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Fundamental Theorem of Calculus: Cognitive Demands and Learning Limitations on Mathematical Tasks

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ABSTRACT

Background: Mathematical tasks for university teaching are generally of great cognitive demand, without thinking about the limitations they imply for their students. **Objectives**: To develop a theoretical analysis of the teaching limitations and cognitive demand of four tasks proposed for teaching calculus, specifically, the Fundamental Theorem of Calculus. **Design:** Qualitative paradigm, with a descriptive-interpretive approach, according to the nature of the data collected. **Setting and Participants**: The study is framed in a Colombian University, in the subject "Calculus II" for second-semester engineering students, where a professor designs the tasks to be implemented in this course. **Data collection and analysis:** The data correspond to the professor's lesson plans the statements of the main mathematical tasks within them. These plans were chosen based on availability and accessibility. A content analysis was conducted, considering as units of analysis the paragraphs or sets of paragraphs of the statement of each school mathematics task. **Results**: Most of the proposed tasks correspond to high cognitive demand (procedures with connections and mathematical construction) and only one was of low demand (memorisation). Moreover, each of the tasks presents its own cognitive demand and several learning constraints that, some of them, agree with the exposed literature. **Conclusions**: The work aims to have implications for higher education, since to think of a didactic proposal for a better approach to teaching is necessary to configure lesson plans that mobilise the learning of mathematics in engineering, but from the use of tasks with different cognitive demands, in which will vary from less to more, and lead to meaningful learning for the approach of new tasks.

Keywords: Cognitive demand; Mathematical tasks; Fundamental theorem of calculus; Integrals; Derivatives.

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Teorema fundamental do cálculo: exigência cognitiva e limitações na aprendizagem das tarefas matemáticas

RESUMO

Contexto: as tarefas matemáticas para o ensino universitário são geralmente de grande exigência cognitiva, sem pensar nas limitações que implicam para os seus alunos. **Objetivos**: desenvolver uma análise teórica das limitações do ensino e da exigência cognitiva de quatro tarefas propostas para o ensino do cálculo, especificamente, o Teorema Fundamental do Cálculo. **Design**: do paradigma qualitativo, com uma abordagem descritiva-interpretativa, de acordo com a natureza dos dados recolhidos. **Ambiente e participantes**: o estudo é enquadrado numa Universidade Colombiana, na disciplina “Cálculo II” para estudantes de engenharia do segundo semestre, onde um professor concebe as tarefas a serem implementadas neste curso. **Coleta e análise de dados**: os dados correspondem aos planos de aula do professor, em particular, as declarações das principais tarefas matemáticas dentro deles. Estes planos foram escolhidos com base na disponibilidade e acessibilidade. Foi realizada uma análise de conteúdo, considerando como unidades de análise os parágrafos ou conjuntos de parágrafos da declaração de cada tarefa de matemática escolar. **Resultados**: A maioria das tarefas propostas corresponde a uma elevada procura cognitiva (procedimentos com ligações e construção matemática) e apenas uma foi de baixa procura (memorização). Além disso, cada uma das tarefas apresenta a sua exigência cognitiva e várias restrições de aprendizagem que, algumas delas, concordam com a literatura exposta. **Conclusões**: O trabalho pretende ter implicações no ensino superior, uma vez que para pensar numa proposta didática para uma melhor abordagem do ensino é necessário configurar planos de aula que mobilizem a aprendizagem da matemática na engenharia, mas a partir da utilização de tarefas com diferentes exigências cognitivas, nas quais variará de menos a mais, e conduzirá a uma aprendizagem significativa para a abordagem de novas tarefas.

Palavras-chave: Demanda cognitiva; tarefas matemáticas; teorema fundamental do cálculo; integrais; derivadas.

INTRODUCTION

Teachers at different educational levels express concern about identifying and proposing mathematical tasks for teaching that are not of the same level of cognitive demand or its edges. Itzcovich (2005), Ruiz-Olarría (2015) and Valer (2017) state that the faculties’ class plans we see at university include mathematical tasks levelled with the cognitive demand, which do not claim a variability in the development of mathematical thinking for the knowledge of the topics or the development of mental flexibility. Thus, they are used to exemplify the typical errors and difficulties in explanations or exercises. This causes the typification of styles of teaching mathematics that does not specify in didactics the contents or procedures to be taught, “but rather how professors prefer to teach and what are their typical or predominantly used procedures in the act of teaching” (Ventura, 2013, p. 11).

Consequently, we observe a style of mathematics teaching marked in practices that reveal static levels, focused on practices whose decisions and teaching actions correspond to other epistemological models used throughout history (Gascón, 2001) and influence the didactic apparatus of the construction of teaching proposals at the elementary and high school levels, where there is greater diversity in the use of tasks of different cognitive levels (Cárdenas, & Blanco, 2016; López, 2013). First, however, it is worth asking: What happens in higher education? Studies support that this school level is marked by teaching styles focused on a high level of demand for the development of tasks, where the proposed activities tend to require from students a high cognitive demand (Kessler, Stein, & Schunn, 2015; Planas, 2004; Smith & Stein, 1998; Ursini, & Trigueros, 2006).

Considering that achieving teaching objectives entails implementing mathematical tasks of different cognitive levels (Smith & Stein, 2016), it is necessary to identify other elements that project a type of university teaching that considers those levels of cognitive demand in the mathematical tasks proposed in the higher education classroom, in order to support the regulated learning of their students (De la Fuente-Arias et al., 2008; García & Benítez, 2013; Penalva, Posadas, & Roig, 2010), and not to repeatedly frustrate the approaches of university students towards the knowledge of the theoretical bases, especially in the training of engineers (Álvarez, & Ruíz-Soler, 2010; Cortés, Arellano, & Vázquez, 2019; García Retana, 2013).

