



Mathematics Teaching and Subjects that Include Mathematics in Pedagogy Courses

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ABSTRACT

Background: Research in initial education and professional performance of pedagogues in mathematics is increasing, but they are still few compared to other topics in mathematics education and point to the need for more studies. **Objectives:** To present a reading of the speeches of beginning mathematics teachers in early childhood education and the early years of elementary school about teaching mathematics, addressing its difficulties and facilities and how they work in classrooms, establishing relationships with the school modules involving mathematics in the degree in pedagogy courses they attended. **Design:** Qualitative research. **Setting and Participants:** Six teachers from the municipal schools of São José do Rio Pardo, who agreed to participate in the research through a questionnaire sent to schools in this municipality, the environment being stipulated by them. **Data collection and analysis:** Data were produced through semi-structured interviews and analysed based on the assumptions of the model of semantic fields. **Results:** the main result was that the pedagogues had many difficulties in how to teach mathematics, even though their training was more focused on the methodological aspects of the modules that involved mathematics. **Conclusions:** Although the pedagogues had doubts about how to teach mathematics, we observed that the mathematical contents, practically not addressed in initial education, do not concern them in professional practice, as most of them work in early childhood education; this suggests research on what mathematics are necessary for the initial and even continuing education of pedagogues who teach mathematics.

Keywords: Education of teachers who teach mathematics; Professional practice; Pedagogy; Model of semantic fields; Mathematics education.

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A docência em Matemática e as disciplinas de Matemática na Pedagogia

RESUMO

Contexto: pesquisas na área de formação inicial e de atuação profissional de pedagogos (as) no âmbito da matemática (ou matemáticas) estão aumentando, mas ainda são poucas se comparada a outras temáticas na Educação Matemática, e apontam para a necessidade de mais estudos. **Objetivos:** apresentar uma leitura de falas de professoras iniciantes de matemática da Educação Infantil e dos anos iniciais do Ensino Fundamental sobre a docência em matemática, abordando suas dificuldades e facilidades e como atuam em salas de aula, estabelecendo relações com as disciplinas envolvendo matemática nos cursos de Licenciatura em Pedagogia que realizaram. **Design:** pesquisa qualitativa. **Ambiente e participantes:** seis professoras da rede municipal de São José do Rio Pardo que aceitaram participar da pesquisa por meio de questionário-convite enviado às escolas deste município, sendo o ambiente estipulado por elas. **Coleta e análise de dados:** os dados foram produzidos através de entrevista semiestruturada e analisados a partir dos pressupostos do Modelo dos Campos Semânticos. **Resultados:** o principal resultado foi que as pedagogas tiveram muitas dificuldades em como ensinar matemática, apesar da formação ter sido mais voltada para os aspectos metodológicos nas disciplinas envolvendo matemática. **Conclusões:** apesar das pedagogas terem ficado com dúvidas sobre como ensinar matemática, nota-se que os conteúdos matemáticos, praticamente não abordados na formação inicial, não as preocupam na prática profissional, pela maioria atuar na Educação Infantil; isso sugere pesquisas sobre que matemática (ou matemáticas) são necessárias para a formação inicial e até mesmo continuada de pedagogos(as) que ensinam matemática.

Palavras-chave: Formação de Professores que Ensinam Matemática; Prática Profissional; Pedagogia; Modelo dos Campos Semânticos; Educação Matemática.

INTRODUCTION

Teaching early childhood education or the early years of elementary school is one of the many different professional possibilities for graduates with a degree in pedagogy course. This teaching involves different contents from different areas, including mathematics, which justifies its presence in the specific knowledge modules in the pedagogues' initial education.

Research has increasingly focused on the initial or continuing education of teachers who teach mathematics in the early years of elementary school and early childhood education and on the professional practice of pedagogues in mathematics (Fiorentini et al., 2002; Fiorentini, Passos & Lima, 2016). Santos and Lima (2011), for example, carried out analyses of dissertations and theses on the education of mathematics teachers for the early years of elementary school and stated that research has increased and created

the following categories of analysis that express themes of greater focus: initial teacher education, teachers' conceptions, teaching methodologies, historical study, teaching profession, teacher knowledge, and teaching practice. Most of the works addressed more than one category, with the first two and teacher's knowledge being the predominant ones. Even with this increased interest and research scenario, there are still gaps and possibilities for new investigations, reflections, and discussions. As Curi (2020, p. 16) points out: "What mathematics should be proposed in pedagogy courses and how should it be treated, considering the number of hours allocated to the subject is still small?"

