the autonomy to act and think for themselves. This design causes freedom in several aspects, which is considered one of the principles of the created method, in line with the environment. This freedom does not involve the sense of releasing or liberating. It is not just a principle for the classroom but an understanding that the author brings, and that has been revealed little by little in her trajectory about the objective of education as a whole. Montessori used to study and explain as much as possible each principle she listed, showing that it was not just a mere rule to be followed but a mission of educators who perhaps adopted her methodology (Montessori, 2018).

There is an appreciation of the discrete but sensitive and delicate teaching action, based not only –but mainly– on observation; this position assumed by the educator reflects directly on how Montessori conceived learning, since it points out that students are also expected to have this posture. "Some of their reasonings reveal an accumulation of observations, a kind of 'touchstone' that we [adults] do not possess. They confront external things with the images that are fixed in their spirit, expressing appraisals of surprising accuracy" (Montessori, 2017, p. 177).

Based on Itard's and Séguin's studies, she proposed the education of the senses to form observant beings. From an early age, she recommended instigating their senses to prepare them for their performance in the environment as a form of awareness/understanding. Such education of the senses takes place through three external axes: the teacher with discreet and attentive performance, the environment prepared as mentioned above, and a resource that Montessori (2019b) called scientific, which enables the students to analyse not only their movements but the experience of order and knowledge involved in each of those manipulative materials. Although these axes are called external, the method strongly highlights the personality and the human spirit. Therefore, the indirect analysis she proposed in her writings on the word "meaning" goes beyond physical perceptions (touch, smell, taste, vision and hearing) and transcends to being, in a peacemaker bias.

The entire research, based on the specificity of being, dialogues with the scientific posture that Montessori proposed to be made possible for students, i.e., that they feel they can, through their senses, experience discoveries and through this motivating bias, seek more information about it,

growing in an environment that increasingly favours the involvement of their being with the knowledge.

Thus, she established two more principles: individuality and activity. This gives us support to understand and resume the idea already mentioned before that her method was all built in the light of the care for oneself and others. This directly communicates with our study of phenomenology, where Heidegger talks about healing and concern: "the existential condition of the possibility of 'care for life' and 'dedication' must be conceived as healing in an original, that is, ontological sense" (Heidegger, 2005a, p. 265). As covered and complemented by Hastenreiter (2011, p. 51):

> Being-there is being-in-the-world as care, which means that human beings only exists in the world if they are cared for, even in impropriety. Care is a precondition of Being, which does not mean that being cared for is already an authentic, correct form of life, because in inauthenticity, care is also present. Care refers to oneself and the world, to life in general.

From the perspective of the concept of being-there, we understand that Montessori uses the education of the senses, which involves almost all pillars of the method, made possible by the use of manipulative materials created by Séguin and Froebel, as well as the creation of a large arsenal of new materials, which were created little by little, from their attentive observation of the needs of their students, a fact that occurred from 1899, in the psychiatric clinic (Montessori, 2017, Almeida, 1984).

There is a notorious appreciation of the education of the senses in her method. But why? The answer is linked to her medical background since her first contact with this study comes from there. First, she sometimes mentions readings from De Vries (2009, 2018, 2019), a researcher who first identified sensitive periods in animals. Based heavily on this author and Séguin and Froebel, Montessori (1939, 2015, 2017, 2018, 2019a, 2019b, 2020, 2021) exposed her understanding of the education of the senses in almost all her books, corroborating its importance in the approach she developed: "the first organs that begin to function are the sensory ones, and the normal child absorbs everything, still not distinguishing each sound, each object; first he apprehends the world, then he analyses it" (Montessori, 2015, p. 53). She considers using their senses a child's first achievement, evidencing it to be a psychic activity and an important stage.

Montessori incorporated studies from psychiatry and psychology for her method to provoke the student to advance not only in formal contents of schooling but also psychically and spiritually, with the senses being the way of contact with the environment and the mind (Montessori, 2021). It is in the education of the senses that she aims to print and stimulate activity, another principle of the method, which counterposed what she observed in the clinic, that children were prevented from staying active during their stay in the psychiatric hospital, later also observed in the regular schools of her time.

Considering that the baby is born completely dependent on their guardians, Montessori points out that the achievement of child independence happens according to their development, since it gradually gives them subsidies to act for themselves and, increasingly, condemns the servile attitude of the adult in their activities. "Whoever is *served*, instead of being *helped*, is, in a sense, *harmed* in their independence. (...) To be effective, a pedagogical activity must help children advance on the path of independence" (Montessori, 2017, p. 61, emphasis added).

In 1909, due to the growing number of schools adopting the method she developed, she felt the need to publish *Il método dela pedagogia scientífica*[4], which deals with her theories and experiences, which were quickly popularised and drew attention in several countries. She began, in 1913, educating teachers to work in Montessori classes very strictly since she wanted teachers different from those of the traditional teaching trend. She stated that

> We must bear in mind that the phenomenon of inner discipline is something that must be fulfilled and not something preexistent. Our duty is to guide you on the path of discipline. Discipline will be born when the child has focused their attention on the object that attracts them and allows not only a useful exercise but the control of error. [...] Only when the teacher has acquired the power of discernment can she become an observer and guide (Montessori, 2021, pp. 244-245).

Her courses for teachers were taught in several countries, which ended up deploying her pedagogical method in their nations. One of them was the

[4]Corresponding to the current book "The discovery of the child" (Montessori, 2017).

