Geometry in the Curriculum Pedagogical Projects of Mathematics Degree Courses at Parana State Universities

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ABSTRACT

Background: Geometry is an important branch of mathematics and there are many arguments in favour of teaching it at all educational levels. **Objectives**: To analyse the pedagogical projects of the degree in mathematics courses (PPC) of Paraná state universities, to identify how geometry contents are organised for the degree students' education from the perspective of the current regulations. **Design**: Exploratory research developed from bibliographical and documentary studies, focused on the National Common Core Curriculum (BNCC), the Paraná Curriculum Reference (RCP) and the PPCs of several mathematics degree courses. **Setting and participants**: We examined the fifteen PPCs of the state universities of Paraná, obtained from the institutional websites or by email via course coordination. **Data collection and analysis**: We explored the resources and subsequently developed the categories based on the learning objectives of the BNCC, the RCP, and the regulations for initial teacher education courses, supporting the procedures mobilised on the content analysis. **Results**: We found that the geometry learning objectives in the regulations align with the profiles identified in the PPCs and the contents presented in their syllabuses. **Conclusions**: We admit the relevance of a complete and broad curriculum for the initial education of mathematics teachers. However, we believe it should not be imposed or inappropriate but the basis for qualifying good professionals who, in the future, will favour the development of students' capacities such as reflection, autonomy, and collaboration.

Keywords: Geometry; BNCC; RCP; Pedagogical curriculum projects; Degree in mathematics.

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A geometria nos projetos pedagógicos curriculares dos cursos de licenciatura em matemática das universidades estaduais paranaenses

RESUMO

Contexto: A Geometria é um importante ramo da Matemática e muitos são os argumentos a favor do seu ensino em todos os níveis educacionais. **Objetivos**: Analisar os Projetos Pedagógicos dos Cursos (PPC) de Licenciatura em Matemática das universidades estaduais paranaenses, a fim de identificar como estão organizados os conteúdos de Geometria para formação do licenciando à luz dos desígnios propostos pelas atuais regulamentações. **Design**: Essa pesquisa caracteriza-se como exploratória e foi desenvolvida a partir de estudos bibliográficos e documentais, com ênfase nas normativas vigentes, na Base Nacional Comum Curricular (BNCC), no Referencial Curricular do Paraná (RCP) e nos PPC das Licenciaturas em Matemática. **Ambiente e participantes**: Foram pesquisados os quinze PPC das universidades estaduais do Paraná, obtidos nos *sites* institucionais ou por *e-mail* via coordenação dos cursos. **Coleta e análise de dados**: Foi realizada a exploração do material e, posteriormente, a elaboração de categorias baseadas nos objetivos de aprendizagem constantes na BNCC, no RCP e normativas para os cursos de formação inicial de professores, e os procedimentos mobilizados apoiaram-se na Análise de Conteúdo. **Resultados**: Verificou-se, sobretudo, que os objetivos de aprendizagem vigentes nas regulamentações, relacionados a Geometria, estão em consonância com os perfis identificados nos PPC e os conteúdos apresentados em suas ementas. **Conclusões**: Admite-se a relevância de um currículo completo e amplo para formação inicial do professor de Matemática, porém, que não seja imposto ou descabido, mas a base para formar bons profissionais que no futuro favoreçam o desenvolvimento de capacidades como reflexão, autonomia e colaboração em seus alunos.

Palavras-chave: Geometria; BNCC; RCP; Projetos Pedagógicos Curriculares; Licenciatura em Matemática.

INTRODUCTION

Geometry is a branch of mathematics with a strong and constant presence in everyday life. It can develop, among other aspects, reasoning and visual perception and enable the understanding and resolution of issues in other areas of knowledge. It also forms and makes available tools to assist in understanding and interpreting the world, and in favouring abstraction and generalisation processes, with many arguments defending its effective teaching at all educational levels.

However, major challenge teachers face relates to teaching the subject. Lorenzato (1995) already warned that geometry was going through a difficult situation, one of omission, favoured by the delicate position it occupied in the

curriculum of teacher education courses. Pereira da Costa, accordingly, points out that, generally, future basic education teachers, students of the mathematics and pedagogy degree courses, "have little (or no) contact with geometry, or else experience formative experiences that explore geometric concepts quite disjointedly from their future pedagogical practices" (Pereira da Costa, 2020, p. 130).

In this regard, authors such as Pavanello (1989), Passos (2000), Pereira (2001), Silva and Silva (2014), Moretti (2017), Pereira da Costa and Rosa dos Santos (2017), and Pereira da Costa (2019), among others, have been carrying out research in recent decades. Discussions on how geometric concepts are addressed in basic education and mathematics teacher education courses have revealed that, despite the progress achieved with debates on the subject, a certain fragility still prevails in this teaching area. This may be linked to different factors, such as teachers' lack of content mastery for their pedagogical practices and the university curriculum, which does not articulate academic geometric concepts and elementary school.

Considering that the curriculum provided by the initial education courses is essential for the future triggering of teaching actions, this text aims to present a study carried out on the pedagogical projects of the mathematics degree courses (PPCs), of the seven state public universities in Paraná, to identify how geometry contents are organised in the formation of the licensee in the light of the objectives proposed by the current regulations. Thus, understanding that the official norms that govern educational processes have changed, especially since the definition of the National Common Core Curriculum (BNCC), the questioning consisted of investigating how these documents portray the teaching and learning objectives for geometry and whether the competencies that must be acquired by the future professor who will work with this subject are in accordance with the request.