In this regard, Acero (2019) studies the cognitive demand in university texts related to linear algebra, concluding that four of the five texts studied exceed the average regarding high cognitive demand activities. On the other hand, the fifth book is below average in all components of high cognitive demand that they define, except for algorithmic calculation. This conclusion clarifies that, at the university level, high cognitive demand activities seem prioritised over others.

On the other hand, García and Benítez (2013) present a proposal to carry out the assignment of tasks that support mathematics learning in first-year university students. Their results show that students’ learning is favoured when there is congruence between the cognitive demand of the tasks, the mathematical content of the problems and the curricular objectives.

In turn, Penalva, Posadas, and Roig (2010) characterised the problem-setting activity in the probability domain by university students based on the cognitive demand put into play. In their findings, they affirm that, based on the comparative analysis of the characteristics of the mathematical activity of groups of students who have raised problems with a high level of cognitive demand and those who have done so with a low level of demand, it was not possible to establish a relationship between the type of approach the students used and the way they solved problems as a group with the concepts dealt. Even so, the authors consider that providing a proper balance between tasks of problem setting and problem solving has a positive effect on mathematics teaching at the university level.

A relevant topic in the teaching of integral calculus is the relationship between the antiderivative and the defined integral within the fundamental theorem of calculus (FTC), in which conceptual elements of differential calculus and the bases of integral calculus converge (Larson & Edwards, 2016). Therefore, to interpret them, students must know the basic concepts of FTC (Larson & Edwards, 2016), such as: continuous functions, derivation and its properties, antiderivative, and calculations with antiderivatives. This variety of elements to consider in its treatment can generate a presentation of the topic based on high cognitive demand, leaving aside proposals with different cognitive levels, which becomes relevant when it comes to achieving teaching objectives (Smith & Stein, 2016).

Given the above, this article aims to develop a theoretical analysis of the cognitive requirement of four tasks proposed for the teaching of FTC and its limitations for learning (errors, difficulties, and obstacles). To this end, we intend to answer the questions: What levels of cognitive requirement are put into play from the task statements for the teaching of FTC? What learning limitations does the FTC teaching presuppose from the mathematical tasks statement?

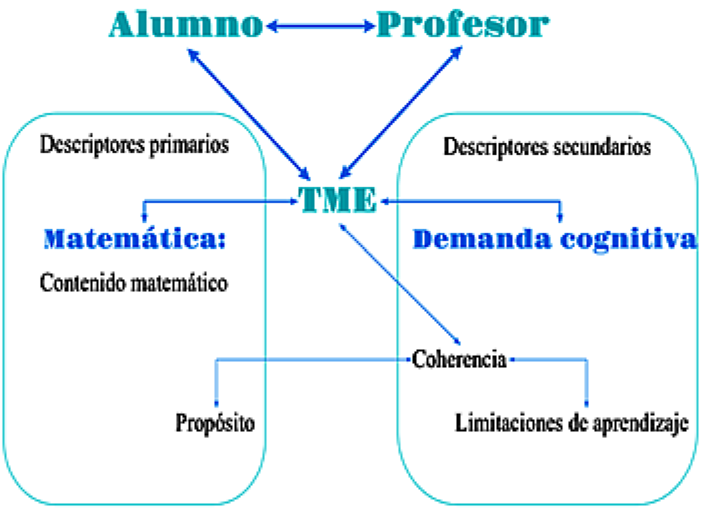
THEORETICAL FRAMEWORK

Our theoretical frame of reference is based on three constructs: the notion of school math work, the notion of cognitive demand for it, and the notion of learning limitations. In addition, we will delve into the FTC in order to have theoretical references on its applicability.

We will understand the notion of school math tasks as the one proposed for the student that implies his/her action (an activity) when facing mathematics and that the teacher plans as an instrument for learning or learning evaluation (Moreno & Ramírez-Uclés, 2016). These can be analysed from different perspectives, one of which is presented by Ramos-Rodríguez, Valenzuela, and Flores (2019) from the student-teacher-content triad involved in the model illustrated in Figure 1. The model proposes a way to approach school math tasks considering the analysis and discussion stimulated by their primary and secondary descriptors. Primary descriptors have to do with what is observed at first sight in school math tasks, specifically, their purpose and the mathematical content put into play in their development.

Figure 1

Characterisation of school math work. (Ramos-Rodríguez, Valenzuela, and Flores, 2019)



Secondary descriptors require a deeper analysis of the task, i.e.: i) studying the coherence between the task statement instruction and the task purpose, ii) investigating the learning limitations (errors, difficulties, and obstacles) and iii) analysing the cognitive demand that the task involves (Smith, & Stein, 2016) and that must be considered in its implementation.

Learning limitations refer to the possible errors, difficulties, and obstacles that arise during mathematics teaching and learning. These mathematical tasks involve mathematical modelling, problem solving, and other issues related to reading comprehension and appropriation of symbolic language, low appropriation of previous concepts such as function mastery (Gallardo & Galindo, 2015; Guzmán & Vallejo, 2004; Plaza-Galvez, 2016), aiming to support faculty in choosing suitable tasks for calculus teaching. In this regard, Bachelard (2000) and Brousseau (1983) categorised the obstacles into three types: ontogenic (neurophysiological and learning limitations), epistemological (related to the origin of the concepts), and didactic (related to the teaching of the concepts, the scenarios, and the educational system).

Meanwhile, Autino et al. (2011) characterise the obstacles faced by engineering students, such as: understanding mathematical objects and their use in problem situations, and using mathematical and everyday language in both senses (epistemological); lack of study methods, motivation, interpretation, and continuous attention in academic (ontogenetic) commitments; and organising the contents of the course, the curricular structure, identifying characteristics of the group being taught and inadequate class materials (didactic). Other authors, such as Hein and Biembengut (2006), reveal students' difficulties in not knowing the interpretation of a context linked to modelling phenomena or processes.

