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Statistical Reasoning Process of Students in Decision Making Using Commognitive Framework

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ABSTRACT

Background: Statistical reasoning plays an important role in decision making, as the latter requires the individual to be able to connect and relate various aspects and ideas. **Objectives**: This study aimed to describe the student's statistical reasoning process using a commognitive framework based on the task of making decisions. **Design**: This study used a qualitative approach with a descriptive exploratory design. **Setting and Participants**: Respondents in this study were students majoring in mathematics in the fourth and sixth semesters. There were 58 students involved; three of them were research subjects. **Data collection and analysis**: The authors gave students a task containing a question. The question had to do with choosing one out of six employees who were most successful at their jobs. **Results**: The findings in this study indicate two main aspects when carrying out the statistical reasoning process using a commognitive framework in making decisions: 1) the visual mediator created when representing data is the foundation for analysing and interpreting data. Visual mediators that were irrelevant to data had an impact on the emergence of ritualised routine, resulting in an illogical decision; 2) substantiations narrative, in the form of involving various aspects of decision making, and memorisation narrative, in the form of rules related to mathematical or statistical concepts in decision making, were tools to minimise the risks arising from the decisions made. **Conclusions**: Commognitive framework could be used as a tool when students carried out statistical reasoning process in making complex decisions. It was important to make a visual mediator matching the data display when students represented data.

Keywords: Statistical reasoning; Commognitive; Decision making.

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Processo de Raciocínio Estatístico de Alunos na Tomada de Decisão Usando Estrutura Commognitiva

RESUMO

Contexto: O raciocínio estatístico desempenha um papel importante na tomada de decisão. Isso ocorre porque a tomada de decisão requer a capacidade de conectar e relacionar vários aspectos e ideias. **Objetivos**: Este estudo teve como objetivo descrever o processo de raciocínio estatístico do aluno a partir de um referencial comognitivo baseado na tarefa de tomar decisões. **Desenho**: Este estudo utilizou uma abordagem qualitativa com um desenho exploratório descritivo. **Cenário e participantes**: Os entrevistados neste estudo eram alunos com especialização em matemática no quarto e no sexto semestre. Participaram 58 alunos, sendo três sujeitos da pesquisa. **Coleta e análise de dados**: Os autores deram uma tarefa contendo uma pergunta aos alunos. A questão tinha a ver com a decisão de escolher um dos seis funcionários mais bem-sucedidos em seu trabalho. **Resultados**: Os achados deste estudo indicam que houve dois aspectos principais na realização do processo de raciocínio estatístico usando uma estrutura comognitiva na tomada de decisões: 1) o mediador visual criado ao representar os dados é a base para a análise e interpretação dos dados. Os mediadores visuais que eram irrelevantes para a exibição de dados tiveram um efeito no surgimento da rotina ritualizada, resultando em decisão ilógica, 2) narrativa de fundamentação, na forma de envolver vários aspectos da tomada de decisão e narrativa de memorização na forma de regras relacionadas à matemática ou conceitos estatísticos na tomada de decisão, foi uma ferramenta para minimizar os riscos decorrentes das decisões tomadas. **Conclusões**: o quadro comognitivo pode ser usado como ferramenta quando os alunos realizam um processo de raciocínio estatístico na tomada de decisões complexas. Era importante criar um mediador visual que correspondesse à exibição de dados quando os alunos representassem os dados.

Palavras-chave: Raciocínio estatístico; Comognitivo; Tomando uma decisão

INTRODUCTION

Statistical reasoning plays an important role in decision making (Ulusoy & Altay, 2017). A wrong decision can be caused by a lack of knowledge or an inability to relate some information linked to decision making. Reasoning about some statistical information well means that the person can use rules and various statistical concepts to make conclusions (Lovett, 2001), connect one concept to another, and combine various ideas about data (Kalobo, 2016). Statistical reasoning can be developed through real data completion (Jones et al., 2004; Garfield & Ben-Zvi, 2009; Neumann et al., 2013). Jones et al. (2004) designed a task related to concert performance with several artists, where students had to choose one of the most successful artists in the performance. Garfield & Ben-Zvi (2009) state that using real-life data is one of

the statistical reasoning learning environment models that can improve students' statistical reasoning. Neumann et al. (2013) conducted research by asking students questions about the use of real-life data during lectures in classrooms. The results showed that cognitive and affective factors were closely related to using real-life data in statistics learning. Completing real data made it easier for students to reason because what they thought followed facts and realities found in the real world.

