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The Concept of Randomness in Chilean Primary School Textbooks

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ABSTRACT

Background: Randomness is a fundamental concept in the formation of citizens because it intervenes in various daily life situations. In addition, it is present in the school curriculum of various countries, such as Chile. **Objective**: To analyse the teaching of the concept of randomness in primary education mathematics textbooks in Chile. **Design:** The research uses a qualitative methodology and uses the content-analysis method. **Setting and participants**: 42 textbooks were analysed, including student texts and activity logs. **Data collection and analysis**: Through content analysis, the sections and segments of the textbooks in which the concept of randomness intervenes are studied, relating them to definitions, problem situations, cognitive levels that should be developed, and the use of technological tools. **Results**: The concept of randomness is introduced through random games and concepts associated with random experiment, chance, random, and sample space, without delving into the definition of some cardinal concepts such as chance and randomness. **Conclusions**: The textbooks partially conform to the Chilean curricular guidelines. In addition, activities related to the concept of randomness generally refer to the use of physical devices and, on a small scale, to the use of technological tools. Finally, the textbooks present a small number of problem situations with an adequate cognitive level that favours the development of skills necessary for primary education students' acquisition of probabilistic thinking.

Keywords: Textbook; Primary education; Mathematics; Statistics; Randomness.

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El concepto aleatoriedad en los libros de texto chilenos de educación primaria

RESUMEN

Contexto: La aleatoriedad es un concepto fundamental en la formación de los ciudadanos, dado que interviene en diversas situaciones de la vida cotidiana. Además, está presente en el currículo escolar de diversos países, como Chile. **Objetivo**: Analizar la enseñanza del concepto de aleatoriedad en libros de texto de matemática de Educación Primaria en Chile. **Diseño**: Se sigue una metodología de corte cualitativa y usando el método del análisis de contenido. **Entorno y participantes**: Se analizaron 42 libros de texto, entre textos del estudiante y cuadernos de actividades. **Recopilación y análisis de datos**: Por medio del análisis de contenido, se estudian las secciones y los segmentos de los libros de texto en que intervienen el concepto de aleatoriedad, relacionándolos definiciones, situaciones problemas, niveles cognitivos que se desean desarrollar, y uso de herramientas tecnológicas. **Resultados**: El concepto aleatoriedad es introducido mediante juegos aleatorios y conceptos asociados a experimento aleatorio, azar, aleatorio y espacio muestral, sin ahondar en la definición de algunos conceptos cardinales como azar y aleatoriedad. **Conclusiones**: Los libros de texto se adecuan parcialmente a las directrices curriculares chilenas. Además, las actividades afines al concepto aleatoriedad, generalmente, hacen mención a la utilización de aparatos físicos y, en pequeña escala, al uso de herramientas tecnológicas. Finalmente, se evidencia en los libros de texto, una exigua presencia de situaciones problemas con un adecuado nivel cognitivo que favorezca el desarrollo de habilidades necesarias para la adquisición de un pensamiento probabilístico en los estudiantes de la Educación Primaria.

Palabras clave: libro de texto; Educación Primaria; matemática; estadística, aleatoriedad.

O conceito de aleatoriedade nos livros didáticos do ensino fundamental chileno

RESUMO

Contexto: A aleatoriedade é um conceito fundamental na formação dos cidadãos, pois intervém em diferentes situações do cotidiano. Além disso, está presente no currículo escolar de vários países, como o Chile. **Objetivos**: Analisar o ensino do conceito de aleatoriedade nos livros didáticos de Matemática do Ensino Fundamental no Chile. **Design**: Apresenta-se a seguir uma metodologia de corte qualitativo, utilizando o método de análise de conteúdo. **Ambiente e participantes**: Foram analisados 42 livros didáticos, incluindo textos de alunos e livros de atividades. **Coleta e análise de dados**: Por meio da análise de conteúdo, estudam-se as seções e segmentos de livros didáticos em que se apresenta o conceito de aleatoriedade, relacionando definições, situações-problema, níveis cognitivos a serem desenvolvidos e o uso de ferramentas tecnológicas. **Resultados**: O conceito de acaso é introduzido por meio de jogos aleatórios e conceitos associados a um experimento aleatório, acaso, acaso e

amostra espacial, sem considerar a definição de alguns conceitos cardinais como acaso e aleatoriedade. **Conclusões**: Os livros didáticos são parcialmente adaptados às diretrizes curriculares chilenas. Além disso, as atividades relacionadas ao conceito aleatório geralmente mencionam o uso de dispositivos físicos e, em pequena escala, o uso de ferramentas tecnológicas. Por fim, há evidência, nos livros didáticos, de uma escassa presença de situações-problema com nível cognitivo adequado que favoreça o desenvolvimento de habilidades necessárias para a aquisição do pensamento probabilístico em alunos do Ensino Fundamental.

Palavras-chave: Livro texto; Educação primária; Matemática; Estatística; Aleatoriedade.

INTRODUCTION

As textbooks are used in classrooms worldwide, researchers are increasingly interested in learning how the contents of school curriculums are treated in these educational resources regarding depth, intelligibility, timeliness, and relevance of the content developed. In this regard, Bergwall (2019) identifies several presentations of mathematical content in textbooks, both within and between countries. Thus, Shield and Dole (2013) suggest analysing the texts and inquiring about content insertion, examining the opportunities available to students during their learning process. In this same line, for Occeli and Valeiras (2013), textbooks play a fundamental role in school education because "they constitute mediating tools that translate and specify those meanings included in the curriculum prescribed by government agencies and that do so through a didactic presentation" (p. 134). In other words, they institutionalise the concepts and knowledge of the official curriculum in the educational process.

In this regard, given the relevance of textbooks as pedagogical and didactic resources in teaching and learning processes, the Chilean Ministry of Education (MINEDUC) attributes them an essential character in the educational tasks of the teacher, guiding the planning, preparation, and development of learning activities in the classroom and other spaces, and playing a fundamentally connecting role in students' learning (MINEDUC, 2008).

Therefore, the entire student body of municipal educational establishments freely and systematically receives textbooks and activity booklets subsided by the State of Chile. Texts that, as stated by the MINEDUC, are aligned with the Chilean school curriculum and offer motivational, challenging, and appropriate activities to the students' cognitive level to develop deep and meaningful learning.