In this way, the aim is to collaborate with reflections on the curriculum and teacher education, bringing up perspectives on geometry exposed in the PPCs of Paraná state universities, recognising and valuing the importance of the teaching degrees, and understanding that, as the curricula align with the regulations, their objectives become feasible, as long as this occurs taking into account the context and history of these degrees.

In the sequence, we discuss some presuppositions of research carried out on the subject and the consultation of the norms that discuss the teaching of geometry at the federal level and, later, in the State of Paraná, regarding its learning objectives, from basic education to teacher education, to understand

whether the regulations are connected and, also, which competencies the teachers who will teach mathematics must acquire.

CONTEXTS INVOLVING GEOMETRY

In Brazil, during the last decades, research in mathematics education advanced much, mainly in geometry. As those studies have been expanding and gaining space for new proposals, projects, and interventions, at the same time, achievements and perspectives in the school context have increased, which signals positive trajectories in improving education. On the other hand, even in the face of all this growth, several results seem not to have reached the basic education student. This problem in articulating the results from geometry academic productions and classroom reality causes an accumulation of reflections that are not properly dealt with in the school environment (Pereira da Costa, 2020).

Thus, geometry teaching in schools is still timid, recalling some contagion with the "geometric omission", a phenomenon initially discussed by Lorenzato (1995). Furthermore, research indicates that the minimum about geometry is addressed in the classroom and that several teachers do not feel comfortable teaching it, especially in the early years of elementary school, with repercussions in the final years and high school (Moretti, 2017; Pereira da Costa & Rosa dos Santos, 2017).

Silva and Silva (2014) showed that teachers are usually not confident when teaching geometric content. Carried out with ten basic education mathematics teachers, the research indicates that the lack of geometry in their pedagogical practices was related to poor academic education and the university curriculum, which did not articulate the geometric concepts addressed during higher education with those of basic education.

Despite the progress generated by discussions about difficulties in geometry teaching, Lorenzato (1995) still signals points, such as the lack of geometric knowledge necessary for pedagogical practices and the great importance given to the didactic book, which dictates how geometry is seen - commonly as a set of formulas and definitions, without many applications or explanations of a historical or logical nature. In fact, there is great emphasis on calculating the measurement of geometric quantities associated with the figures, especially from the use of equations.

This characteristic is maintained in higher education books. Ferner, Soares, and Mariani (2020) analysed the spatial geometry textbook most indicated in the bibliographies of curriculum components of the mathematics degree courses in Brazil and concluded that less than 50% of the analysed activities "provide some kind of figural apprehension and can mobilise a dimensional deconstruction of the figure" (p. 69). Still, in this regard, the authors state that

> The teaching of geometry aimed at teacher education requires the analysis of different aspects related to mathematical and didactic knowledge. One of the challenges for undergraduate degree courses is to provide situations that require articulation between experimental and axiomatic geometry and the analysis of the specificity required by cognitive activity, involving discursive and figural treatments, simultaneously. (Ferner, Soares, & Mariani, 2020, p. 69)

In this sense, knowing how the PPC of the degree in mathematics courses address questions related to geometry brings indications of the possible articulations between experimental and axiomatic geometry and the other aspects indicated above, and allows for broadening the discussions about the profile of the teachers who graduated from these courses.

GEOMETRY IN REGULATING DOCUMENTS

Brazilian educational legislation has undergone several changes in recent years, the most significant being in 2017, when the BNCC was approved by the Ministry of Education (MEC) for elementary school and, in 2018, for high school levels. In 2019, the National Curriculum Framework and Guidelines for the Initial Education of Basic Education Teachers were updated, a document prepared in the hope of redefining parameters for initial teacher education, supported by the BNCC.

According to the MEC, the BNCC is a normative document that determines the general and specific competencies, skills, and essential learning for basic education students. In addition, it intends to help education professionals to overcome the fragmentation of educational policies so that the achievement of basic learning is uniform in all social classes and socioeconomic contexts of the country. Thus, teaching networks and public and private institutions began to have mandatory references for the elaboration or adaptation of their pedagogical proposals (Brasil, 2018).

In this context, the curricular contents are directed to the development of competencies, where essential learning must ensure the development of ten general competencies, which are defined as

> [...] mobilisation of knowledge (concepts and procedures), practical, cognitive, and socio-emotional skills, attitudes and values to solve complex demands of everyday life, of the full exercise of citizenship, and the world of work. (Brasil, 2018, p. 10)

This part of the document leads us to understand that competency is the mobilisation and application of school knowledge and includes knowledge and the ability to use them. The competency model is highly criticised, and, according to Malanchen and Santos (2020, pp. 6-7), reduces individuals' formation to a "merely pragmatic and execution dimension based on a technical rationality of an instrumental nature". Therefore, a strong discourse can be seen guiding a curriculum with so-called more significant learning, in implementing the pedagogy of competencies. In this pedagogy, one can witness the passage from "a teaching centred on disciplinary knowings to a teaching defined by the production of verifiable competencies in specific situations and tasks" (Ramos, 2006, p. 221).