This article is part of this theme. It brings an excerpt of research involving professional practice and initial teacher education. From the perspective of the model of semantic fields (MSF), the research aimed to read the productions of meanings of six beginning teachers of early childhood education and the initial years of the municipal education system in São José do Rio Pardo, São Paulo, Brazil. The research is based on semi-structured interviews about the prospective teachers' training in basic education and the pedagogy degree course, their teaching practices, and the relationships between their teaching practices in mathematics teaching and training.

We defined beginning teachers as those in their first five years of teaching. For example, Huberman (2000) and Gonçalves (2000) describe teachers' professional practice over the years through phases that impact them and contribute to changing how they understand that practice. For Huberman (2000), the initial phase is marked by the struggle for survival and discovery phases, the first being the first confrontation with the complexity of professional practice and difficulties with educational situations and the second the initial enthusiasm for finally being in front of a classroom. In the same direction, Gonçalves (2000) calls this initial phase "the beginning", when there is a "struggle for survival, determined by a shock of reality and the enthusiasm of discovering a new world that is opening up to the young teacher" (Gonçalves, 2000, p. 164). For some teachers, the career entry "resulted in an authentic struggle between the desire to assert themselves and the desire to leave the profession" and showed "the evident lack of preparation, effective or supposed, for the teaching profession they joined, in most cases, difficult working conditions and not knowing how to make themselves accepted as a teacher" (Gonçalves, 2000, p. 164). For other teachers, in turn, the beginning of their careers proved to be smooth, resulting from "self-confidence motivated by the conviction of being prepared for the teaching profession" (Gonçalves, 2000, p. 164), even if later, when recalling moments of their career, they pointed out that "this was not so easy as it seemed to be back then" (Gonçalves, 2000, p. 164).

The MSF is not only a theoretical reference for the elaboration and accomplishment of interviews. Viola dos Santos and Lins (2016) have also theorised about the training of teachers who teach mathematics. However, productions focused on the initial or continuing education of teachers who teach mathematics in the early childhood education and the early years of elementary school are still outnumbered (Paulo, 2020, Zanetti, 2020).

The research excerpt that we brought to this article is a reading of the interviewees' speeches about the initial teaching of mathematics in early childhood education and the early years of elementary school, addressing the challenges found in this process and the contents and methodologies with which they work. We will sometimes pause our readings on initial teaching and intersperse them with reading about elements of the modules linked to the type of mathematics the prospective teachers attended in the degree in pedagogy. With this action, we intend to bring thoughts about the initial education, especially about the contents of mathematics, teaching methodologies, and the educational assumptions of the teacher educators in those modules. However, before that, we will approach the MSF, which is our theoretical reference throughout the research process.

MODEL OF SEMANTIC FIELDS

The MSF was created by Romulo Campos Lins (Lins, 1999, 2012) and has the notions of object, meaning, and knowledge as its core (Lins, 2004). According to Lins (1999), an object is anything a person is talking about; the meaning of an object is what can effectively be said about it within an activity. Thus, producing meaning is constituting objects.

It is not that the objects are there and I am here so that I can discover their meanings from there; on the contrary, I constitute myself as a cognitive being through the production of meanings that I make, at the same time that I constitute objects through these utterances. (Lins, 1999, p. 86)

The notion of production of meaning is fundamental because “all production of meaning implies the production of knowledge” (Lins, 1999, p. 87) and, for Lins (2012, p. 12), knowledge consists of “a statement-belief (the subject states something in which he/she believes) together with a justification (what the subject understands as authorising him/her to say what he/she says)”. Justification is not “an explanation for what I say. It is only what the subject of knowledge (the one who produces it, enunciates it) believes that authorises

him/her to say what he/she says” (Lins, 2012, p. 21). We can say that two people who say the same thing produce different knowledge if their justifications differ. A classic example of this is the statement that $2+3=3+2$. A child justifies this by alternating two fingers of one hand with three fingers of another, while the mathematician resorts to the commutative property of the set of natural numbers. The justifications are different; therefore, the child’s and the mathematician’s knowledge production is different, as the objects constituted are different, even though the text $2+3=3+2$ is the same (Lins, 1999).

In the context of the interviews, for example, when we asked the teachers what their maths classes were like, they constituted this object from their production of meanings. However, in other conversations or interviews, they could produce meanings in other ways about how their maths classes are, constituting other objects, producing different knowledge.

The MSF enables fine readings of meaning production processes, called positive and plausible reading. In the positive reading, there is a search for “mapping the terrain while trying to know where the other is” (Lins, 2012, p. 24) and not where one could be or what one lacks. This search can take place through the attempt to share interlocutors, which “is a direction in which one speaks. When I speak to an interlocutor, it is because I believe that this interlocutor would say what I am saying and would accept/adopt the justification that authorises me to say what I am saying” (Lins, 2012, p. 19). According to Lins (2012), positive reading is useful in interaction situations, such as in interviews. Therefore, we chose a semi-structured interview script – approved by the Research Ethics Committee (CEP) of the Federal University of Alfenas (UNIFAL-MG), opinion number 2.815.989. This instrument was essential to let us know the teachers’ location, understand them at that moment, and try to share interlocutors.