In particular, several authors point out limitations in the learning of calculus, highlighted from the imbalance between the conceptual and algorithmic treatment of integrals (Muñoz, 2000; Zavala, Vera, & Ruiz, 2017). Also, the contexts of application of calculus are 'stereotyped' with indiscriminate use of techniques and procedures, privileging algorithms over geometric treatment and its meaning in the teaching of the FTC (Artigue, 2002), or lack of moments of 'discovery' among the student body at university level (Gordon, & Gordon, 2007).

Moreover, authors such as Zavala, Vera, and Ruiz (2017) highlight that university students present difficulties regarding the use of representations (and transit between them) in the teaching of calculus, which are due to (i) the complexity in their use; (ii) the time spent to propose various records (if specialised programs are not used to model such as GeoGebra); (iii) they are not considered significant within the plans; or (iv) simply because the curriculum is delimited by a textbook that teaches only the algebraic and operational aspects of the integrals.

On the other hand, the cognitive demand of a school math task refers to the cognitive requirement that it involves, for which we will use the taxonomy presented by Smith and Stein (2016) (Table I).

Table 1

Cognitive Demand Taxonomy. (adapted from Smith and Stein's description (2016, p.16-17))

Type	**Indicators**
	Low-level cognitive demand
Memorisation (MEM)	**(MEM1)** The task involves reproducing or developing previously learned rules, formulas or definitions.
	(MEM2) The task does not have an immediate resolution through procedures, due to their absence or because of the little time the teacher allocates for the task to be solved with the presented procedure.
	(MEM3) The task is unambiguous. This implies that the task is an exact reproduction of the previously seen procedure or concept and the instruction clearly invites to reproduce these directly.
	(MEM4) The task is unrelated to the concepts or meanings that underlie the facts, rules, formulas, or definitions that you are learning or reproducing.
Unconnected Procedures (PWoC)	**(PWoC1)** The task is algorithmic and uses the procedure specifically mentioned or evident from previous instructions, experiences or locations of the task.
	(PWoC2) The task requires limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it.
	(PWoC3) The task has no connection or relationship to the procedures to be used.
	(PWoC4) The task focuses on producing correct answers rather than developing an understanding of the mathematical object.
	(PWoC5) The task does not require explanation or only focuses on the description of the procedure used.
	Low-level cognitive demand
Procedures	**(PWC1)** The task fixes the student's attention on employing procedures to develop deeper levels of understanding regarding mathematical ideas and concepts.

	(PWC2) The task explicitly or implicitly suggests the paths to follow, even if they are general procedures proposed superficially, which are closely linked to the underlying conceptual ideas, unlike rigid algorithms that are opaque with respect to implicit concepts. **(PWC3)** The task requires some degree of cognitive effort. Although general procedures can be followed, this is not done thoughtlessly. Students must engage with the conceptual ideas underlying the procedures to successfully complete the task, which develops understanding. **(PWC4)** The task is often represented in multiple ways, such as visual diagrams, manipulative materials, symbols, and problem situations. That which invites the student to make connections between different representations helps elaborate meaning.
Construction of Mathematics (DM)	**(DM1)** The task requires non-algorithmic and complex thinking so that the task, its directions, or a (previously) solved example does not explicitly suggest predictable or studied paths or approaches. **(DM2)** The task requires students to explore and understand the nature of mathematical concepts, processes, or relationships. **(DM3)** The task requires the student to self-monitor or self-regulate the cognitive processes. **(DM4)** The task urges the students to access their knowledge and recall relevant experiences in which they make appropriate use of it while working on the task. **(DM5)** The task requires the student to analyse it and to actively examine its restrictions, which allow him to identify, in time, possible limitations of the solution strategies and/or the solutions themselves. **(DM6)** The task requires considerable cognitive effort and may involve a level of anxiety for the student due to the unpredictable nature of the required solution process.

Table 1 presents the two subdivisions that characterise the taxonomy of cognitive demand. In the first part, the indicators correspond to the low-level cognitive requirement (memorisation and procedures without connections) and in the second part, they correspond to the high-level cognitive requirement (procedures with connections and construction of mathematics).

Finally, it seems important to say that before addressing the treatment of the FTC, we must refer to the theorem, which is presented in Figure 2.

Figure 2

Statement Fundamental Theorem of Calculus (Larson, & Edwards, 2018, p.36).

Theorem. Fundamental Theorem of Calculus If a function f is continuous in the closed interval $[a, b]$ and F is an antiderivative of f in the interval $[a, b]$, then: $\int_a^b f(x)dx = F(b) - F(a)$

Its presence in Calculus is vital since it relates two important contents of it, the derivative and the integral. Its demonstration leads us to a complex visualisation from the mathematical point of view, which, at the same time, makes it difficult to teach it.

In relation to learning limitations in the teaching of the FTC, some studies point to its identification. Among the most common obstacles in the understanding of the FTC are the fuzzy memories about previous mathematical objects and the basis for the understanding of others typical of the theorem, such as: function, continuity, derivative, and integral, the reason for change and accumulation, and the lack of relationships between representations of the integral, by not identifying the link of the integral with the area that represents or not variability of the limits of the integral), or also dismissing the recognition of the importance of the FTC in engineering education programs (Muñoz-Villate, 2021). These ideas are confirmed by Reyna Segura, (2019) who points out that the learning difficulties of the FTC lie in the complexity of the notions of Calculus and the language used.

METHODOLOGY

This work follows the qualitative paradigm, focusing on the data's richness and depth over quantity (Hernández-Sampieri, Fernández-Collado, & Baptista, 2010). It is considered a descriptive-interpretative approach, according to the nature of the data collected.

