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Geometric Constructions in the Current Math Teacher Training Courses at the Federal University of Mato Grosso do Sul

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ABSTRACT

Background: Math teacher training in Brazil has been the subject of numerous studies under the most diverse approaches. From a historiographical perspective, we can highlight the performance of the Oral History and Mathematics Education Group and the History of Mathematics Education in the Research Group, both acting more directly with the narratives. Disciplines with the content of Geometric Constructions have historically been part of the training of Mathematics teachers. At issue in this text is the contribution of these disciplines to the formation of future teachers. **Objective:** This paper adds elements to mapping Mat teacher training and performance in the state of Mato Grosso do Sul and in Brazil, expanding discussions about the state and the possible role of geometric disciplines in undergraduate courses. **Design:** We present the normative documents of such courses and disciplines and the narratives, in the end, comparing the relevant literature for their analysis based on the researchers' questions. **Environment and participants:** Courses of Math Teacher Training at the Federal University of Mato Grosso do Sul – Brazil. The time frame of the research ranged from 2004 to 2019, the period in which the seven interviewees taught the subjects mentioned earlier. **Data collection and analysis:** The narratives of the interviewees, the teaching materials presented by them, and the normative documents are analyzed. **Results:** The narratives and pertinent literature allowed us a dispersive look, which turned outside the narratives themselves, having in them the triggering movement of reflections. Thus, we could produce questions and possible answers to such inquiries. **Conclusions:** Among the final notes, the following stand out: the importance of these disciplines, identified as "basic" in the course, to recover Geometry contents that students should have studied in Basic Education; the relevance of failure as a stimulus for the study; and the choice

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of materials that focus on other aspects of geometry. Such subjects are often intended for the substitute teacher, and there is no prior training to work in this discipline with a specific focus on teacher training.

Keywords: Teacher Training; Geometry; Oral History.

Construções Geométricas nas Licenciaturas em Matemática da Universidade Federal de Mato Grosso do Sul

RESUMO

Contexto: A formação de professores de Matemática no Brasil tem sido alvo de inúmeras pesquisas sob as mais diversas abordagens. Em uma perspectiva historiográfica podemos destacar a atuação do Grupo de História Oral e Educação Matemática e o Grupo História da Educação Matemática em Pesquisa, ambos atuando mais diretamente com as narrativas. Disciplinas com conteúdo de Construções Geométricas tem feito parte da formação de professores de Matemática historicamente. Em questão neste texto está a contribuição destas disciplinas para a formação do futuro professor. **Objetivo:** Este texto tem por objetivo acrescentar elementos no mapeamento da formação e atuação de professores de Matemática no estado de Mato Grosso do Sul e no Brasil, ampliando as discussões sobre esta formação no estado e o possível papel das disciplinas de caráter geométrico na formação de professores de Matemática. **Design:** Apresentamos os documentos normativos de tais cursos e disciplinas e as narrativas, ao final, cotejando a literatura pertinente para suas análises a partir de questionamentos dos pesquisadores. **Ambiente e participantes:** Cursos de Licenciatura em Matemática da Universidade Federal de Mato Grosso do Sul - Brasil. O recorte temporal da pesquisa vai, de 2004 até 2019, período em que os sete depoentes ministraram as referidas disciplinas. **Coleta e análise de dados:** Analisam-se as narrativas dos depoentes, os materiais didáticos apresentados por estes e os documentos normativos. **Resultados:** As narrativas e literatura pertinente nos permitiram um olhar dispersivo, que se voltava para fora das próprias narrativas, tendo nelas o movimento disparador de reflexões. Assim, pudemos produzir questões e possíveis respostas para tais indagações. **Conclusões:** Entre os apontamentos finais, destacam-se: a importância destas disciplinas, apontadas como "básicas" no curso, para recuperar conteúdos da Geometria que os alunos deveriam ter estudado na Educação Básica; a relevância da reprovação como estímulo para o estudo; e a escolha de materiais que privilegiam outros aspectos da Geometria. Tais disciplinas são muitas vezes destinadas ao professor substituto; e não há formação prévia para atuar nesta disciplina com enfoque específico para a formação de professores.

Palavras-chave: formação de professores; geometria; história oral.

FIRST TRACES

This research addressed the role of the disciplines that deal with the subject Geometric Constructions (GC) in Mathematics undergraduate courses at the Federal University of Mato Grosso do Sul[1] (UFMS) through document analysis and interviews with teachers (Souza, 2021). This theme emerged from the Scientific Initiation: "Production and Dissemination of Historical Documentation regarding the discipline of Geometric Constructions in the Mathematics Degree Course at UFMS", carried out for two years -2016 and 2017-, when we could explore locally some aspects of the discipline so named in the Mathematics Institute (Inma) of the Cidade Universitária,

In these two years of Scientific Initiation, we worked with the methodology of Oral History, treating official documents, a handout prepared by teachers of the subject, and an interview with one of these teachers, also the author of the handout. This first movement allowed us to reflect about the role of this subject in the formation of mathematics teachers, which, in this material, linked two aspects to it: revision of Geometry contents from Basic Education and the introduction of logic and mathematical writing. We could also evidence some changes over time in this discipline in the course studied, especially with the change of the teaching staff, not because of the norms, but because of the postures and teaching positions in the conduct of the discipline.

Once the Scientific Initiation was over, we extended our contours to delineate notes about this discipline in all courses of Mathematics Licensing at UFMS (6). In spite of the University's attempts to unify these courses - culminating in 2023 with the compulsory equivalence of 50% of the curricular matrix -, the differences are potentially rich for the academic discussion about teacher education.

Mato Grosso do Sul is a fairly recent state from an administrative perspective. Until 1979, the territory was part of Mato Grosso, with the city of Cuiabá as its capital, which distanced the population from the decision making spaces. Mato Grosso do Sul today has fewer than eighty municipalities, being one of the least populated states in the country (21st) and one of the largest territorial areas (6th). There is a large population concentration in the state capital - Campo Grande - followed by three

[1] In Brazil, since 1996, teacher training must take place in higher level courses in the Licentiate modality. Currently, licentiate degrees must have a minimum of 3200 hours of activity, generally distributed over four years.

medium-sized cities (more than 100 thousand inhabitants), the other cities are small, scattered, and distant from each other throughout the territory. UFMS is present in ten cities in the state and has six BS in Mathematics and one BA in Mathematics.