United States, where she gave a lecture and organised a group of people interested in her ideas in the 1910s, in the presence of Graham Bell, Thomas Edison, Helen Keller, and John Dewey, in a demonstration of support and admiration. In the following decade, the group broke with the publication of negative reviews of William Heard Kilpatrick (1914), Dewey's follower. In 1960, Nancy McCormick Rambusch, one of Montessori's students, founded the American Society Montessori (AMS).

Although Montessori's ideas were widespread on Italian soil at the time, with the end of World War I, when Italy became a fascist country under the direction of Benito Mussolini, the freedom of Italians began to fade, and some Montessori schools began to close. Nevertheless, to not antagonise people so much, Mussolini invited Montessori for an interview, in which she remained reserved. This conversation/interview resulted in permission to operate her schools (Almeida, 1984). However, in 1926, four years after this conversation, she was forced to leave her homeland accompanied by her son, as Mussolini began to impose barriers to the continuation of her research. She remained in the Netherlands for a while and, in 1929, founded the Association Montessori Internacionale (AMI), intending to safeguard her legacy and promote, among other activities, congresses and teacher training.

Mussolini reappeared in her life when he used her international prestige for political propaganda, a fact reprimanded by Montessori, and in reprisal, he ordered all Italian institutions that used her method to be closed. Nazi Germany also forbade schools with this methodology, as it did not agree with the theories of the dictator (Almeida, 1984). She then went to live in Barcelona, where she produced academic results, publishing important books for understanding mathematics through her method.

With the arrival of the Spanish Civil War, Montessori was trapped in the country. King George VI, a great admirer of her works, used an international protocol that ordered one of the Royal English Navy boats to rescue her and her family. However, she goes to live in the Netherlands. In the 1930s, Montessori gave seven lectures on the theme "Educating for Peace" (years later compiled and published as a book) in Denmark and continued talking about the topic in other places just before World War II broke out.

When invited to give a course in Adyar, India, in which about 300 teachers from all parts of the country participated, she was surprised that Italy joined the war and was in an enemy position to India, which, at the time, it was part of the British Empire. She was confined to the International Theosophical Society, where she taught her courses, while her son Mario was

held in a civilian camp in Ahmednagar (Almeida, 1984). It seems ironic that someone who had spent almost ten years giving conferences about peace was being deprived of hers.

When she turned 70, she received a letter from the viceroy of India granting her the right to see her son again. In 1943, she gave two teacher education courses, which resulted in two books: *Education for a New World* and *To Educate the Human Potential*. "In the second course, Maria Montessori illustrates her cosmic education plan. According to her, each creature belongs to matter or life and obeys an unconscious work in respect to the requirements that regulate the general economy of the cosmos" (Almeida, 1984, p. 27). This plan was created by observing and admiring Indian culture.

In 1946, she returned to Europe, where she was able to better disseminate her latest research and understandings and was received with great honours. Durhan University, in England, granted her the title of Doctor of Letters *Ad Honorem*. She was appointed an honorary member of the Educational Institute of Scotland in Edinburgh. She was awarded the Legion of Honor in Paris, received the Orange Nassau Order and the *Ad Honorem* Laureate in Letters and Philosophy from the University of Amsterdam.

In 1952, her son Mario, to preserve his mother's health, convinced her not to travel anymore due to her advanced age, as she always kept the impetus to help educate teachers wherever she went. She died that year in Nordwjik, in the Netherlands, where she had lived for a long time. Her legacy lives on in the associations and organisations that bear her name, but mainly in AMI and AMS.

METHODOLOGICAL FRAMEWORK

This study aimed to explain perspectives of Montessori pedagogy for mathematical literacy, finding the possibility of being developed in the form of qualitative research, assuming the phenomenological posture of investigation, which seeks to refrain from value judgments so that the phenomenon shows itself (Bicudo, 2011).

With the focus of the study on literacy-mathematics-in-Montessori-perspective (Alves, 2019), the path undertaken was to investigate the phenomenon of mathematics-literacy-from-Montessori-perspective, revealed and explained in the bibliography left by Montessori. Hermeneutics proved appropriate for the study of texts, since we intended to know the Montessori

perspective through the scientist's writings. For this, we follow the baseline of GEForProf, which forwards studies and research from the works of Martin Heidegger (2005a, 2005b), Maurice Merleau-Ponty (1996), and Hans-Georg Gadamer (1999). The study of the author's works requested movements of interpretation-comprehension in attention to the phenomenon "mathematical-literacy-from-Montessori-perspective" when pursuing the question: "What is this, mathematical-literacy-from-Montessori-perspective?" The option for the "What is this" covers and expands the senses and ontological dimensions of the envisioned phenomenon, making it possible to also pay attention to the "how" and other aspects that may emerge throughout the understandings. In this way of researching, the phenomenon

> does not allow itself to be imprisoned at the moment of its event; it is not static; it always brings with it what it anticipates in terms of possibilities of happening and what it has accomplished in past events retained in memory and its social, historical, and cultural expressions. In a word: it is by being (Bicudo, 2011, p. 13).

In this sense, the position of interpreters of this author's work became pertinent. Interpreting, in this case, encompasses the lexical technique but surpasses it by considering that who is interpreting and what is available for interpretation carry with them marks of experience. Thus, the interpretation encompasses the encounter between what comes from the historical tradition and the present time on the researched. As an interpreter, we understand that "thinking historically now means granting each epoch its right to exist and even to its own perfection" (Gadamer, 1999, p. 311). Assuming this posture was, at first, to safeguard the work and then to gauge meaning about where we were at the time.