The BNCC is structured in such a way as to explain the competencies that must be developed throughout basic education and at each stage of schooling, and elementary education is organised into five areas of knowledge: Languages, Mathematics, Natural Sciences, Human Sciences, and Religious Education. According to this regulation, each field of knowledge establishes specific competencies to be developed over the nine years that explain how the ten general competencies are expressed in that area.

Regarding high school, the BNCC maintains an organisation in four areas of knowledge: Languages and their Technologies, Mathematics and their Technologies, Natural Sciences and their Technologies and Applied Human and Social Sciences, being that:

> it does not necessarily exclude the subjects, with their specificities and historically constructed knowings [...], but rather, it implies the strengthening of the relationships between them and their contextualisation for apprehension and intervention in reality. (Brasil, 2018, p. 32)

Concerning mathematics, the document states that, although it is a hypothetical-deductive science, i.e., supported by a system of axioms and postulates,

> [...] it is also fundamental to consider the heuristic role of experiments in learning mathematics [...] [to relate] empirical observations of the real world to representations (tables, figures, and diagrams) and to associate those representations with a mathematical activity (concepts and properties). (Brasil, 2018, p. 265)

Then, in addition to the conjectures and tests developed at the heart of this science, students are expected to be able to relate mathematical concepts and procedures to everyday situations, managing to internalise knowledge in a meaningful way, expanding their ability to identify situations and opportunities for using it daily.

In the early years of elementary school (from the 1st to the 5th grade), the thematic unit geometry consists of the "study of a wide set of concepts and procedures necessary to solve the problems of the physical world and different areas of knowledge" (Brasil, 2018, p. 271). In general, students are expected to develop the concepts of spatial perception, location, relationships between objects and geometric elements (polygons), and identify symmetries using software of dynamic geometry that favours argumentation, language, and hypothetical-deductive reasoning.

In the final years of elementary school, this unit should serve to consolidate and expand the concepts studied, besides providing the development of new skills. They are: analysing and constructing transformations, enlargements and reductions of flat geometric figures; identifying their elements and properties to recognise concepts of congruence and similarity. In addition to these skills, students are expected to be able to perform simple demonstrations and develop hypothetical-deductive reasoning (Brasil, 2018).

According to what is exposed in the document, in the final years, following the consolidation of the concepts studied since the beginning of schooling, students should develop skills that allow them to acquire thought processes and attitudes, detaching themselves from the simple repetition and application of algorithms. In this way, "geometry cannot be reduced to the mere application of area and volume calculation formulas nor to immediate numerical applications of theorems [...]" (Brasil, 2018, p. 272), but it needs to

be related to one of the objectives of competencies for mathematics, i.e., the use of mathematical processes, tools, and digital technologies to model and solve everyday problems.

The BNCC presents, in specific tables, objects of knowledge and skills in mathematics from the 1st to the 9th grade and high school, from different fields, including geometry. For the final years of elementary school, it indicates several skills, noting that among them, geometric aspects are very interrelated with algebra and analytical geometry through the study of the Cartesian plane. In addition, several geometric constructions evolve from 'simple' skills in building figures with a ruler and a compass to the subsequent development of algorithms, which use a ruler and a compass or software to develop some tasks.

We can notice that the document is structured aiming to organise the contents, i.e., the objects of knowledge so that students can achieve the skills gradually throughout elementary school. According to the association made with the various contents, the intention to familiarise and equip students with computational software is also clear.

Regarding high school, the BNCC claims that until then, schools had an excess of curriculum components and pedagogical approaches that were distant from youth cultures, the labour market, and contemporary issues (Brasil, 2018). According to the document, in high school, its focus is constructing an integrated view of mathematics and reality.

Thus, it is necessary to think about contents that allow the use of digital technologies due to technological advances and that materialise the development of skills that enhance the processes of investigation and problem solving. In this sense, the document establishes a regulation in which priorities with a more comprehensive and humanistic character are removed from the scene to let room for the instruction of knowledge aimed at the labour market, with suggestions of articulations between arithmetic, algebra, geometry, probability and statistics, and quantities and measures (Brasil, 2018).

Regarding the skills and competencies related to geometry, we noticed that it emphasises the methods of obtaining surface areas, calculation of total areas, and volumes of prisms, pyramids, and round bodies in real situations, tiling problems, and projections used in cartographies, etc., "with or without digital technologies", confirming the use, whenever possible, of technological resources in solving more complex problems.

Also, when mentioning that the acquired skills must meet "the demands of the region, preferably for its community", according to EM13MAT201

(Brasil, 2018, p. 545), we can see that the skills reinforce more practical aspects of this science, indicating the work on contents in various contexts, such as proposing actions involving measurements and calculations, application of methods for calculating areas (surface) and volumes of spatial figures in real situations.

With the enforcement of the BNCC, and based on its competencies and abilities, the Reference Curriculum of Paraná (RCP) was developed (Paraná, 2018), a normative document that has state coverage and serves as a reference for the review and reorganization of curricula of schools in Paraná, which seeks to meet regional specificities.

In RCP, concerning mathematics, the following thematic units are determined: Numbers and Algebra, Geometries, Quantities and Measurements, and Information Treatment, which must be correlated and receive different emphases according to the school year, expanding and/or restructuring what the BNCC proposes, to subsidise teachers in the development of their classes. The unfolding of the specific skills mentioned in the BNCC originated the RCP learning objectives, which maintained the original symbology.