Plausible reading can be done when there is no effective interaction; it is what I believe someone else has already said, in other words, that it can be defined as: “every attempt to understand an author must go through the effort to look at the world through the author’s eyes, to use the terms the author uses in a way that makes the whole of their text plausible” (Lins, 1999, p. 93).

After the interviews, we transcribed them, which makes them no longer the teachers’ speeches or our interaction with them. They can be called residues of enunciations, which is what was left of the interview process, i.e., “Something that I come across and I believe someone has said” (Lins, 2012, p. 27), and that demands reading. The reading that we tried to carry out of the transcripts was a plausible reading to get to know those teachers, their formative

and teaching practices and, from there, make considerations about the initial education of pedagogues.

THE INITIAL TEACHING CHALLENGE

To learn about their teaching practice, we asked the respondents, fictitiously called Ana, Maria, Luciana, Patrícia, Vera, and Angélica, about their maths classes and the challenges they met in teaching the subject. We started our analysis with the setbacks and satisfactions because the interviewees eventually talked about the beginning of their careers, as did teachers Patrícia, Angélica, and Luciana, as shown below:

When I started, right, because I've been teaching for two years, [...], I was a little afraid of teaching maths [...], because it is the first experience in the classroom, you do not know it very well, [...], you say: "Wow, will they understand it in the way I am teaching?" But what was difficult, like [silence], was knowing how I was going to get through that. [...] I researched, I talked to other teachers to find out how I could apply that kind of thing [...]. (Excerpt from an interview with teacher Luciana, 2019)

Then when I went to do this [the math writing part] I had to research, I had to learn it for myself first [laughs] and then teach, which is the hardest part, because when you master it, it's super easy [...], but when you don't master it, it's more complicated, [...]. The first time I took a class, and I had to develop the content, I found it really hard, my first practice of how to disclose it to children, right, [...], so it was sharing ideas, even [a teacher] helped me a lot [...]. The difficulty, then, was that, the impact [...]. (Excerpt from an interview with teacher Angélica, 2019)

[...] I think that understanding how I would impart it, because knowing for you is one thing, knowing how to teach a child is another thing, [...]. (Excerpt from an interview with teacher Patrícia, 2019)

The three teachers addressed the difficulty in how to disclose, or pass on, the mathematical content. The expression "to disclose the content" refers to understanding teaching as an act of transmitting, which seems to indicate the educational posture E1 discussed by Lins (1999). From this viewpoint, the

teachers think they know the students; thus, they develop their classes based on these students, whom teachers anticipate, looking for efficient ways to make what they already know happen “naturally”. E2 stance, which we advocate and differs from E1, is the one in which the teachers do not know the students. They must go and talk to the students to know where they are, and create and interact, sharing interlocutors. In this way, instead of passing on the content or transmitting it, the teachers find ways of approaching, for example, the mathematical notions to foster students’ production of knowledge (Lins, 1999).

The difficulty in how to disclose the mathematical content to children may be related to teaching methodologies, which made us question how (or whether) the modules that included mathematics in the teachers’ pedagogy course dealt with aspects related to mathematics teaching methodologies.

PAUSE 1: METHODOLOGIES IN MODULES INVOLVING MATHEMATICS IN INITIAL EDUCATION

Modules involving mathematics are part of the initial education of pedagogues. However, as Curi (2005) and Cunha (2010) pointed out, the few modules offered focus more on mathematics teaching methodologies than mathematical content. For example, Curi (2005, p. 61) stated that “about 90% of pedagogy courses choose methodological issues as essential to the formation of multipurpose teachers”. As a result, pedagogues may be trained without knowing enough mathematics or feeling confident about the knowledge of the mathematical content they will teach. For this research, we wanted to know how those modules occurred to (or with) the respondents, who graduated from two higher education institutions we called University A and College B.

Teacher Luciana, who studied at University A, said that her professor “tried to disclose the subject the best he could” and that the methodologies were more theoretical:

[...] the methodology was worked on, but they talked much about that. How to work with maths, they didn’t tell us. They worked with mathematics in a constructivist way, the scholars, but they said a lot that we had to use games, put more meaningful things, show in practice how it works, [...], we set up some games. He showed this theoretical part. (Excerpt from an interview with teacher Luciana, 2019)

Teacher Vera said that the professor's class was good and interesting, and that it had didactics and methodology, remembering some practical moments:

We had both: methodologies and the didactics, of how to teach. I remember the videos, the teacher teaching, working with golden material, the classes were focused on how to teach, but I also had methodologies, [...] which methodology I would use. I remember the games, I remember there was the teaching of multiplication tables with music [...]. (Excerpt from an interview with teacher Vera, 2019)

Through Vera's words, we noticed that, in her view, didactics is related to how to teach, which seems to us to be the professor's posture in the classroom; and methodologies, the use of games, manipulative material, and everyday practice. Vera's differentiation and Patrícia's relationship between mathematics didactics and how to work/teach, shown below, suggest that using the term "methodologies" as a synonym for how to teach must be revised or problematised in future research.

