ISSN: 2178-7727
DOI: 10.17648/acta.scientiae.7061

Knowledge and Competencies of Prospective Teachers for the Creation of Proportionality Problems

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Received for publication 22 Mar. 2022. Accepted after review 21 Sep. 2022
Designated editor: Claudia Lisete Oliveira Groenwald

ABSTRACT

Background: Problem posing is a fundamental competence that enhances the didactic-mathematical knowledge of the mathematics teacher, so it should be an objective in teacher education plans. **Objectives**: This paper describes and analyses a training intervention with prospective teachers to develop such competence using proportionality tasks. **Design:** This qualitative and interpretative study adopts a methodology characteristic of didactic design or engineering research. The design of the intervention and the content analysis of the participants' answers use theoretical and methodological tools from the onto-semiotic approach to mathematical knowledge and instruction. **Context and participants**: The training action was carried out with 127 undergraduates attending a primary education teaching degree in the framework of the Design and Development of the Mathematics Curriculum in Primary Education subject in a Spanish university. **Data collection and analysis:** The prospective teachers, organised in 33 working teams, were asked to create two problems based on a given situation and to identify objects and difficulties, the solution of which was analysed by the research team. **Results:** The results show that the participants encounter difficulties in elaborating relevant proportionality problems from the given situation, identifying the associated level of complexity, recognising the mathematical objects interacting in the solution to their problems and the difficulties that these could cause to primary school pupils. **Conclusions:** It is mandatory to reinforce problem creation competence and proportional reasoning in teacher education.

Keywords: Problem posing; Proportionality; Teacher education; Didactic-mathematical knowledge, Onto-semiotic analysis.

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Conocimientos y competencias de futuros maestros para la creación de problemas de proporcionalidad

RESUMEN

Antecedentes: La invención de problemas es una competencia fundamental que potencia los conocimientos didáctico-matemáticos del profesor de matemáticas, por lo que debe ser un objetivo en los planes de formación de profesores. **Objetivos**: Este trabajo describe y analiza una intervención formativa con futuros maestros, dirigida a desarrollar la competencia mencionada, usando tareas de proporcionalidad. **Diseño**: Es un estudio cualitativo e interpretativo que adopta una metodología propia de las investigaciones de diseño o ingeniería didáctica. El diseño de la intervención y el análisis de contenido de las respuestas de los participantes usan herramientas teóricas y metodológicas del Enfoque Ontosemiótico del conocimiento y la instrucción matemáticos. **Contexto y participantes**: La acción formativa se llevó a cabo con 127 estudiantes del grado de Educación Primaria en el marco de la asignatura Diseño y Desarrollo del Currículum de Matemáticas en Educación Primaria, en una universidad española. **Recolección de datos y análisis**: Se propuso a los futuros maestros organizados en 33 equipos de trabajo, crear dos problemas a partir de una situación dada e identificar los objetos y dificultades, cuya solución fue analizada por el equipo investigador. **Resultados**: Los resultados muestran que los participantes encuentran dificultades para elaborar enunciados de proporcionalidad pertinentes a partir de la situación dada, identificar el nivel de complejidad asociado, reconocer los objetos matemáticos que interactúan en la solución a sus problemas y las dificultades que éstos podrían ocasionar a los alumnos de primaria. **Conclusiones**: Es necesario reforzar la competencia de creación de problemas y el razonamiento proporcional en la formación de profesores.

Palabras clave: invención de problemas; proporcionalidad; formación de profesores; conocimiento didáctico-matemático; análisis ontosemiótico.

INTRODUCTION

It seems clear that formulating problems is as important for mathematics as solving them. There is no problem to solve if one has not been raised before. However, while in mathematics teaching and learning, the development of problem-solving skills has been at the centre of curricula and educational practice, problem posing has received less attention (Ellerton, 2013). In addition, its use in mathematics classes is not usual (Espinoza et al., 2014).

Posing problems is not only closely related to problem solving; in fact, both should be seen as complementary proposals that allow students to increase their mathematical skills (Espinoza et al., 2014; Mallart-Solaz, 2019; Pino-Fan et al., 2020; Silver, 2013). On the one hand, it contributes to the development and evaluation of mathematical knowledge, given that it stimulates a high level of abstraction and requires a significant mastery of the content being studied, as well as proper use of mathematical language, concepts, processes, and procedures (Ayllón et al., 2016; Fernández-Millán & Molina, 2016; Kwek, 2015). However, on the other hand, it reduces errors, increases creativity and motivation, and decreases students' anxiety and fear of mathematics (Ayllón et al., 2016; Fernández & Carrillo, 2020; Tichá & Hošpesová, 2013).

For teachers to design suitable problem-solving tasks for their students and manage difficulties in that context, they, too, need to be qualified to pose problems (Singer et al., 2013). Teachers must be able to choose, modify, or create problems and solve them with a didactic purpose (facilitate or deepen their students' learning and stimulate their mathematical reasoning). They must also critically evaluate the quality of the mathematical activity they promote (Malaspina et al., 2015, Malaspina et al., 2019). However, even experienced teachers find it challenging to propose relevant problems for their students' learning. Sometimes, they use statements that are not adapted to the educational level they are teaching, sometimes the wording is incorrect or incomplete, and sometimes they are mostly considered only academic (Ellerton, 2013; Mallart et al., 2018; Singer & Voica, 2013).

The discussion above has recently raised the interest of many researchers in mathematics education in the invention of problems, explicitly pointing out its close link with teaching skills and highlighting the importance of promoting their development in teacher education programmes (Ellerton, 2013; Espinoza et al., 2014; Felmer et al., 2016; Malaspina et al., 2015; Malaspina et al., 2019; Mallart et al., 2018; Milinković, 2015; Silver, 2013; Tichá & Hošpesová, 2013). According to Singer et al. (2013), problem posing must be seen:

> Both as a means of instruction (aimed at engaging students in genuine learning activities that produce a deep understanding of mathematical concepts and procedures) and as an object of instruction (focused on developing the competence to identify and formulate problems from unstructured situations). (p. 5)

Incorporating the creation of problems in teacher education means developing theoretical-methodological tools that guide teachers through a complex task with which they are unfamiliar (Ellerton, 2013; Mallart-Solaz, 2019; Mallart et al., 2018).

The interventions developed with pre-service teachers (Burgos et al., 2018; Burgos & Godino, 2020b, 2021a, 2021b, 2022; Malaspina et al., 2015; Malaspina et al., 2019; Mallart et al., 2018), using the constructs of the onto-semiotic approach (OSA) to mathematical knowledge and instruction (Godino & Batanero, 1994; Godino et al., 2007), show the close link between the ability to create problems that facilitate students' learning and their competence in didactic analysis. The latter involves, in particular, "analysing the mathematical activity when solving problems, identifying the practices, objects and processes put into play, and the variables that intervene in the statements, in order to formulate new problems and adapt them to each educational circumstance" (Godino et al., 2017, p. 92).

This research describes the design, implementation, and results of a formative intervention with prospective primary school teachers aimed at developing their competence to create mathematical problems for didactic purposes, following the scheme developed by Malaspina et al. (Malaspina, 2013; Malaspina et al., 2015; Malaspina et al., 2019; Mallart et al., 2018) within the OSA framework. Thus, starting from considering the creation of problems to enhance the articulation of skills and knowledge of the mathematics teacher, the core of this work is the invention, by teachers in training, of problems that involve proportionality.

Despite its great importance and the fact that it has been a fruitful research topic in recent decades, the notions of ratio and proportion continue to present difficulties for teaching and learning (Obando et al., 2014). Specifically, research focused on the analysis of the mathematical knowledge necessary for teaching proportionality shows that both pre-service and in-service teachers find it hard to teach concepts related to the theme (Balderas et al., 2014; Buforn & Fernández, 2014; Buforn et al., 2018; Burgos et al., 2018; Burgos & Godino, 2020b, 2022; Lo, 2004; Rivas et al., 2012; Weiland et al., 2020).