Regarding the learning objectives related to geometry in the early years, we observe that it strongly encompasses issues of the child's spatial location, evolving into the location of other objects in the Cartesian plane. And it gradually works by recognising the basic characteristics of elements of flat and spatial geometry, polygons, and solids.

In the final years of elementary school, the RPC continues the initial years. In this schooling phase, students are faced with greater challenges, which involve systematized knowledge, specific to each curriculum component, which, as in the BNCC, are progressive, and the levels of complexity related to them rise annually. Below, we give an example of this statement.

Concepts of parallelism and perpendicularity (EF06MA18) are in the 6th grade, where, in addition to understanding the definition, construction skills of these elements are required with rulers and squares, or software (EF06MA22). In the 7th grade, those contents are implicitly addressed when determining geometric figures associated with symmetry axes (EF07MA20) and studying the measurement of angles formed by parallel lines and a transversal one (EF07MA23). Finally, in the 9th grade, the concepts are implicitly resumed through the Thales theorem (EF09MA10) and determining the midpoint of a segment (EF09MA16).

Thus, like the BNCC, the RCP emphasises the use of algorithms, with or without the use of software, for the resolution of specific problems and, together with its various learning objectives, serves as a guideline for the structuring of school pedagogical projects.

HIGHER-LEVEL REGULATORY RESOLUTIONS

The mathematics degree courses have as their main objective the education of teachers to work in basic education. Opinion CNE/CES n. 1302/2001, which enacts the National Curriculum Guidelines for Mathematics for research and degree courses, states that the degree students are expected to understand their social role as educators, the contribution of mathematics to the exercise of citizenship and, furthermore, that mathematical knowledge can and should be accessible to all (Brasil, 2001).

According to this opinion, the curriculum content common to all degree courses are differential and integral calculus; linear algebra; fundamentals of analysis; algebra fundamentals; geometry and analytical geometry fundamentals, including, among others, "mathematical contents present in basic education in the areas of algebra, geometry and analysis" (Brasil, 2001, p. 06).

Therefore, the incorporation of geometry contents from different perspectives is incorporated into the students' formation, from those related to the geometry fundamentals, with its axioms and theorems that express the logical-deductive aspects of this area, to analytical geometry, with the study of vectors, of the Cartesian plane and its various algebraic relationships, and also the geometry contents present in basic education.

On the other hand, Resolution CNE/CP 02/2015, which defined the curriculum guidelines for the initial education of degrees and continuing education, is the basis for most of the PPC analysed in the research. It proposes that initial education be aimed at developing professionals for teaching functions in basic education from a broad and contextualised understanding of education, aiming to contribute to the area of knowledge, collaborate with the political-pedagogical project of the institution and guarantee rights and learning objectives and their development (Brasil, 2015).

According to Zaidan et al. (2021, p. 15), Resolution CNE/CP 02/2015 sought to "update the formation from the perspective of universal, diverse, and inclusive basic education", which represents diligence to cover training inserted

in the current contemporary debate, which respects differences and seeks to understand heterogeneity.

Still, according to this guideline, the courses must guarantee the approach of specific contents, in the respective area of knowledge or even interdisciplinary, in addition to those related to the fundamentals of education and its methodologies, which must comply with educational policies. Thus, we emphasise the need to insert the contents of geometry indicated in the regulations in the curriculum of mathematics degrees.

Finally, CNE/CP Resolution N. 02/2019 (Brazil, 2020) revoked the 2015 Resolution, and after the BNCC approval, it enforced the National Common Core for the initial education of basic education teachers (BNC-Formation). This Resolution, based on the principles of general competencies established by the BNCC, signals the development of general and specific competencies that teachers in training must acquire.

This Resolution caused much resistance from the academic community and several movements were organized requesting the suspension of the BNC-Formation since, in this framework, teaching differs, among many other points, from a "training conception of teaching that articulates theory and practice as inseparably linked, within a socio-historical, emancipatory, and inclusive vision defended by academic entities in the field of education" (ANFOPE, 2019). Thus, the reform prominently affects teacher education and moves away from discussions that concern the basic purposes of education to discussions of a more technical and pragmatic nature (De Freitas, 2019).

METHODOLOGICAL FRAMEWORK

The methodology used for the development of this work was based on the precepts of qualitative research, according to which one of the objectives "is to promote the confrontation between the data, the evidence, the information collected on a certain subject and the accumulated theoretical knowledge about it" (Lüdke & André, 1986, p. 1). To this end, we developed first a bibliographical survey which, according to Tozoni-Reis (2009, p. 25, emphasis added), "has as its main characteristic the fact that the field of data collection is the very *bibliography* about the topic or object to be investigated". Thus, we sought in the selected authors and works data that allowed a better understanding of the problem involving geometry.

The documentary research modality was adopted because the data source is the PPCs of mathematics degrees. For data analysis, we used procedures supported by the content analysis theory that offered support to the systematisation of the material obtained, as it is defined as a

> Set of communication analysis techniques aimed at obtaining indicators (quantitative or not) for the procedures and objectives of describing the content of messages that allow the inference of knowledge related to the conditions of production/reception (inferred variables) of those messages. (Bardin, 2011, p. 48)

The corpus of the research, the PPCs, was extracted from the official pages of the universities or through contact with managers, coordinators, and/or secretariats of the courses of the state public higher education institutions (universities) of Paraná (table 1).