Statistics and probability is one of the contents of school mathematics in primary education recently included in Chilean textbooks, considering that the curriculum guidelines of different countries seek to favour the development of probabilistic thinking of students from the first educational levels (Ruiz, 2015).

In the Chilean case, the cognitive dimension they aim to develop in primary education is oriented toward the knowledge and understanding of reality to favour the development of analytical, research, and theorising skills so that the students can face problem situations critically and positively (MINEDUC, 2012a; 2012b).

Likewise, they attribute technological tools a facilitating role in learning, contributing to the exploration and search for patterns that allow proposing and validating conjectures, organising and displaying data, facilitating representation, visualisation, modelling and problem solving, as well as the development of intuitive, deductive, and logical thinking and argumentation.

Regarding the Data and Probabilities axis of the Chilean school curriculum, the above is reflected in the development of reading skills, recording, and classification of daily-life information, among which is the concept of randomness and the use of different representations (pictograms, tables, graphs) to interpret, make predictions and communicate results. In addition, students are expected to study random phenomena by developing the ability to "Describe the possibility of occurrence of an event based on a random experiment, using the terms certain – possible – not quite possible – impossible" (MINEDUC, 2012a, p. 249).

On the other hand, teachers' efficient utilisation of textbooks for learning management depends on the level of their acquired probabilistic skills. In this regard, primary education teachers must: know in depth the contents of the Data and Probabilities axis of the school curriculum; be able to analyse and describe random phenomena, quantifying and representing the probability of occurrence of events; present the content of probabilities as a mathematical tool that models chance, contributing to decision-making; and conduct the learning of the concepts of probability, through playful and daily situations that illustrate how to quantify chance, identifying the difficulties of their learning at each level (MINEDUC, 2012a). This implies that teachers must have a solid formation in statistics and probability at the primary level to effectively guide the teaching of conceptual knowledge about the cardinal notions of probability, such as randomness and chance. Likewise, Estrella et al. (2015) mention that

for primary school teachers to acquire the capacity to criticise, produce, and analyse statistics, they must have a deep understanding of appropriate tools and representations, using an extensive repertoire of tasks, questions, and contexts; which they should eventually find in textbooks.

From this perspective, this research aims to analyse the teaching of the concept of randomness in textbooks of primary education in Chile. To respond to this general objective, the following specific objectives are defined: 1) Analyse the concept of randomness in textbooks at the different levels of primary education, and its adequacy with the curricular guidelines; 2) Classify the activities proposed in textbooks regarding the development of the concept of randomness, according to the level of cognitive demand that is declared; and 3) Identify the use of technological tools in textbooks for the learning of the concept of randomness.

RATIONALE

Probabilistic thinking and randomness

One of the objectives pursued by the Chilean school curriculum is that primary education students, from the first years of schooling, acquire statistical and probabilistic thinking to understand reality and make decisions, which implies knowing why and how statistical research is carried out to help us understand the phenomena that surround us (Chance, 2002). In this way, to develop probabilistic thinking, we must familiarise students with the research process: question formulation, data collection, analysis, testing assumptions, etc. Moreover, it is essential to understand how models are used to simulate random phenomena, recognise how, when, and why existing inferential tools can be used, and be able to understand and use the context of a problem to plan and evaluate investigations and communicate conclusions (Wild & Pfannkuch, 1999).

In relation to the concept of randomness, according to Renelle et al. (2021), people are often in contact with random phenomena, however "humans, in general, have a poor understanding of randomness" (p. 56), often applying erroneous ideas about this concept.

Although the term randomness is not unknown to most students, as they have experienced it in daily life inside and outside the school classrooms, Ramírez and Batalha (2019) recognise that there are differences between informal ideas and previous judgments about what they understand by

randomness in everyday life and what is formally transmitted in the classroom. Likewise, at the level of empirical research, Zisimos and Tasos (2021), in their study with 15-year-old students, identify that they conceive the notions of randomness, depending, to a large extent, on social, personal, and/or institutional aspects of the places they live.

In this way, the complex nature associated with randomness is evident, which tends to generate difficulties in student learning (Ortiz et al., 2001). In a way, this complexity is due to its direct relationship with the concept of probability, which also has a historical burden that has hindered a simple definition of it to this day (Batanero et al., 2012).

In this regard, the specialised literature brings several definitions. One of them, by Ayer (1974), relates randomness with probability calculation, i.e., a phenomenon is random if it behaves according to probability calculation. On the other hand, for Liu and Thompson (2002), the concept of randomness moves along an epistemological continuum, in which factors such as human ignorance intervene, being characterised by not being objective. In this context, Zisimos and Tasos (2021) observe that there are different interpretations of randomness at the school level, such as, for example, *randomness as equiprobability*, randomness as frequency stability, and the subjective view of randomness. In this regard, Batanero (2016) exposes different conceptions that are held about randomness: *randomness as equiprobability*, where an object behaves randomly, if it is selected from a set of objects where all of them are equally likely to be selected. This approach assumes conditions such as: that the number of objects is finite and they behave symmetrically, which consolidates the equiprobability; randomness as frequency stability, which relates the probability with the relative frequency of each member of a long-term random class, which, from Martins' (2011) perspective, becomes a means to study probability as a property associated with random phenomena and, therefore, is related to patterns that are governed under the law of large numbers; a *subjective vision of randomness, in which randomness* depends on the subjective knowledge one has of the phenomenon studied; and *complexity approach*, which corresponds to a mathematical perspective of randomness that relates it to the complexity that a computer program presents to model a random phenomenon.

The concept of randomness in the Chilean school curriculum

According to Inzunza (2014), probability as an area of knowledge has taken on greater importance in the educational system globally, where randomness plays an important role. This situation is not foreign in the Chilean case, which introduces this concept from the second grade of primary education (7 years of minimum age), with a proposal that has implicit Piaget's educational model regarding the acquisition of content from prelogical reasoning (from 2 to 7 years), concrete operational, through the operation of objectives (from 7 to 11 years) and formal operational, where the subject operates on operations or the results of said operations (12 years onwards).