Our work is part of the projects of the History of Mathematics Education in Research Group (Hemep [2]), which has in its scope the production of research about the history of the formation of mathematics teachers in this region. We are also linked to the project "Mapping the Training and Performance of Teachers who teach/teach Mathematics in Brazil", of the Oral History and Mathematics Education Group (Ghoem[3]). In contributing to these projects, we aimed at producing historiographic sources for this and other researches, sources that would help to trace the understanding of how the formation of mathematics teachers in Mato Grosso do Sul occurred.

Several works have already been produced, especially focusing on some cities or regions of the state or even on a specific course. However, few were those so far that focused on a broader scale, on a set of courses. We can highlight here the doctoral work of Carla Regina Mariano da Silva (2015) and that of Katia Guerchi Gonzales (2017), on the bachelor's Degrees in Science and Mathematics and Associate Degrees in the state, respectively.

Because it is a master's research and a theme that already came from the Scientific Initiation, we chose to operate a cut[4], addressing a particular discipline (or its equivalents) in the six undergraduate courses in Mathematics at UFMS.

Besides producing a historical panorama of these courses, we wanted to discuss the role of this discipline (or its equivalents) in the formation of Mathematics teachers, and, for this, Oral History seemed to us to be an adequate approach. The official documents about the course provided little

[2] Research lines and defended papers are available at: www.hemep.org

[3] Research lines and defended papers are available at: www.ghoem.org

[4] Cut here comes inspired by the philosophy of Deleuze and Guatarri (Thousand Platots, 2011), which deals with flows and cuts. The impossibility of apprehending the past as a totality imposes on us other movements, restrictions, temporal or geographic cuts, versions about an occurred fact, etc. Our movement, thus, operates a cut, not in the courses, but in our look, putting us to compare the differences and the conceptions from what was found in the production of new angles, new possibilities of thinking such courses and even the training of mathematics teachers.

information about this issue, pointing, at most, to general aspects of education, ways of separation of the disciplines (specific, general, etc.) and workloads, as investigated by Souza (2022). Thus, interviews would be a promising way to know more directly the ways of working with this subject "in the classroom" and of understanding the training of mathematics teachers, as well as the function of these subjects/contents in this training.

We accessed the Pedagogical Course Projects (PCP) and the current Curricular Grades, available on the web pages of the courses. At UFMS, each campus has its own course, which is different from other institutions that have a single course and several offerings. Thus, not all courses have a discipline entitled "Geometric Constructions [5] ", however, all of them have some discipline that addresses the contents of Geometric Constructions. We opted to take as a criterion of similarity to the contents of the disciplines. In Scientific Initiation, we had the indication that this was a discipline at the beginning of the course and, now, we reproduce in Table 1 the information about the offering period of each discipline that presents geometric constructions contents.

Table 1

Subjects of the various campuses that present contents of geometric constructions (Souza, 2021, p. 28)

Campus/City	**Subject**	**Program**	**Semester**
AQUIDAUANA	Plane Geometry and Geometric Design (68 hours)	Axiomatization of Geometry. Congruence of triangles. The Axiom of Parallels and its consequences. Similarity of triangles. Areas. **Elementary geometric constructions. Constructible segments. Problem solving of geometric places**.	1.st
CAMPO GRANDE	Geometric Constructions (68 hours)	**Elementary constructions. Algebraic expressions. Approximate constructions.**	1.st
CORUMBÁ	Geometric Design (51 hours)	**Elementary Constructions.** Construction of triangles and quadrilaterals. **Constructions of Polygons and Circumferences.** Squaring a Polygon. Equivalence of some plane figures.	Optional
PARANAÍBA	Euclidean Geometry and	Axiomatic study of plane geometry. **Geometric constructions with ruler and compass.**	1.st

[5] The work of Oliveira and Lisboa (2015) shows how, in the Mathematics Degree course at the Federal University of Juiz de Fora, the discipline Geometric Design was absorbed into the discipline Geometry starting in 2006.

	Geometric Design I (68 hours). Euclidean Geometry and Geometric Design II (68 hours)	**Congruences**. Similarities. Areas.	2.nd
PONTA PORÃ	Geometric Constructions (68 hours)	Elementary constructions; algebraic expressions; areas; approximate constructions; geometric transformations; **some possible constructions using ruler.**	Optional
TRÊSLAGOAS	Elements of Geometry (68 hours)	**Primitive concepts**. Angles. Polygons. Number of diagonals. Classification of polygons. Euler's theorem. Inscribed and circumscribed polygons. Congruence of triangles. Main postulates. Circumferences. **Elementary geometric constructions (method of geometric places), geometric construction of the capable arc**, geometric construction of triangles, geometric construction of quadrilaterals, geometric construction of regular polygons and **geometric construction aimed at solving tangency problems using ruler and compass.** The postulate of parallels; Quadrilaterals; Proportionality Theorem and Thales' Theorem. Similarity of triangles.	1.st

Note. The bold correspond to the contents related to Geometric Constructions.

Observing the Table, more recurrent than Geometric Constructions is the Geometric Design, this with vast literature on the subject in various levels of education, such as, for example, "Learning proposal on the importance of Geometric Design and Descriptive Geometry" (Silva, 2006) and "The importance of Geometric Design to improve the quality of Geometry Teaching" (Guarnieri, 2011). However, our scientific initiation research pointed to a difference in the use of these nomenclatures for teachers of the Campo Grande course since Geometric Design may be more linked to the discipline of Technical Drawing, which participates in the curriculum of other courses, including those unfocused on teacher training and those unrelated to Mathematics. Of course, our data may include Geometric Design courses with identical content and functions as Geometric Constructions, but we believe it is important to highlight and advocate our option of maintaining the nomenclature "Geometric Constructions contents" in our work, to focus on the education of Mathematics teachers. Thus, we searched the menus of these disciplines for contents such as: elementary geometric constructions,

constructions involving ruler and compass, primitive concepts, congruences, able-arc constructions, approximate constructions, graphed in bold in Table 1.

At this first glance, we can observe divergences regarding the workload of the disciplines, especially if we notice that some of them include other contents in the same workload. Another notorious point is in relation to the compulsory nature or not of the course. In Corumbá and Ponta Porã, it is optional, so it is not possible to know from this document how often it is offered or even if it has been offered at any time. Optional courses provide a complementary formation to the course, beyond the ordinary and common flow for all academics. We can also note that, in the Três Lagoas course, the contents are inserted in a discipline of Elements of Geometry, similar to what occurs at the Federal University of Juiz de Fora, belonging to the state of Minas Gerais, and researched by Oliveira and Lisboa (2015) and, in two other courses (Aquidauana and Paranaíba), the title of the discipline is divided into Geometry and Geometric Design.