Table 1

Identification of the PPCs, the institution and campus

PPC	Higher Education Institution
01	State University of Western Paraná (Unioeste) - campus Foz do Iguaçu
02	State University of Western Paraná (Unioeste) – campus Cascavel
03	State University of Paraná (Unespar) - campus Campo Mourão
04	State University of Maringá (UEM) - campus Maringá
05	Midwest State University (Unicentro) - campus Guarapuava
06	Midwest State University (Unicentro) - campus Irati
07	State University of Northern Paraná (UENP) – campus Cornélio Procópio
08	State University of Northern Paraná (UENP) – campus Jacarezinho
09	State University of Ponta Grossa (UEPG)- campus Ponta Grossa
10	State University of Ponta Grossa (UEPG) - distance education - several municipalities
11	State University of Paraná (Unespar) - campus Apucarana

12	State University of Londrina (UEL) - campus Londrina
13	State University of Paraná (Unespar) - campus Paranaguá
14	State University of Paraná (Unespar) - campus Paranavaí
15	State University of Paraná (Unespar) - campus União da Vitória

Once the selection was completed, we went on exploring the resource to produce the data for the study, i.e., we read the documents carefully in view of the specifications of the resolutions/opinions (02/2015, 02/2019, and 1302/2001) and guidelines for Brazilian education (BNCC and RCP). Thus, we proposed four categories for analysis. Based on the contents requested in Opinion 1302/2001, we established: Fundamentals of Geometry (C-FG); Contents of Basic Education Geometry (C-CEB). And, due to the emphasis given to the use of technologies in the BNCC and RCP, Technology and Information Technology (C-TI) was instituted. Also, the Resolutions category (C-R) was created.

Regarding the Fundamentals of Geometry category (C-FG), we decided that the term *geometry* should be the descriptor. This category encompassed general education subjects, such as Euclidean geometry (plane and/or spatial), analytical geometry, geometric drawing, and others.

The category related to the contents of the basic education geometry (C-CEB) aims to identify the subjects that address basic mathematics topics that can be taught in different subjects, including specific education subjects that could mention geometry. Thus, we settled the following quantifiers: *basic education; basic mathematics; mathematical contents*. During the analysis, we restricted the data to only those dealing with geometry teaching, eliminating subjects that mentioned basic education without intending to explore skills and/or competencies of the contents of this level.

Another category analysed is related to the Resolutions (C-R), in which we sought to identify which deliberations are included in the PPCs. Finally, due to the emphasis in the BNCC and the RCP on the use of technological resources, we scrutinised this subject through the Technology and Information Technology (C-TI) category, both in the contents (curriculum syllabus) and in the theoretical-methodological foundations that guide the PPCs (justification, conception, purposes, objectives, professional profile, methodology, and assessment). The BNCC and the RCP indicate, in certain learning objectives

(EF01MA11; EF03MA16; EF04MA16; EF04MA18; EF04MA19; EF05MA14. EF05MA17; EF05MA18; EF06MA21; EF06MA22; EF07MA21; EF07MA23; EF08MA15; EF08MA18; EF09MA11; EF09MA15; EM13MAT307; EM13MAT309; EM13MAT505; EM13MAT509), the use of technological resources, which shows an effort to build/expand proposals that use dynamic geometry software. Thus, as quantifiers related to C-TI, we have*: technolog(y)(ogical); software; algorithm; computing.*

The subjects that make up this category are those that use techniques or software that help to develop the logical-deductive processes of specific tasks. Therefore, we identified two classes: one for those that address geometry content and another for those that work with technologies/algorithms, but do not mention geometry content.

As an exclusion criterion for the units of analysis (subjects or sections of the PPC) identified through the quantifiers, we used the non-approximation to the topic of geometry teaching, or because it is an optional subject since, in the latter case, the offer is not continuous and not all academics can be contemplated with these studies. Thus, we created two situations: 1st situation – the descriptors select units that fall within the scope of the analysis; 2nd situation – the descriptors identify units that are excluded from the analysis for the reasons mentioned above.

Next, to exemplify the work done in the 15 PPCs, we expose the process of extracting data from PPC 01, from the mathematics degree course at Unioeste, campus Foz do Iguacu.

Unioeste – campus Foz do Iguaçu offers the mathematics course, degree modality, lasting four years, offered in the morning period. According to its PPC (Unioeste, 2016), its curriculum structure is composed of 2006 hours of general formative content (forms the national profile, in accordance with the National Curriculum Guidelines), 510 hours of differentiated formative content (forms the profile specific to each course), 408 hours of the supervised teaching practice, 136 hours of the endo-of-course work and 200 hours of complementary academic activities, totalling 3,260 hours. Organised in a serial, annual manner, its purpose is to qualify professionals to work as mathematics teachers in the final years of elementary school and high school.