On the other hand, the learning objectives proposed in the Chilean school curriculum related to the concept of randomness (Table 1) guide its development at different levels, focusing essentially on methodologies for the use of games related to situations in students' daily lives, based on simple stochastic experiments with dice, tokens, or coins (Alsina & Salgado, 2019), and from an epistemological view of the concept: from the intuitive to the prediction of results.

Table 1

Learning objectives about randomness in primary education, by level (MINEDUC, 2012b, 2015).

Level	Learning Objective	Code
2	Collect and record data to answer statistical questions about coin and dice games, using blocks and counting tables and pictograms. Register in simple bar charts and graphs the results of random games with dice and coins.	OA2
3	Register and sort data obtained from random games with dice and coins, finding the smallest, the largest and estimating the midpoint between these.	OA3
4	Perform playful and daily random experiments, and manually tabulate and represent through graphs and/or with educational software.	OA4
5	Describe the possibility of the occurrence of an event based on a random experiment, using the terms certain - possible - not quite possible - impossible. Compare probabilities of different events without calculating them.	OA5
6	Conjecture about the trend of results obtained in repetitions of the same experiment with dice, coins or others, manually and/or using educational software.	OA6
7	Explain the probabilities of events obtained through experiments manually and/or with educational software: estimating them intuitively. Use relative frequencies. Relate them to reasons, fractions or percentages.	OA7
8	There are no learning objectives related to the concept of randomness.	-

Cognitive level of randomness activities in textbooks

The current mathematics curriculum in Chile (MINEDUC, 2012a; 2012b) states that mathematics education involves the development of skills to solve problems, model, represent, argue and communicate. In this context, the level of cognitive demand for an activity presented in textbooks should be closely related to the cognitive processes necessary to respond to it (Doyle, 1988); processes that can vary, as mentioned by Estrella et al. (2020), from memorisation, the application of procedures and algorithms, to the understanding of the use of complex thinking and reasoning strategies, such as modelling, arguing, communicating, and inferring results; those that, according to Stein and Smith (1998), are classified as processes of low cognitive demand and high cognitive demand, respectively. These are determinants of student learning (Penalva & Llinares, 2011).

In this context, Franklin and Garfield (2006) suggest that textbooks should consider the following aspects, essential for the acquisition of stochastic topics: 1) activities should facilitate the acquisition of statistical knowledge, which allows the development of statistical reasoning; 2) the use of information from real data whose contexts would allow reflection and make sense of statistical summaries; 3) activities that allow developing a conceptual understanding of statistical concepts, rather than a simple procedural knowledge; 4) activities should facilitate active learning, i.e., promote analysis, synthesis, and evaluation; and 5) the use of technology, since its dynamic scope promotes the understanding of statistical concepts, in particular, that of variability.

On the other hand, according to Wild and Pfannkuch (1999), during the educational process, textbooks should incorporate research activities related to the concept of randomness, going beyond the throwing of dice and cards, i.e., moving away from *the cult of numbers* (López-Mojica et al., 2018), guiding the understanding of the student body through modelling, graphic representations, making intuitive inferences, requesting arguments, and making use of technological tools.

Technological tools for learning probabilistic concepts

Currently, there is a consensus that information and communication technologies (ICT) are more present than ever in the school system (Montiel & Gómez-Zermeño, 2021). They can enhance the skills students must develop, where simulation plays a fundamental role in problem solving, transforming a

problem from formal science to factual science (Huerta, 2020). In this sense, D'Angelo et al. (2014) mention that simulation can be considered an essential educational tool because it facilitates interaction and experimentation in various scenarios, allowing students to observe, perform, and test their conjectures.

Likewise, Homa-Agostinho and Oliveira-Groenwald (2020), in a study on the didactic possibilities of digital technologies for mathematical education in school and higher education, show that digital resources are positive in the teaching and learning processes, both in school and in teacher education. Likewise, Grisales-Aguirre (2018) indicate them as highly effective resources to motivate students to experiment, promoting a leading role in their learning.

In this context, authors such as Cullen et al. (2020) propose a model for the use of technology in the teaching of mathematics, through four components: 1) promotion of test cycles, a phase that allows exploring mathematical or statistical phenomena, by formulating guesses based on exploration and tests, the calculator and software being a technological tool that, if well used or instructed by the faculty, would allow students to check the type of situations, beyond seeing it as an immediate resolution tool (MC1); 2) generation and connection of representations, which consists of creating multiple graphical representations, dynamically and through the use of the computer, as a means for students to interact with various ways of capturing a mathematical situation (MC2); 3) evidence-based reasoning, which allows generating, collecting, organising, representing and analysing data, using statistical software (MC3); and 4) as in the student's case, it consists of the teacher learning from the different technological resources, and that, from them, the teacher can build their own resources, related to the teaching of each of the contents of mathematics (MC4).

METHODOLOGY

In line with the proposed objectives, we decided to carry out a qualitative research (McMillan & Schumacher, 2011) focused on the content analysis of activities related to the statistical object randomness (Krippendorf, 1997) in three series of textbooks (student textbooks and activity notebooks) of primary education, delivered by the MINEDUC, free of charge and open access, during the years 2016, 2020, and 2021. A non-probabilistic sampling of the intentional type, composed of forty-two books, was considered, labelling the student's texts as TE and the activity logs as CA. To facilitate writing, Table

2 identifies only the study sample books used as references and evidence in the results section.

For content analysis, we follow Mayring's (2000) proposal: first, to define the textbooks to be analysed; second, to identify and select the chapters related to statistics and probability, the sections and segments of information related to the concept of randomness; third, to identify emerging codes according to the object of randomness, related to units of analysis associated to definitions, problem situations, cognitive levels that should be developed, and use of technological tools. Finally, the findings were contrasted with empirical studies and the guidelines given by MINEDUC (Franklin & Garfield, 2006; Will & Pfannkuch, 1999; Cullen et al., 2020.)