This work shows a methodology to develop and evaluate prospective teachers' didactic-mathematical knowledge and competencies on proportionality by creating problems. During the training, prospective teachers are invited to design proportionality problems from a given situation to grade their complexity and identify, based on the mathematical elements involved in their resolution, the students' potential difficulties.

THEORETICAL FRAMEWORK

The OSA assumes an anthropological and pragmatist vision of mathematics as a human activity, which leads it to consider people's actions in solving problems as a central element in constructing mathematical knowledge, in its double institutional and personal facet (Godino et al. al., 2007).

Pragmatic meaning and onto-semiotic configuration

Given the plurality and relativity of mathematical activity, the meaning cannot be summed up in a purely mathematical definition; rather, it is closely linked to the practice and the object. In the OSA, *mathematical object* is any entity that intervenes in some way in the practice or mathematical activity, and that can be separated or individualised, while *mathematics practice* is "any action or manifestation (linguistic or not)[1] performed by someone to solve mathematical problems, communicate the solution to others, validate the solution, and generalise it to other contexts and problems" (Godino & Batanero, 1994, p. 334). The *meaning* of a mathematical object is defined as a system of practices (institutional or personal) associated with the field of problems from which the object emerges at a given moment. If the systems of practice are shared within an institution, the emerging objects are considered institutional objects. In turn, if these systems are specific to a person, they are considered personal objects (Godino & Batanero, 1994). This duality allows us to talk about the institutional and personal meaning of mathematical objects.

In mathematical practices, or a system of mathematical practices, different types of mathematical objects participate and emerge. They are classified according to their nature and function in the following categories: *problem situations* (exercises and more or less open problems, intra-mathematical or extra-mathematical applications, defined as tasks that induce mathematical activity), languages (mathematical terms and expressions; notations, symbols, graphic representations in their various registers), *concepts* (mathematical entities that can be introduced by description or definition), *propositions* (statements about concepts, properties or attributes) *procedures* (calculus techniques, operations and algorithms), and *arguments* (required to prove propositions or explain procedures).

Mathematical activity is modelled in terms of systems of operative and discursive practices, from which emerge the different mathematical objects (the

[1]Verbal, graphic, etc.

“structure”), languages, concepts, propositions, arguments, procedures, and problem situations, through the processes (the “operation”) of communication, definition, enunciation, argumentation, algorithmisation and problematisation, respectively. In this way, the *onto-semiotic configuration of practices, objects, and processes is determined,* understood as an articulated network in which objects and processes perform a specific function within the mathematical practice that originates them. This tool is key to the analysis of mathematical activity from its two interpretations: from the epistemic or institutional perspective, the analysis allows characterising institutional knowledge; in the cognitive or personal interpretation, it describes personal knowledge (Font et al., 2013). Recognising the configurations of objects and processes involved in mathematical practices at stake in the resolution of problem situations allows the teacher to anticipate potential learning conflicts, assess students’ mathematical skills, and identify objects that must be remembered in a timely manner in the learning process (Godino et al., 2017).

Model of knowledge and didactic-mathematical competencies of the mathematics teacher

The model of the teacher’s didactic-mathematical knowledge and competencies (DMKC) developed within the OSA framework articulates the categories of the mathematics teacher’s didactic-mathematical knowledge and competencies through the facets and components of the mathematical study processes considered in this framework (Godino et al., 2017). Thus, it is assumed that the teacher must have *mathematical knowledge per se* that allows them to solve the problems and tasks proposed in the curriculum of the educational level where they teach, articulating it with the higher levels. Moreover, as some mathematical content is put into play, the teacher must have a didactic-mathematical *knowledge* of the different facets that affect the educational process: *epistemic* (didactic-mathematical knowledge about the content itself, institutional meanings of reference), *ecological* (relations of the mathematical content with other disciplines, curricular and socio-professional factors that condition the processes of mathematical instruction), *cognitive* (how students reason and understand mathematics and how they progress in their learning), *affective* (affective, emotional, attitudinal aspects and beliefs of students about mathematical objects and their study), *mediational* (technological, material and time resources suitable to enhance student learning) and *interactional* (knowledge of the teaching of mathematics, selection and organisation of tasks, addressing students' difficulties, management of interactions that can be established in the classroom).

Specifically, creating problems, solving them, analysing the knowledge put into play, and changing them taking into account that knowledge or students' difficulties, constitute an essential part of the epistemic, cognitive, and interactional facets of the model insofar as they allow the teacher to grade the complexity of the tasks they propose to their students, understand learning conflicts, and manage the institutionalisation of knowledge.

Moreover, the DMKC model proposes that teachers must also be *competent* in addressing the basic didactic problems they face in the teaching and learning processes. In particular, the *competence of analysis and didactic intervention* allows teachers to describe, explain, and judge what has happened in the study process and suggest improvements (Godino et al., 2017). The theoretical and methodological tools of the OSA allow the development of said competence, which is articulated through the subcompetencies: analysis of global meanings, onto-semiotic analysis of practices, management of didactic configurations, normative analysis, and analysis of didactic suitability (Godino et al., 2017). Given the interest of this research, we will focus on the competence of *analysis of global meaning,* based on the identification of problem situations and the operative, discursive, and normative practices involved in their resolution and on the competence of *onto-semiotic analysis of practices*, which involves the ability to recognise the configuration of objects and processes involved and emerging from mathematical practices. Both competencies are essential in creating problems to respond to specific requirements. Reciprocally, the creation of problems serves as a means to develop these competencies since it requires reflecting on: the global structure of the problem, the objectives it pursues; whether the information provided is sufficient to solve the problem and how to address it; the analysis of the objects and mathematical processes involved and how they are related to solve the proposed problem; and the recognition of students' potential difficulties that and how to address them when approaching new situations.

Problem creation and teachers' competencies

Although there are different positions on which strategies or methodologies are considered in problem creation (Akay & Boz, 2010; Chapman, 2012; Contreras, 2007; Silver, 1994; Stoyanova, 1998), in this work we adopt Malaspina's proposal (2013). According to this author, problem creation is a process by which one obtains a new problem, which is determined by four fundamental elements: *information*, i.e., the quantitative or relational data given in the problem; the *requirementt*, i.e., what is required to be found, examined, or concluded, which can be quantitative or qualitative, including graphs, and demonstrations; the *context*, which can be intra-mathematical or extra-mathematical; the *mathematical environment* or global mathematical framework in which the mathematical concepts that intervene or may intervene to solve the problem are located, for example, linear functions, number theory, analytical geometry etc.

New problems can be created through the variation of a given problem or by elaboration (Malaspina, 2013). The *variation of a problem* is a process by which a new problem is constructed by modifying one or more of the four elements of the initial problem. On the other hand, the *elaboration of a problem* is a process in which a new problem is built freely from a situation (given or configured by the author) or by a specific requirement, which may have a mathematical or didactic emphasis. In the *elaboration of a problem from a situation*, the context originates in the situation, the information is obtained by selecting or modifying what is perceived in that situation, the requirement is a consequence of the relationships between the elements of the information implicit in the statement, and the mathematical environment can be determined by the author or by the ways to solve the problem. In the *elaboration based on a specific requirement* (mathematical or didactic), the context or information must be established to respond to that requirement adequately. Didactic-mathematical knowledge about the content, in our case, proportionality, is crucial to appropriately respond to the requirement.

Didactic-mathematical knowledge of proportional reasoning

From an epistemic perspective, i.e., from institutionalised mathematical knowledge, proportionality has been studied fundamentally based on three approaches: *arithmetic,* focused on the notions of ratio and proportion (where problems of comparison or missing value stand out); the *algebraic-functional*, based on the notion and properties of the linear function; and the *geometric,* focused on the similarity of figures. Although the first of these meanings predominates in most curricular proposals and research (Ben-Chaim et al., 2012; Lamon, 2007), many authors defend the importance of beginning the study of proportionality with an informal approach, before formalising the concepts of ratio and proportion (Cramer & Post, 1993; Ruiz & Valdemoros, 2004). Based on perceptive comparison and qualitative analysis of the multiplicative relationships between particular numbers, this intuitive-type perspective is proposed as the first approach to proportionality (Burgos & Godino, 2020a; Fiol & Fortuny, 1990).