Many theorists argue that reasoning also plays a critical role in decision making(e.g., Evans et al., 1993; Johnson-Laird & Shafir, 1993; Moore, 2010), as the latter involves cognitive processes (Wang & Ruhe, 2007). Moore (2010) says that decision making skill depends on reasoning. Furthermore, a person makes decisions based on the information on which their reasoning is founded (Johnson-Laird & Shafir, 1993). Reasoning in the real world supports a person in making decisions (Evans et al., 1993). Meanwhile, decision making is a mental process involving all humans throughout their lives, and decision making is a problem-solving process that ends when the desired solution has been reached (Shahsavarani & Abadi, 2015). Furthermore, Evans et al. (1993) state that reasoning can support decision making and is aimed at achieving goals. Therefore, reasoning and decision making are high-level skills (Johnson-Laird & Shafir, 1993). In this connection, statistical reasoning also plays an important role in decision making (Ulusoy & Altay, 2017).

Statistical reasoning is not only used in statistical topics, but it can also be used in non-statistical topics, such as topics in mathematical problem solving (Hidayanto & Rahmatina, 2020) and topics in biology (Deane et al., 2016; Fiedler et al., 2019). Hidayanto & Rahmatina (2020) investigated the characteristics of students' statistical reasoning, in which one of the characteristics of statistical reasoning was a relation. In this regard, students could connect several mathematical concepts, including linking the concepts of volume, space, numbers, and one-variable linear equation. Deane et al. (2016) examined statistical reasoning in inventories, and the results showed that in an inventory of biological concepts statistical reasoning was an effective tool for assessing students' conceptual abilities. Meanwhile, Fiedler et al. (2019) examined the relationship between statistical reasoning, evolutionary knowledge, and evolutionary acceptance. Their research findings indicated a significant relationship between statistical reasoning and the understanding and acceptance of major biological ideas such as the ideas of natural selection and macroevolution. That study shows that statistical reasoning does not only include statistical contexts, but it can also involve non-statistical contexts and activities.

In statistical reasoning, a person is expected to be able to explain statistical processes (Kalobo, 2016). The ability to explain statistical processes is closely related to the ability to communicate. The delivery of ideas from the results of thoughts is accompanied by this ability. Difficulty in explaining ideas is due to low communication skills, both in the form of interpersonal and intrapersonal communication (Sfard, 2007), so it may result in difficulties in making decisions. In addition, communication skills include not only listening and speaking, but also reading and writing (Khan et al., 2017). Thus, communication skill is closely related to statistical reasoning in decision making.

Several studies examining students' statistical reasoning in decision making have been conducted by Holt & Scariano (2009) and Frischemeier (2019). Holt & Scariano (2009) investigated students' activities in exploring the mean, median, and mode of decision making. The students explored the reasons for choosing a suitable concentration size in decision making. In this case, the reasons expressed by the students were a form of endorsed narrative to strengthen their arguments in making decisions. Narrative is an expression in the form of a spoken or written text to describe the relationship among objects (Sfard, 2008). Meanwhile, the term "endorsed narrative" indicates that the narrative expressed is true or false (Sfard, 2008). Frischemeier (2019) introduced a framework for a video study to evaluate students' statistical reasoning processes and their proficiency in using the TinkerPlots software in comparing several groups. The graphic appearing in the TinkerPlot software was a visual form of the mediator to communicate the relationship among groups. Visual mediator is one of the commognitive frameworks (Sfard, 2008). The series of activities carried out by these students in comparing several groups was a form of "routine", which is a procedure or regulation in discourse (Sfard, 2008).

Thus, the research conducted by Holt & Scariano (2009) and Frischemeier (2019) allowed the use of commognitive in the student's statistical reasoning process. However, these researchers did not delve deeper into how the statistical reasoning process was in making decisions based on commognitive theory. In fact, commognitive terms used were the entire fabric of human development and a lens for research (Presmeg, 2016). Furthermore, Zayyadi et al. (2019) state that a commognitive framework can help students solve mathematical problems. In this case, the commognitive framework in the form of narrative plays an important role in reasoning, problem solving, and building knowledge (Saletta et al., 2020). Rahmatina et al. (2020) found that

narrative consistency in statistical reasoning played a crucial role in making the right decision.

On the other hand, narrative inconsistency when carrying out the statistical reasoning process can cause errors in decision making (Rahmatina et al., 2020). In this case, A narrative is generated from word use and visual mediator, while routine is a regulation in word use, visual mediator, and narrative (Sfard, 2008). This shows that there is a relationship between commognitive and statistical reasoning. Thus, the commognitive framework, in the form of word use, visual mediator, narrative, and routine, is thought to also play an important role in statistical reasoning.