Table 2

Students' books and activity notebooks selected from the sample

Code	Authors/Title/Publisher	Year
TE4	Alfaro, S., Espinoza, Y., Cano, S., Student Text, Matemática 4o. Básico [Mathematics 4th Basic], Houghton Mifflin Harcout.	2016
TE5	Alfaro, S., Espinoza, Y., Riquelme, M., Ainardi, V., Aldunate, V., Falconi, P., Olivares, C., Student Text, Matemática 5o. Básico [Mathematics 5th Basic], Houghton Mifflin Harcout.	2016
TE7	Merino, R., Muñoz, V., Pérez, B., Rupin, P., Student Text, Matemática 7o. Básico [Mathematics 7th Basic], SM.	2016
TE11	Urra, A., Córdova, C., Quezada, C., Student text, Matemática 3° Básico [Mathematics 3th Basic], Santillana.	2020
TE14	Maldonado, L., Castro, C., Student text, Matemática 6° Básico [Mathematics 6th Basic], Santillana.	2020
TE15	Iturra, F., Manosalva, C., Ramírez, M., Romero, D., Student Text, Matemática 7o. Básico [Mathematics 7th Basic], SM.	2020
TE22	Alvarado, J., Rojas, M., Soto, P., Villalobos, N., Student text, Matemática 6° Básico [Mathematics 6th Basic], Santillana.	2021
TE23	Iturra, F., Manosalva, C., Ramírez, M., Romero, D., Student Text, Matemática 7o. Básico [Mathematics 7th Basic], SM.	2021
CA14	Castro, C., Cuaderno de Actividades [Activity Log], Matemática 6° Básico [Mathematics 6th Basic], Santillana.	2020

RESULTS AND DISCUSSION

Regarding the concept of randomness in the textbooks analysed and their relationship with the declared learning objectives, we observed no direct relationship between some objectives and their approach at the corresponding level. To clarify, Table 3 summarises the presence of learning objectives in textbooks at each school level.

Table 3

Learning objectives and their presence in the textbooks of the years 2016, 2020, and 2021, by level

Learning	2016							2020							2021						
objective.	2	3	4	5	6	7	8	2	3	4	5	6	7	8	2	3	4	5	6	7	8
OA2									x	x		x	x				x			x	
OA3				x					x	x		x				x		x			
OA4			x	x	x	x			x	x	x	x	x			x	x	x	x	x	
OA5				x	x	x					x							x			
OA6				x	x	x				x		x	x					x	x	x	
OA7						x							x							x	

We identify that, at most levels, the activities related to the OA4 objective prevail. This fourth-grade learning objective of primary education aims to develop the concept of randomness through playful and daily experiments, data tabulation, and graphic representation. Likewise, the learning objective corresponding to the second level of schooling (AO2), aimed at the collection and recording of data based on games with coins and dice, is not addressed in any of the texts at the corresponding level. The teaching of this objective started at the third level, in the text of the year 2020, and at the fourth level, in the text of the year 2021. Finally, it is noteworthy that, in the texts of the years 2020 and 2021, the AO5 objective, corresponding to level 5, is not approached at subsequent levels, contrary to what is proposed in the 2016 textbook. This objective aims to describe an event based on a random experiment, using terms such as certain, possible, not quite possible, and impossible, tending to the development of an intuitive probability, being the basis for the development of probabilistic thinking (Batanero, 2005).

Concerning the inclusion of the concept of randomness in textbooks, concepts such as experiment, random, chance, equiprobable, result, and probability are mostly mentioned, and it is predominantly approached on the basis of routine games with coins, dice, and roulette (OA4). This is a methodological strategy that allows students to interact with non-deterministic experiments to provoke learning (Zabala-Vargas et al., 2020) and promote the development of probabilistic thinking (Kamii & Rummelsburg, 2008). Likewise, from the fifth to the seventh grade of primary education, the presence of questions related to formulating hypotheses or guessing about the trend of the results obtained in randomised experiments (OA6) is less evident.

On the other hand, we observed that in the series of books analysed, only the book for the third grade of primary school, year 2020, mentions the methodology of random games as a learning resource (Gal, 2005). It states:

> Random games are those that depend on chance. If a game is not random, you know in advance the result that you will achieve, that is, chance does not intervene. (TE11, 2020, p.253)

In this regard, Franklin et al. (2006) observe that play is a didactic strategy for students to understand probabilistic concepts, which is in line with the OA4 learning objective.

Now, in all the text series analysed, random experiments are defined, as shown by the following extracts:

> A randomised experiment is an activity with a likelihood of different outcomes. (TE5, 2016, p.270)

> An experiment is random if, by performing it under the same conditions, you are not sure what will happen. Therefore, its outcome cannot be predicted. (TE14, 2020, p.247)

> An experiment is random if, performed under identical conditions, it produces different results. Its outcome cannot be predicted, although its possible responses are known. (TE22, 2021, p.185)

The definitions of the 2020 and 2021 textbooks are relatively similar, mentioning that in a random experiment, it is not possible to predict the results, i.e., uncertainty is mentioned implicitly. In contrast, in the 2016 book, the definition of a random experiment is approached from the concept of probability.

Now, in the three series of books for the seventh grade of primary education, the concept of randomness is related to sample space, equiprobability, probability, and chance, as follows:

> An equiprobable random experiment is one in which all possible results are equally likely to occur. The prefix “equi” means equal. (TE15, 2016, p.344)

> A randomised experiment is equiprobable if the events in your sample space are equiprobable, that is, they are equally likely to occur. (TE23, 2021, p.210)

In addition, the 2016 book for the seventh grade includes a definition of randomness, mentioning the etymological origin of the concept, indicating:

> Random means hazardous, relative to chance, and refers to all uncertain knowledge. It comes from the Latin *aleas*, which means, originally, dice and then, by extension, chance. (TE7, 2016, p.338)

In summary, the notion of randomness in the analysed books is initially presented in the primary education curriculum through methodologies related to routine random games and records of results. From the fifth grade onwards, the notion of randomness is related to random experiment and equiprobability, a path that could lead students to assume that the results of a random experiment are always equally likely to occur (Huerta, 2020) and that, if there is a certain tendency towards any of the results, the phenomenon would no longer be random Konold (1991).