Teacher Maria spoke about the methodological approaches of the course and recalled using the golden material in practice:

They just disclosed the content and said how you could do it, they didn't do it in practice with us [...], they practically only offered us the theoretical part. We bought a golden material, what I remember from practice was just that. They tell us that we have to work in a constructivist way, but the college itself is still very traditional. [...] there must be more of this practice, manipulation of objects, working more with golden material, working more with the abacus. We only saw the drawing of the abacus and the teacher explained more or less how it was supposed to be worked and that was it. (Excerpt from interview with teacher Maria, 2018)

Maria's speeches make us question the coherence of the educational postures of the educators who teach constructivism —whether Vygotskians or Piagetians— but do not practice it in their classes, placing future pedagogues as spectators, not as the centre of the educational process. Teacher Ana's speeches depict a different attitude from her educator. Ana liked mathematics and remembers a more practical methodology in her classes, being the subject

she liked the most. Her recalling those times may be because of that preference of hers:

She taught us to play math games, explained what her class was like, explained how she did it, what games she used, and what tool she used [...]; it gave us an idea of what it would be like in the classroom. We produced the games, she taught us how to work with them, what goal we wanted to achieve with them, it was pretty cool, every class we ended up playing a little game at the end to use later. [...] she worked more with the methodology. (Excerpt from an interview with teacher Ana, 2019)

We read different speeches from the interviewees who attended University A. Some speeches indicated a more theoretical educator who only showed materials and told them what should be done. Others reported that, in class, the educator proposed the construction of teaching materials and the use of golden material, approaching which objective to achieve with a given material. Probably each respondent saw the performance of their educators differently, based on their ways of seeing mathematics and the course itself, but they might have had different professors. We see the same scenario in the speeches of teachers Angélica and Patrícia, who attended College B.

Patrícia, who has no love for mathematics, said that the educator was inexpressive in her education, remembering some moments more focused on practice with the substitute and the physical education professors:

[...] I met a lot of theorists, a lot of theory, I met a lot of capable people and many books to look for things, but something practical for me to work with was very little. There was no didactics of mathematics, they said that it exists. You can work with games, you can work with it, but it was not built, [...]. They went on with theory. Then, who gave us some ideas on how to work was the [substitute teacher]. She sometimes presented some games, some things that you could understand something, how it worked. I think that as she finds mathematics easy, she explained it to us. And the [physical education teacher], [...], sometimes he would set up some games and in that he would say that you could work on mathematics, the relay game, I learned from him, for Portuguese and mathematics, games with which we could work with balance, weight and measurement, because he did it and he explained it,

and the [substitute professor] also confirmed those things. So, I think it really needs to be shown, [...] having a game class, [...], for the practice to go better, because everything you learn in practice is much better. (Excerpt from an interview with teacher Patrícia, 2019)

Angélica said she had a good professor, reporting how the course was:

[...] a lot of theory, but at the end of the last semester, he started with the games. He worked with mathematics and its mathematical content, and he set an example of how to apply it with the child, it was the theory itself, the importance of mathematics, [...] and the assessments based on theory. He showed some games, and then during the rest of the semester each one had run after it, then we learned a lot, and each one brought something different that we didn't even know existed, an activity, and then we had to do it for the class. (Excerpt from an interview with teacher Angélica, 2019).

From the statements of the interviewees who attended University A and College B, the methodology was central to training. However, the methodology used was more theoretical than practical, leading some teachers to state that although the professors “preach” constructivism, teaching is still very traditional, they still use traditional teaching strategies. Based on the interviews, we considered it important to discuss the epistemological presuppositions that sustain the educational postures of the teachers' educators because they can give the foundations for the pedagogues' postures in their professional practice. Different educational postures such as E1 and E2 may imply using methodologies and teaching materials in different ways.

While in E1 the development of material for the classroom can be defined as engineering and put ahead of the concerns of mathematics education, in E2 one must first start with the construction of a shared communicative space, and the material for the classroom must serve, above all, this purpose (Lins, 1999, p. 86).

An example is the golden material, which can be used only to exercise a list of arithmetic operations in the direction of the E1 assumptions or to discuss how students operate and build operational strategies from a perspective more focused on E2.