According to this PPC, a qualified professional's profile must cover

> [...] solid formation and mastery of specific mathematical contents necessary for understanding and efficient use of mathematics in the apprehension of reality, with pedagogical

mastery inherent to the teaching and learning process and also with conditions to understand their role as a professional with autonomous and critical performance in the school process and the social context, thus contributing to the development of citizenship. (Unioeste, 2016)

Regarding the Resolutions category (C-R), this PPC presents Opinion 1302/2001 and Resolution 02/2015 in its legislation, not yet complying with Resolution 02/2019, since the year of implementation of the current PPC 01 is 2017, therefore, before this last regulation. After reading the document, we inserted in Table 2 the subjects that had in their syllabi some of the quantifiers mentioned in the C-FG and C-CEB categories.

Table 2

Subjects related to C-FG and C-CEB categories of PPC 01

PPC	Situation	C-FG	C-CEB
01	1st	- Analytical Geometry - Euclidean Geometry - Geometric Drawing	- Didactics Applied to Mathematics Teaching - Mathematics Teaching Laboratory - Trends in Mathematics Education
	2nd		- Basic Mathematics - Supervised Teaching Practice I

With regard to the Technology and Information Technology (C-TI) category, related to the theoretical-methodological foundations that guide the PPC 01, Table 3 presents the summary of the notes.

Table 3

Units related to the C-TI category of PPC 01

Subject	C-TI

1	Conception, purposes, and objectives: principles from the perspective of the development of the PPC.	Establishment of curriculum implementation projects, with a view to improving curriculum activities in line with **technological** development.
2	Professional profile: general and specific education: professional profile of the mathematics student.	[...] the ability to understand, criticise and use new ideas and **technologies** for solving problems.
3	Professional profile: general and specific education: professional profile of the mathematics degree student.	Develop teaching strategies that favour creativity, autonomy, and flexibility in students' mathematical thinking, seeking to work with more emphasis on concepts than techniques, formulas, and **algorithms**.

Of the items identified through the descriptors, two of them (1 and 2) fit the objectives of the study, while item 3 is not related to the use of technological resources and, therefore, was excluded from the analysis (2nd situation). Also, from the analysis of the PPC 01 curriculum, there is the subject "Applied Computing to Mathematics Education" linked to the C-TI.

After we finished exploring the resource, after we developed the indicators and collected the data from the information obtained in the PPC of the mathematics degree courses supported by the BNCC and RCP and by the other legislations, we interpreted and treated the results, both based on several charts/summaries. Detailed descriptions of this work can be found in Freitas (2022).

RESULTS AND ANALYSES

The first point of analysis concerns the C-R category, as we found that, of the 15 PPCs investigated, 14 comply with Opinion 1302/2001, except for PPC 11, which does not explicitly express the regulation, but generally meets its regulations; 14 follow the guidelines of Resolution 02/2015, except PPC 10, which appears in the year of publication before 2015 and does not meet the minimum 3,200 hours of workload. We could verify that, for the most part, the Resolutions of the years 2001 and 2015 supported the elaboration of the PPC, and that the projects are still in a process of transition and adaptation to

Resolution 02/2015. As a result, none of the analysed PPCs complied with Resolution 02/2019 (BNC-Formation).

This fact reinforces the non-acceptance of Resolution 02/2019, since most of the PPCs, reformulated according to the requirements of the 2015 Resolution, had their implementation completed in 2020 and, therefore, there was no time to examine the possible advances that such PPCs triggered, "[...] not knowing the propositions that have been constructed by educators" (De Freitas, 2019, p. 517). Furthermore, the 2019 Resolution, as it was idealised and formulated in the first years of the insertion of the previous deliberation, without waiting for the implementation of the process and subsequent review, did not respect all the collective work that the professors undertook in this activity, disregarding the progress from this period.

In the C-FG category, after reading the syllabuses selected in the 15 PPCs, we highlighted some points on the same contents that appear in similar subjects, which were reorganised into four subcategories: analytical geometry, Euclidean geometry, geometric drawing, and non-Euclidean geometries.

Table 4 highlights the contents related to each subcategory, followed by the number of PPCs in which this topic appears and registers some of the constant learning objectives in the BNCC and the RCP associated with the themes. We observe, however, that in specific syllabuses, some subjects are very detailed, while others are succinct in explaining the contents to be taught, making the analysis difficult. Therefore, it was subjective, for example, to consider that a certain subject, when working with "vectors", would be supported by the study of various elements related to vectors and, thus, include this PPC with regard to the topic of scalar, vector, and mixed product.

Table 4

Subcategories: associated contents and objectives of the BNCC and RCP

Category Fundamentals of Geometry (C-FG)		
Subcategory	**Contents[a]**	**Learning objectives**
Analytical Geometry[b]	Vectors (14); Study of lines – equations (14); Study of planes – equations (14); Dot Product, Vector, Mixed (14); Conics (14); Cartesian Plane (10); Quadric Surfaces (13); Distance between points/lines/planes (9); Relative	EF05MA14; EF05MA15; EF06MA16; EF06MA21; EF07MA19; EF07MA20; EF09MA16. Overall, there are: characterisations of the plane, relative positions of straight lines (parallelism,