The reporting subject of the study is a university professor at the Universidad Industrial de Santander (Colombia), who teaches in the various engineering courses chosen according to availability and accessibility criteria. She has seven years of teaching experience at the university level, is 41 years old, and has a disciplinary degree (master's degree in mathematics teaching) and a didactic degree (PhD in education, in the line of training of pedagogical practices in mathematics).

The data collection instruments correspond to four class plans of the university professor, who designed a didactic proposal for the teaching of the FTC for students of the basic cycle in higher education, which includes the Engineering careers of the university where she works. From them, we extracted statements of the main mathematical tasks of the classes.

The general objective that the professor proposed for the mathematical tasks was models of the geometric representation of the FTC so that students could identify essential properties in the representation and use of the theorem through the dynamic geometry offered by the GeoGebra software.

After collecting the data, we conducted a content analysis (Flick, 2004), considering as units the paragraphs or groups of paragraphs of each mathematical task statement. The categories of analysis were extracted from the theoretical elements presented in the previous section, namely the levels of cognitive demand and learning limitations, in this case, regarding the FTC. Table 2 illustrates the categories used.

Table 2

Study Analysis Categories

Category	Subcategories
Cognitive demand, low level requirements	Math Memory Assignment
	Unconnected Procedures (PWoC)
Cognitive demand, high-level requirements	Procedures with connections (PWC)
	Construction of Mathematics (DM)
Learning Limitations	Student errors in the face of the proposed math task for the TFC

We used the declared categories of cognitive demand (low or high-level requirements) specified in Table 2 to analyse the statements proposed in the four school math tasks and to classify the tasks. Likewise, we identified the learning limitations in all the tasks analysed to observe elements in which they recurrently restrict the total understanding of the mathematical object or implicit elements from the same conception of the design of the tasks for the teaching of the FTC, which contribute to the low or no understanding of the mathematical objects. The learning limitations observed in previous studies were considered.

In the following section, the four school mathematical tasks selected for the analysis of cognitive demand and learning limitations in relation to the teaching of the FTC will be presented.

RESULTS AND ANALYSIS

We present a sequence of mathematical school assignments that point to different moments in the teaching of the FTC.

Before addressing the analysis of the secondary descriptors of each school task (cognitive demand and learning limitations), we will specify the tasks' primary descriptors, the declared purposes, and the mathematical contents put into play in them, which are detailed in Table 3.

Table 3

Declared purposes and contents put into play in each task

Task	Purpose	Mathematical Contents
1	Explore the meaning of the FTC through modelling the behaviour of a linear function and the meaning of the area under the curve as an accumulation function, where its	Integration, derivation, rules of derivation and continuity, graphical representation of a function.

	function behaviour is analysed for FTC applicability	
2	Evaluate the conditions of the FTC, and the existence or not of particular conditions for a function.	Integration, lateral limits, and continuity.
3	Infer image values of the accumulation function F of the function f for the interval [0,2□□] through special characteristics of continuity and derivability of the cosine function.	The area under the curve, integration of a function, evaluation of the integral in an interval.
4	Example on a function with particular continuity characteristics, in which the dynamic geometry of the GeoGebra software is used to recreate the behaviour of the functions, under continuity characteristics (or not) and its implications of applicability of the FTC.	Function continuity, derivative, derivation rules, integration, area under the curve.

This highlights the primary descriptors of each school mathematical task (Ramos-Rodríguez, Valenzuela, & Flores, 2019), which have to do with what is observed at first sight in school mathematical tasks, specifically, its purpose and the mathematical content that is involved in its development.

Secondary descriptors will then be addressed as school math task statements are presented for FTC teaching.

School Math Task 1. Figure 3 illustrates the statement of the first proposed mathematical school task at the university level, whose objective is to explore the meaning of the FTC through the modelling of the behaviour of a linear function and the meaning of the area under the curve as an accumulation function, where its function behaviour is analysed for the applicability of the FTC.

For this, the teacher properly chooses a continuous function *f*, with derivable *F*, as described in figure 3.

Figure 3

Statement of the first school math task.

For function f defined by $f(x) = \frac{4}{3}x - 1$, with $x \in [0.1,4]$, using your graph and GeoGebra tools:

a) Check if the function is continuous;

b) Calculate the area under the curve described by f;

c) Determine the integral that represents the area under the curve in the interval $[0.1,4]$.

d) Visualise the trace values of the function that identifies the accumulated area F, as the value of x increases (towards the end 4), for what values of x, F turns 0?

e) Perform the calculation of F and verify using the FTC.

(Tip: Use *Applet 1: https://www.GeoGebra.org/m/hcvdbkt8*)

In this challenge, students are expected to provide an answer such as the one illustrated in Figure 4.

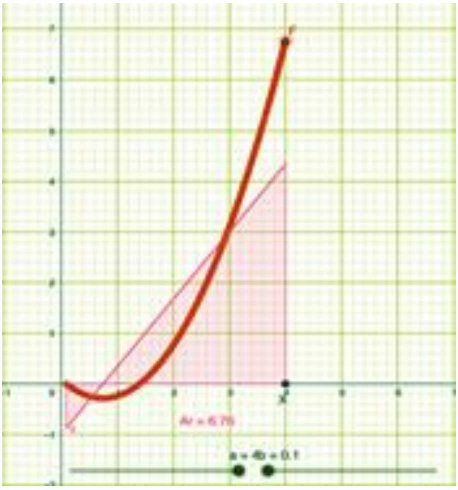
This task starts from a linear function highlighted in red, which at first glance can be checked for continuity in the closed interval $[0.1,4]$. Because it is a linear function, it is “smooth” throughout its path.