CHALLENGES/SATISFACTIONS WITH THE MATHEMATICAL CONTENT IN THE TEACHERS' CLASSES (OR FUTURE CLASSES)

The beginning of our respondents' careers was marked by the difficulty they found in disclosing the content to the students, as we have already discussed. Teacher Luciana's speech reveals her fear of teaching mathematics at this initial phase and that she overcame it through the expression "to understand better". At another point in the interview, teacher Patrícia said she does not feel confident teaching mathematics. We believe that Patrícia's attitude is another mark of this initial teaching period and, also, a mark of the difficulty with mathematics she pointed out. This seems to suggest that the difficulty in teaching mathematics may be related to liking or disliking mathematics, something commonplace in discussions involving pedagogues and mathematics (Zanetti & Julio, 2020). However, teacher Luciana, who prefers mathematics to Portuguese, also reports difficulties in "how to impart" the content.

We believe that teachers' difficulties in teaching mathematics for the early years do not necessarily end there, because they may be requested to teach other stages of schooling throughout their professional years. It is important to emphasize Angélica's and Luciana's speeches about getting help from other teachers, pointing out training as a continuing process that can take place through contact with other teachers, contributing to teaching, insecurities, and fears regarding mathematics teaching.

As for the mathematical content, most teachers said that in early childhood education, they have no hardships because the content is simpler. However, they state that in the early years, they would have problems and would have to research because the content is more complex or because they do not remember it (given that they studied that content back in their school days, not in the pedagogy course). The contents they mentioned were:

[...] percentage, which is a relatively easy content, but I always had problems with it when I was studying, so if I am an elementary school teacher and I have to pass it on to my students, I will find it tough, I will have to remember this content, study, elaborate it all over again. (Excerpt from interview with teacher Maria, 2018)

[...] geometry, it's a challenging part, I don't know if it's a problem of mine, it is from my experience that I have this vision. (Excerpt from an interview with teacher Ana, 2019)

[...] those things there, we work more in elementary school, I found it very hard, not so much to develop problems, but this part of reasoning and geometric shapes, I have a lot of problems passing it on to them [the students]. (Excerpt from an interview with teacher Angélica, 2019)

Teachers Maria and Ana talked about some content they would find difficult to impart if they were to work in the early years of elementary school, because they are related to the difficulties they experienced before. In fact, experiences with mathematics before the pedagogy course can impact the course itself and teaching, as Nacarato and Passeggi (2013), Julio and Silva (2018), and Zanetti and Julio (2020) have pointed out. Something important for the courses to focus on is the ways of producing meaning for mathematics and for teaching mathematics. An exemplary speech related to this discussion is Maria's:

[...] sometimes, it even interferes with what I transfer to the students because then I focus more on reading, on stories, on songs, on spoken language, and I end up leaving numbers aside. I focus more on Portuguese, and I end up leaving math a little aside, even that I see that I sin a little, but because this is my difficulty, whether we like it or not, we end up passing this to the students. (Excerpt from interview with teacher Maria, 2018)

Regarding content, when talking about the reasons for the problems with the division, teacher Patrícia, who works in the early years of elementary school, says that she can do it. However, when she wants to explain it, she seems confused and that something is missing. She attributes this difficulty to the fact that she still has not found a playful path. Patrícia says she finds it easy to work with magnitudes, measures, geometry, and counting because she deals more playfully with that content.

In the beginning, teacher Luciana thought about how to disclose the content, but she has no problems with mathematics. Luciana and Ana are the only teachers to state that they do not find mathematics teaching difficult:

I love maths [...]. I think it's very easy, I find Portuguese more difficult, [...], now in maths I don't have anything like that, that

I think I find hard. (Excerpt from an interview with teacher Ana, 2019)

They reported that division, percentage, reasoning, which we can call logical-mathematical reasoning, and geometry were the most difficult topics for them to teach. In turn, the respondents stated that numbers, counting, magnitudes, and measures were easier. However, we want to draw attention to the following statements: that in early childhood education, the contents are easier; their need to study if they were to work in the early years, and the difficulties they consider related to the contents. This information leads us to look at mathematical education in terms of the mathematical contents studied in the undergraduate course.