The PCPs also briefly inform us about the history of the courses. Of those that are active, the oldest is the one in Campo Grande, of the current Mathematics Institute (Inma), implemented in 1981 as a course of Sciences with qualification in Mathematics and recognized in 1984, already as Degree in Mathematics. Next, we have a course in Mathematics in Corumbá (Pantanal Campus - Cpan), which started its activities in 1985 as a degree course in Sciences - Mathematics, at the then Instituto Superior de Pedagogia de Corumbá (ISPC). The undergraduate Mathematics course at the Três Lagoas (CPTL) campus, in the eastern region of the state of Mato Grosso do Sul, has existed since 1986[6]. The course of Mathematics - Degree/ Aquidauana (Cpaq) was created in 1996 by Resolution No. 55, Coepe /UFMS[7], of November 13, 1996, and authorized to operate in the 1997 school year. In Paranaíba (Cpar), the Degree in Mathematics was created and implemented in the second semester of 2001. In Ponta Porã (CPPP), the Degree in Mathematics and Information Systems was opened and offered in 2009.

Before venturing into the interviews, already planned initially because we work directly with Oral History, it was also necessary to study the theme, including to qualify the construction of the research project and, also, the

[6] There was on campus the Degree in Science course, as well as in Corumbá, being a possible precursor of the Degree in Mathematics (Silva, 2015).

[7] Council for Teaching, Research and Extension.

interview scripts. We carried out an extensive literature review, from which we will present here a small section with the most significant works to understand this article.

Although commonplace, it is necessary to highlight for readers new to the subject that several studies show little importance, neglect, or even abandonment, to the teaching of Geometry, both in older texts, such as those by Pavanello (1989[8]) and Imenes (1989), but also in more recent texts, such as those by Pais (2019), or Sena and Dorneles (2013), which show the little emphasis given to the teaching of Geometry and the lack of teacher preparation in this area of knowledge.

Relative to Geometric Constructions, we have the dissertation by Gilson Bispo de Jesus (2008), entitled "Geometric Constructions: an alternative to develop knowledge about the demonstration in a continuing education", which tells us that the author, who is also a teacher, had interest in this topic when he realized that, as elementary school students were having difficulties regarding the development of geometric concepts, he went after answers. Jesus considers Geometric Constructions as a valuable tool for the teaching and learning process of Geometry, more particularly of demonstrations.

The dissertation of Pimentel (2013), "Teaching Geometry through Geometric Constructions", points out that geometric constructions compose essential knowledge connected to Geometry and that the deepening of its concepts contributes enormously for the student who is entering High School (Pimentel, 2013).

The text by Oliveira and Lisboa (2015) produced a history of Geometric Design in the course of Mathematics Licentiate at the Federal University of Juiz de Fora, since its creation, still in the 1960s, having even a Department of Geometric Design, until its extinction with the absorption of the contents in the Geometry discipline, in 2006. This text is in dialogue with our research, especially regarding the creation of a specific textbook, used for years on end, having the labor market as a motivator for the permanence of

[8] Silva (2010) emphasizes how delicate it is to point to the Modern Mathematics Movement (MMM) as a vector of this abandonment. According to the author, the Movement was concerned with Geometry, even proposing important changes in its teaching. Silva (2010) also emphasizes the need for further research on the effectiveness of the MMM proposals for Geometry in the classroom.

the subject in the curriculum of mathematics teacher training and the absorption of these contents into other disciplines, as we saw earlier.

Machado (2012) investigated the teaching of the discipline Drawing in the Application College of the Federal University of Santa Catarina from 1960 to the mid-2000s. The author points out that due to the utilitarian focus of the teaching, linked to the industrial process, the discipline that had four strands: Drawing of the Natural, Geometric Drawing, Conventional Drawing, and Artistic Drawing had greater emphasis than Geometric Drawing. For the author, the introduction of Artistic Education also contributed to the decline of Drawing as a subject in Basic Education.

In "The importance of Geometric Drawing to improve the quality of Geometry Teaching" (Guarnieri, 2011), the author indicates some reasons for the low performance of elementary school students in solving problems involving geometry. Some of these reasons are strongly linked to teacher training, such as: a large part of the teachers did not receive an adequate teaching of Geometry, having a precarious basic training, due to the very influence of Modern Mathematics in the 1960s and 1970s (Guarnieri, 2011, p. 68); and the fact that the initial training courses, both magisterial and undergraduate, continue, for the most part, without discussing with students elements that could be used in their respective classrooms to have an efficient teaching of Geometry.

In "Historical Fragments of Geometric Drawing in the Brazilian Mathematics Curriculum" (Costa & Rosa, 2015), we can see the need for the Geometric Drawing subject (this one that studies certain early geometric concepts such as Geometric Constructions) to return as a mandatory subject in the Brazilian school curriculum because with it, students could also better understand the relationships between Geometry and Algebra and other more teaching and learning concepts.

DRAWING A SUPPORT LINE

This Master's study had Oral History as its qualitative methodology, the same used in the Scientific Initiation. This methodology relates historical aspects, documents of the most varied kinds, as well as the intentional production of historical sources from narratives. Outlining a project in Oral History implies structuring a series of procedures: 1) selecting interviewees, 2) elaborating interview scripts, 3) conducting interviews, 4) conducting transcriptions, textualizations, and transcription [eventually], and 5) observing

ethical care and the letter of assignment[9] (Silva, 2016). We chose to work with interviews because we believe they can subsidize our initial questions.

As much as we know that official documents and norms effectively guide the teacher's work in the classroom, we sought, beyond guidelines, the perceptions of these teachers about their work in these disciplines and how they participated in the training of mathematics teachers. As Machado (2012) states, the existence and maintenance of a school subject depend on numerous factors, including historical ones. At the university, this is no different. Eventually, we find a list of disciplines that seem to have little dialogue with the training of mathematics teachers for Basic Education, as Souza (2022) states. Moreover, technological advances may have introduced significant changes in professional practice regarding Geometric Constructions.