In this connection, commognitive as a lens in reasoning research has been carried out by previous researchers, such as research on mathematical reasoning (Jeannotte & Kieran, 2017; Rabin et al., 2013), geometric reasoning (Toscano et al., 2019; Wang, 2016; Wang & Kinzel, 2014), and statistical reasoning (Lampen, 2015; Park & Lee, 2014; Rahmatina et al.,2020). This shows that commognitive reasoning has become a trend in reasoning research. However, not many researchers have studied how the commognitive framework has been used as a tool in the statistical reasoning process. Lampen (2015) and Park & Lee (2014) investigate the use of commognitive framework in statistical reasoning. Lampen's research (2015) focused on analysing Sfard's commognitive theory (2008) on teachers' narrative in making sense of the average algorithm as a discourse basis of variability measure. His research indicates that the mean is fundamental to determining the measure of variability. Park & Lee (2014) used Sfard's (2008) commognitive framework to investigate how teachers implemented formative assessments that could be applied in statistical education and explored how the phase changed in students' discourse statistics. His research indicated that there were two impacts on the teacher's attention to students' discourse statistics, namely that they could increase students' discourse statistics and implement formative assessments through teacher and student interactions. Meanwhile, the research focus of Rahmatina et al. (2020) was to examine students' statistical reasoning from a narrative perspective regarding variability in the process of analysing and interpreting data. Thus, research conducted by Lampen (2015) and Park & Lee (2014) investigated how to analyse data to get a conclusion, but they did not examine in more detail how the statistical reasoning process was before reaching the conclusions. Unlike the research case of Rahmatina et al. (2020), they studied the process of statistical reasoning from a commognitive perspective, but the use of the commognitive framework they studied was limited to the use of narrative in data analysis and interpretation.

For this reason, the authors needed to further investigate the statistical reasoning process using a commognitive framework in decision making. Commognitive is a combination of cognition and communication (Sfard, 2008). Sfard (2008) states that the commognitive framework comprises four components: word use, visual mediator, narrative, and routine.

Word use includes the types of words used in discourse (Sfard, 2008). Another term for word use is keywords. The term keywords in Sfard (2008) is a word that denotes numbers, variables, and functions. Furthermore, Fernández-león and Gavilán-Izquierdo (2019) state that word use involves using mathematical terms (such as "topology", "polygon", "prism") and colloquial words that have special meanings in mathematics (such as "limit", "open", "continuous", and "group"). There are also colloquial words with special meanings in statistical discourses (such as the words "error", "centre", and "estimate"), in which the words "error", "centre", and "estimate" used in everyday life have different meanings when used in statistical discourse. Mpofu & Pournara (2018) categorise everyday words as colloquial word use, where colloquial word use is a combination of mathematical and non-mathematical language. Meanwhile, all words used in mathematics are called literate word use (Mpofu & Pournara, 2018). Thus, word use literate in statistical discourse is all words or terms used in the context of statistics, such as the words population, sample, and variable.

The visual mediator is a visible object to communicate relationships and operations with mathematical objects (Roberts & le Roux, 2019). The visual mediator can be in the form of numerals, algebraic symbols, and graphs (Sfard, 2020) and diagrams (Pratiwi et al., 2020), in which numbers, algebraic formulas, algebraic notations, graphs, pictures, and diagrams are examples of visual mediators that are most widely used in mathematics (Sfard, 2007; 2008). Furthermore, visual mediators can be classified into two categories, namely iconic visual mediator and symbolic visual mediator. In this case, the iconic visual mediator contains graphics and tables, while the symbolic visual mediator contains equations/formulas (Mpofu & Pournara, 2018). For example, the symbol $\bar{x}$ is a symbolic visual mediator for viewing the mean of the sample.

Narrative is a spoken or written text, which is formed as a description of the object, or the relationship between objects (Sfard, 2008). Furthermore, Mpofu & Pournara (2018) classify narrative into two parts, namely substantiations and memorisation narrative. Substantiation narrative is a justification and reason for a certain action, while memorisation narrative involves formulas/rules (Mpofu & Pournara, 2018). For example, A

substantiation narrative is in the form of reasons used by students to create a system of linear equations based on the features in the table, and Amemorisation narrative is in the form of using rules related to average scores in making decisions.

Meanwhile, routine is a regulation in the use of word use, visual mediator, and narrative (Sfard, 2008). Nardi et al. (2014) state that routine includes well-defined and regularly applied practices used clearly and specifically by the community (e.g., defining, proving, conjecturing, estimating, generalising, and abstracting).