PAUSE 2: MATHEMATICAL CONTENT IN THE DEGREE IN PEDAGOGY COURSES

Continuing our reading about the initial education of pedagogues involving mathematics—in the aspect of mathematical content—, teachers Angélica and Patrícia, who studied at College B, said:

[Interviewer: Has your knowledge increased in mathematics?] No, it remains the same. [...] I learned more when I had to help a teacher than in college. (Excerpt from an interview with teacher Angélica, 2019)

[...] I don't even remember the subject, I don't even remember the teacher. [Interviewer: Did you learn more about mathematics in college regarding concepts?] No [silence]; the contents have not changed at all. (Excerpt from an interview with teacher Patrícia, 2019)

At University A, from the speeches of professors Luciana and Ana, we also see that the mathematical contents were not covered:

When I say theory, I'm talking more about scholars. The maths concepts? No, not that. Maths contents, you mean? Not that, because mathematics was not taught, it was only shown that, right, how it will be worked, the history [...]. (Excerpt from an interview with teacher Luciana, 2019)

The contents were not addressed much, it was more the methodology, how to teach, I don't remember the contents; I

don't even remember having maths content at graduation, but maths content, maths subject I don't remember having anything that added to me. (Excerpt from an interview with teacher Ana, 2019)

Teacher Maria said:

So they just delivered the content, and said how you could do it, they didn't do it in practice with us [...]; some content that I don't remember, right, because I studied it when I was in school, later in college I couldn't review it [...]. (Excerpt from interview with teacher Maria, 2018)

The excerpt from Maria's interview draws our attention to some confusion when we talk about content. It was not always seen as mathematical content, which suggests that we can be in different places from Maria, as well as with Vera, in which we do not manage to interact on this issue: "I learned more during college, yes, my knowledge increased. [...] now, I won't be able to tell you exactly, but I believe that my knowledge has increased" (Excerpt from an interview with teacher Vera, 2019).

Regarding the mathematical contents, based on the residues of the interviewees' enunciations, it seems that they were not addressed in the courses, and when they were, they seem to have had no impact on their memories because at no time did they mention anything more specific. At another point, teacher Angélica said that she had to learn first for her and then teach, being an exemplary speech in the sense of the problem of mathematical content in initial education.

Teacher Maria brings up an interesting point: she saw mathematical content in basic education and did not see it again during graduation. Viola dos Santos (2008) and Cunha (2010) want to know where new graduates will see mathematical or other content necessary for teaching that they did not see during their initial teaching education. Teachers Angélica and Patrícia seem to answer this question by stating that they learned mathematics in practice. Thus, we ask whether this learning-in-practice contributed to making them like mathematics, their views on it, and the impact it can have on their teaching.

Even though Lins (2005) discussed modules with mathematical content in mathematics undergraduates' education, he brings elements that can contribute to the discussion about the pedagogues' training in mathematics. According to him,

the teacher needs to know more mathematics, not less. However, I always clarify that this ‘more’ does not mean more content, it means understanding, having more lucidity, and this necessarily includes the understanding that even within the mathematics of the mathematician we produce different meanings for what seems to be the same thing. And I have always defended, too, that we create many of the difficulties our students face, for example, when, early in life, we deny them access to specific ideas. (Lins, 2005, p. 122)

We say that this discussion can contribute to the formation of pedagogues because Lins and Silva (2008), for example, produced material on fractions aimed at the mathematics learning of the teacher of the early years, in which we see concern with the mathematical contents. However, it is not about the content itself but about thinking about the students’ mathematics education and also about what can be called mathematical lucidity:

A greater understanding of how the contents are organised and constitute a module, going through problematisations of those contents that also involve the teaching practice in elementary schools, and considering that, even within this module, different meanings can be produced for what may seem to be the same thing. Mathematical lucidity also involves mathematical confidence—which would be an attitude of not running away from situations involving mathematics or taking them for granted (Viola dos Santos, 2012)—in teaching work. (Julio & Oliveira, 2018, p. 120)

As mathematical lucidity involves readings of productions of meanings, we agree with Santos and Viola dos Santos (2018), who state that the teachers must build repertoires to carry out these readings in formative spaces, in conversations with colleagues, and in the attentive reading of how their students operate in the classroom.

For the construction of this repertoire, Santos and Viola dos Santos (2018) say that it is necessary to think about producing/building it with students, with teachers and students on the same side, expanding their ways of producing meanings and problematising mathematical and non-mathematical meanings—which characterise, according to Lins (2004), the mathematics of the mathematics teacher (MMT):

The central aim is to broaden the scope of meanings acceptable, readable – that is, the centre is in the reading capacity of the teacher, which is directed towards the students –, not to narrow the content – that is, the centre is not in the reproductory capacity of the teacher. And, in didactical terms, we must always bear in mind that the student has the right to know when meaning production by the teacher changes. (Lins, 2004, p. 13)

Completing Lins's (2004) citation, MMT:

has this potential, in the direction of reading the processes of production of mathematical and non-mathematical meanings of students; interacting and intervening in how they operate; explaining differences and different meanings produced by them; thematising and expanding other ways of producing meanings. (Santos & Viola dos Santos, 2018, p. 49)

Based on these discussions, we consider that the education courses for pedagogues, in particular the educators, can try to work in the direction of what has been characterised as mathematical lucidity and MMT, although we consider it important to carry out further research based on the MSF, which discusses mathematics (street mathematics, school mathematics, mathematics of the mathematicians, and MMT, among others) in the initial education of pedagogues.