Having a vast bibliography of Maria Montessori, we found three works that were closer to the phenomenon and the question: *The Discovery of the Child* (Montessori, 2017), *Psychoarithmetic* (Montessori, 2020) and *Psychogeometry* (Montessori, 2019a). The last two books were consulted in their original versions of 1934 to preserve the author's voice and not that of her translators and/or later collaborators.

When we address the selected works of the method's precursor (Montessori, 2017; 2020; 2019a) with the fundamental question: "How does mathematical literacy appear in Montessori's work?" we rely on Bicudo (1994; 2011) for the analyses that, according to phenomenology, occur in two great non-watertight moments: ideographic analysis and nomothetic analysis.

The first talks of the analysis based on the particularity of ideas expressed in each of the works, and the second refers to the encounter of generalisation aimed at the structure of the phenomenon under study.

We began the analysis by reading those books more than once to get acquainted with them. The option was for an analysis in chronological order of her writings, whereby we sought to highlight, in the author's "voice", excerpts that answered the question, calling them Units of Meaning (UM). For this reason, we organise an Ideographic Framework (IF) with three columns. In the first, we identified the UM highlighted, in the second, we brought elements that helped us understand what was said and, in the third column, we exposed our understanding of what was said. Thus, we call the second column a hermeneutic graft, because the interpretation of UM was due to the return to the book as a whole, and the dialogue with Montessori scholars found in the Organização Montessori no Brasil (OMB) [Montessori Organization in Brazil], the Associação Brasileira de Educação Montessoriana (ABEM) [Brazilian Association of Montessori Education] and in reading indications in the courses taken, what these entities publish, including regarding the training of literacy teachers.

At the end of the ideographic analysis movement, which originated the IFs, each UM was resumed. We wrote them in our language, with the interpretations allowed by the hermeneutic graft. Thus, we went back to looking for meanings, the structure of what was articulated in the works, asking ourselves what they had said, intending to highlight what was nuclear in each UM, which in this study is being called Core Ideas (CI), added to the table highlighted in gray, as shown in Table 1.

Table 1

"Ideographic framework of Book 3 Psychoarithmetic (Montessori, 1934b)". (Alves, 2019).

UM	Hermeneutic Graft	Hinged UM
L3.4: "Esta imposibilidad material de error es el control de error colocado en los mismos objetos, razón por la cual el niño, una vez conocido el uso de aquellos, puede trabajar sin necesidad de maestro." (p. 19). [This material impossibility of error is the control of error placed on the objects themselves, which is why once the child knows it, he/she can work without the need for a teacher."]	Using as an example the material of the geometric molds, she begins the explanation of the error control: "En efecto todas las figuras deben estar construidas de tal modo que tengan la misma extensión lineal: 10 centímetros. El triángulo equilátero tiene 10 centímetros de lado y el círculo 10 centímetros de diámetro. De este modo no podrá entrar en el círculo el triángulo por no ser inscrito, ni con mayor razón, el círculo en el triángulo." (p. 19). [In fact, all the figures must be built in such a way that they have the same linear extension: 10 cm. The equilateral triangle is 10 cm on a side and the circle is 10 cm in diameter. In this way, the triangle will not be able to enter the circle because the circle is not inscribed, nor with greater ratio, in the triangle.] In UM L1.4 there is the study of the understanding of error control to Montessori.	The importance of manipulative material intentionally prepared so that the child can, by realising what they are doing, control their mistakes and emancipate themselves from the teacher. Manipulative material Intentionality of teaching

In all, 84 UMs were evidenced, which converged to 15 CIs. This process characterises the transition between ideographic and nomothetic analyses. We understand that nomothetic analysis seeks generalisations, establishing the referred CI and syntheses that can show, in the case of this research, how each idea unfolded in the works through successive phenomenological reductions. The way to do this is by putting all these CIs side by side, to ask what they said in the light of the guiding question of the study. Thus, new convergence movements were made possible, increasingly highlighting basic characteristics of the phenomenon under study. The impossibility of new convergences revealed open categories, so called

"because they are given to the understanding and interpretation of the phenomenon in the region of inquiry investigated" (Bicudo, 1994, p. 22). For each one, a theoretical discussion was developed with the bibliography of Montessori and other phenomenology and mathematics education authors. The "Movement of understanding of Geometry" was one of the three categories pointed out in the introduction of this text to gain prominence in this article.

GEOMETRY UNDERSTANDING MOVEMENT

Montessori (2015) considered geometry the abstraction of an abstraction. The analyses of her writings (Alves, 2019) point out that to her, this aspect of mathematics is made possible by the understanding, interest, and enthusiasm in unveiling this area, which she comments has usually been seen as painful to be learned and taught. The author also emphasises a certain erroneous understanding that "It was always believed that everything proceeded according to a straight line, from the simple to the complex, from the concrete to the abstract, from the known to the unconnected, from the imperfect to the perfect, from the bad to the good"[5] (Montessori, 2019a, p. 8).