Working with interviews demanded a previous selection of possible interviewees and a serious discussion about scope and time cut, in order for the research to be feasible and, simultaneously, as broad as possible. Thus, we chose to restrict ourselves to the UFMS's active Undergraduate Mathematics courses. Certainly, studying how other institutions in the state approach this issue would also be relevant, but not very feasible for a master's degree. The time frame was based on how long the courses have existed, the oldest active course being the one in Campo Grande (1981) and the newest one being the one in Ponta Porã (created in 2009). However, during the interviews with professors who taught or had taught the selected subjects, this scenario was restricted, according to their performance, which varied from 2004 to 2019.

After the initial selection of professors - at least one from each course/campus - we proceeded to the preparation of the interview scripts, which aimed at allowing the professors to present their ideas, involving the conduction of the subject and its importance for the education of mathematics teachers. During the interviews, the script was not followed rigidly. Many questions were answered without being asked, they arose from the flow of the interviewee's narrative, and others were asked based on what they reported. However, having a guiding document in hand made both the deponent and the

[9] Our Informed Consent Form (ICF) are the letters of cession signed by all the deponents and analyzed by the board of qualification and defense of the work. We explicitly assume to exonerate Acta Scientiae from any consequences arising therefrom, including full assistance and eventual compensation and any resulting damage to any of the research participants, in accordance with Resolution No. 510, April 7, 2016, of the National Health Council of Brazil.

interviewer feel more secure during the interview process. We follow Garnica (2007), who proposes that interviews in Oral History should be dialogues in a climate of complicity between interviewer and interviewee, listening attentively, openly, and simultaneously, qualified, and that it is important not to maintain, in relation to the interviewee, a posture of silent detachment, causing the impression of disinterest. In contrast, the researcher must not remain neutral, but must interact with him, captivate him, question him about his answers, when necessary.

With the seven interviews conducted, we transcribed them, from the oral to a written record, trying, however, to maintain typical aspects of the oral record. According to Souza (2006, p. 114), the transcription exposes the impossibility of neutrality of the researcher because this process, as reliable as we may wish it to be, imposes choices on us. What to manifest and what to make explicit in our texts? Which gestures deserve to be mentioned? What tone of voice? This is, already, a first interpretation.

Next, we move on to the textualization of the interviews, which is the moment in which the researcher transforms the transcription more radically, elaborating a more concise text, reducing verbal lapses, grammatical mistakes, and some oral language vices. The textualization is the historian's text that respects the deposition data, but imprints his style on it (Garnica, 2007). Moreover, at this stage, as Bom Meihy (1991) highlights, this resource increases the reader's involvement. This is a delicate process, for ample interpretative possibilities emerge that will later be shared with the audience. The silences and the unspoken of the interviewee can also be interpreted in a survey.

> The ethical issue involving scientific research would not be an afterthought to the analysis process, but part of it? what should or should not become public is something that should have a space in the middle of the negotiations of the interviews: it is the interviewee's right to hide information already said at the moment of the recording or to add it when he/she deems it necessary. (Souza, 2006, p. 96)

Although the whole process described above is loaded with subjectivity and decision making on the part of the researcher, there is still a separate topic in these works, directed to the interpretations produced by the researcher, either in the construction of a plausible narrative on the theme, or in the development of categories of analysis. Initially, we thought of producing a paradigmatic analysis, inspired by the work of Martins-Salandim

(2012). The data analysis performed by the author was built in two moments: in the singularities and divergences of the written sources and their fifteen narratives, thus evidencing reflections on the training of mathematics teachers. Our movement, however, did not seem as productive, and there were difficulties in writing, when our themes intersected and merged, making it impossible to separate categories. The narrative mode also did not seem adequate to our objectives, mainly because we wanted, besides building a history, to problematize this discipline for the formation of math teachers in the current scenario.

In our Research Group, other works have already exercised modes that do not fit exactly in a narrative perspective, nor in a paradigmatic perspective. For example, Pinto's (2013) thesis: "Minerva Projects: kaleidoscopic game box", bets on the multiplicity of visions and interpretations, with the creation of nine disjoint volumes, each one discussing specific themes, generating problematizations concerning the themes and producing, at the end, a fictional radio debate that, in some way, presents several positions on the theme, put in check, one in front of the other.

The work of Moreira (2018), "Language Games and Euclidean Plane Geometry: a Wittgensteinian therapeutic look to two textbooks used in Mathematics undergraduate courses", with a Wittgensteinian perspective, presents the language games of each textbook and bets on the explicitness of the differences, indicating a comparative model of the axiomatic of these books. With this, he manages to show different axiomatics operating in two manuals of Euclidean Plane Geometry that coexist, currently, in mathematics teacher training courses.

Silva's (2019) research, "Non-Euclidean Geometries in Mathematics Education: a grammatical analysis", exercises a grammatical analysis, of Wittgensteinian inspirations, of a scientific article strongly referenced in his survey of articles published on non-Euclidean geometries in Mathematics Education journals, producing a possible discourse from the constructed conceptual scheme. Silva looks at the grammar - in the Wittgensteinian sense - of this text and elaborates possible discourses through a graph.

Silva (2020), " A dispersive and narrative look at the creation of the Computer Science Course at UFMS", investigated the creation process of the Computer Science Course at UFMS through Oral History. This fact involved many processes, negotiations among professors of the then Mathematics Department, including the migration of many professors to the area then being created. The term "a dispersive look" in the title is justified, as it points

"outside" the elements presented in his statements. Their gaze turns, for example, to the creation of the area - Computing at a national level. Even though their deponents did not talk abundantly about it, focusing on local movements, they revealed this aspect as important at that moment and without which the movement at UFMS would not occur.

In all researches presented so far, each one has its own way of operating its analysis. In this sense, we also traced in this research a *third* way with our proposal of analysis, not fitting, thus, neither with the narrative mode, nor with the paradigmatic mode fully. Our analysis was composed of guiding questions that aimed not to close, but to broaden the discussions, sometimes by collating our statements more directly, sometimes by going beyond them, as Silva (2020) did. This fluid movement conforms to the theoretical perspectives adopted in our research group that point to the impossibility of full knowledge of the world or the phenomena, nor of its imprisonment in scientific texts[10]. Our analyses are, above all, an exercise of opportunities to think, produce questions, and, also, find some answers, always locally situated and provisional.