Subsequently, the student is expected to calculate the area under the curve using the “trace” tool of the accumulation function in GeoGebra, resulting in $A = 6.76$.

One should not find difficulties in determining algebraically the integral requested in paragraph c) since it only requires simple calculations after evaluating the integral in the closed interval $[0.1,4]$. That is, the trace can show what the area under the curve will be, by pointing to the ends of the interval, to finally obtain 6.76. One of the hypotheses that students must establish about the trace function is to identify if the function is derivable.

Figure 4

Expected answer with application from Applet 1;
(https://www.GeoGebra.org/m/hcvdbkt8)



For paragraph d) the accumulation function F will be defined as $F(x) = \frac{2}{3}x^2 - x$, which is highlighted in the graph in pink. This area travelled is highlighted by the trace that the curve leaves (with the values of the area) when "passing" through the points of x, along the interval [0.1,4].

Finally, in section e), when deriving the accumulation function $F(x) = \frac{2}{3}x^2 - x$, we obtain: $F'(x) = 2\left(\frac{2}{3}\right)x^{2-1} - 1 = \frac{4}{3}x - 1$, which coincides with the function f, satisfying the FTC.

In particular, it is important for the student to check, through graphical modelling of the function f and the trace of the accumulation function F, that the value of the accumulated area varies. That is, its accumulated area is in the negative values, by the values of x, until it reaches $x = 1.4$. From this point of view, the graph of the accumulation function takes positive values up to 6.76.

We can observe three learning limitations for this task. First, to check continuity in function, students are expected to engage the concepts of lateral boundaries of the linear function or to deduce it from the graphical representation. One of the difficulties that the student may have at this point is about defining the interval of the function $[0.1, 4]$, since they are not necessarily openly defined, but for a specific interval, which would imply a lack of understanding of the mathematical object and its specificity for the given interval. This difficulty coincides with Hein and Biembengut's (2006) findings.

Second, students are expected to identify the linear function and the area under the curve of the linear function, $[0.1, 4]$. However, they may not consider the area under the curve for the interval $[0.1, 0.75]$, since it is an area that is not explicitly under the curve and is not related to the value of an area in a negative zone of the plane by the values of x for the function. For this, it is possible to investigate the values of x, for which the function F will take positive or negative values or 0. This leads to restrict the understanding of the geometric treatment and meaning of the integral, as pointed out by Artigue (2002).

Finally, a difficulty may lie in relating the value obtained from F at a point x to the area under the curve. That is, recognising the function F as an area accumulation function. Finally, the area equivalent to the pink zone is (6.76) equivalent to the stored value of the area represented by the accumulation function F. In this case, this trouble is a consequence of the geometric understanding of the accumulation function and its use in a particular situation within the FTC, as alluded to by Autino et al. (2011)

On the other hand, this task is classified as high cognitive demand task of the type "procedures with connections" (PWC4), since it requires the use of different representations (algebraic, numerical, graphical, and tabular), with the handling of different concepts, to be able to verify the applicability of the FTC in this challenge. So, the students are involved in solving the tasks from the connections between the representations that lead to the meaning of the concept.

School Math Task 2. Figure 5 illustrates the second task, whose objective is to evaluate the conditions of the FTC from the analysis of the existence (or not) of particular conditions for a function f, from the representation of the function accumulation and verification of minimum conditions of the functions.

For this task, the teacher appropriately chooses a non-continuous function f, with F derivable in $[-5,0]$ and $(0,5]$.

Figure 5

Statement of the second school math task

Given the function: $f(x) = \begin{cases} 3, si\ x \in [-5,0] \\ -x, si\ x \in (0,5] \end{cases}$

a) Draw the accumulation function F for each part of the function domain f.

b) Verify the condition of existence algebraically.

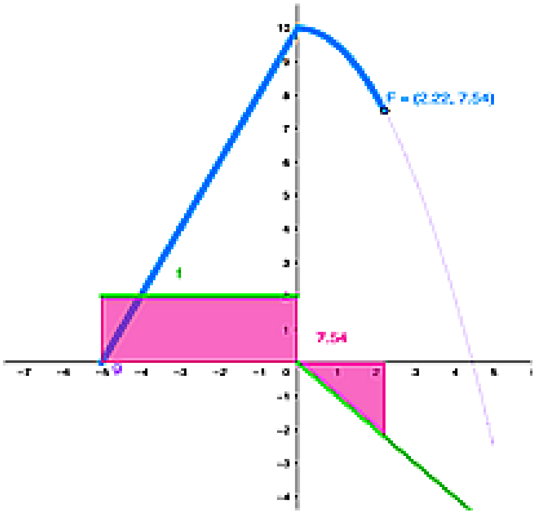
c) Does f the TFC satisfy? Argue.

(Tip: Use Applet 2: https://www.GeoGebra.org/m/thh5dnwv)

From the use of Applet 2, students can provide answers such as the one illustrated in Figure 6.

Figure 6

Discontinuous function representation from Applet 2
(www.GeoGebra.org/m/thh5dnwv)



According to the graph of the function, it is possible to observe that it presents a discontinuity when $x = 0$, which implies that f is not a continuous function, for $\lim_{x\to 0^+} f(x) = 0$ and $\lim_{x\to 0^-} f(x) = 3$. Despite this, F exists at every point of the function (see *Applet* 3: https://www.GeoGebra.org/m/rqe7trjt: GeoGebra) because, under the curve, the area continues to accumulate. However, despite the discontinuity condition in f. In f this it does not affect the continuity of F at every point of the interval $[-5,5]$. That is, in the interval [-5,0] would be $F(x) = 3x$, and in the interval $(0,5]$ would be $F(x) = \frac{-x^2}{2}$.

The "abrupt change" that the function F makes in$x = 0$ is notorious since its graph changes slope and decreases until it reaches $x = 5$.