In the works, we find that Montessori (1939, 2017, 2019a) subtly showed to be rethinking the logic of this idea when she highlighted that human growth is marked by different psychic interests and not always outlined in a "straight line", i.e., by the same direction. She used an analogy with the acquisition of the mother tongue as an example since it is acquired spontaneously. However, learning a second language after adulthood triggers different psychic processes, according to Almeida and Silva (2014) and Silva (2017), paying attention to each child's potential and bringing them to the centre of the teaching process.

Thus, Montessori (2019a) states that access to geometry at school must be initiated in early childhood education, not aiming at analyses of particularities of geometric entities, but in the process of familiarisation and, mainly, sensitisation to forms and possible relationships with the context in which one lives. We noticed that the author suggests that it is as if the child receives "permission" to access the perception of geometric entities, being

[5] Siempre se creyó que todo procedía según una línea recta, de lo sencillo a lo complejo, de lo concreto a lo abstracto, de lo conocido a lo desconocido, de lo imperfecto a lo perfecto, de lo malo a lo bueno.

able to verify their possibilities as they manipulate, observe, and connect them with their own experiences.

We understand that sensitisation is in the field of the sensitive, considering that

> The sensitive is what one learns #*with* the senses, but we now know that this "with" is not simply instrumental, that the sensory apparatus is not a conductor, that even the periphery of the physiological impression is involved in relations once regarded as central. (Merleau-Ponty, 1996, p. 32, emphasis added)

The knowledge that will enable a specific understanding of the world will be closely linked to each person's experience, evidencing, once again, that this is an aspect dear to the present study and Montessori's perspective. Regarding the experience, when exposing the ideas of Gadamer (1999), Kluth (2011, p. 93) states that

> The etymological attributes of the word experience contribute to understanding what it is to live the manifest. These reside precisely in the mediation of both meanings given to the word experience: immediacy and transmission, which allow us to understand experience as a productive connection; something becomes an experience to the extent that it was not only experienced but that its being experienced had a special emphasis, which gives it a lasting character.

Where do we start the work with geometry? With what? We understand from the author that from the moment the child observes the environment that surrounds them, they will already be getting familiar with geometry. For the child's observing action will guide the work with this mathematical content (Montessori, 2017, 2019a). This surrounding world that appears to us throughout life seems to sustain the teaching of geometry, not to the point that the child modifies it, but that they can learn from it (or in it).

The question that remains is: How does the child experience the world so that their perception is "filled" by the knowledge of geometry? According to Montessori (2019a), sensory-motor exploration permeates the whole idea that knowledge will reach the child through their perception of the world.

> No se trata solamente pues de un conocimiento que penetra en la mente del niño. En él se *desarrolla* algo que entra a formar

> parte de su vida mental, es un *sentido geométrico* que se identifica con su organismo psíquico en camino de activa creación. Los ojos del niño se sienten atraídos por la parte geométrica del ambiente que le rodea; se sienten cautivados por una luz que les penetra de manera natural. (...) Sus conocimientos son intuiciones de conjunto recibidas a través de una experiencia activa.[6] (Montessori, 2019a, pp. 15 – 17, emphasis added by the author)

The "education of the senses" that Montessori (2017) speaks of, based on her studies by Edouard Séguin, meets this idea, since it puts the possibility of exploring, which keeps alive the interest and the search for understanding the environment, which can be made possible by observing nature.

Familiarisation with geometric entities begins in early childhood education, with works categorised in the sensory area.[7] Our analyses point out that the following materials show approximations to geometry, either directly or indirectly (Figure 1):[8] Solid fittings (a), Red bars (b), Brown ladder (c), Pink tower (d), Coloured cylinders (e), Geometric solids (f), Cabinet of flat geometric shapes (g), Constructing triangles (h), Binomial cube (i), Trinomial cube (j), and Power-of-two cube (k). Most of them point to the geometric view of mathematical ideas, while others present the entity by itself and the Constructor Triangles (or Triangulation Boxes), in particular, show the possibility of geometric composition and decomposition, as well as the

[6] It is not just a matter of knowledge that penetrates the child's mind. Something develops in them that becomes part of their mental life, it is a geometric sense that identifies with their psychic organism in the process of active creation. The child's eyes are attracted to the geometric part of the environment that surrounds them; they feel captivated by a light that penetrates them naturally. (...) their knowledge is general intuitions received through active experience.

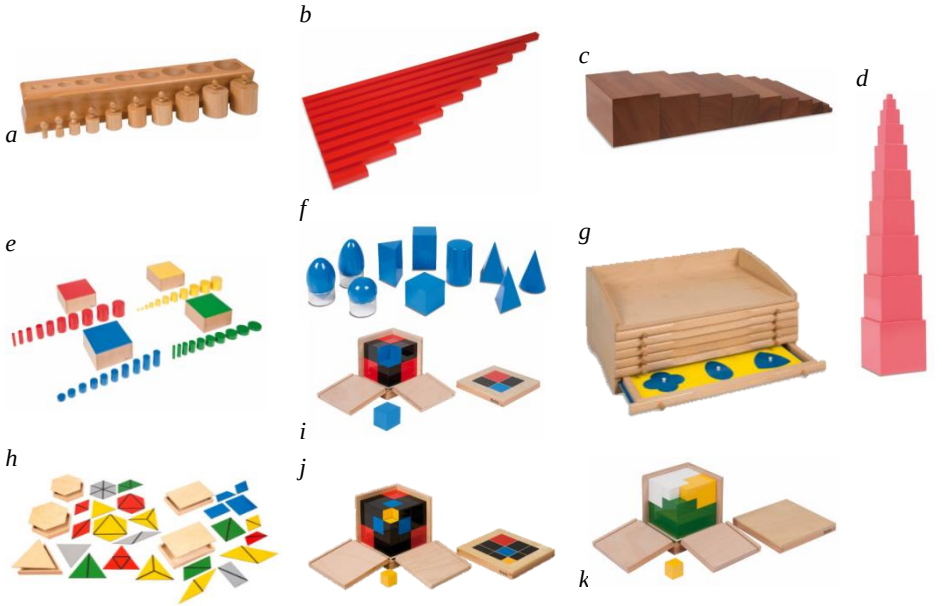
[7] The author under study classifies her materials by areas and by direct objectives: practical life, sensory, language, History, geography, sciences and mathematics. We emphasise that the sensory ones gained their own area, which does not mean that the others are not sensory as well, but not with a direct objective.