RESULTS AND ANALYSIS: INTERSECTIONS, GEOMETRIC LOCATIONS, AND NEW ROUTES

Our movement of analysis of the interviews and written documents uncovered aspects unknown to us until then - as systematized research - about this discipline and its importance for educating mathematics teachers. We sought this always taking as a starting point the speeches/texts produced throughout the work and, based on them, problematize such questions or modes based on the pertinent literature. Thus, we opted for a **dispersive** look,

[10] We could point here to the rhizomatic thought of Deleuze and Guatarri (A Thousand Platots), the impositions that language makes on us, when we question the world with Friedrich Nietzsche (particularly in Gaia Science), or, still, the multiplicity of language games and ways of life of Ludwig J. Wittgenstein (Philosophical Investigations). We are not here in this paper "using" these concepts, but rather, adopting a less dogmatic stance of the world, taking it as a production, always, locally situated, enabled and constrained by our language games. Thus, the production of questions aims, among other things, to fray our language games, to search for other ways (and even languages) to "attack" such problems. It is thus not a game of questions and answers, but a fluid, rhizomatic movement of walking through a field, engendering new thoughts.

as Pinto (2013) and Silva (2020), by taking questions that would lead us both to discuss what had already been seen during the research process, bringing together speeches, evidencing divergent points of view, and to look "outside" the research done up to that moment, moving other sources and theoretical references. Our discussion was based on five questions elaborated at the moment of analysis:

1. Geometric Constructions: an elementary discipline?
2. Geometric Constructions: change to improve?
3. Geometric Constructions: adapt to survive?
4. Geometric Constructions: fail, a necessary filter?
5. Geometric Constructions: what materials guide the teacher in conducting his subject?

For this article, however, we have chosen to address only three of them: (1) Geometric Constructions: an elementary subject? (4) Geometric Constructions: Fail, a necessary filter? and (5) Geometric Constructions: what materials guide the teacher in conducting his subject?

GEOMETRIC CONSTRUCTIONS: AN ELEMENTARY DISCIPLINE?

An interviewee told us that this subject presents very elementary concepts, as do other subjects in the first semester of the course in which he works. He points out that they are necessary since it is in these more elementary subjects that the work of a Basic Education teacher is developed. Another deponent classifies it in the same way, in opposition to advanced subjects, creating a hierarchy among the subjects of the course, and, consequently, among the teachers who teach them: temporarily hired in the first group - the "basic" ones - and permanent in the second - the "advanced" ones. Thus, the referred subject in this specific campus is commonly assigned to contracted/substitute teachers.

Based on these statements, some questions arise: why is this subject assigned to substitute teachers, if it is precisely the subject that approaches the work in Basic Education (the focus of undergraduate education)? Wouldn't this be a primordial subject for the future teacher, considering the neglect of Geometry already mentioned by many historical researches (Casado, 2011; Pavanello, 1989)? What conceptions of teacher education and mathematics

are associated with these statements? Would the effective teachers have difficulty working with these disciplines, preferring others, perhaps due to a training too specialized in other subjects? We will not attempt to answer these questions here, but we will trace some possibilities of understanding.

Teachers interviewed by Leda (2015) show indignation about the role they are assigned because they are substitutes. Generally, they are not allowed to participate in extension or research projects, having their workload entirely devoted to teaching. With a large workload, they end up taking on a diversity of subjects from different fields, at times and in classes that, for some reason, have been overlooked by the permanent professors.

Alternatively, most of our respondents said they had not had this discipline (or its equivalent) in their undergraduate degrees. In some cases, the interviewee's "Geometric Constructions" was not even taught by a professor of the Mathematics area, which shows a strong connection with Technical Drawing, ignoring possible discussions for teacher training, either at a pedagogical level or at a conceptual level - as to the geometric/axiomatic justifications of the constructions performed. In other institutions, as mentioned by Oliveira and Lisboa (2015), in the early days of the Federal University of Juiz de Fora (UFJF), there was even a Department of Design, disconnected from the Department of Mathematics.

Only one interviewee mentions having had Geometric Constructions in his undergraduate studies and that this was an inspiration to continue his studies in the perspective of Geometry. He also highlights the fact that he had excellent teachers, which made him feel prepared to teach this subject, replicating to the students the knowledge acquired during his graduation.

Our interviewees, in general, indicate that this subject is essential for the future professional practice of the graduate because it makes it possible to teach Geometry in a better way in schools, making it more "palpable", concrete, as it is possible to see geometric concepts in practice and not only in an axiomatic method. In this sense, it would be important for the teacher's approach to be adequate to this purpose, since the work on Geometric Constructions can often be close to Technical Drawing and to a language that is not very appropriate to Basic Education.

An interviewee mentions the relevance of the subject to recover Geometry contents that students practically do not see in Basic Education, and to lead them to acquire a more formal language. She also points out that GC is a subject that best makes connections with the Mathematics of Basic

Education, due to its concreteness and because it is content of Basic Education and, simultaneously, due to the formal language used in the justification processes based on the axiomatic of Euclidean Geometry. The teacher also reinforces the significant possibility of breaking the dichotomy between theory and practice in this subject.

> For another respondent, it is not very difficult to understand Geometric Constructions. And since it is not a very deep subject, it becomes more accessible to students who are lagging behind. From what she said, it seems to us that she considers it elementary, like many students in their first semester. It is expected that students should arrive at the university with many concepts already well established in high school, so these contents should not be new to them. The course would serve, in this sense, as a kind of "leveling" or review. This interviewee also opens a discussion that, for us, is very pertinent, about the problems of a degree course in mathematics and how unattractive it is for the "best students" of high school. Tatto and Scapin (2004) explain very well the rejection of Mathematics at the Undergraduate level, which, according to them, is related to the preconceived idea that mathematics is difficult because of past negative experiences, the lack of interest and a negative self-image that the student has of himself, the lack of family support, the lack of motivation due to the contents not having a practical application, the lack of incentives from some teachers and non-specific training, conflicting human relationships, conditioning, passivity and the use of memory to the detriment of reasoning, these causes may be extrinsic or intrinsic to the students. (p. 13)

The evasion in Mathematics courses is quite considerable, as shown in the text by Bittar et al. (2012), which addresses the thirty years of the course at UFMS in Campo Grande (DMT/Inma). The article describes the worrisome difference between the amount of students who enter the course of Licentiate in Mathematics and those who leave at the end of the course. The dropout rate is very significant. Only 20% of the entrants in the last thirty years have managed to graduate from the undergraduate mathematics course studied, and, to make matters worse, there are also a considerable number of graduates who have not pursued a career in education. The article points out several reasons that would justify these situations and could be discussed.