Regarding the development of proportional reasoning in schoolchildren and their difficulties when facing situations of proportionality, several studies (Fernández & Llinares, 2011, 2012; Silvestre & Ponte, 2011; Tournaire & Pulos, 1985; among others) show that the greater or lesser success in proportionality tasks depends on factors such as the relationship between the numbers involved, the use of integer and non-integer ratios, the units of the magnitudes involved in the situation, the format in which the task is presented, and the familiarity of the content, among others.

Finally, to face the difficulties that students meet in developing proportional reasoning, it is important to explicit the multiplicative relationship in proportional situations, allow them to distinguish multiplicative comparisons from additive ones, and involve both internal (relationships between different values of the same magnitude) and external ratios (between values of different magnitudes) in the proposed situations (Fernández & Llinares, 2011; Lamon, 2007; Ruiz & Valdemoros, 2004).

Having this didactic-mathematical knowledge in the epistemic, cognitive, and instructional facets will allow teachers to create problems in which the various meanings of proportionality are involved. To do so, they need to know the configurations of characteristic objects (Burgos & Godino, 2020a) and the relationships established, identify how they contribute to an adequate development of their students' proportional reasoning and the difficulties they may find when solving them.

METHODOLOGY

The study is part of descriptive research with a mixed approach: qualitative, since it allows describing and interpreting the didactic-mathematical knowledge of prospective teachers and the difficulties they present in creating problems, and quantitative, since it facilitates the treatment of data by categorising, measuring, and describing the characteristics and profiles of the group of participants.

Taking into account the research problem, the methodological framework will be didactic engineering, understood in the general sense proposed by the OSA (Godino et al., 2014) that follows the phases of design research: *preliminary study*[2] (institutional meanings of reference, interpreted through systems of practices and configurations of mathematical objects and processes; students' personal meanings, anticipated difficulties and beliefs in relation to mathematical content, analysis of anticipated technical and temporal resources), *design of the didactic trajectory* (selection of problems, sequencing and their a priori analysis, teachers' planning of controlled interventions), *implementation* of the didactic trajectory (observation of the interactions between people, resources, and evaluation of the achieved learning), *retrospective analysis* (derived from the contrast between what was foreseen in the design and what was observed in the implementation). In addition, we will employ content analysis (Cohen et al., 2018) to examine the answering protocols of the pre-service primary school teachers that participated in the training experience.

Next, we describe the context of the research and its design, paying attention to the selection of the tasks proposed.

Context and participants

The training action was carried out with two groups of 61 and 66 students from the third year of a primary education teaching degree within the framework of the subject Design and Development of the Mathematics Curriculum in Primary Education [Diseño y Desarrollo del Currículum de Matemáticas en Educación Primaria] in a Spanish university.[3]

Proportionality is one of the contents of the course Mathematical Bases for Primary Education [Bases Matemáticas para la Educación Primaria] the students took in their first university year. At the end of the course, the prospective primary teachers (PPTs, from now on) should know and relate the main concepts, properties, and procedures that make up the topics of school mathematics in primary education and be able to state, pose, and solve mathematical problems through different strategies in a variety of situations

[2]It supposes, in particular, the study of the referents on the creation of problems and didactic-mathematical knowledge in relation to the proportionality that prospective teachers require to pose pertinent problems.
[3]The Informed Consent Form (ICF) was not signed to preserve the participants' identity. In any case, Acta Scientiae is exempt from the consequences that may arise, including exhaustive assistance and possible compensation for damages that may be caused by any of the participants in the research, according to Resolution No. 510, of April 7, 2016, of the National Health Council of Brazil.

and contexts, effectively communicating mathematical arguments. In the second year, the PPTs received specific education on the fundamentals of the didactics of mathematics in cognitive aspects (mathematical learning, errors, and difficulties) and didactic aspects (tasks and activities, materials, and resources). In the course Design and Development of the Mathematics Curriculum in Primary Education [Diseño y Desarrollo del Currículum de las Matemáticas en Educación Primaria], the prospective teachers should deepen and apply the knowledge acquired in previous courses to design and base teaching units. In particular, one of the fundamental aims of the course is the analysis, planning, and sequencing of mathematical tasks according to specific content and learning expectations.

Design and implementation of the intervention

The intervention was developed through four different moments. In the first training session, carried out during the theory class (two-hour long), we presented the notions of mathematical homework, practices, and objects. Next, we highlighted that the teachers were expected to select, design, and sequence tasks that effectively promote their students' learning, presented as elements[4] that guide the task search, selection, and change, as follows:

- *Mathematical content* that involves: mathematical objects, meanings, contexts.
- *Purpose*: Learning expectations (specific objectives and competencies) that can be developed.
- *Learning limitations:* Possible difficulties and errors that may appear in its resolution.
- *Complexity level.* In the context of the Design and Development of the Mathematics Curriculum in Primary Education course, we consider the levels of complexity of the problems established in the PISA framework (OECD, 2003): a) *reproduction level*, relatively familiar tasks that basically require the reiteration of the knowledge studied, the use of simple algorithms or the performance of simple operations; b) *connection level*, in addition to the necessary

[4] These elements are contemplated in the programme of the course Design and Development of the Mathematics Curriculum in Primary Education in whose frame the intervention is developed.

competencies in the previous level, requires integrating and linking the main ideas, establishing relationships between different representations of the same situation, or linking different aspects to reach the solution to non-routine problems but in familiar or nearby contexts; c) *reflection level,* mobilises capacities that require understanding and reflection on the processes necessary or used to solve the problem, identify concepts, or properties that are not always explicit, to plan strategies in different scenarios, argue and justify results, and generalise to new contexts.

This session was organised around the analysis of tasks, some of which were of proportionality, recalling the epistemic configurations that emerge from the practices associated with the different meanings and some of the difficulties and errors on that content that were identified in research on the didactics of mathematics (preliminary study).

In the following practical session (one-hour long), the PPTs worked with their usual working teams in the analysis of a mathematical task (focused on the scale construction of a puzzle and proportional allocation of prices to its pieces) following the elements described. The third session (two-hour long) focused on making the PPTs aware of the methodology for problem creation described in the previous section, reinforcing the importance of problem creation, the role it plays within the curriculum and its development, and the need for teachers to acquire the ability to create math problems to properly guide the development of such ability in their students. In this formative session, some examples of proportionality tasks were also used to show the creation of problems by variation or by elaboration, reminding students again of the necessary knowledge of proportional reasoning. In the fourth session (one-hour long), the students worked collaboratively again, forming a total of 33 working teams (19 teams in one group and 14 in another, which we will refer to as T1, T2, etc.) to respond to three instructions on problem creation by variation (task 1), problem creation from a given situation (task 2), and problem creation from a didactic-mathematical requirement (task 3). Figure 1 describes the instructions on task 2 proposed to the PPTs, whose results are reported in this article.

Figure 1

Instructions on creating problems from a situation

> From the situation presented below, please create two proportionality problems you think have different degrees of complexity. Identify in each case the mathematical objects involved and the possible difficulties that primary school students may find, indicating the course for which they should be intended.
>
> > *Two friends have gone to an amusement park. Julio has ridden on eight attractions, costing him €17.80. Clara has ridden on ten attractions and has paid €21.*

As established in the instructions, the problems must be created around the mathematical environment of proportionality. By itself, the situation given in Figure 1 does not determine a problem since there is no requirement (the students are not asked to solve anything). Creating the problem from the situation means, in particular, "adding" questions to the situation. We observe that when requesting that the problems respond to the proportionality surroundings and have a different level of complexity, didactic-mathematical requirements must also be met. However, the starting point for the elaboration of the problems is given by the established situation. The additional conditions allow the task to be delimited according to the interest of the investigation and to have its specific characteristics analysed.