[8] Here are the names of the materials adapted to Brazilian contemporaneity, that is, how the Montessori community refers to each of them today.

equivalence between plane geometric figures, from the various types of triangles.[9]

Figure 1

Mosaic with materials from the Montessori sensory area (Nienhuis, 2022a).



Maria Montessori (2019a) emphasised the possibility of extension and dialogue with art through geometry, initially as a preparation of the hand for writing, then as a way of sensitising the child to the world that surrounds them and finally as a way of highlighting essential aspects of knowledge, in this case, geometry. For example, to study the types of triangles, she asks students to embellish the equal sides with ornaments, which is already done in the early years of elementary school.

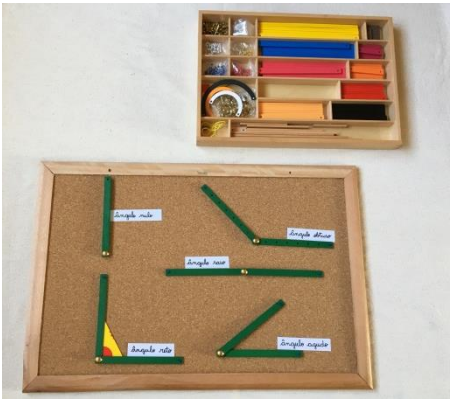
Another suggestion of the author is that, subtly and indirectly, one leads children to study the lines and their positions by forming a geometric

[9] All the materials cited in this paragraph are found in Maria Montessori's books (1939, 2017, 2019a, 2020).

entity and the types of angles, which expands through the manipulation of specific material, Geometric Rods (Figure 2).

Figure 2

Study of angles with geometric rods.



Montessori (2019a) suggested that the children themselves should manipulate the rods for the study, getting gradually familiar with the terminology of each type of line and angle until they could register and name each of them. This material continues to be taken up from time to time, sometimes for the study of parallels, perpendiculars etc., sometimes for more specific geometric analyses, with the support of the Centesimal Circle, which consists of a black iron circle in low relief with a hundred divisions drawn in white.

Concomitantly, this study extends with the material of the Equivalence Figures (or Geometric Equivalences – Figure 3), which allows the analysis of the possibilities of decomposing and composing the geometric shapes, which can also be glimpsed from the perspective of the idea of a fraction. The manipulative material of Fraction, with which this idea is introduced, is composed of circles ranging from the whole to the tenths, which can also inaugurate understandings about equivalences.

Figure 3

Equivalence figures. (Nienhuis, 2022b).

On this theme, the author stated: "The equivalence cannot be perceived with the senses, it is only a reasoning about the construction of the figures that leads us to that conclusion"[10] (Montessori, 2019a, p. 110), which can show us that at this point of development the senses contribute, but what stands out is the reasoning about the mathematical idea involved. However, she pointed out: "We, however, want to do other work; a work of reasoning instead of the simple physical verification"[11] (Montessori, 2019a, p. 104), which adults no longer always need. For this reason, one must be careful when teaching geometry to elementary school children so they can be respected in their particular movement of understanding.

This interpretative movement of the Montessori work is also composed of an articulation between arithmetic and geometry, mediated by algebra. Montessori (2017, 2019a, 2020) exposed this understanding several times, showing it not only in her classwork descriptions but also with schemes that demonstrate her intention. A good example of this articulation is the additions and multiplications with numbers from 1 to 10 with the Multiplication Board and with material from the Colored Beads, which make

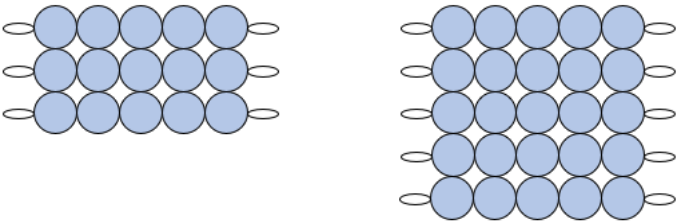
[10] La equivalencia no puede percibirse con los sentidos, es sólo un razonamiento sobre la construcción de las figuras el que nos lleva a dicha conclusión.

[11] Nosotros, sin embargo, queremos hacer otro trabajo; un trabajo de razonamiento en vez de la simple comprobación material.

up rectangles or squares (Figure 4). The intention is first to understand the idea of multiplication, then its memorisation, but indirectly, the author already indicates the geometric sensitisation.