Generally, the activities related to randomness and cognitive skills that textbooks address refer to routine problem situations or what authors such as Manouchehri et al. (2012) have called *exercise*. Activities associated with games of chance such as roulette, dice, cards, and coins prevail, as seen in Table 4, those that model random experiments with essentially equiprobable responses, which could generate in the student body the bias of equiprobability, and with it, the mechanistic application of the Laplace Rule in the calculation of probabilities (Huerta, 2020). A similar trend is evident in the activity logs.

On the other hand, the three series of books used the tree diagram to model random phenomena, such as the throwing of dice and coins. For example, in the sixth-grade book, TE14 (p. 247), the use of a tree diagram to simulate possible outcomes by throwing two legitimate dice is shown. This representation is relevant to model and explain all the responses associated with a random experiment, allowing us to know the cardinality of the sample space and "helping to understand the structure of an experiment and, therefore, to solve the problems" (Roldán et al., 2018, p. 56) associated with it. However, its use is not explicitly indicated in the learning objectives of primary education in Chile, described in Table 1. In addition, both specialised literature and specifically Roldán et al. (2018) state that the tree diagram is a didactic resource that allows representing and studying the behaviour of a random experiment and the sampling space associated with it, concepts that are the basis for the calculation of probabilities.

Table 4

Activities related to the concept of randomness in the student's texts, in the years 2016, 2020, and 2021, by level.

Activity	2016							2020							2021						
	2	3	4	5	6	7	8	2	3	4	5	6	7	8	2	3	4	5	6	7	8
Roulettes			x	X			x		x	x		x	x	x			x	x	x	x	x
Dices			x	X	x	x	x		x	x		x	x		x	x	x	x	x	x	x
Cards						x				x		x						x		x	
Coins	x			X	x		x				x	x	x	x	x	x		x	x	x	x
Bags				x			x		x		x						x				
Tombola									x		x	x									
Ballot boxes						x	x						x					x	x	x	x
Cards											x								x	x	
Internet												x	x						x	x	
Excel														x					x		x
Mobile phone												x									

Regarding the cognitive skills related to randomness present in textbook activities, the type of activities proposed is shown in Tables 5 and 6. In this regard, differences are observed in the number of activities present in the years 2016, 2020, and 2021, where the skill related to the simulation of random experiments related to coins, dice, raffles, and roulette prevails.

Table 5

Declared skills in the activities on randomness in the student texts, in the years 2016, 2020 and 2021, by level

Skills	2016						2020						2021					
	3	4	5	6	7	8	3	4	5	6	7	8	3	4	5	6	7	8
Simulate		2	6	5	2		8	8	7	17	7	11		4	7	15	7	11
Classify		4			4			3		8	7				16	9	7	
Present		2	1			6	6	3	1	3	1	3		4	4	6	1	3
Interpret								1								1		
Exemplify		1			4		3							3				
Conceptual						4		7		4	1					1	1	
Conclude			2				4	6	1	4	1			1	7	13	1	1
Infer		4		4			2	1		6	6						6	
Conjecture											1					4	1	

We also identified that student texts and exercise books promote procedural work, offering little cognitive challenge, as observed by Van Zanten

and Van den Heuvel-Panhuizen (2018). However, the school texts and activity logs of 2020 present a greater variability in skills, in contrast to those of 2016 and 2021, which are displaced between the procedural, the conceptual and the inferential. These, eventually, would allow students to delve into the concept of randomness, presenting a greater cognitive demand; a scenario that is consistent with what was mentioned by Franklin and Garfield (2006) and by Will and Pfannkuch (1999) about the development of skills that allow conceptual understanding, graphic representation, and inferring results and, to a lesser extent, conjecture about the results obtained, respectively.

Table 6

Declared skills in randomness activities in the activity notebooks, in the years 2016, 2020 and 2021, by level

Skills	2016						2020						2021					
	3	4	5	6	7	8	3	4	5	6	7	8	3	4	5	6	7	8
Simulate			9	5	13	6	5	6	9	9	11	8		5	10	19	5	13
Classify					5		2	10	9	7	6	3			13	8	5	
Present			4	1	15	1	2	4		9	5	5		2	6	2	8	4
Determine			4	4	7		1		4	7					4	2	5	
Interpret											3			1				
Exemplify																3		
Conceptual																3		
Conclude			1					2	2	1		3		3		14	3	5
Infer				2				4		5	3	2						
Conjecture																2		

As an example, in one activity of the third-grade primary student, TE11 (p.258), about the extraction of cards, with images of fruits, of a bag, the skills of simulating and inferring the result are explicit, proposing the question "Is it possible to know which fruit will come out in advance?", thus facilitating the development of a statistical thought (Franklin & Garfield, 2006). However, although these authors and the GAISE report (2005; 2016) have highlighted the importance of the context in problem situations, this activity does not mention a context related to simulation to allow students to present arguments about the importance of the fruits present in the cards in their daily diet, according to the policies of a healthy diet promoted in Chile.

On the other hand, with higher cognitive skills, there are usually few activities aimed at inferring and formulating conjectures in the texts of years 2016 and 2021, which would eventually impair the development of the ability

to infer and, therefore, enhance an informal inference in the student body at the basic education level (Makar & Rubin, 2009).

According to the above, one of the few activities related to the ability to guess presented in the sixth-grade textbook TE22 (p. 189) asks students to make guesses about the behaviour when throwing coins and legitimate dice. This activity invites students to analyse and study the variation of results and, therefore, to develop probabilistic thinking (Garfield & Ben-Zvi, 2008).

Similarly, in the TE22 textbook (p. 185), students are asked to classify the type of experiment carried out and represent through a tree diagram the probability of obtaining a prize under specific conditions. This representation, according to Sánchez and Ruiz (2020), makes "the use of data emerge as evidence that allows generalisation" (p. 14). In addition, this activity implicitly has a component related to probabilistic thinking, the interpretation of the results in the context of the problem, which entails communicating the results with the use of probabilistic language (León, 2020), processes that have associated activities with high cognitive demand (Stein & Smith, 1998).

Likewise, the activity notebook CA14 (p. 128), for the sixth-grade of primary school, presents an activity in which the results of the throwing of a dice are given summarised in a distribution table. This is an activity in which one must simulate and infer the results, i.e., go beyond the empirical evidence making inferences about their behaviour, and making the students get closer to the law of large numbers, a topic that is introduced in the textbooks of secondary education.