TALKING AGAIN ABOUT TEACHERS' MATHEMATICS CLASSES AND INITIAL EDUCATION

So far, we have discussed our respondents' statements about what they found to be difficult and what they found to be easy when they started teaching mathematics. We have also discussed some aspects of the mathematical content in their teaching practices (or future practices), which allowed us to problematise their initial education processes in the scope of the modules that included mathematics.

Finally, we want to discuss how the teachers talked about their mathematics teaching, bringing what we consider to be traces of initial education and emphasising some positions we outlined throughout this article.

Ana's and Vera's speeches about classroom routines were similar, as both works with children aged 0 to 3. About mathematics teaching, Ana said that she works with:

countings, songs that have countings, day of the week, how many students came, [...], because they still don't have an idea of the number [...], they count along with me, [...] material type of mathematics [...], it is that in this age group it is hard, I do not work specifically with mathematics, because they are very small. There are the geometric shapes that we work with, but it really is a brushstroke, very playful, very playful, relating objects to everyday things, geometric shapes to things in the room or something I take to them. I use matching games, jigsaw puzzles, [...] some stamps that I take in the format and we cut out, they play stamping. (Excerpt from an interview with teacher Ana, 2019).

Regarding games, this teacher said that the game does not come first, it depends on how students think and what didactic goals they want to achieve. This trace may have come from initial education, as Ana mentions that she had an educator who taught games, how to use them, and the goal one wanted to achieve. Teacher Ana's speeches about her and her professor's practice seem to be closer to the E2 posture (Lins, 1999), in which the reading of the students is essential for the educational process because, from this, the teacher can decide what to do, what methodology to use. He/she can go to where the student is and talk to him/her, so that together they can go to other places, making the students' modes of production of meanings legitimate, at the same time as the modes of production of meanings of teachers may also become legitimate for students.

Teacher Vera talks about mathematics in her teaching as follows:

[...] in a story, playing, in a game, there is mathematics, at various times, mathematics is very present in all the things we do. [silence]. So, mathematics ends up staying, but today we see that it is very present, we use it a lot, sometimes it can be in a kind of intrinsic, kind of hidden way. Many things we believe contribute, so it is present a lot in early childhood education. [...] Mathematics we work every day, because [...] the numbers on the wall, we always count how many children there are in the room and there is the calendar. (Excerpt from an interview with teacher Vera, 2019)

At first, this teacher had problems speaking more specifically about her teaching, saying she could not answer. Then, she said that she made some materials to work with mathematics, citing as an example one that the child had to associate the number of caps with a number.

Teachers Luciana, Maria, and Angélica work in early childhood education (Phase 1 or 2, with children aged 4 or 5, respectively). They said they have a daily routine that involves maths. The first teacher stated that she worked with:

quantity, counting [...], starting to insert sum, right, and in the case of subtraction. I did maths every day, at least one counting; when it wasn't the sheet, it was the handout. (Excerpt from an interview with teacher Luciana, 2019).

When asked about the materials she uses in her classes, Luciana said that she uses toothpicks, balls, and bottle caps to work on counting and summing in a more practical way. She also uses the handout adopted by the municipality, games, and recipes. Teacher Luciana mentioned that she even showed the golden material to students, but she thinks it a bit complicated. She also said that she tries to be more constructivist in her classes, “because I believe in constructivism, in parts, right, as there are positive things in traditionalism, there are negative things in constructivism”.

In our reading, pointing out positive things in the traditional approach to teaching may be related to their good teachers in basic education who used this approach. Even if it has negative aspects, their adherence to constructivism seems to be a trace of the education, when their educators encouraged or worked on mathematics in this trend. This teacher seeks to show in practice how some things work, which may be inspired by some of her basic education and higher education teachers.