Previous research (see the review by Stahnke et al., 2016) shows that pre-service and in-service teachers find it difficult to analyse mathematical tasks (following their own beliefs), differentiate routine from non-routine tasks, identify their didactic potential, and choose the appropriate formats to promote students' learning. For this reason, we propose to the PPTs that, as part of the analysis, they recognise the mathematical objects involved in their solution, the possible associated difficulties, and the course for which they would be suitable. Thus, we articulate the creation of problems with the competencies for the analysis of meanings and onto-semiotic analysis of practices, objects, and processes (Godino et al., 2017). Asking the PPTs to identify the difficulties that their students may have with the problems they pose allows us to diagnose and reinforce their didactic-mathematical knowledge in the cognitive aspect (Godino et al., 2017). Moreover, reflecting on the course to which they would be destined helps them to specify the complexity level according to the knowledge that is more or less familiar to the students, according to school planning.

Information analysis categories

In this section, we present the categories used to assess the participants' answers to the statement.

Relevance of the problems and associated complexity level. For the problem to be considered relevant, it must first be significant, meaning that the proposed statement must actually establish a mathematical problem with a solution that is not implicit in the statement. Furthermore, the wording must be intelligible and unambiguous (not redundant information), and the several elements that characterise it are explicitly identified. If the problem does not have any of these characteristics, it is considered a non-significant problem. In this sense, a statement is not considered significant if it proposes inventing a problem, if the requirement implies only beliefs, a merely conceptual domain, or if its solution does not involve any type of mathematical procedure.

The problem is considered *relevant* if it is significant, it starts from the given initial situation without altering it, and it is of proportionality. In this way, the following categories appear a priori:

- None of the problems created is pertinent.
- Only one problem raised is pertinent, but it does not imply a change in the complexity level with respect to the other.
- Only one problem posed is pertinent and supposes a change in the complexity level with respect to the other.
- The two problems raised are pertinent, but both correspond to the same complexity level.
- The two problems raised are pertinent, and their complexity levels differ.

Recognition of mathematical objects. The PPTs must identify the mathematical objects involved in each of the problems created. We expect them to reflect on the onto-semiotic configurations in the proportionality problems they themselves created and intend to help them diagnose possible difficulties in mathematical activity. According to the purpose of the work, we focus on the configurations associated with the pertinent statement problems. In this case, within each problem, the following object recognition analysis categories were used according to the OSA classification:

- *No answer*. The PPTs do not identify the mathematical objects in the problem.
- *Incorrect*. All the mathematical objects pointed out by the PPTs are incorrect.
- *Most incorrect*. The PPTs correctly identify at least one of the mathematical objects in the problem, but less than half.
- *Partially correct*. The PPTs correctly identify at least half, but not all, of the mathematical objects in the problem.
- *Correct.* All mathematical objects in the problem are correctly identified.

Identification of anticipated difficulties in solving the problem. On the other hand, the PPTs must point out the difficulties primary school students could face when solving the problems. According to the categories of OSA objects, they are classified as:

- *Situational*, associated with understanding the problem statement.
- *Conceptual*, related to concepts, their descriptions or definitions.
- *Propositional*, associated with propositions or properties that relate concepts.
- *Procedural*, linked to the development of calculation techniques, operations, and algorithms.
- *Argumentative*, relative to the justification of propositions and procedures.

RESULTS

Next, we present the results of the content analysis of the answers given by the working teams to the proposed task (Figure 1), following the categories established in the previous section.

Pertinence and complexity level of the problems elaborated by the PPTs

As can be seen in Table 1, the PPTs had difficulties elaborating pertinent problems based on the given situation; only five teams answered adequately to the task by creating two pertinent problems with different complexity level.

Table 1

Distribution of the PPT teams according to the pertinence and complexity level of the problems created (n=33)

Categories	Frequency (Percentage)
None of the problems created is pertinent.	16 (48.48)
Only one problem raised is pertinent, but it does not imply a change in the level of complexity with respect to the other.	5 (15.15)
Only one problem posed is pertinent and supposes a change in the level of complexity with respect to the other.	7 (21.21)
The two problems raised are pertinent but have the same level of complexity.	0 (0.00)
The two problems raised are pertinent and have different levels of complexity.	5 (15.15)
Total	33 (100)

Of the 66 problems that were analysed (given that each team had to create two problems in the task), 44 were graded as non-pertinent. Of these 44 non-pertinent problems, 28 are also not significant, mainly because the statements are ambiguous, lack clarity, or cannot be solved. For example, in Figure 2, the T19 team proposes a problem in which the student is asked to compare the weight of two cakes and decide whether the price is proportional to the weight (here, they include an erratum in their statement), which one was more expensive. However, based on the dimensions of the cake, its weight cannot be determined, so the question cannot be answered.

Figure 2

Non-significant statement. Cannot be solved (T19)

> *Ana finds two cakes to give as a birthday present. She knows that one weighs 500 g, but the second one does not specify the weight, although it does bring the measurements, and we find that it is 10 centimetres long, 10 centimetres wide, and 6 centimetres thick. Which cake is the heaviest? If the weight were proportional to the weight, which would be more expensive?*

Figure 3 shows one of the problems proposed by T25. In this statement, the initial situation is preserved; however, it is not clear whether Pablo's tickets are of the same type as Julio's (so they will have a determined price) or as Clara's (they will have a different price), or how its unit value can be determined.

Figure 3

Non-significant problem, ambiguous statement (T25)

> *Two friends have gone to an amusement park. Julio has ridden on 8 attractions, costing him €17.80. Clara has ridden on 10 attractions and has paid €21. In the late afternoon, their friend Pablo joins them. Clara decides that, as it is his birthday, she will invite him to as many attractions as he wants. Pablo rides on 6 attractions. Knowing that Clara has paid €21 for riding on 10 attractions, how much will she have to give Pablo?*

Of the 16 problems considered significant but not pertinent, 11 maintain the environment of proportionality (at least implicitly), changing partially (Figure 4) or totally the information given in the starting situation.

Figure 4

Non-pertinent problem. Partially modifies the given situation (T24)

> *Two friends have gone to an amusement park. Julio has ridden on 4 attractions and it has cost him €10. Clara has ridden on 10 attractions and has paid €20. If each spent €25, how many rides can they take?*

The other five significant but non-pertinent statements partially maintain the situation. However, they pose additive problems of comparison

(4) or combination (1). For example, Figure 5 shows the problem proposed by T1 team, which, although starting from the given situation, needs to calculate the difference between Julio's and Clara's total expenses, which does not refer to a proportional situation but to an additive comparison problem.

Figure 5

Non-pertinent problem, created outside the proportionality environment (E1)

> *Two friends have gone to an amusement park. Julio has ridden on 8 attractions and it has cost him €17.80. Clara has ridden on 10 attractions and has paid €21. How much more than Julio has Clara spent?*

The 22 (of a total of 66) problems created in a pertinent way by the PPTs respond fundamentally to these types:

a) Two pairs of directly proportional magnitudes are considered: the number of tickets–the price paid for them for each child. A new known value is given for the number of tickets to be purchased for each of them (common or not), and they must determine the new price, or else they are asked to determine the number of tickets of each type that each child can buy (see Figure 6). This category is found in ten of the pertinent issues created.

b) Two pairs of directly proportional magnitudes are considered: the number of tickets–the price paid for them for each child. They are asked to compare the unit values (the price of a single ticket) in each case, usually to decide which child obtained a better price for his ticket (Figure 7). Of this type, you will find eight pertinent problems.

c) The two magnitudes number of tickets (understood as all of the same type) and price are considered to be directly proportional. The total price difference between the children is because a voucher or another item was purchased or a discount was applied (in this case, to Clara). One must find out the price of the bond or of the applied discount–ask whether they are directly proportional. It supposes accepting that both Julio and Clara should have paid the same unit price (Figure 8). In this category, four problems are proposed.