Similarly, in the fourth-grade textbook TE4 (p. 213), two students are asked to physically simulate the experiment of throwing a die 20 times, register the results and represent them in a bar graph. In this regard, this activity has essentially procedural skills associated, such as representing the frequency of the results obtained in a table and graphing it, which shows a low cognitive demand. This situation could focus on the development of higher-order skills, such as interpretation and decision-making, based on information summarised in a tabular and graphical way, coming from a non-deterministic phenomenon, which could eventually be strengthened by incorporating the use of technological tools.

In short, it is evident that the activities in the texts usually show a low cognitive demand, essentially focused on procedural skills, according to what was mentioned by Ruiz-Reyes et al. (2017). These results are similar to those found by Salcedo (2012; 2015) in primary education textbooks in Venezuela,

and by Vásquez et al. (2019), in the case of Chilean textbooks from the first to the fifth grade of primary education when studying probability-related topics. However, we notice that the textbooks of the year 2020 incorporate activities that follow the guidelines given by Franklin and Garfield (2006), including activities that would promote higher-order cognitive skills, such as concluding and inferring based on results.

Regarding the use of technological tools in textbooks for teaching the concept of randomness, these were analysed using the model of Cullen et al. (2020), summarised in Table 7. In general, we observe that these can be classified in the category *reasoning based on evidence*, i.e., they allow generating and analysing random phenomena through computers and Apps. A scarce use of technological tools is also identified, even though there is an increase in the years 2020 and 2021, which could be a product of remote education that was implemented in response to the health contingency product of the Covid-19 pandemic.

Likewise, we did not observe any activities that aim to dynamically generate and connect multiple graphical representations through the computer (MC2). Finally, about making use of the calculator and software to explore statistical phenomena, through the formulation of assumptions based on exploration (MC1), only activities for the sixth-grade are evidenced, in the years 2020 and 2021, being valuable for the development of probabilistic thinking, which should be implemented at all grades.

Table 7

Activities related to the model proposed by Cullen et al. (2020) on the use of technology in student texts

Categories	2016						2020						2021					
	3	4	5	6	7	8	3	4	5	6	7	8	3	4	5	6	7	8
MC1										x						x		
MC2																		
MC3					x					x	x					x	x	
MC4					x					x	x	x				x	x	x

As an example, we point out two activities for sixth-graders, where they are asked to simulate a random phenomenon using technological tools, in texts TE14 (p. 254) and TE22 (p. 191). In the first activity, they are asked to simulate the flipping of a coin a specific number of times through an application of the mobile phone and record the results. The second activity mentions the random

function of Excel to simulate the throwing of a die and validate the conjectures and results obtained when performing the game in physical form, enabling the student body to develop higher-order skills such as guessing the behaviour of a non-deterministic phenomenon.

Finally, the delivery of an access link to the MINEDUC platform is identified in TE15 (p. 218) to simulate the throwing of two dice. This activity requests students only to record the results obtained and quantify the sample space. We note that this activity is essentially procedural, leaving the challenge for teachers to enhance these activities so that higher skills such as argumentation, inference, and conjecture are developed.

On the other hand, it is also evident that the activity notebooks do not present problem situations where technological tools intervene to simulate random experiments. This reveals a gap between theory (textbooks) and practice (exercise books), evidencing a lack of connection between them.

CONCLUSION

According to the results obtained, regardless of the series of books analysed, the concept of randomness is inserted by the use of teaching methodologies related to random games and concepts associated with random experiment, chance, random, and sample space without delving into the definition of some cardinal concepts such as chance and randomness, in order to promote the development of skills that allow students "to have a sufficient command of language to understand the problems they face, solve the tasks, communicate the solutions found and justify them" (Hernández-Salmerón et al., 2017, p.306).

In addition, the tasks present routine problems related to games of chance, such as the throwing of dice, flipping coins, and roulette, among others, which allow teachers to take advantage of the students' personal experiences (Makar, 2009), in coherence with what is mentioned in the Chilean school curriculum for primary school education (OA2, OA3, and OA4). However, this view is essentially biased since a large part of the random phenomena we must interact with daily do not behave in this way (Huerta, 2020). Moreover, the presence of activities from real contexts is scarce, which would allow the students to reflect, argue, and make sense of the findings.

Likewise, textbooks introduce tree diagrams, representations that are not mentioned in the learning objectives, which, according to Roldán et al.

(2018), allow for a brief description of the behaviour of a random phenomenon and go "from the abstract to the particular" (p.62). From our experience, incorporating them at the primary level books is a hit.

On the other hand, the activities generally show a low cognitive demand since, essentially, the students face activities that focus on knowing definitions, carrying out established procedures, and replicating or simulating the same experiments mentioned in the examples, frequently promoting procedural and algorithmic work. This lack of cognitive demand eventually prevents students from acquiring the skills and knowledge of randomness and becoming citizens with probabilistic thinking since the concept of randomness is related to the understanding of the concept of probability. However, 2020 and 2021 textbooks include tasks that could allow students to develop high-level cognitive skills, such as concluding and inferring (Stein & Smith, 1998). In this regard, we believe that textbooks should play a more preponderant role in teaching-learning topics related to probability, to respond fully, not partially, to OA5, as evidenced in the results delivered.

On the other hand, the activities in the textbooks refer to a small number of activities related to the use of technological tools declared in the learning objectives OA4, OA6, and OA7. Those that generally promote the student body's active learning, encouraging the understanding of cardinal concepts related to probability. However, in the 2020 and 2021 textbooks, we identified more activities using technological tools, which, from our point of view, should allow teachers to facilitate the approach of probabilistic concepts and their understanding.

Finally, this study gives evidence on how one of the fundamental statistical objects in probability is being addressed in school texts: randomness; those that could guide reflection around thematic nuclei, to delve into conceptual aspects related to probability in such a way that it is oriented toward teacher education, their planning designs, and implementation, related with the activities proposed by the study programmes and textbooks.

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AUTHORSHIP CONTRIBUTION STATEMENT

F.R.A. conceived the idea of the research presented. F.R.A. collected the data. The three authors (F.R.A., M.A., and D.D.L.) actively participated in the development of the theory, methodology, data organisation and analysis, discussion of results and approval of the final version of the work.