Furthermore, Mpofu & Pournara (2018) classify routine into two parts, routine based on kinds and routines based on properties. Routine based on the kinds is classified into ritualised routine and exploratory routine. Ritualised routine occurs when students can perform the necessary procedures but cannot justify the answers obtained. Routine is categorised as exploratory when it is used to verify endorsed narrative. The exploratory routine occurs when it is used to verify endorsed narrative guiding students to use specific steps or procedures; for example, the steps used by students to select elements in the table to produce a Linear Equation System. Roberts & le Roux (2019) state that sequential steps to solve an equation are called a realisation routine. Furthermore, properties-based routine is classified into three categories: applicability, flexibility, and reliability routines. In this case, the applicability routine is when some specific routine procedures may be generated. For example, students complete the Linear Equation System to be able to generate value for each variable.

The four commognitive frameworks are tools in discourse (Sfard, 2020). In turn, discourse can occur when someone is given a problem and tries to solve it (Zayyadi et al., 2019). Thus, to solve statistical and mathematical discourses, word use, visual mediator, narrative, and routine are relevant to the problem context in the discourses. This study aimed to describe the process of students' statistical reasoning in decision making using a commognitive framework. To achieve this goal, we assigned students tasks with data presented in tabular form. The table in this task contained a variety of complex information related to real-world data, so that it required students to do a statistical reasoning process. Statistical reasoning, in this study, is the students' ability to connect several concepts and ideas (Kalobo, 2016; Hidayanto & Rahmatina, 2020), use statistical concepts (Lovett, 2001) and mathematics, understand and be able to explain statistical processes and interpret statistical

results (Garfield, 2002) for decision making. The commognitive framework was used as a lens to describe the student's statistical reasoning.

There are four statistical reasoning processes according to Jones et al. (2004): describing data, organising data, representing data, and analysing, and interpreting data. However, in this study, we focused on describing the two processes of statistical reasoning, i.e., representing data, and analysing and interpreting data. In this case, representing data played an important role in analysing and interpreting data (Jones et al., 2004). In this case, analysing and interpreting data are the core processes in statistical reasoning (Jones et al., 2004). In this context, representing the data in this study is the way students construct the table in the questions into another form of data display. Meanwhile, analysing and interpreting data are how students think rationally when they must decide on one of the employees who are successful in the work.

The commognitive framework used in this study referred to Sfard's (2008) commognitive framework, which consists of Word Use, Visual Mediator, Narrative, and Routine. The classification of those terms in this study refers to the classification of the commognitive components of Mpofu & Pournara (2018). We focus on describing how students use a commognitive framework, which consists of: 1) literate word use and colloquial word use, 2) iconic visual mediator and symbolic visual mediator, 3) substantiation narrative and memorisation narrative, 4) ritualised routine, exploratory routine, and applicability routine when representing, analysing, and interpreting data.

METHODOLOGY

Research Participants and Subjects

This study involved 58 students (male: 15; female: 43; age: 19-22 years old) in two classes, that is, 29 fourth-semester students (male: 9; female: 20; age: 19-21 years old), and 29 sixth-semester students (male: 6; female: 23; age: 20-22 years old) majoring in mathematics at the State University of Malang, Indonesia. Three out of 58 students were the research subjects. The processes of obtaining research subjects were carried out in two stages. In the first stage, the authors selected students who could carry out statistical reasoning processes based on the results of written answers. Of the 58 students, eight could do statistical reasoning processes. In the second stage, the authors selected three out of eight students to be interviewed. The three students were used as subjects in this study. The reason for choosing these three subjects was the similarities in their written answers. In this case, there were three similar forms of written answers in completing tasks. First, there are the answers given by subjects S1,

S4 and S5. Second, the answers by subjects S2 and S5. Third, the answers by the subjects S3, S7 and S8. Therefore, the authors grouped the subject's answers into three groups, as follows:

Group I	: S1	S4	S5
Group II	: S2	S5	
Group III	: S3	S7	S8

For this reason, the authors chose three subjects who could represent each group to investigate how the students' statistical reasoning process used a commognitive framework. The three subjects were named S1, S2, and S3. The reason for taking the three subjects was that they could do statistical reasoning using a commognitive framework.

Instruments

There were two instruments used in this study, specifically, a written assignment and an interview guide. The task involved solving a problem by making a decision (Figure 1). The question on this assignment was inspired by research conducted by Jones et al. (2004), in which Jones et al. (2004) gave students the task of choosing the artist who was the most successful in his concert. While the interview guidelines contained semi-structured questions, in which the subject was given the freedom to express his thoughts in carrying out a statistical reasoning process based on a list of interview questions.