Despite this condition, F'will always exist in this type of graphic, if one were to restrict the function to the points at which f is continuous. That is, in this example, we can notice that $F$is not derivable in 0, in the interval $[-5,0]$ would be $F'(x) = 3$, and the interval $(0,5]$ would be $F'^{(x)} = \frac{-2x}{2} = -x$. That is: $F(x) = \begin{Bmatrix} 3x, si\ x \in [-5,0] \\ -\frac{x^2}{2}, si\ x \in (0,5] \end{Bmatrix}$ and its derivative: $F'(x) = \begin{Bmatrix} 3, si\ x \in [-5,0] \\ -x, si\ x \in (0,5] \end{Bmatrix}$

Therefore, although the function *f* does not satisfy continuity, *F* does, and it satisfies that *F*' $= f$ in each sub-interval $[-5,0]$ and (0,5].

We have set this example of school math task to evaluate the conditions of the FTC and the existence or not of particular conditions of a function.

We could observe three limitations in relation to the implementation of the task. First, a common error in this activity is that the student considers the function f as continuous when relating it to the accumulation function. Not noticing the characteristics of each function, and dismissing continuity, this action can lead to a conceptual error where they indicate that $F'(x) = f(x)$. One of the questions that will arise here will be: Why, if there is the accumulation function F, do you need to ensure the continuity of f? In this way, it would be related to a difficulty of a conceptual type, and its application in a situation of discontinuity, such as those indicated by Hein and Biembengut (2011).

Second, students may consider two functions instead of a single, part-defined, function. When considered as two functions, it will allow us to find the F functions of the intervals $[-5,0]$ and $(0,5]$. So, it meets the given condition of the FTC and may come to propose one $F'(x) = f(x)$ of each interval. In this case, it could be interpreted as a challenge from the geometric treatment and its adequate meaning (in Artigue's words, 2002) of the graphed function as isolated parts.

Finally, as the third limitation of learning, we observed that we must pay attention to the 0 when analysing the continuity in the function. The displacement through different representation records could, for example, lead students to contemplate the 0 as a value in both intervals, although it is only contained in the first domain of the function. That is, it corresponds to a problem of representation records, as stated in Zavala, Vera, and Ruiz (2017).

On the other hand, from the point of view of the cognitive demand of the task, we can see that its statement presents a task whose cognitive demand is high, where procedures with connections (PWC4) stand out, since it aims for the student to mobilise algebraic and graphic representations to verify the existence of the function F that is understood from its verbal-algebraic representation, for the meaning of the concept.

School Math Task 3. The statement proposed in Figure 7 illustrates the third task, whose purpose is to infer image values of the accumulation function F of function f for the interval $[0,2\pi]$ through special characteristics of continuity and derivability of the cosine function. These characteristics reveal both the initial function$f$ and the accumulation function.

For this school math task, the teacher properly chooses a function where the student can identify function F from modelling with the GeoGebra software and the FTC application.

Figure 7

Statement of the third school math task

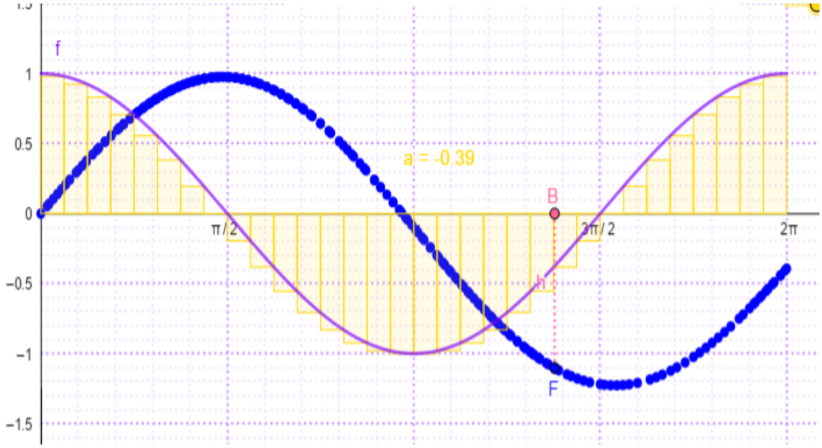
Let function f be defined as $f(x) = \cos(x)$ in the interval $[0,2\pi]$. (Tip: Lean on GeoGebra *Applet 3*: https://www.GeoGebra.org/m/rqe7trjt,to verify that function F, indicated in the TFC, exists)

In Figure 8, the trigonometric function f defined as$f(x) = cos(x)$ is highlighted in purple. This function is continuous in the closed interval [0,2[□□]. Moreover, the function is derivable.

The accumulation function $F(x) = sen(x)$, highlighted in the graph in blue, arises from the Riemann rectangles. This travelled area is demarcated by the images of the point x, included in the closed interval $[0,2\pi]$.

Figure 8

Modelling the sine function in the interval [0.2 π] from Applet 3 (https://www.GeoGebra.org/m/rqe7trjt)



This is, for the point $\frac{11}{9}\pi rad$, equivalent to $x = 3.84$. The axis y will be given by $y = -0.66$. This indicates that the area that is in the negative part, accumulated to that point, is greater than the positive one, in 0.66 units. That is, the cumulative area of the highlighted function $f(x) = \cos(x)$, is-0.66 , when they have been almost traveled $\frac{11}{9}\pi rad$.

At this stage of the mathematical task, we should propose to the students two representations for their interpretation of the model and analysis of its behaviour for the applicability of the FTC.

This school math task also allows us to evaluate the conditions of the FTC and the existence or not of particular conditions of a function.