Figure 4

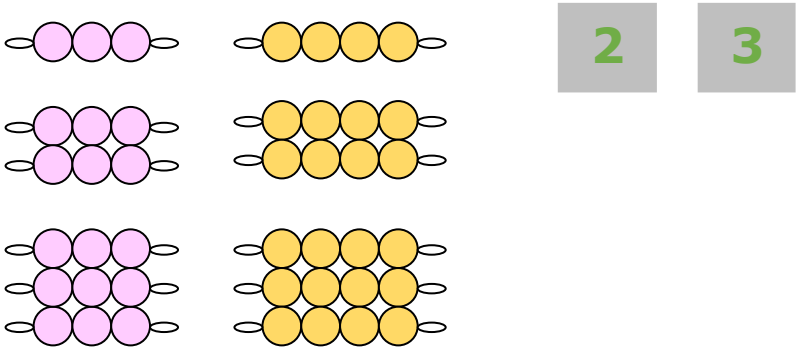
Multiplication with Colored Accounts



In this way, knowledge will expand to the point of demonstrating the square of binomials and trinomials, initially tying elastic bands in the hundred of the golden material and studying what the new sections demonstrate mathematically. This reasoning path leads to understandings such as that shown in Figure 5 in the study of multiplication between binomials. The example is one of the processes of the execution of: (3+4)(2+3), which can be applied by a child of 7 or 8 years (who is in the Montessori movement of mathematical understandings).

Figure 5

Multiplication between binomials



The steps after Figure 5 would be: check the partial results, that is, of each product (in this case: 6, 8, 9 and 12), write them on small papers and position them above each set of pieces. Then, perform the sum of all to reach the final result, in which the author comments that the student can apply the commutativity to check what he obtained. We noticed that Montessori (2020) observed the knowledge already acquired by the child and the variety of possibilities to advance in this regard.

The example shown in Figure 4 requires children to understand addition and multiplication and, from there, expand to the binomial. Many teachers could say that teaching multiplication between binomials can be simpler if we told them that we should only add what is in parentheses to do the multiplication, but what is evident in the Montessori way is the processes and personal meanings, because it uses students' previous knowledge to enable contact with content commonly applied with students twice their age. We understand that this enables them, over the years, to perceive this rule in work performed spontaneously. This is a highlight of the author's idea: not to give the ready-made ideas but to enable them to walk step by step to abstraction with a certain independence, which corroborates the ideas exposed by Domenico (1988).

Algebraic thinking seems to pulsate between the geometric and arithmetical initiatives of Montessori works. In Figures 4 and 5, it is possible to verify the multiplication presented with elements that make up rectangular arrangement, pointing to the geometric visualisation of this content that is forwarded to arithmetic with processes that glimpse algebra and involve algebraic thinking. This was perceived throughout the analyses, given that the constant reading of the author's writings allowed us to interpret-understand that it is something recurrent in the methodological logic brought by Montessori to teach mathematics. We can observe it in her other suggestions, which are also in other works by Montessori that we did not select.

The suggestions of Montessori (2019a, 2020) for the following steps would be to demonstrate the binomial and trinomial cubes (Figure 1 i-j), a material with which the child is already familiar since early childhood education as a kind of puzzle and that can and should be resumed for study. Montessori (2020), when noting children's interest in requesting

quadrinomials and other polynomials, establishes as a game the construction of the Decanomium [12] (Figure 6), with understandings confluent to the Pythagorean table.[13] Composed of phases, this work (indicated to be done in small group) leads to abstraction over the years.

Figure 6

Vertical and horizontal constructions of the decanomium

A suggested sequence of work:

1) The constructions expressed in Figure 6, regardless of which of the two is made first, provided that they have an interval period for free student experience;

2) Positioning of paper with the corresponding operation on each rectangle (Figure 7), such as “4 x 5 = 20”;

[12] Decanomium is nothing more than a polynomial of ten terms.

[13] The Pythagorean table is a table of two entries, where through the intersection of information, it is possible to practice the basic facts of multiplication.

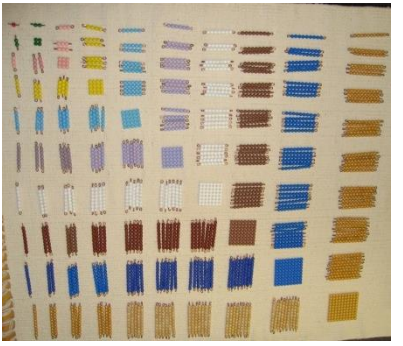
Figure 7

Building the decanomial square. (Princeton Montessori School, 2019)

3) Apply the commutative property by transforming this Decanomium of one of the constructs of Figure 6 into an angular view;

Figure 8

Decanomial work – the colourful bead bars add beauty to the classroom (Virtually Montessori, 2014)



3.1) The square of each number is evidenced, replacing the loose parts that formed this shape with another in which they are fixed in the configuration of the square (Figure 8);

3.2) The pieces that have remained loose are joined to verify how many squares can be formed in each number, and this exchange is made as in 3.1.

3.3) This empties into the cube of the numbers 1 to 10, because the squares that remained on the carpet, when they are superimposed, form cubes for each number.

A close relationship to the Rose Tower (contained in the sensory area of early childhood education – Figure 1d), as a more detailed study of the dimensions of that Tower. This game can extend to the algebraic construction of the decanomium. This example shows arithmetic, geometry and algebraic thinking dialoguing and being constantly involved in the processes of abstraction of mathematical contents.