Next, we consider what the real degree of complexity was in those problems considered pertinent and how the PPTs evaluated them. Of the 22 pertinent problems, eight correspond to the degree of reproduction, eight to connection, and six to reflection. Of these, only eleven were classified by the PPT teams: ten, according to the PISA levels, six of them correctly (one of reproduction, two of connection and three of reflection), and another as "context and familiarity" (T6). Those who did it incorrectly gave a higher level of complexity than was really implied (connection by reproduction and reflection by connection). In addition, few justified the evaluation of that degree. This shows that, although most of the teams (thirteen out of eighteen) that created relevant problems did so with different levels of complexity (Table 1), they had difficulties in adequately identifying the true complexity degree of their problem. These limitations may be because the PPTs: (a) do not clearly know what characterises each level, for example, they consider that a problem is of the level of reflection whenever it requests a justification (whether it includes "justify your answer"), independently of the processes involved; (b) they do not take into account the students' knowledge in the courses in which they pose the problems; and as we will see, (c) they have difficulties in identifying the network of objects that intervene in proportionality problems, even when they themselves have created them.

To assess whether the course for which the PPTs propose the problems is appropriate, the knowledge required by the students who must solve it should be taken into account in relation to the curriculum contents and the programming of the primary education courses. As can be seen in Table 2, it is adequate in 13 of the 22 pertinent problems.

Table 2

Relationship between the complexity level and course in pertinent problems

		Course to which it is directed			
		Correct	**Incorrect**	**Not specified**	**Total**
Complexity level	**Correct**	4	1	0	5
	Incorrect	4	2	0	6
	Not specified	5	3	3	11
	Total	13	6	3	22

In addition, we observe that when the PPTs correctly identify the complexity level, most of them choose the course appropriately. When they correctly assign the course and complexity level, the problems proposed for the third cycle respond to the three levels of complexity. In the case of incorrectly identifying the course, the PPTs propose problems that involve the division of decimal numbers for the third or fourth grade of primary school, so they are not considered pertinent according to the curriculum. Let us also mention that, of the 44 irrelevant problems, 37 did not indicate the complexity level and nine did not indicate the course for which they would be intended.

In Figure 6, a problem corresponding to the reproduction level is included.

Figure 6

Reproduction problem (T31)

> *Two friends have gone to an amusement park. Julio has ridden on 8 attractions, costing him €17.80. Clara has ridden on 10 attractions and has paid €21. How much will it cost Julio to get on five more attractions than he has already ridden? And Clara?*

The T31 team justifies this degree of complexity by indicating that students "must use routine processes and carry out simple operations. In this case, a rule of three for each of the questions". This is correct, as long as the problem is set for the 6th grade, when the rule of three is usually taught as a procedure for solving proportionality problems. In another case (for example, in the 5th grade for which T31 poses the statement), the students could use other procedures that would go through obtaining the unit value in Julio's case or even, in Clara's case, obtaining half the price (given that the number of attractions being asked about now is exactly half of what was previously purchased).

Figure 7 presents one of the problems proposed by T29 for the 6th grade of primary school.

Figure 7

Connection level problem (T29)

Two friends have gone to an amusement park. Julio has ridden on 8 attractions, costing him €17.80. Clara has ridden on 10 attractions and has paid €21. Who do you think has chosen better, taking into account the quantity-price ratio? Explain your answer.

In this case, the team assigns a complexity level of reflection indicating that “the student is asked to carry out complex reasoning and collect it in writing in a justified manner”. However, we consider his/her correct level to be one of connection. The student must decide the best option based on the relationship between the quantity (of tickets) and the price (paid for them). It supposes a relative comparison, which happens by comparing the ratios or by obtaining the unit price. However, the 6th-grade students know these concepts. Finally, Figure 8 includes a problem T26 proposed for the third cycle of primary education (the team does not indicate the course) and that, although they do not classify it, we consider that it would be close to the level of reflection.

Figure 8

Reflection level problem (T26)

Julio and Clara have gone to an amusement park together. Julio has been on 8 attractions and has spent €17.80. Clara has ridden on 10 attractions and has spent €21. One of the two remembered that he/she had a discount ticket. Which of the two had the ticket? What percentage of discount does the ticket offer?

The approach involves accepting that Julio’s and Clara’s tickets have the same unit price, so the difference owes to the fact that an initial discount was applied to one of them. This supposes recognising and distinguishing the proportional part of the additive to decide to whom the discount was applied.

Identification of mathematical objects

With the identification of mathematical objects, we expect the PPTs to pay attention to those elements of the mathematical activity that can explain the possible difficulties that schoolchildren face with problems they, as pre-service teachers, have proposed.

Next, we analyse the mathematical objects identified by the working teams in the 22 relevant problems of proportionality they created in the previous instruction. Table 3 shows these results from 17 pertinent problems since, in five cases, the PPTs mentioned some mathematical objects but did not specify to which of the two problems they belonged nor the type of object. For example, T14 indicates as objects "ratio part-whole, symbolic values (euros, €), greater or lesser, multiplication, division" but does not clarify whether they are concepts or procedures; neither do they indicate whether they are from the first or the second problem.

Table 3

Frequency of identification of mathematical objects in the pertinent problems (n=17)

Objects	NR	IN	MI	PC	CO
Languages	3	0	0	11	3
Concepts	2	2	5	7	1
Propositions	9	8	0	0	0
Arguments	10	5	2	0	0
Procedures	2	3	0	4	8

Note: NR= no response, IN= incorrect, MI= mostly incorrect, PC= partially correct, CO= correct.

According to these data, we observed that, as in previous research (Burgos et al., 2018; Burgos & Godino, 2021a; Mallart et al., 2016), the PPTs present difficulties in identifying mathematical objects that could intervene or emerge in the solution of their problems on proportionality. This is especially noticeable in the case of propositions and arguments that are mostly either not identified or done incorrectly.

In the case of concepts, there is more significant variability of success in their identification. The concepts that most frequently identify optimally are those of proportionality (seven teams in nine of the pertinent problems) and magnitude (six teams in six pertinent problems, although in three of them, they refer only to the price paid, omitting the number of attractions as one of the quantities involved). As in Burgos et al. (2018), Burgos and Godino (2020b), and Rivas et al. (2012), several PPT teams (six in total) indistinctly consider the rule of three as an associated concept in the problem of proportionality or as a procedure to solve it.

Languages and procedures are the objects the PPTs identified with the highest degree of pertinence. In the case of the procedures, they correctly indicate division and comparison of decimal numbers, reduction to the unit or rule of three. In the case of languages, although they correctly recognise natural or diagrammatic language, they ignored symbolic language, which is why most of the identification of language types was partially correct.

Figure 9

Solution and mathematical objects indicated by T26 to the problem in Figure 8

To determine which of the two has a discount on the attractions, we need to see how much it is for them to ride on the same number of attractions. To do this, we will calculate the price of tickets for Julio if he rode on ten attractions. Knowing that it cost him €17.80 to ride on eight attractions, to know the price of the ten, we proposed the following rule of three:

We compare the prices paid by both people (22.25 > 21) and see that Clara is the one who pays less, so she is the one who has the discount. Knowing that the price without a discount for 10 trips (what Julio would pay) is €22.25 and that the price with a discount for 10 trips (what Clara pays) is €21, we can draw the following:

TOTAL PRICE OF TEN TRIPS: €22.25	
PRICE WITH A DISCOUNT: €21	DISCOUNT

If €22.25 is 100% of the price, the discount (€22.25 - 21 = €1.25) will be a part (?) of the price; therefore, we can propose the following rule of three:

$$\frac{22.25}{100} = \frac{1.25}{?};\ \frac{?\cdot 22.25}{100} = 1.25; \frac{?}{100} = \frac{1.25}{22.25}; \frac{1.25 \cdot 100}{22.25} = ?\,;\, ? = 5.61797753\%$$

<u>Solution</u>: Clara has the discount, and it is 5.62%.