Regarding mathematics in the practice of teacher Angélica, she was working on quantity, body exploration, and counting (of the body, toys, and elements of nature). To carry out this work, she seeks to:

associate everything, not just make this illusory world, associate it with the child's practice. [...], the blocks also help a lot, then we also do the separation of colours, quantity [...], and hand activities. [...] the first time they have contact with a number, I make it on the chalk floor, and they go over it, I make it bigger, to also explore the body issue [...], and then move on to the paper sheet. I used to use the numeral games [...], related to the identification of birthdays. These things, then the child sees that mathematics is in everything. (Excerpt from an interview with teacher Angélica, 2019)

Maria said she works with the calendar, counting the days and saying they are numbers, and then counting slowly. She said that she tries to use what she has in the classroom and only uses paper for spelling the numbers:

[...] balls, toys, backpacks; [...] I use the numerals that I have on the wall along with the calendar. [...] the older and younger children, we compare the students, making the growth ruler; [...] I took a pot of coloured chalk and we put it on the table and distributed it, first, second, third, fourth, fifth. (Excerpt from interview with teacher Maria, 2018)

Teacher Patrícia, who was teaching in the early years, said she works Portuguese and mathematics every day, and that in geography, for example, there is mathematics. This teacher walks in the direction of what we pointed out about the production of meanings for mathematics and, also, about mathematical lucidity. A specific school maths class is not required to address maths. In a geography class, it can be approached as well. Being more specific about working with mathematics, Patricia said that she worked:

[...] numbers up to 100, the counting, right, addition, subtraction, and the beginning of multiplication, the beginning of division. So, I try to make them think about how they can solve problems and set up the countings, showing how to do it, then after I do that, I move on to practice, the simplest part, so I work with the problems for them to think and each one finds their solution. I work a lot with textbooks, plays, games, little problems. I take the measuring tape, then we play, and we measure; the school games, from the PNAIC, I make some, for example, because they are simple things; the golden material, the never ten, for them to have an idea to work the decimal, the question of base 10; there is a game with a hula hoop that I like to play with them to add up, a card game, which works, playing blackjack. The relay game also works very well [...] then we set up two groups, and each one has a counting list. (Excerpt from an interview with teacher Patricia, 2019)

Counting appeared in every interview, making it seem to us that working maths in kindergarten and early years means working with numbers. At no time in the interviews was there any talk about numbers and operations in the initial education. Incidentally, no specific content was mentioned.

Other contents emerged, such as: greater or lesser, geometric shapes, and the four operations. This makes us question how pedagogues see mathematics in early childhood education and in the early years and how initial education can expand the modes of production of meanings for it, with MMT being a possibility.

The teachers use everyday and classroom things, showing us the richness of the school space for approaches involving mathematics. Working with recipes was also mentioned. Other methodological resources such as music and storytelling emerged. Some of them were mentioned in the interviews when the pedagogues talked about the methodologies used in the subjects involving mathematics in initial education. Patrícia, for example, mentioned the relay game, which we see as a trace of this initial education.

The golden material was mentioned in two ways in the interviews: with teacher Luciana saying that it is somewhat complicated, and she only showed it; teacher Patrícia also mentioned it. They mentioned blocks, perhaps the logical blocks by their relation to colours. Except for teacher Maria, all emphasised that they use games, as they are part of childhood and with the potential to articulate imagination and imitation of reality.

Lins (1999, p. 84) discussed the uses of teaching materials in the two epistemological conceptions we approached, E1 and E2, as already seen. Except for teacher Ana, in the other interviews, it was impossible to explain the teachers' epistemological postures, which could help us to read how those materials are used in their classes, even though Luciana said that she tries to be more constructivist. We emphasised that the degree in pedagogy courses can also work in this direction of explaining epistemological postures.

CONCLUDING REMARKS

In this article, we read, based on the assumptions of the MSF, the transcripts of semi-structured interviews carried out with beginning teachers of early childhood education and the initial years of elementary school in the municipal education network of São José do Rio Pardo, São Paulo, about initial teaching in mathematics. The teachers stated that their most difficulty at the beginning of their career was “how to disclose” the content. However, they also affirmed that the mathematics content was simpler for early childhood education, therefore, it was not challenging for them. Moreover, counting was remarkable in the teachers' classes and the use of different day-to-day resources for working with mathematics at school. In this process, we interspersed our

readings on their initial education in subjects involving mathematics in terms of content and methodologies, because the discussions on this theme emphasise the prevalence of methodologies in the degree in pedagogy courses and the intention of establishing some relationships with teaching practice.

Based on those readings, we problematised both teaching practice and initial education – such as the importance of explaining epistemological assumptions that underlie the teachers' educational posture and their educators – , we addressed the need for greater clarification of terms such as didactics, methodology, content, and mathematical content, which seem consensual but can allow different productions of meanings. Besides, we question how teachers see mathematics, as they focus their teaching practices on counting.

Finally, we want to point out that the discussions by Lins (1999, 2004, 2005) on epistemological assumptions, mathematical lucidity, mathematics of the mathematician, and mathematics of the mathematics teachers can be a way to think about the modules that involve mathematics in the initial education of pedagogues even if the time allotted to them is short.

AUTHORSHIP CONTRIBUTION STATEMENT

RSJ and MZ conceived the research and participated collaboratively in the construction of this article.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the corresponding author, RSJ, upon reasonable request.

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