This lack of interest by students, especially those who are more prepared to compete for positions in a university entrance exam, has generated statements from university professors, among whom are some of our interviewees. They point out that students reach a higher education level with mathematical knowledge that is less than desired. This is true for the geometric disciplines, most probably because they have been neglected or little worked on historically (Pavanello, 1989). For an interviewee, the failure in Geometry at the higher education level is because, in Elementary Education, geometric entities have a strong visual appeal, linked to concreteness, unlike what happens in Higher Education, generating several problems. According to Rogenski and Pedroso (2014), this gap in relation to Geometry comes from Elementary School, where it is not related to the other structuring contents, such as algebra and numbers, there is no real understanding of the properties and concepts, being only mere illustration and exemplification.

Because of this, the "elementary" aspect of the subject is quite controversial: at times it points to ease, very much linked to its "concrete" aspect, or because it is a Basic Education content; at other times it points to a connection between Basic Education and Higher Education, as a possibility of introducing a more hermetic symbolic language and of justifying and/or exemplifying the contents of an Axiomatic Geometry (proper of Higher Education). On the other hand, this "facility" in working with the subject is in counterpoint with other disciplines of the course, kept under the supervision of effective professors of the University. Again, it is necessary to point out that our discussion operates a cut in teacher education, so, it is up to us, besides enunciating these aspects of the work with these disciplines, the existence of disciplines in a Mathematics Undergraduate course that fulfill such role: to resume contents from Basic Education, to connect the two levels of education and to be passed over by effective teachers under the justification of being easier.

GEOMETRIC CONSTRUCTIONS: FAIL, A NECESSARY FILTER?

It is noticeable in the speech of some interviewees that "failing" this subject is valued as a mechanism to draw the student's attention to its importance. This is because, as students tend to find it easier, they devote more time and attention to other subjects with higher failure rates, pointing to a relationship between appreciation and failure rates. Many teachers resort to

discourses related to assessment, as a subsidy for the teaching process, as in Gomes (2013)

> The true meaning of evaluation is to provide subsidies seeking to guide the current pedagogical practices, overcoming problems and consecutively favoring the educational process so that the teaching learning process occurs. Therefore, evaluation and teaching learning must go hand in hand, as both are responsible for the success or failure of an educational system. (p. 3)

The traditional evaluation, available in the teaching media, usually generates a grade, which says whether the student is approved or not, that is, it does not always result in improved learning or significant changes in teaching methodology. It is a momentary and partial evaluation, and can interfere very little in the daily practices of students and teachers, materializing only results that, in the last instance, release the students to take the subject again.

It becomes a challenge for teachers, especially in the case of Geometry, to stimulate students to appreciate this area. An interviewee, who, even though he has a Bachelor's degree, worked academically in Applied Mathematics, says that sometimes students have no desire to continue their studies in Geometry because it is treated axiomatically, which ends up causing a repulsion for Mathematics in general. Imenes (1989) pointed out how the axiomatic model of Geometry permeated the other branches of mathematics and even the textbooks at the time of his research. Rigor, formalism and mathematics are words that go together and, generally, it is in assessments that this rigor shows itself most pronounced. It is interesting to see how even the rigor, although present, is changing in school and even in Geometry textbooks, as Carvalho (2022) points out.

There is the possibility of "memorizing" procedures even without the proper understanding, especially in disciplines where there is only one way to solve certain questions, only one way to demonstrate a theorem. According to the latest edition of the OECD[11]'s Pisa (Program for International Student Assessment), memorizing math formulas helps to solve simple questions, but not to tackle more complex ones. When faced with one of the most difficult questions, which required more reasoning, with different steps to reach the conclusion and inspired by real situations, students who reported studying by

[11] Organization for Economic Cooperation and Development.

"memorizing" were four times worse than the others (Gazeta do Povo, 2016). This is the conclusion of an analysis of the performance of 15-year-old students from sixty-four countries. Nevertheless, for the OECD consultants,

> memorization is not without its essentials in learning. It is the basis for reflection, helps reduce anxiety when solving problems, and provides the necessary fluidity of reasoning for difficult problems. But to achieve an excellent performance, it is necessary to study the subject in a more reflective way, finding different solutions to the same problems and making connections - and in this case, memory can be used, but only as a good tool. (Gazeta do Povo, 2016)

Mathematics, like other sciences, has several positions regarding its ontology and epistemology. One way of conceiving mathematical knowledge is in the form of a ladder (Carvalho, 2022). Each step must be climbed to access the others. Thus, when ascending higher rungs, the subject must necessarily master the lower (elementary) ones.

Thus, when ascending higher rungs, the subject must necessarily master the lower (elementary) ones. Thus, by prioritizing a more advanced mathematics content in the last semesters of the course, institutions would be guaranteeing quality in the training of these future teachers since they would have already passed the most elementary steps. In other words, by passing these higher levels, they would prove that they had acquired the most elementary knowledge. We believe that the meaning of high flunking may often be tied to this idea of mathematical knowledge.

Maria Laura de Oliveira Gomes (2016) shows us how students in a subject she taught in 2015, Numbers in Basic Education, when confronted with possible questions from a Basic Education classroom, involving rational and irrational numbers, presented several errors regarding the use of language and the proposed mathematical concepts. Perhaps these data would be commonplace, if not for that:

> for the most part, [these students] had previously passed Calculus, Analytic Geometry, Linear Algebra. Moreover, among the specific disciplines of mathematical training in the UFMG course eleven of these students had previously passed Fundamentals of Analysis and/or Analysis I and/or Fundamentals of Algebra. ... As we tried to point out in a previous article (Gomes, 2016), mathematics education in

> undergraduate degrees is still centered on the contents of academic mathematics, treated from a point of view that prioritizes their values, without analyzing them from the demands of school mathematics [emphasis in the original].(Gomes, 2016, p. 1086)

Gomes' text (2016) allows us to see how this assumption can lead to numerous difficulties for future teachers who often do not realize in their training the comparison of issues of school mathematics, called by many "basic". Here, the adjective is confused because although it is a mathematics present in Basic Education, it may contain countless difficulties and sophistication that are not guaranteed by the study of a "higher" mathematics. This perspective can be explored, from the view of Lins (1999, 2004), when demarcating the mathematics of the mathematician and the mathematics of the street as diverse to the mathematics of the mathematics teacher from their legitimacies. Vilela (2007) also highlights the different adjectivities of Mathematics in the literature, specifying different ways of operating with all of them.