	position between points/lines/planes (6); Angles (4).	perpendicularity, orthogonality, etc.), reference points, transformations of polygons in the Cartesian plane, midpoint of a segment in the Cartesian plane, etc.
Euclidean Geometry	Axioms of Euclidean geometry (12); Congruences (9); Polygons (8); Similarity of triangles (8); Circumference and circle (8); Areas (8); Point, line and plane in three-dimensional space (4); Intersection of lines and planes (3); Parallelism and perpendicularity of lines/planes (8); Dihedrons, trihedrons (3 – PPC 02, 05 and 06); Polyhedrons (9); Sphere (7); Cylinder and cones (6); Euler's theorem (3); Prisms (4); Pyramids (4); Trunks (4); Axiom of parallels (5); Congruence of triangles (6); Thales theorem (6); Trigonometry (4).	EF06MA18; EF06MA19; EF06MA20; EF07MA23; EF07MA24; EF07MA27; EF08MA15; EF09MA12; EF09MA13. Linked are issues of similarity of triangles, parallelism, Thales' Theorem, polygons, and geometric solids, among (several) other subjects.
Geometric Drawing	Elementary Geometric Constructions (7); Homotheties and similarities (4); Constructions with angles (2); Constructible segments and algebraic expressions (2); Equivalence of areas (3); Approximate processes (1); Isometries and congruences (2); Applications of the Pythagorean theorem (1); Golden segment (1); Triangles, quadrilaterals (2); Translation (2); Symmetry (2); Circumference (2); Geometry and aesthetics and geometric patterns (1); Notions of descriptive geometry (3); Scales (2); Tangency and concordance (3); Ovals; arches and spirals (3); Conic curves (3); Types of	EF07MA21, EF08MA18: Compositions of geometric transformations (translation, reflection, rotation, etc.) EF07MA27, EM13MAT505: Plane tiling, with or without the support of apps. EF09MA17: Recognition of orthogonal views of spatial figures and drawing objects in perspective. EM13MAT105: Use of notions of isometric transformations (translation, reflection, rotation, and their compositions) and homothetic transformations to build figures.

| Non-Euclidean Geometries[c] | projections. (two); Descriptive methods; (two); Intersection of solids; solids section (2); Development of surfaces (2); Study of point (1); Study of the line (3); Study of the plan (1); Relative positions/intersections of lines and planes (3); Point common to three planes (1). Notions of Non-Euclidean geometry (9); Non-Euclidean geometry (2); Spaces with the inner product (1); Isometries (1); Orthogonal groups (1); Spherical and elliptical geometry (5); Spherical trigonometry (1); Hyperbolic geometry (5); Hyperbolic trigonometry (1); Projective geometry (3); Topological geometry (2); Geometry of fractals (4); The emergence of Non-Euclidean geometries (2); Axioms of parallels (2); The models of Poincaré and Klein (1); Taxicab geometry (2); Affine geometry (1). | EM13MAT509: Deformation of angles and areas caused by the different projections used in cartography.

EM13MAT105: Mentions work with the geometry of fractals. EF06MA18: Notions of topology, EF09MA17: Study of the basic concepts of projective and fractal geometry. |

Note. [a] Details of the contents worked on by each PPC can be obtained from Freitas (2022). [b] PPC 08 is the only one that does not include this subject. [c] PPC 01, 02, and 11 do not include content related to this subcategory.

Most projects include the basic contents of *Analytical Geometry*; however, some very specific topics are rarely mentioned, for example: Equations of the circumference (3 – PPC 03, 05, and 06); Triangle area (1 – PPC 03); Polar coordinates (1 – PPC 01) etc. This subcategory stimulates, due to the characteristics of analytical geometry, the articulations between algebra and geometry, and this theme appears in Resolution n. 1302/2001 (Brasil, 2001) as mandatory for the formation of the mathematics degree student, which, in isolation, does not imply staying away from dealing with geometric concepts to the detriment of their more algebraic and arithmetic aspects.

The subcategory *Euclidean Geometry* emphasises the constructible and axiomatic topics of geometry, as shown in Table 4. Again, topics covered in more descriptive PPCs stand out due to the low frequency presented. They are:

Exterior angle theorem (2 – PPC 01 and 05); Perimeter, area and study of the rectangle triangle (2 – PPC 03 and 06); Notable quadrilaterals (2 – PPC 06 and 11) etc. On the other hand, in poorly detailed PPCs, there are comprehensive indicators, where general subjects allow for wide flexibility in the choice of contents associated with the theme, jeopardizing the categorisation work, such as the syllabus of the Euclidean Geometry discipline of PPC 04, which is succinct: "Euclidean Geometry as a model for the systematisation of mathematics: origin and history".

The subcategory of *Geometric Drawing* highlights low frequencies of items associated with it and, consequently, a diversity of topics addressed on the subject. This shows no unanimity regarding the possibilities of approaching Geometric Drawing. However, it is evident that the institutions work with themes aimed at understanding geometric figures and their handling in three-dimensional space, i.e., with the study of their characterisations (vertices, sides, flattening, etc.) and transformations (rotations, translations, reflections, symmetries, projections, etc.), contents contained in the BNCC and RCP.

For the subcategory *Non-Euclidean Geometries*, it is important to consider that this type of geometry in basic education generates opportunities for valuing the historical construction of geometric knowledge, which is a content that is timidly present in curriculum regulations, and, therefore, relevant in the curricula of teacher education. In RCP and BNCC, non-Euclidean geometries are related to the basic concepts of projective geometry and fractal geometry and, in a brief footnote, the RCP suggests: "In **geometries**, in addition to Euclidean geometry, notions of non-Euclidean geometries are addressed, given the pedagogical potential of the relationship between them" (Paraná, 2018, p. 808, emphasis added).