In relation to learning limitations, we observed two arising from this mathematical task. The first has to do with the representation of the trigonometric function and the scale units that the radians represent in this trigonometric function, since they may lead to students' erroneous representation. When passing through the equivalences of the measurements of the contemplated angles (in radians), both for the ffunction and for the accumulation function F, they could become limitations of the task, since the time foreseen for its development would be invested mostly trying to graph properly if the suggestion of using Applet 3 is not used. This leads to one of the learning limitations mentioned by Zabala, Vera, and Ruiz (2017) regarding the transit between representations, and above all, their minimum use within the classroom when trying to represent adequately and not involving adequate resources for the graph, which optimise the time in the correct interpretation of the graph, based on the previously given conditions.

The second limitation of learning is related to the fact that since the function f is continuous in the defined interval $[0,2\pi]$, it is possible to identify that there is an accumulation function F that is continuous in the same interval, and also derivable, so by deriving the accumulation function given by $F(x) = sen(x)$, we obtain: $F'(x) = \cos(x) = f(x)$. In this particular function, you have to focus your gaze on the characteristics of the resulting accumulation function for this function. Since they are trigonometric functions, it is possible that students confuse the value of derivatives and integrals, since by linking a wrong sign to the integral of the function f , it will lead to the image of the sine function, reflected on the axis x, which will not allow him to see the relationship with the accumulation function. Inadequate understanding of the concept, its meaning, and its geometric treatment (Artigue, 2002) are involved in the correct interpretation and representation of functions.

In this task, we proposed that students move between two representations to interpret the model and analyse its behaviour for the applicability of the FTC. In this way, this task is considered to be of high cognitive demand, but focused on the construction of mathematics (DM1 and DM2). The use of trigonometric functions in the examples of representations leads to having some parameters in measurements in radians on the plane, which require an effort of the cognitive work that this entails in relation to what has already been previously performed. This cognitive work required of the student is non-algorithmic and complex, and verification does not require previously reviewed paths.

School Math Task 4. The statement of the task in Figure 9 aims to exemplify a function with particular characteristics of continuity, in which the dynamic geometry of the GeoGebra software is used to recreate the behaviour of the functions, under characteristics of continuity (or not) and its implications of the FTC applicability. In particular, for this task, the teacher chooses appropriately the interval [0,9] over the function intended to identify the primitive, given a function f modelled by Applet 4.

Figure 9

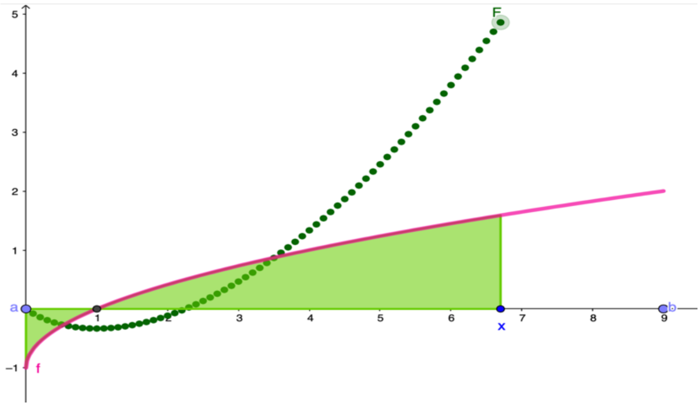
Statement of the fourth mathematics school task.

Let the function f be defined as $f(x) = \sqrt{x} - 1$ in the interval [0,9], find the function F, primitive of f. (Tip: Use Applet 4, https://www.GeoGebra.org/m/abwfysf8).

In Figure 10, the trigonometric function f is defined algebraically as $f(x) = \sqrt{x} - 1$, is plotted alongside its primitive.

Figure 10

Picture of $f(x) = \sqrt{x} - 1$ *next to its primitive from Apple 4*
(https://www.GeoGebra.org/m/abwfysf8)



The function f (highlighted in Figure 10 in red) is continuous in the closed interval [0,9] and is also derivable in that interval. The accumulation function F is highlighted in the graph in dark green. This travelled area is demarcated by the images of the point x, included in the closed interval[0,9].

We set this example of school math task to evaluate the conditions of the FTC and the existence or not of particular conditions of a function.

In relation to the learning limitations that students may have when facing this school math task, we emphasise three. The first is that students could consider the non-existence of the function for intervals with lower limits than 0, so it would not guarantee the condition of continuity. However, this function is defined for the interval $[0,9]$. The function defined in this interval is continuous at all points. In this way, the limitation would be related to the definition and its understanding in the application of a particular situation (other values not defined in the interval), as indicated by Autino et al. (2011).

The second learning limitation is related to the fact that, in this task, it seems that the students could not identify the function that describes the accumulation function, i.e., they did not know the rules of integration of the radical function; however, a conversion from root to power $\left(\frac{1}{2}\right)$ could be useful when finding the integral defined for that interval. Finally, as a learning limitation, students may have problems with the application of arithmetic-type algorithms that could interfere with the development of this task when finding the function F. Likewise, they could interfere in the inverse process of identifying the derivative of F as the initial function f. Despite the imbalance between the conceptual and procedural, seen as the application of integration rules, problems still persist in understanding the mathematical object, as well as the concept and its meaning (Artigue, 2002).

The statement of this school math task can be classified as of low cognitive demand, memorisation type (MEM1), since it is easy to apply algorithms already learned. For example, the school math task 1, in which the functions were in similar conditions, of an area under the curve, and it would be easier if the student had already lived a previous experience, despite having a radical function in this task.

CONCLUSIONS

We intended to present and analyse school mathematical tasks for the treatment of the FTC from its learning limitations and the cognitive requirement that puts its statement at stake. With this, it is possible to advance in the understanding of the interpretation of the geometric representation and the algebraic verification of the theorem and how to bring it into the classroom.