We emphasise that the suggestions to teach geometry from the Montessori perspective are vast and can extend to the contents destined for the final years of elementary school, according to BNCC (Brasil, 2018). We must also bear in mind that the detailed specifications of how to use each material are mainly a responsibility of Montessori education courses recognised nationally and internationally. The objective of this article is to present Montessori's work and materials to foster teaching and academic interest.

FINAL CONSIDERATIONS

Montessori's perspective showed us indirectly that the geometric view of mathematical contents links the arithmetic view with algebraic thinking as a bridge, which is closely related to the studies brought by Almeida and Silva (2014) and Silva (2017). We understand that this intimate relationship has as mediators the "observe-action" and approximations to the idea of perception in Merleau-Ponty (1990).

The mentioned observe-action is a term brought by Alves (2019) as a result of her movements of interpretation-comprehension throughout her research. As observation is one of the pillars of Montessori's (1939, 2017) educational perspective, the analyses attentive to the phenomenon "mathematics-literacy-from-Montessori-perspective" emphasises the idea that we observe to act and act to observe. This #*observe* does not rely only on the functions of the sight, but on the whole complex of organs destined to feel the object that we touch and our environment. *#Action*, on the other hand, comes from the idea of impetus and the intention to accomplish something.

In the superficial reading of Montessori's works addressed to geometry, we can mistakenly think that knowledge is reduced to the senses. However, we understand that, agreeing with Merleau-Ponty (1990), it would be the #observing-action of the origin of knowledge, understanding that the personal experience of knowledge is of paramount importance to inaugurate knowledge, not to give it by itself.

The finding that some of the published books were more directed to mathematics and the understanding of the ideas that inaugurate this area of knowledge made the books *The Discovery of the Child* (Montessori, 2017), *Psychoarithmetic* (Montessori, 2020) and *Psychogeometry* (Montessori, 2019a) stand out before the others by the potential to answer the question: How does mathematical literacy appear in Montessori's work? Data analysis was outlined using phenomenology and hermeneutics as a methodological approach to enable a reading based on the hermeneutic circle (Gadamer, 1999), ensuring that Montessori's "voice" was valued and used to expand the understandings that were made throughout the analysis and discussion of the categories.

In dealing more directly with what the study answered about geometry, its teaching and aspects that address teaching knowledge, principles for mathematical literacy stand out, which proved inseparable for the processes involved to move towards the independence of learning, determining the teaching posture as a guide and facilitator. The firm intention confirms the importance of the prepared environment, structured manipulative materials, #observe-action, enhancement of the physical senses, and arithmetic-geometry articulation. All this seems to hover in balance through the figure of guides previously prepared and willing to accompany the children entrusted to them with respect for the processes and understandings they express.

We reiterate that, to Montessori (2015), geometry is understood as the abstraction of an abstraction, which appears concomitantly with arithmetic and algebra, establishing solid bases for the understanding of students since the period of mathematical literacy. The present study revealed that the arithmetic-algebra-geometry triad is not addressed only to understanding mathematics but to understanding the world and its own being-in-the-world, an understanding latent in Montessori's (2003) idea of cosmic education, a theme that remains open to the continuity of research and studies.

AUTHORSHIP CONTRIBUTION STATEMENT

LDA and LFM conceived the idea presented. LDA carried out the analyses, guided by LFM, while LDA elaborated on the historical bias under JOB's supervision. The final text was written by LDA, JOB, JSB and LFM when actively participating in the discussions and adaptations to the context and proposal of the thematic dossier.

DATA AVAILABILITY STATEMENT

Data sharing does not apply to this article because no new data was created or analysed in this study.

REFERENCES

Almeida, D. H. de & Silva, M. C. L. da (2014) Resolução de problemas na formação de professores primários em tempos de Escola Nova. In: *Acta Scientiae*, *16*(1), 57-71.

Almeida, T. de. (1984). *Maria Montessori: uma história no tempo e no espaço*. OBRAPE.

Alves, L. D. (2019) *Alfabetização matemática na perspectiva montessoriana*. [Dissertação de Mestrado, Universidade Federal do Paraná]. Acervo Digital UFPR. https://acervodigital.ufpr.br/handle/1884/66489

Andrade, S. P. (2016). *Alfabetização matemática*: o professor em forma-ação. [Dissertação de Mestrado, Universidade Federal do Paraná]. Acervo Digital UFPR. https://acervodigital.ufpr.br/handle/1884/47422

Batista, J. O. de., Mocrosky, L. F.& Mondini, F. (2019). Why speak about hermeneutics? In: *Acta Scientiae*, *21*(4), 49-62.

Bicudo, M. A. V & Esposito, V. H. C. (Org.) (1994) *A pesquisa qualitativa em educação: um enfoque fenomenológico*. Editora Unimep.

Bicudo, M. A. V. (Org.) (2011) *Pesquisa qualitativa segundo a visão fenomenológica*. Cortez.

Brasil (2018). Ministério da Educação. Secretaria da Educação Básica. *Base Nacional Comum Curricular*. BNCC. 600 p.

Cardoso, V. C. Paulo, R. M. & Dalcin, A. (2014) Beleza da matemática: uma

proposta pedagógica de sensibilização estética para o ensino da matemática. In: *REnCiMa*, *5*(2), 56-74.

Danyluk, O. S. (1998). *Alfabetização matemática: as primeiras manifestações da escrita infantil*. Sulina, Ediupf.