Mathematical objects.

Concepts: proportional magnitudes (money - attractions), percentage, discount.
Procedures: rule of three/reduction to unity.

> Propositions: “When we apply a discount, what happens is that we calculate the discount percentage on the total price, and then we rebate it from the total price”.
> Arguments: none.
> Languages: natural, symbolic, diagrammatic.

In Figure 9, we include the proposed solution and the objects identified by T26 to the problem described in Figure 8. To solve the problem, T26 obtains the price that Julio should pay to get on ten attractions, assuming that the price of each attraction is the same, which allows him to decide that Clara is the one who has received the discount. Next, determine what percentage it corresponds to.

T26 team uses the rule of three as a procedure (although it also indicates the reduction to unity), although it does not base it on the relationship of direct proportionality. What they indicate as a proposition is more a sequence of procedures, and we observe how they explicitly indicate the absence of arguments in their solution**.**

Recognising difficulties

After analysing the mathematical objects involved in the practices necessary to solve the problems on proportionality that they created, the PPTs had to identify the potential difficulties for their elementary students. In this case, the classification was based on the typology of primary objects established in the theoretical framework (Section 3.5). Of the 33 teams, three did not indicate difficulties, and another five indicated them generically (they did not distinguish according to statements). Thus, of the total number of statements proposed by the PPTs, there are difficulties specifically associated with 50 problems (17 pertinent and 33 non-pertinent ones). The types of difficulties most frequently identified were procedural (in 12 of the 17 pertinent problems and 20 of the 33 non-pertinent ones) and situational (in nine of the 17 pertinent problems and in 17 of the 33 non-pertinent problems). All the difficulties pointed out by the PPTs in the pertinent problems were correct. They were also so in most non-pertinent problems, where the PPTs only incorrectly pointed out some difficulties (three procedural, one situational, and one argumentative).

Table 4 summarises the frequencies of statements in which difficulties of each type are identified.

Table 4

Number of problems in which difficulties are identified by category (n=50)

Difficulties	Number of pertinent problems (*n* =17) with difficulties identified by the PPTs	Number of non-pertinent problems (*n* =33) with difficulties identified by the PPTs
Argumentative	1	1
Propositional	1	1
Conceptual	4	9
Situational	10	17
Procedural	12	20
Total	27	48

The situational difficulties that most PPTs point out have to do with recognising or adequately interpreting the proportionality relationship that is established between the magnitudes of the problem ("Difficulty in relating the data", T3; "Not reasoning the proportional relationship price –the times it is mounted", T6). When the requirement of the problem they pose leads to determining the price that Julio and Clara must pay for riding on a given number of attractions, they indicate difficulty in distinguishing two different multiplicative relationships in the same statement ("that the student is confused when seeing that it will not cost Clara the same as Julio when riding on the same number of attractions", T28). Similarly, when they must obtain the unit price to decide which attraction was cheaper, they pertinently point out interpreting an absolute and not a relative comparison ("the main difficulty that can arise is that they directly relate the fact that Clara has less money left over meaning she has ridden on more expensive rides, without taking into account that Julio has ridden on fewer rides than she did", T23).

The procedural difficulties are mainly related to the application of the rule of three ("that the student does not know well the procedures for applying a rule of three correctly and adequately", T28), the determination of prices units ("divide the price by the number of times instead of the other way around", T22) and with arithmetic calculations in relation to the division of decimal numbers ("in this case, they must work the division with decimal numbers and with two figures, which can cause difficulties and lead to mistakes if they are learning it or have not yet mastered it fluently", T1). The greater emphasis on the difficulties linked to the rule of three is related to the fact that this continues to be the procedure prospective teachers prefer the most to solve problems of proportionality. Opinions similar to those of the T20 team, for whom "students may have difficulties when performing the operations since, instead of performing the rule of three to calculate the operations, they can multiply", show that, for the PPTs, use other different strategies can be a reason or source of the difficulty.

The conceptual difficulties are fundamentally related to the concept of proportionality ("not understanding the concept of proportionality", "difficulty with the constant concept of proportionality", T3).

Finally, the participating PPTs found it complex to identify propositional and argumentative difficulties in their own problems. This may be motivated by the PPTs' limited knowledge of the properties of the proportionality relationship and its limitations to argue in situations of proportionality (Balderas et al., 2014). Propositional-type difficulties are essentially those that have to do with the properties of operations with decimal numbers. In contrast, argument-type difficulties are associated with students' limitations in justifying the procedures (obtaining the proportionality constant or price by unit). This becomes a worrying situation, given that argumentation is one of the processes and competencies that must be developed in different mathematics curricula (Balderas et al., 2014; Ministerio de Educación Pública, 2012).

SUMMARY AND CONCLUSIONS

Besides solving mathematics problems, a teacher must be able to select, modify and pose problems for specific educational purposes (Tichá & Hošpesová, 2013). Creating problems, not only in the design and planning of classes but whenever necessary during their implementation, is one of the features of the teacher's didactic analysis and intervention competence (Godino et al., 2017). Also, creating problems with didactic purposes appears as a means to enhance and articulate other didactic-mathematical skills and knowledge of the teacher; it requires reflecting on the elements that characterise the problem (information, requirement, context, and mathematical environment), the way or ways it can be solved, analysing the mathematical practices and objects involved, and identifying the possible difficulties that students may have in each case.

In this article, we have reported on the design, implementation, and analysis of an intervention with prospective primary school teachers focused on creating proportionality problems from a given situation. The results described contribute, on the one hand, to understand pre-service teachers' difficulties in creating proportionality problems and, on the other, to diagnose their knowledge and didactic-mathematical skills through their own analysis.

From the point of view of didactic-mathematical knowledge in the instructional facet, the PPTs find it hard to create proportionality problems that respond to a given situation, highlighted when more than 66% of the problems they propose are considered non-pertinent. They also pose ambiguous, meaningless statements that impaired solving the problem or the solution was implicit in the statement itself, meaning that more than 42% of their problems were insignificant. Our requirement to create problems from a specific situation may have put them in a weird, unexpected, and more restricted position (Tichá & Hošpesová, 2013).

In the epistemic facet, we could note the PPTs' limitations in identifying the mathematical objects involved in the problems they themselves created (Burgos et al., 2018; Burgos & Godino, 2022) and distinguishing proportional from non-proportional situations by creating problems that do not respond to a situation of proportionality but that they consider to be typical of that environment (Fernández et al., 2012). The limitations in identifying mathematical objects could be a consequence of the lack of experience with this activity (Rivas et al., 2012), since their previous training in the degree and the workshop taught seem not to have been sufficient for them to understand the nature and functionality of the mathematical objects (Burgos & Godino, 2020b, 2021a).

In the cognitive aspect, the PPTs scarcely identify the difficulties that the resolution of their proposed problems can generate in schoolchildren, and when they do so, it is not always correct; they are usually focused on procedural difficulties, which coincides with the results obtained by Burgos and Godino (2020b). As Lamon (2007) and Riley (2010), among others, show, the pre-service teachers focus on the operational aspect and justify their proportionality problem-solving strategies based on procedures, which leads them to not identify objects as propositions or arguments and ignore them as sources of difficulties for students. Anticipating the difficulties that students may have in solving tasks allows selecting more appropriate strategies to adapt them to the learning purposes (Burgos & Godino, 2020b), for which the prospective teachers must assume that knowledge of the procedures alone is not enough, delving into the concepts, properties, and their relationships in the primary school curriculum (Tichá & Hošpesová, 2013).

The identification and justification of the associated level of complexity were also complex for the participants, despite being an analysis that the PPTs should be familiar with due to previous courses. However, the instruction received in this regard is based on problems present in curriculum resources (textbooks, among others) or primary school assessment tests and not on the problems that they themselves create. Therefore, it seems necessary to reinforce, in teacher training, the study of the complexity level of the tasks based on the analysis of objects and mathematical processes involved in their resolution. Likewise, although the PPTs know the curriculum guidelines and the curriculum units of primary school programmes, they showed important limitations in determining or adapting their problems to a specific educational level (for example, considering the use of the rule of three in courses prior to those stipulated in the mathematics curriculum).