Regarding the Contents of the Basic Education Geometry category (C-CEB), we found that 12 PPCs (except PPC 12, 13, and 15) include one or more subjects in their curriculum that meet the first criterion: they used the quantifiers to describe the intention to work with elementary and secondary education contents in a subject of teacher education. However, no detailed specifications were found on the topics, which made it difficult to generate a broad analysis of those issues.

With this, the degree courses include not only the common part of the contents of academic mathematics, but mathematical contents present in basic education, so that teacher education courses can reduce gaps in basic schooling. Thus, we intend to articulate the view of the formation of higher education for basic education, which, according to the data analysed, shows that, in general,

the degree courses of Paraná are making an effort to meet those requirements, reducing the distance between the academic and school knowings, in the sense of resolving the dichotomy between specific and pedagogical knowledge, moving towards working, in the same way, with pedagogical content knowledge (Shulman, 1986; Fiorentini, 2005).

Finally, in C-TI, Technology and Information, we identified points related to the development of the professional profile in line with new technologies and, of the 15 PPCs investigated, 14 mention the use of technological resources (except for PPC 14). In general, the PPCs present the following topics: Criticizing/using new technologies (8 – PPC 01, 02, 03, 04, 05, 08, 11, 15); Using technological means to teach mathematics (7 – PPC 02, 03, 05, 08, 10, 11, 12); Competently using technological and computational means (3 – PPC 02, 08, 13); Mastering new technologies (3 – PPC 04, 08, 10); Recognising the relationship between mathematics and technology (2 – PPC 04, 13); Knowing technologies (2 – PPC 02, 04); Providing learning of contents related to computing (2 – PPC 06, 07); Complying with technological development (1 – PPC 01).

Fourteen PPCs (except PPC 08) presented subjects associated with the C-TI, whose emphasis is not primarily on the geometry content but on understanding the use of techniques or *software* that help to develop logical-deductive processes for the elaboration of algorithms/specific tasks. Of these, eleven have subjects that meet the first criterion (11 – except PPC 07, 08, 11, 15), and 12 have at least one subject that fits the second criterion (12 – except PPC 01, 08, 12).

Therefore, we realised, according to the association of different subjects and contents in C-TI, the intention of familiarising students with software for geometry teaching. This is significant, as the BNCC and RCP emphasise the use of software and digital technologies to solve specific problems, because "[...] working from the perspective of education in digital culture makes it possible to combine new ways of learning and teaching with educational processes and practices" (Paraná, 2018, p. 14). In this way, the main allusions made to technological resources focus on the importance that the use of those means can bring to teaching, given that, by operating all available mechanisms, students can use this technology to the search, selection, analysis, and articulation between information and, thus, build their knowledge.

CONCLUSIONS

This work intended to present the objectives related to the teaching of geometry contained in the current regulations for basic education and, concomitantly, look at the official documentation that governs the organisation of the curriculum of the mathematics degree courses of Paraná to verify if their PPCs satisfy such regulations.

When investigating the objects of knowledge and skills related to geometry, in the BNCC and the RCP, after interpreting and analysing the PPCs, we found that the topics covered in the categories and subcategories align with what is expected to be taught about this topic. Far beyond that, the PPCs, in general, are fulfilling their role and meeting what is expected of training courses, with the inclusion of a professional profile that knows how to use new technologies, as well as working with teacher education subjects that seek to base the contents of basic education.

In fact, when working with basic education contents in a higher education subject, we expect that mathematics degree students can discuss issues related to teaching and learning, where they can have contact with the contents that will be part of their future praxis, seeking to eliminate the omission of geometry from the school curriculum.

In this sense, so that the formation of mathematics teachers has its conception guided in the context of the acquisition of professional knowledge essential to the exercise of the profession, we verified the importance of thinking about a well-designed curriculum that prepares academics to face the challenges of the current professional scenario.

In addition, the rigorous analysis of the PPCs made it possible to portray the various mathematics degree courses at the seven state universities in Paraná, especially concerning the teaching of geometry. With this, this work also contributes towards identifying profiles, specific contents (described in the subcategories), and distinct subjects that work directly with basic education contents or with technologies to contribute to discussions that involve future changes in those projects. It is worth mentioning that the PPCs support and guide pedagogical actions. However, it is up to the faculty to implement such proposals.

Finally, we recognise the relevance of a complete, broad curriculum but not an imposed or unreasonable one, as proposed by BNC-Formation, which disqualifies the advances achieved in the formulations of the current PPCs to the detriment of an unjustified adjustment to the BNCC that, as this research

shows, is already being in force, but that it be elaborated and promoted through discussions based on the heart of the local, regional and national academic community, subject to improvements and updates, so that, in due course, it is structured on a basis to qualify good professionals, who in the future, in action, favour the students' development of skills such as reflection, autonomy, and collaboration.

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AUTHORSHIP CONTRIBUTION STATEMENT

GDF, KRML and ML conceived the idea, and actively participated in writing; critical reading; and discussion of ideas and reflections textualised in the article.

DATA AVAILABILITY STATEMENT

The data supporting the result of this study are openly available at the TEDE (Digital Library of Theses and Dissertations) site of Unioeste: https://tede.unioeste.br/. These data were derived from resources available in the public domain of the seven state universities in Paraná mentioned in the work.

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