DATA AVAILABILITY DECLARATION

The data supporting the results of this investigation will be made available by the correspondent F.R.A., upon reasonable request.

REFERENCES

Alsina, Á. & Salgado, M. (2019). Ampliando los conocimientos en Educación Infantil: la incorporación de la probabilidad. *Revista de Estudios y Experiencias en Educación, 18*(36), 225-240.

Ayer, A.J. (1974). El Azar. In M. Kline (Ed.), *Matemáticas en el mundo moderno* (pp. 172-181). Blume.

Batanero, C. (2005). Significados de la probabilidad en la educación secundaria. *RELIME, 8*(3), 247-264.

Batanero, C. (2016). Understanding randomness. Challenges for research and teaching. In K. Krainer & N. Vondrova (Eds.), *Proceedings of the ninth congress of the European Society for Research in Mathematics Education* (pp. 34–49). ERME.

Batanero, C., Gómez, E., Serrano, L., & Contreras, J.L. (2012). Comprensión de la aleatoriedad por futuros profesores de Educación Primaria. *Journal of Research in Mathematics Education, 1*(3), 222-245.

Bergwall, A. (2019). Proof-related reasoning in upper secondary school: characteristics of Swedish and Finnish textbooks. *International Journal of Mathematical Education in Science and Technology, 52*(5), 731-751. https://doi.org/10.1080/0020739X.2019.1704085

Chance, B.L. (2002). Components of Statistical Thinking and Implications for Instruction and Assessment. J*ournal of Statistics Education, 10*(3). www.amstat.org/publications/jse/v10n3/chance.html

Cullen, C., Hertel, J., & Nickels, M. (2020). The roles of technology in Mathematics Education. *The Educational Forum*, *84*(2), 166-178. https://doi.org/10.1080/00131725.2020.1698683

D'Angelo, C., Rutstein, D., Harris, C., Bernard, R., Borokhovski, E., & Haertel, G. (2014). *Simulations for STEM learning: systematic review and meta-analysis executive summary*. SRI

Doyle, W. (1988). Work in mathematics classes: the context of student's thinking during instruction. *Educational Psychologist*, *23*(2), 167-180.

Estrella, S., Zakaryan, D., Olfos, R., & Espinoza, G. (2020). How teachers learn to maintain the cognitive demand of tasks through Lesson Study. *Journal of Mathematics Teacher Education, 23*, 293-310. https://doi.org/10.1007/s10857-018-09423-y

Estrella, S., Olfos, R., & Mena-Lorca, A. (2015). El conocimiento pedagógico del contenido de estadística en profesores de primaria. *Educação e Pesquisa, 41*(2), 477-493.

Franklin, C. & Garfield, J.B. (2006). The GAISE Project: developing statistics education guidelines for Pre K-12 and College Courses: 2006 NCTM Yearbook. In G. Burrill (Ed.), *Thinking and Reasoning with Data and Chance: 2006 NCTM Yearbook* (pp. 345-376). NCTM.

Garfield, J. & Ben-Zvi, D. (2008). *Developing students' statistical reasoning: Connecting reseachs and teaching practice*. Springer.

Gal, I. (2005). Towards "probability literacy" for all citezens: Building blocks and instructional dilemas. In G. Jones (Ed.), *Exploring probability in school* (pp. 39-63). Springer.

GAISE (2016). *Guidelines for Assessment and Instruction in Statistics Education: College report*. ASA.

GAISE (2005). *Guidelines for assessment and instruction in statistics education (GAISE) report: A curriculum framework for PreK-12 statistics education*. ASA

Grisales-Aguirre, A. (2018). Uso de recursos TIC en la enseñanza de las matemáticas: retos y perspectivas. *Entramado, 14*(2),198-214.

Hernández-Salmerón, E., López-Martín, M.M., & Batanero, C. (2017). Estudio exploratorio sobre el lenguaje del azar en Educación

Secundaria Obligatoria. In J.M. Muñoz-Escolano, A. Arnal-Bailera, P. Beltrán-Pellicer, M.L. Callejo, & J. Carrillo (Eds.), *Investigación en Educación Matemática XXI* (pp. 305-314). SEIEM.

Homa-Agostinho, I. & Oliveira-Groenwald, C. (2020). Digital information and communication technologies as a didactic resource in the mathematics curriculum. *Uniciencia, 34*(2), 153-170.

Huerta, P.M. (2020). Hipótesis y conjeturas en el desarrollo del pensamiento estocástico: retos para su enseñanza y en la formación de profesores. *RELIME, 23*(1), 79-102. https://doi.org/10.12802/relime.20.2313

Inzunza, S. (2014). Geogebra: Una herramienta cognitiva para la enseñanza de la probabilidad. In J. Asenjo, O. Macías, & J.C. Toscano (Eds.), *Actas del Congreso Iberoamericano de Ciencia, Tecnología, Innovación y Educación* (pp. 1-11). OEI.

Kamii, C. & Rummelsburg, J. (2008). Arithmetic for first graders lacking number concepts. *Teaching Children Mathematics, 14*(7), 389-394.

Konold, C. (1991). Understanding students' beliefs about probability. En E. Von Glasersfeld (Ed*.), Radical constructivism in mathematics education* (pp. 139-156). Kluwer.

Krippendorff, K. (1997). *Metodología de análisis de contenido. Teoría y práctica*. Paidós.

León, N. (2020). Alcances de la enseñanza de la estadística a través de la investigación en la educación media en Venezuela. *Paradigma, 41*, 657-684.

Liu, Y. & Thompson, P. (2002). Randomness: Rethinking the foundation of probability. In D. Mewborn, P. Sztajn, E. White, H. Wiegel, R. Bryant, & K. Nooney (Eds.), *Proceedings of the Twenty Fourth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 1331-1334). PME-NA.

López-Mojica, J.M., Ojeda-Salazar, A.M., & Salcedo-Prado, J. (2018). Ideas fundamentales de estocásticos en libros de texto de educación primaria: una alternativa de enseñanza. *IE Revista de Investigación Educativa de la REDIECH, 9*(17), 87-102. https://doi.org/10.33010/ie_rie_rediech.v9i17

Manouchehri, A., Zhang, P., & Liu, Y. (2012). Forces hindering development of mathematical problem solving among school children. *Proceedings of the 12th International Congress on Mathematical Education*, Seoul, Korea.