Creating problems is a fundamental part of the teaching task, a challenge to their didactic-mathematical knowledge and competencies (Malaspina et al., 2019). Deficiencies in the knowledge of mathematical content limit the teachers when posing problems. In this sense, teachers end up using problems proposed in textbooks, which, in some cases, bring errors or do not respond to students' contexts or needs (Salazar, 2017). A biased knowledge of proportional reasoning can cause a deficiency not only in creating problems but also in recognising the difficulties involved in solving proportionality tasks (Burgos & Godino, 2020b). Taking as a starting point the need to reinforce proportional reasoning in teacher education, the limitations found by the participating PPTs suggest the need to improve the research instrument and develop new specific training actions:

- Although some teams solved the problems, the solution to each problem created was not requested in the instruction of the training intervention. Maybe the results would be different if they had been explicitly asked, since such a requirement could have improved their ability to identify the mathematical objects and the difficulties involved in solving them.

- On the other hand, creating problems can also make prospective teachers improve a previous proposal or its results. Thus, after solving the first problem created, the participants can be asked to create a new problem that can help them overcome the difficulties diagnosed in the previous one, that contributes to their understanding, or that has a higher degree of complexity or cognitive demand for the student.

– The analysis of complexity could be complemented with the study of the mathematical processes involved and the degree of formalisation and mathematical abstraction required.

Our instrument can be adapted and expanded, modifying the situation, the didactic-mathematical requirement, or the environment to meet new knowledge. This flexibility also allows the design and implementation of new interventions with prospective secondary education teachers. We are also interested in developing training actions on creating problems with in-service teachers in primary and secondary education, with whom we hope to obtain better results.

ACKNOWLEDGEMENTS

This work is part of the research project PID2019-105601GB-I00 / AEI /10.13039/501100011033 (Ministry of Science and Innovation), with the support of the Research Group FQM-126 (Junta de Andalucía, Spain), the AUIP and the Ministry of Economic Transformation, Industry, Knowledge, and Universities of the Junta de Andalucía (the last two, sponsors of the AUIP Academic Mobility Scholarship Programme).

AUTHORSHIP STATEMENT

MB: conceptualisation, research, methodology, supervision, validation, visualisation, writing - review and editing. JJC: conceptualisation, data processing, validation, visualisation, writing – original draft. Both authors actively participated in the analysis and discussion of the results, reviewed, and approved the final version of the work.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the corresponding author, MB, upon reasonably previous request.

REFERENCES

Akay, H. & Boz, N. (2010). The Effect of Problem Posing Oriented Analyses-II Course on the Attitudes toward Mathematics and Mathematics Self-Efficacy of Elementary Prospective Mathematics Teachers. *Australian Journal of Teacher Education, 35*(1), 59-75. https://doi.org/10.14221/ajte.2010v35n1.6

Ayllón, M. F., Gallego, J. L., & Gómez, I. A. (2016). La actuación de estudiantes de educación primaria en un proceso de invención de problemas. *Perfiles Educativos, 38*(152), 51–67. https://doi.org/10.22201/iisue.24486167e.2016.152.57588

Balderas, R. G., Block, D. & Guerra, M. T. (2014). “Sé cómo se hace, pero no por qué": Fortalezas y debilidades de los saberes sobre la proporcionalidad de maestros de secundaria. *Educación Matemática, 26*(2), 7–32.

Ben-Chaim, D., Keret, Y., & Ilany, B. S. (2012). *Ratio and proportion: Research and teaching in mathematics teachers' education*. Sense.

Buforn, A. & Fernández, C. (2014). Conocimiento de matemáticas especializado de los estudiantes para maestro de primaria en relación al razonamiento proporcional. *Bolema, 28*(48), 21–41. https://doi.org/10.1590/1980-4415v28n48a02

Buforn, A., Llinares, S., & Fernández, C. (2018). Características del conocimiento de los estudiantes para maestro españoles en relación con la fracción, razón y proporción. *RMIE, 23*(76), 229–251.

Burgos, M., Beltrán-Pellicer, P., Giacomone, B., & Godino, J. D. (2018). Conocimientos y competencia de futuros profesores de matemáticas en tareas de proporcionalidad. *Educação e Pesquisa, 44*, 1-22.

Burgos, M. & Godino, J. D. (2020a). Modelo ontosemiótico de referencia de la proporcionalidad. Implicaciones para la planificación curricular en primaria y secundaria, *AIEM - Avances de Investigación en Educación Matemática*, *18*, 1–20. https://doi.org/10.35763/aiem.v0i18.255

Burgos, M. & Godino, J. D. (2020b). Prospective primary school teachers' competence for analysing the difficulties in solving proportionality problem. *Mathematics Education Research Journal.* https://doi.org/10.1007/s13394-020-00344-9

Burgos, M. & Godino, J. D. (2021a). Conocimiento didáctico-matemático de la proporcionalidad en futuros maestros de educación primaria. *Profesorado, 25*(2), 281–306. https://doi.org/10.30827/profesorado.v25i2.8725

Burgos, M. & Godino, J. D. (2021b). Prospective primary school teachers' competence for the cognitive analysis of students' solutions to proportionality tasks. *Journal für Mathematik Didaktik.* https://doi.org/10.1007/s13138-021-00193-4

Burgos, M. & Godino, J. D. (2022). Assessing the epistemic analysis competence of prospective primary school teachers on proportionality tasks. *International Journal of Science and Mathematics Education 20,* 367–389. https://doi.org/10.1007/s10763-020-10143-0

Chapman, O. (2012). Prospective elementary school teachers' ways of making sense of mathematical problem posing. *PNA, 6*(4), 135–146.

Cohen, L., Manion, L., & Morrison, K. (2018). *Research methods in education*. Routledge.

Contreras, J. (2007). Unraveling the Mystery of the Origin of Mathematical Problems: Using a Problem-Posing Framework with Prospective Mathematics Teachers. *The Mathematics Educator, 17*(2), 15–23.

Cramer, K. & Post, T. (1993). Connecting research to teaching proportional reasoning. *Mathematics Teacher*, *86*(5), 404–407.

Ellerton, N. F. (2013). Engaging pre-service middle-school teacher-education students in mathematical problem posing: development of an active learning framework. *Educational Studies in Mathematics*, *83*(1), 87–101. https://doi.org/10.1007/s10649-012-9449-z

Espinoza, J., Lupiáñez, J., & Segovia, I. (2014). La invención de problemas y sus ámbitos de investigación en educación matemática. *Revista Digital: Matemática, Educación e Internet, 14*(2), 1–12. https://doi.org/10.18845/rdmei.v14i2.1664

Felmer, P., Pehkonen, E., & Kilpatrick, J. (Eds.). (2016). *Posing and Solving Mathematical Problems: Advances and New Perspectives*. Springer. https://doi.org/10.1007/978-3-319-28023-3

Fernández, M. E. & Carrillo, J. (2020). Un acercamiento a la forma en que los estudiantes de primaria formulan problemas. *Revista de Educação*

Matemática, 17, 1–19. https://doi.org/doi.org/10.37001/remat25269062v17id257

Fernández, C. & Llinares, S. (2011). De la estructura aditiva a la multiplicativa: efecto de dos variables en el desarrollo del razonamiento proporcional. *Infancia y Aprendizaje, 34*(1), 67–80. https://doi.org/10.1174/021037011794390111

Fernández, C. & Llinares, S. (2012). Características del desarrollo del razonamiento proporcional en la Educación Primaria y Secundaria. *Enseñanza de las Ciencias, 30*(1), 129–142. https://doi.org/10.5565/rev/ec/v30n1.596

Fernández, C., Llinares, S., & Valls, J. (2012) Learning to notice students' mathematical thinking through online discussions. *ZDM. Mathematics Education, 44*(6), 747–759. https://doi.org/10.1007/s11858-012-0425-y

Fernández-Millán, E., & Molina, M. (2016). Indagación en el conocimiento conceptual del simbolismo algebraico de estudiantes de secundaria mediante la invención de problemas. *Enseñanza de las Ciencias, 34*(1), 53–71. https://doi.org/10.5565/rev/ensciencias.1455

Fiol, M. L. & Fortuny, J. M. (1990). *Proporcionalidad directa. La forma y el número.* Síntesis.