Again, we should direct our attention not only to the contents of Geometrical Constructions, but to the ways of conceiving the education of mathematics teachers. The discourses presented here about failing grades call our attention to the dispute of space and dedication of Mathematics undergraduate students because of the number of failures in each subject, and to the way these evaluations dialogue with sequential models of Mathematics itself. Rigid assessments on advanced contents would guarantee control over learning about "basic" contents, which diverges from Gomes (2016) and from the discussions of Lins (1999, 2004) and Vilela (2007).

GEOMETRIC CONSTRUCTIONS: WHAT MATERIALS GUIDE THE TEACHER IN THE CONDUCT OF HIS SUBJECT?

During the interviews, we were introduced to some technological materials and devices that the interviewees use in their classes in these disciplines, to support and guide the teacher's work, leading the classes to certain actions: ruler and compass, dynamic geometry software, manuals or textbooks, etc. Table 2 shows an inventory of this information.

Table 2

Materials used in the disciplines separated by course/campus (Souza, 2021, p. 180)

Discipline and Campus	Materials used
Geometric Constructions - Campo Grande Campus.	Handout of Geometric Constructions prepared by two teachers of the course (Maura Candolo Marques and Marilena Bittar) Barbosa, J. L. M. (s.d). Euclidean Plane Geometry. SBM. Lima, E. L. (2002). Plane Coordinates. SBM. Euclid, (2009). The elements (I. Bicudo, Trad.). Unesp Publisher.
Plane Geometry and Geometric Design - Aquidauana Campus.	Dolce, O., & Pompeo, J.N. (2019). *Foundations of elementary mathematics - Plane geometry* (Vol. 9.). Current. Barbosa, J. L. M. (1997). *Euclidean Plane Geometry. SBM*. Handout by Marcos Luiz Lourenço about Cabri Géomètre.
Euclidean Geometry and Geometric Design - Campus Paranaíba.	Dolce, O., & Pompeo, J.N. (2019). *Foundations of Elementary Mathematics - Plane Geometry (Vol. 9.). Actual.*
Geometric Design - Corumbá Campus.	Carvalho, B. A. (1970). Geometric Drawing. To the Technical Book Publishing House.
Euclidean Geometry and Geometric Design - Campus Paranaíba.	Barbosa, J. L. M. (1997). Euclidean Plane Geometry. SBM. Euclides, (2009). The elements (I. Bicudo, Trad.) Editor Unesp. Lima, E. L. (2002). Plane Coordinates. SBM.
Geometric Constructions - Ponta Porã Campus.	Wagner. E. (2007). Geometric constructions. SBM. Downs, F. L., & Moise, E. E. (1971). Modern mathematics series - Geometry. Editorial Norma. Putnoki, J. C. (1989). Elements, Geometry and Geometric Design. Scipione.
Elements of Geometry - Três Lagoas Campus.	Lima, E. L. (1977). Areas and Volumes. To the Technical Book Publishing House. Dolce, O., & Pompeo, J.N. (2019). Foundations of Elementary Mathematics - Plane Geometry (Vol. 9.). Current.

João Lucas Barbosa's book, Euclidean Plane Geometry (1997), is cited in some statements and is present in the basic or complementary bibliography of almost all courses under study. Moreira (2018) compares this book with *Plain Euclidean Geometry and Geometric Constructions* by Eliane Quelho Frota Rezende and Maria Lúcia Bontorim de Queiroz (2000) based on Wittgenstein's language games (1999). In this comparison, he highlights differences from the viewpoint of the language adopted and the axiomatic sequence. For this text, it precisely matters the axiomatic viewpoint of the work, highlighted by Moreira (2018), but quite visible in the presentation of the work on the publisher's page:

> This book gives the teacher an expanded view of what they teach in the classroom. The author shows Euclidean plane

> geometry from a point of view that goes beyond elementary and high school topics. The book can be used in the geometry course for undergraduate mathematics students. It allows familiarity with geometric facts from the presentation of theory, exercises, problems and comments by João Lucas Marques Barbosa. The axiomatic exposition of geometry allows one to begin and deepen learning about the axioms of incidence and order, axioms of segment and angle measurement. The method of axiomatic geometry allows a convincing demonstration of the power of pure thought. Euclid's books, by the rigor with which they explore this field, were references for the philosopher Spinoza, in his Ethics demonstrated in the manner of geometers, and for the physicist Newton, in his work Principia (Mathematical Principles of Natural Philosophy). (SBM, n.d.)

However, this work does not address aspects of Geometric Constructions, serving, perhaps, as a theoretical foundation for the constructions or, in a joint work, as Oliveira and Lisboa (2015) point out, for the absorption of Drawing by Geometry, the drawing as a "concrete" support to the demonstrations, too abstract. At a campuse, the subject is annual and, in the case of the professor who was once a professor at that location, the book by João Lucas Barbosa was used all year round, half of it in the first semester and the other half in the second semester, but there was also the support of others, such as Euclid's *Elements* and books on Technical Drawing.

In another course, a professor interviewed uses textbooks that deal with geometric constructions, adapting these to use dynamic geometry software, such as *Geogebra and Cabri Géomètre*. One of our interviewees was aware of João Lucas Barbosa's book, but did not use it in his classes. He had more contact with the book, *Volume 9: Plane Geometry*, by Gelson Iezzi, Osvaldo Dolce, and José Nicolau Pompeo. Ramassotti (2015) interviewed higher education teachers of a certain institution about issues related to teaching Geometry for Undergraduate, and one of them thus analyzed the collection::

> it is an old collection, which is a collection that was... that was used in high school a long time ago, but it is a book that although it doesn't have this axiomatic character, it has many exercises that give ideas to the students. Many exercises that I think are interesting. It also works some concepts in a lighter

> way, in a softer, more malleable and more accessible way. So, this book of "Foundations of Elementary Mathematics", I guide my students, some specific topics to look for this book or start doing some exercises from this book, but thinking about what was discussed in class. And then go to the exercises part of Barbosa. (Ramassotti, 2015, pp. 168-169))

A first comparison of the materials shows great divergence in the axiomatic character and in the language used, besides, of course, the layout, use of figures, and colors. Although older, the book of the Iezzi Collection has greater visual appeal, and the proofs and demonstrations are present, but not in a sequence and sequence similar to Euclid's book (2009). Even this book does not come close to Geometric Constructions, using mostly exercises related to the calculation of measures. This work seems to connect, as previously mentioned and in the promotion of the book itself, Elementary Education to Higher Education.