A variety of proposals for the classroom that point to the treatment of mathematical university concepts can be found in the literature (Castelló & Monereo, 1999; Espinosa, 2008; Peñalosa, Sonia, & Roa, 2013; Robles, Tellechea & Font, 2014). We highlight the work of Robles, Tellechea, and Font (2014) and Monroy and Riveros (2020), who propose an alternative approach to the FTC from sequence designs, which, in turn, include virtual environments or other approaches that value inverse relationships between integrals and derivatives. In this scenario, we observed that, although the existing proposals favour the understanding of the FTC object, they do not consider the levels of cognitive requirement that the present classroom proposal poses. Therefore, from this point of view, this work contributes to the university community with a proposal of specific school mathematical tasks for the teaching of FTC focused on the diversity of cognitive demand that must be present in them, of low and high level of requirement, which involve memorisation tasks, procedures without connections, procedures with connections, and construction of mathematics.

On the other hand, and in the same line, we agree with what was stated by García and Benítez (2012), who suggest that students' learning is favoured when there is congruence between the cognitive demand of the tasks, the mathematical content of the problems, and the curricular objectives, which, in one way or another, we have wanted to consider in this proposal, being careful regarding the objective of the task and the demand required.

In relation to learning limitations, we must take into account that the university professor may not be aware of them, which may result in obstacles in the teaching and learning process of the various mathematical concepts. In the case of the FTC, we found that the limitations detected in the literature (Muñoz-Villate, 2021; Reyna-Segura, 2019) were part of those detected in this study for the proposed mathematical tasks. In particular, we noted that in the university reality, an algebraic and/or algorithmic treatment predominates in the teaching of the FTC, i.e., an imbalance between the conceptual and the algorithmic, which privileges the automation of operating techniques of the mathematical object (isolated/disintegrated) and not the formation of future generations of engineers capable of transcending the use and relationships between integrated mathematical objects, as pointed out by Muñoz (2000) and Zavala, Vera, and Ruiz (2017). In particular, the four tasks presented here reveal difficulties related to the understanding of the mathematical object and its application in particular situations, mostly delimited by intervals, or the understanding of continuity. Likewise, some of these tasks reveal possible difficulties related to the scarce appropriation of the mathematical object and application to other representation records, not necessarily algebraic. The lack of use of these records, and the little transit between them, reveals possible implications of the deduction of characteristics of the integrals, for their analysis and treatment, or also, for a correct interpretation and application of the FTC. This also shows the difficulty previously pointed out by Muñoz-Villate (2021) in relation to the lack of previous knowledge about the FTC.

From the above, it follows that it must be a purpose of the university professor to be constantly instructed about it, in order to favor the learning that arises in the classroom and implement the necessary resources to make mathematical modelling a transversal process also at the higher education level.

Various proposals for activities are presented in university textbooks, where there seems to be no previous analysis of the cognitive demand that it entails for students, and, sometimes, they are set considering the developmental time they involve for the teacher, which can lead to contemplating times outside the reality of the students' level, which coincides with the study of Acero (2019).

According to Smith and Stein (2016), it is necessary to propose to students math tasks that involve diversity in the cognitive demand put into play. In this sense, we agree with Penalva, Posadas and Roig (2010) on the need to provide an adequate balance between different types of mathematical tasks, since it produces a positive impact on the teaching of mathematics at university level. Therefore, we maintain that at higher education, this need must also be taken into account, whether they are low-level (memorisation or procedure without connections) or high-level (procedures with connections or mathematical constructions), so that university students can pass through the various requirements that a task can provide and be able to gradually mature the abstract and complex concepts that are involved in it.

Based on the study, we maintain that at the university level, it is advisable to begin with a *staff* of mathematical tasks that involve a low-level requirement, so that there is an adequate familiarisation of the concepts so that students gain confidence in the process of learning increasingly complex and abstract concepts. However, requirements should not be kept at this level of demand, but rather, based on new mathematical tasks, they should evolve to those of a high level of demand, to ensure a deep understanding of the concepts studied and advance in the development of high-level cognitive skills, such as inquiry and inference.

On the other hand, these requirements are relativised according to the students' individual characteristics. It is the desire of the university professor to recognise its characteristics, where he/she can search for or design mathematical tasks that provide various types of cognitive challenges to students, and that lead them to move between the different levels, so that students' learning is favoured when there is congruence between the cognitive demand of the tasks, the mathematical content of the problems and the proposed curricular objectives, as stated by García and Benítez (2013). This will allow university students to solve tasks that are not necessarily repetitive in memorisation, procedural and operative skills, but transcend the applicability of the concepts, in this case, the applicability of the FTC.

In this line, a projection of this study is to provide university faculty with effective professional development programmes (Ramos-Rodríguez, Bustos, & Morales, 2021) that allow them to develop skills to face their teaching performance from a broad perspective in relation to the cognitive demand that is at stake in their classroom and the learning limitations that may arise in it, in order to have future engineers with more appropriate knowledge to face their future profession.

On the other hand, based on the study, we consider relevant the inclusion of modelling in the Calculus courses, as proposed in the mathematical tasks studied, since it can allow students to formulate hypotheses based on the use of tools, such as GeoGebra in this case.

Finally, there is no doubt that an extension of this study is to investigate how university professors implement these mathematical tasks, in order to understand them, and support them to maintain the cognitive demand that the mathematical task requires.

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AUTHORS' CONTRIBUTIONS STATEMENTS

The data collection was carried out by JPAR in 2019. The article is based on a preliminary analysis, carried out by JPAR and ERR. Subsequently, both authors, JPAR and ERR, discussed the planning of the article and actively participated in the discussion of the results and their analysis, in addition they reviewed and approved the final version of the work.

DATA AVAILABILITY DECLARATION

The data produced and supporting the results of this study may be provided by the authors upon reasonable request.

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