Font, V., Godino, J. D., & Gallardo, J. (2013). The emergence of objects from mathematical practices. *Educational Studies in Mathematics*, *82*, 97–124. https://doi.org/10.1007/s10649-012-9411-0

Godino, J. D. & Batanero, C. (1994). Significado institucional y personal de los objetos matemáticos. *Recherches en Didactique des Mathématiques, 14*(3), 325–355.

Godino, J. D., Batanero, C., & Font, V. (2007). The onto-semiotic approach to research in mathematics education. *ZDM. The International Journal on Mathematics Education, 39*(1-2), 127–135. https://doi.org/10.1007/s11858-006-0004-1

Godino, J. D., Giacomone, B., Batanero, C., & Font, V. (2017). Enfoque Ontosemiótico de los Conocimientos y Competencias del Profesor de Matemáticas. *Bolema, 31*(57), 90–113. https://doi.org/10.1590/1980-4415v31n57a05

Godino, J. D., Rivas, H., Arteaga, P., Lasa, A., & Wilhelmi, M. R. (2014). Ingeniería didáctica basada en el enfoque ontológico - semiótico del conocimiento y la instrucción matemáticos. *Recherches en Didactique des Mathématiques, 34*(2–3), 167–200.

Kwek, M. L. (2015). Using problem posing as a formative assessment tool. En F. Singer, N. Ellerton & J. Cai (Eds.), *Mathematical problem posing: from research to effective practice* (pp. 273–292). Springer.

Lamon, S. (2007). Rational number and proportional reasoning. Toward a theoretical framework for research. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 629–667). Information Age.

Lo, J-J. (2004). Prospective elementary school teachers' solution strategies and reasoning for a missing value proportion task. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education, 3,* 265–272.

Malaspina, U. (2013). La creación de problemas de matemáticas en la formación de profesores. En Sociedad de Educación Matemática Uruguaya (Ed.), *VII Congreso Iberoamericano de Educación Matemática* (pp. 129–140). CIBEM.

Malaspina, U., Mallart, A., & Font, V. (2015). Development of teachers' mathematical and didactic competencies by means of problem posing. In K. Krainer & N. Vondrová (Eds.), *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education* (pp. 2861–2866). CERME.

Malaspina, U., Torres, C., & Rubio, N. (2019). How to stimulate in-service teachers' didactic analysis competence by means of problem posing. In P. Liljedahl & L. Santos-Trigo (Eds.), *Mathematical Problem Solving* (pp. 133–151). Springer. https://doi.org/10.1007/978-3-030-10472-6_7

Mallart, A., Font, V., & Diez, J. (2018). Case Study on Mathematics Pre-service Teachers' Difficulties in Problem Posing. *Eurasia Journal of Mathematics, Science and Technology Education, 14*(4), 1465–1481. https://doi.org/10.29333/ejmste/83682

Mallart, A., Font, V., & Malaspina, U. (2016). Reflexión sobre el significado de qué es un buen problema de en la formación inicial de maestros. *Perfiles educativos, 38*(152), 14–30.

Mallart-Solaz, A. (2019). Interés de los futuros maestros en saber crear problemas de matemáticas para enseñar a resolverlos. *Psicología Educativa*, *25*(1), 31–41. https://doi.org/10.5093/psed2018a17

Milinković, J. (2015). Conceptualizing Problem Posing via Transformation. En J. Cai, N. Ellerton, & F. M. Singer (Eds.), *Mathematical Problem Posing: From Research to Effective Practice*, (pp. 47–70). Springer.

Ministerio de Educación Pública (2012). *Programas de estudio de Matemáticas.* San José, Costa Rica. https://www.mep.go.cr/sites/default/files/programadeestudio/programas/matematica.pdf

Obando, G., Vasco, C. E., & Arboleda, L. C. (2014). Enseñanza y aprendizaje de la razón, la proporción y la proporcionalidad: un estado del arte. *Relime, 17*(1), 59–81. https://doi.org/10.12802/relime.13.1713

OCDE (2003). *The PISA 2003. Assessment Framework. Mathematics, Reading, Science and Problem Solving Knowledge and Skills*. OCDE.

Pino-Fan, L. R., Báez-Huaiquián, D. I., Molina-Cabero, J. G., & Hernández-Arredondo, E. (2020). Criterios utilizados por profesores de matemáticas para el planteamiento de problemas en el aula. *UNICIENCIA, 34*(2), 114–136. https://doi.org/10.15359/ru.34-2.7

Riley, K. J. (2010). Teachers' understanding of proportional reasoning. En P. Brosnan, D. B. Erchick, & L. Flevares (Eds.), *Proceedings of the 32nd annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 6, pp. 1055–1061). The Ohio State University.

Rivas, M. A., Godino, J. D., & Castro, W. F. (2012). Desarrollo del conocimiento para la enseñanza de la proporcionalidad en futuros profesores de primaria. *Bolema, 26*(42b), 559–588. https://doi.org/10.1590/S0103-636X2012000200008

Ruiz, E. F. & Valdemoros, M. (2004). Connections between qualitative and quantitative thinking about proportion: The case of Paulina. En M. J. Hoines & A. B. Flugestad (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 201–208). PME.

Salazar, L. (2017). Invención de problemas contextualizados de probabilidad: una competencia a desarrollar en profesores de matemática. *Revista comunicación*, *26*(2), 38–48.

Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics, 14*(1), 19–28.

Silver, E. A. (2013). Problem-posing research in mathematics education: looking back, looking around, and looking ahead. *Educational studies in mathematics, 83*(1), 157–162. https://doi.org/10.1007/s10649-013-9477-3

Silvestre, A. I. & Ponte, J. P. (2011). Una experiencia de enseñanza dirigida al desarrollo del razonamiento proporcional. *Revista Educación y Pedagogía*, *23*(59), 137–158.

Singer, F. & Voica, C. (2013). A problem-solving conceptual framework and its implications in designing problem-posing tasks. *Educational studies in mathematics*, *83*(1), 9–26. https://doi.org/10.1007/s10649-012-9422-x

Singer, F., Ellerton, N., & Cai, J. (2013). Problem-posing research in mathematics education: new questions and directions. *Educational studies in mathematics, 83*(1), 1–7. https://doi.org/10.1007/s10649-013-9478-2

Stahnke, R., Schueler, S., & Roesken-Winter, B. (2016). Teachers' perception, interpretation, and decision-making: a systematic review of empirical mathematics education research. *ZDM. The International Journal on Mathematics Education, 48*(1), 1–27. https://doi.org/10.1007/s11858-016-0775-y

Stoyanova, E. (1998). Problem posing in mathematics classrooms. In A. McIntosh & N. Ellerton (Eds.), *Research in Mathematics Education: a contemporary perspective* (pp. 164–185). MASTEC.

Tichá, M. & Hošpesová, A. (2013). Developing teachers' subject didactic competence through problem posing. *Educational Studies in Mathematics, 83*(1), 133–143. https://doi.org/10.1007/s10649-012-9455-1

Tourniaire, F. & Pulos, S. (1985). Proportional reasoning: A review of the literature. *Educational Studies in Mathematics*, *16*, 181–204. https://doi.org/10.1007/BF02400937

Weiland, T., Orrill, C.H., Nagar, G.G., Brown, R., & Burke, J. (2020). Framing a robust understanding of proportional reasoning for teachers. *Journal of Mathematics Teacher Education, 24*(2), 179–202. https://doi.org/10.1007/s10857-019-09453-0