Makar, K. & Rubin, A. (2009). A framework for thin- king about informal statistical inference. *Statistics Education Research Journal, 8*(1), 82-105.

Martins, M.E. (2011). Como estimar a probabilidade de um acontecimento por simulação. In *Actas ProfMat 2011* (pp. 1-16). APM.

Mayring, P. (2000). Qualitative content analysis. *Forum Qualitative Social Research, 1*(2), 20. https://doi.org/10.17169/fqs-1.2.1089

McMillan, J. & Schumacher, S. (2011). *Investigación educativa*. Pearson-Adisson Wesley.

MINEDUC (2012a*). Bases Curriculares de Primero a Sexto Básico*. Unidad de Currículum y Evaluación.

MINEDUC (2012b). *Bases Curriculares para la Educación Básica*. Ministerio de Educación.

MINEDUC (2008). *Política de textos escolares*. Ministerio de Educación.

MINEDUC (2015). *Bases Curriculares de 7° básico a 2° medio*. Unidad de Currículum y Evaluación.

Montiel, H. & Gomez-Zermeño, M.G. (2021). educational challenges for computational thinking in k–12 education: a systematic literature review of "scratch" as an innovative programming tool. *Computers, 10*(6), 69. https://doi.org/10.3390/computers10060069

Occeli, M. & Valeiras, N. (2013). Los libros de texto de ciencias como objeto de investigación: una revisión bibliográfica. *Enseñanza de las Ciencias, 31*(2), 133-152.

Ortiz, J.J., Batanero, C., & Serrano, L. (2001). El lenguaje probabilístico en los libros de texto. *Suma, 38*, 5-14.

Pea, R.D. (1985). Beyond amplification: Using the computer to reorganize mental functioning. *Educational Psychologist, 20*(4), 167-182.

Penalva, M.C. & Llinares, S. (2011). Tareas matemáticas en la Educación Secundaria. En J.M. Goñi (Ed.), *Didáctica de las Matemáticas* (pp. 27-51). GRAO.

Pfannkuch, M. (2011). The role of context in developing informal statistical inferential reasoning: A classroom study. *Mathematical Thinking and Learning, 13*(1-2), 27-46. https://doi.org/10.1080/10986065.2011.538302

Ramírez, G. & Batalha, A. (2019). Aprendizaje de la aleatoriedad y conceptos asociados: un estudio en secundaria con apoyo de Geogebra. *Revista Digital: Matemática, Educación e Internet, 20*(1), 1-13. https://doi.org/10.18845/rdmei.v20i1.4591

Renelle, A., Budgett, S., & Jones, R. (2021). New Zealand teachers' generation problem misconceptions. *Teaching Statistics, 43*(2), 56-61. https://doi.org/10.1111/test.12248

Roldán, A., Batanero, C., & Beltrán-Pellicer, P. (2018). El diagrama de árbol: un recurso intuitivo en probabilidad y combinatoria. *Épsilon, 100*, 49-63.

Ruiz-Reyes, K., Begué, N., Batanero, C., & Contreras, J.M. (2017). Un estudio comparativo de los contenidos del muestreo en la Educación Secundaria Obligatoria en Chile. *Educação Matemática Pesquisa, 19*(3), 67-83. https://doi.org/10.23925/1983-3156.2017v19i3p67-83

Ruiz López, N. (2015). La enseñanza de la estadística en la Educación Primaria en América Latina. REICE. *Revista Iberoamericana sobre Calidad, Eficacia y Cambio en Educación, 13(1),* 103-121.

Sánchez, N. & Ruiz, B. (2020). Análisis de las actividades propuestas en dos programas de estudio chilenos en el eje de Estadística y Probabilidad. *IE Revista de Investigación Educativa de la REDIECH, 11*, e776. https://doi.org/10.33010/ie_rie_rediech.v11i0.776

Salcedo, A. (2012). Análisis de las actividades para el estudiante en los libros de matemáticas. *Investigación y Postgrado, 27*(1), 83-109.

Salcedo, A. (2015). Exigencia cognitiva de las actividades de estadística en textos escolares de Educación Primaria. In J.M. Contreras, C. Batanero, J.D. Godino, G.R. Cañadas, P. Arteaga, E. Molina y M.M. Gea, & M.M. López-Martín (Eds.), *Didáctica de la Estadística,*

Probabilidad y Combinatoria 2 (pp. 307-315). Universidad de Granada.

Shield, M. & Dole, S. (2013). Assessing the potential of mathematics textbooks to promote deep learning. *Educational Studies in Mathematics, 82*(2), 183-199.

Stein, M.K. & Smith, M.S. (1998). Mathematics tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School, 3*(4), 268-275.

Vásquez, C., Pincheira, N., Piñeiro, J. & Díaz-Levicoy, D. (2019). ¿Cómo se promueve el aprendizaje de la estadística y la probabilidad? Un análisis desde los libros de texto para la Educación Primaria. *Bolema, 33*(65), 1133-1154. http://dx.doi.org/10.1590/1980-4415v33n65a08

Van Zanten, M. & Van den Heuvel-Panhuizen, M. (2018). Opportunity to learn problem solving in Dutch primary school mathematics textbooks. *ZDM, 50*(7), 827-838.

Wild, C. & Pfannkuch, M. (1999). Statistical thinking in empirical enquiry. *International Statistical Review, 67*(3), 223-265.

Zabala-Vargas, S.A., Ardila-Segovia, D. A., García-Mora, L. H., & Benito-Crosetti, B. (2020). Aprendizaje Basado en Juegos (GBL) aplicado a la enseñanza de la matemática en educación superior. Una revisión sistemática de literatura. *Formación Universitaria, 13*(1), 13-26.

Zisimos B., & Tasos P. (2021) Understanding 15-year-old students' conceptions of randomness through their 'potential worlds': a qualitative analysis. *International Journal of Mathematical Education in Science and Technology, 52*(2), 237-258. https://doi.org/10.1080/0020739X.2019.1676928