Danyluk, O. S. (1991a). *Alfabetização Matemática: o cotidiano da vida escolar*. 2ª Ed. Educs.

Dayrell, J. T. (1996). A escola como espaço sócio-cultural. In: Dayrell, J. (Org.). *Múltiplos olhares sobre educação e cultura*. UFMG.

De Vries, H. (2009). *Opera e periodicis collata*. vol.1. Bibliolife.

De Vries, H. (2018). *Die Mutationstheorie*. vol.2. Forgotten Books.

De Vries, H. (2019). *Die Mutationstheorie*. vol.1. Forgotten Books.

Domenico, E. C. G. (1988) *Metodologia de ensino para a iniciação matemática fundamentada na pedagogia montessoriana*. [Dissertação de Mestrado, Universidade Federal do Paraná] Biblioteca Central UFPR.

Gadamer, H-G. (1999). *Verdade e método: traços fundamentais de uma hermenêutica filosófica*. Tradução de Flávio Paulo Meurer. 3. ed. Vozes.

Garcia, J., Mandolini, F. & Moretti, E. (2019). *Maria Montessori: uma vida dedicada à educação*. UTP.

Hastenreiter, F. (2011). O cuidado em Heidegger e na psicoterapia. In: *FGR R.*, Belo Horizonte, *5*(6), 50-53.

Heidegger, M. (2005a) *Ser e Tempo: parte I*. Tradução: Marcia Sá Cavalcante Schuback. 15ª edição. Universidade São Francisco, Vozes.

Heidegger, M. (2005b) *Ser e Tempo: parte II*. Tradução: Marcia Sá Cavalcante Schuback. 13ª edição. Universidade São Francisco, Vozes.

Heidegger, M. (2012). *Ontologia: (hermenêutica da facticidade).* Tradução de Renato Kirchner. Vozes.

Inoui, A. Z. & Brehm, C. M. P. (2017). Clínica sensorial especializada no tratamento de portadores do Transtorno do Espectro Autista (TEA). In: *Atas do V Simpósio Nacional de Gerenciamento de Cidades*. pp. 981-986. Várzea Grande/MT. https://www.amigosdanatureza.org.br/eventos/data/inscricoes/3576/for

m199011751.pdf

Kilpatrick, W. H. (1914). *The Montessori System Examined*. Riverside.

Kluth, V. S. (2011). A rede de significação: um pensar metodológico de pesquisa. In: Bicudo, M. A. V. (Org.) *Pesquisa qualitativa segundo a visão fenomenológica*. Cortez.

Merleau-Ponty, M. (1996). *Fenomenologia da percepção*. Tradução de Carlos Alberto Ribeira de Moura. 2 ed. Martins Fontes.

Montessori, Maria. (1939). *Manual práctico del método Montessori*. Araluce.

Montessori, M. (2003). *Para educar o potencial humano*. Tradução de Mirian Santini. Papirus.

Montessori, M. (2004). *A educação e a paz*. Tradução de Sonia Maria Alvarenga Braga. Papirus.

Montessori, M. (2015). *Educação para um mundo novo*. Tradução de Sonia Maria Alvarenga Braga. Comenius.

Montessori, M. (2017). *A descoberta da criança: Pedagogia científica*. Tradução de Aury Brunetti. Kírion.

Montessori, M. (2018). *Formação do Homem*. Tradução de Sonia Maria Alvarenga Braga. Kírion.

Montessori, M. (2019a). *Psicogeometría.* 2. ed. Montessori-Pierson.

Montessori, M. (2019). *O segredo da infância*. Tradução de Jefferson Bombachim. Kírion.

Montessori, M. (2020). *Psicoaritmética*. Tradução de Nury Trigo Boix. Montessori-Pierson.

Montessori, M. (2021). *A mente da criança: mente absorvente*. Tradução de Jefferson Bombachim. Kírion.

Nienhuis (2022a). *Mosaico com alguns materiais da área sensorial montessoriana.* [Uma adaptação do que se encontra disponível no site: https://www.nienhuis.com/int/en/].

Nienhuis (2022b). *Figuras de equivalênica.* https://www.nienhuis.com/int/en/equivalent-figure-material/product/2297/

Pinheiro, J. M. L, Leal Junior, L. C. & Ferreira, B. L. (2017) Escola e aulas de

matemática: ambiente de ser o que se é ou de ser o que está? *Acta Scientiae*. *19*(2). 193-210.

Princeton Montessori School (2019). *Building the Decanomial Square*. https://princetonmontessori.org/uncategorized/building-the-decanomial-square/

Röhrs, H. (2010). *Maria Montessori*. Coleção Educadores MEC. Tradução: Danilo Di Manno de Almeida e Maria Leila Alves. Fundação Joaquim Nabuco, Massagana.

Silva, M. C. L. da (2017) Saberes para ensinar matemática: um olhar para a formação do professor primário. *Acta Scientiae*. *19*(6). 889-901.

Smole, K. C. S. & Diniz, M. I. (2001). *Ler, escrever e resolver problemas: habilidades básicas para aprender matemática*. Artemed.

Soares, M. (2012) *Letramento: um tema em três gêneros*. 4.ed. Autêntica.

Virtually Montessori (2014). *Decanomial work – the colorful bead bars add beauty to the classroom.* http://www.virtuallymontessori.com/spiritual-needs-elementary-child/