In view of the materials presented by the teachers, certain questions emerged: what is the role of the axiomatic method when it comes to Geometric Constructions content? Would the axiomatic discussion be enough to enable future teachers to work with Geometric Constructions/Drawing in Elementary Education? An interviewee talked about her concern with the mathematical language that the student needs to develop in this subject. According to her, a student who leaves the course writing well will not have problems in Algebra, Internship, etc. The subject seems to deal with topics such as "introduction to mathematical language" or "introduction to mathematical logic", present in the syllabus of subjects such as Pre-Calculus (or Introduction to Calculus).

Pavanello and Andrade (2002) point out that axiomatics cannot be disconnected from the construction of concepts through activities, since this construction and axiomatics are not independent. Hence, it is urgent to articulate Geometry with other curricular components of the undergraduate course, and it is relevant to associate the specific contents of Mathematics at a higher level with those of Basic Education.

A teacher said that he found it difficult to work with the axiomatic method with his students because this method had been little explored during his undergraduate studies. This is consistent with his choice of choosing the book of Gelson Iezzi's collection (Dolce & Pompeo, 2020), which works more directly on metric aspects than on axiomatic ones, compared to the work of João Lucas Barbosa, for example..

One teacher, when commenting about her discipline, remembers the time of her graduation, when she studied Geometric Design and Descriptive Geometry. According to her, the content of this discipline was more focused on Technical Drawing because the institution had many Engineering courses on the same campus, sharing teachers and students, thus influencing several aspects of her education. In her undergraduate days, she didn't understand the importance of this subject from the point of view of the Mathematics Degree, but later, during the course, she realized the relationship with other subjects. In this sense, a question arises: wouldn't the Engineering courses be an influence for the Geometric Design (and/or Geometric Constructions) course, giving it a technical aspect and unfocused on teacher training? The work of Oliveira and Lisboa (2015) points us that yes, at least in the early days of UFJF, the place of their research.

In contrast to the idea of Technical Drawing and aiming at teacher training, a handout of Geometric Constructions was created by teachers of the University City (Campo Grande) and used by them (here again our work connects to Oliveira and Lisboa, 2015). This workbook was created with the goal of recovering and reviewing the basic results of Euclidean Plane Geometry and trigonometry in the right triangle, contents supposedly seen in Elementary Education, using, for this, resources of Geometric Constructions. Generally, the workbook presents many examples, seeks to be self-explanatory, brings different approaches to the contents, with demonstrations, use of software, historical discussions and, of course, the use of ruler and compass. Another teacher from the same campus, however, expressed a contrary opinion, saying that the workbook is not justified, and chose not to use it in her class.

In this topic, in which we present the materials that were reported in the interviews, our focus is on the discipline itself, but mainly on how these disciplines and the discussions that take place in them can (or cannot) contribute to the education of mathematics teachers. Thus, the choice of materials is also a positioning in face of the issues that permeate this training: the connection to one area or another, the link to the use of innovative materials, the way of working with problems or with exercises. These are aspects that can be problematized. Thus, we see here Geometric Constructions that are sometimes close to Technical Drawing and sometimes to Axiomatic Geometry, sometimes as a support for understanding the latter. There is the use of computational tools, however, this did not become the focus of discussion or greater explanations from our interviewees, contrary to what we initially imagined.

CONCLUSIONS: REVIEWING THE STEPS AND CLEANING UP THE DRAFT

Based on the official documents of the courses in question and the seven statements, we intend in our research to understand the role played by disciplines that deal with Geometrical Constructions contents in the education of Mathematics teachers. We aimed to problematize this training, operating this cut: the Geometrical Constructions. In view of the wide range of possibilities, we chose these disciplines and to approach, at this moment, the view of the teacher educators, mainly driven by the results of the Scientific Initiation. In taking a certain position, we are aware of the risks of not giving voice to an important group in this formative process, be it the current students or those who have already graduated, who would certainly contribute to these discussions, broadening our view of the problem posed, bringing perhaps the reality of recent graduates in face of the demands of their classrooms.

Even so, we believe that teachers, beyond the norms, end up deciding, most of the times in a monocratic way, the focus they will give to their subject, the materials they will use, and even the books or handouts that will guide their classes - this is what the Scientific Initiation showed us.

We have, then, from their statements, not a supposed "factual reality" about these classes and courses, but the ways in which these teachers read these actions and conceptions to another (in this case us). We are not here, in this text, judging the actions committed by these or those, but only enhancing, through academic debate, the positions that were made explicit to us.

Thus, our list of interviewees strongly affirms the need and importance of such subjects and contents in the education of mathematics teachers, but they also reiterate the need to improve the language/writing of those entering a higher education course; to give Geometry a contextualization beyond the axiomatic method, and to review contents from Elementary Education in Higher Education. They also point out issues such as: the more complex subjects being left to the effective teachers, and other more elementary ones, to the contracted teachers - GCs figure in this last group; most of the teachers not having had a GC education that would enable them to discuss educational issues, focusing either on the formalism of the axiomatics or on the calculations of measures when it comes to Geometry; the choice of books still based, to a great extent, on the axiomatics in detriment of the

constructions with ruler and compass. Surprisingly, technological issues appear, however, they have not been the reason for major questions for these teachers to the point of radically changing their practice. Finally, since there is a potential connection of these contents with Basic Education, it is necessary to rethink their role in the education of mathematics teachers.

We hope that this article has led to a reflection on the role of this subject in teacher education and in the course where they work. We believe that historical aspects are essential in the problematization of our present, not as a simple and direct network of causes and effects, but in the possibility of exploring the multiplicity that makes up the narratives from and about the past, giving new meanings to our own history. Beyond the questions brought here, others are produced in Souza (2021)..

AUTHORS' CONTRIBUTION DECLARATION

The authors M. D. S. and T. P. P. were responsible for the conception, discussion and writing of this article.

DATA AVAILABILITY DECLARATION

The data presented are under the custody of the author M. D. S. and refer to the Master's Dissertation, available on the portal of the Federal University of Mato Grosso do Sul..

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