Method of Collecting Data

The data in this study were collected through written assignments and interview results. All students completed the assignments individually in the classroom. The first author supervised directly while fourth-semester students worked on the assignments on January 29th, 2020 and sixth-semester students on February 4th, 2020. After the students completed the task, the authors reduced the data by selecting answers of students who did a statistical reasoning process for further analysis. Several days after that, the first author asked the students whether they were willing to be interviewed. Interviews were conducted to find out more about the use of a commognitive framework in the statistical reasoning processes. The first authors and the students agreed upon the time and place of the meeting. Interviews were conducted for 30-60 minutes using audio recordings on a mobile phone.

Figure 1

Written task in this study

Two Taylor (Queen Taylor and King Taylor) each have three employees. The employees at Ratu Taylor are Amin, Awis, and Agus, while the employees at King Taylor are Budi, Bayu, and Tono. Each employee works separately to complete the job. The length of time completing the stitches, the number of clothes sewn, and the wages received by each employee are different from one another, as shown in the employee duos in the table below.

Sewing Service	Employee	Finishing Time	Quantity of Clothing	Salary (IDR)
Queen Taylor	Amin and Awis	8 days	31	1,285,000
	Amin and Agus	15 days	42	1,625,000
	Awis and Agus	19 days	51	2,140,000
King Taylor	Budi and Bayu	15 days	52	1,908,000
	Budi and Tono	8 days	33	1,372,000
	Bayu and Tono	13 days	45	1,660,000

From those six employees, which one is the most success in his or her work? Give the reasons.

Data Analysis

This study used a descriptive exploratory qualitative method. In this case, the authors described and interpreted the student's statistical reasoning process using a commognitive framework. Before the data were analysed, the authors tested the validity of the data through triangulation. The authors tested the suitability of the information provided by the subject, from written answers to the results of interviews. The data analysis processes were carried out in four stages, namely 1) describing the student's statistical reasoning in representing data and analysing and interpreting data from written answers, 2) analysing interview transcripts, 3) presenting data, and 4) making conclusions. Indicators of the statistical reasoning process using a commognitive framework in this study are shown in Table 1.

Table 1

Commognitive framework used in statistical reasoning in this study

Commognitive Framework	Classification	Description
Word Use	Colloquial	Using colloquial words that have mathematical or statistical meanings when changing data displays into other forms, and when making decision (such as using bigger, smaller words)
	Literate	Using terms that have special mathematical or statistical meanings when changing data displays into other forms and when making decision (such as Linear Equation System)
Visual Mediator	Iconic	Creating a table of values based on data displays.
	Symbolic	Generating equations from data displays (e.g. generating linear equation system based on table values)
Narrative	Substantiation	Justifying and clarifying why the data displays can be converted into other forms (for example, explaining that the table can be converted into a linear equation system because...)
		Giving reasons for making decisions (for example, explaining that one employee is more successful than other employees because...)
	Memorisation	Using related rules in changing the display data into another form (such as using rules related to the linear equation system)
		Use a formula to make decision

Routine	Ritualised	Creating procedures or steps to complete other forms of data display correctly but cannot justify the results of the solution properly (for example, can solve the Linear Equation System but cannot justify the results obtained)
	Exploratory	Selecting values in the data display to create another data display (for example, selecting values in a table to create a linear equation system)
		Selecting values on the results of solving an equation to make decisions (for example, choosing values or variables on the results of solving a Linear Equation System to make decisions)
	Applicability	Solving an equation to determine the value of each variable (for example, solving a linear equation system)

RESULTS AND ANALYSIS

Based on the analysis of the written answers and the results of interviews, it shows that all subjects had made statistical reasoning in making decisions using commognition. In the interview, the first author asked the subject whether they understood the statements and questions in the assignment. Based on the results, all subjects had the same perception in describing information on tasks, both information on the tasks statements and on data displayed in tables. This was done so that the authors believed that the subject could understand the meaning of the question on the assignment. Then the interview was continued to investigate further the use of the commognitive framework in statistical reasoning.

Before choosing an employee who was successful in the work, all subjects needed to represent data from a table formed into a Linear Equation System. Word use used by the subject included Linear Equation and Linear Equation System. The Word used was a literate word use, because these words have a mathematical meaning. In this case, the Linear Equation System was

interpreted as an equation containing the multiplication of constants with their variables and a combination of several linear equations, such as the interview transcript below.

> *S1: Linear Equation System is a combination of two or more equations that have terms, constants and single variables.*

The subjects used a visual mediator by making the form of a Linear Equation, the Linear Equation System. The Linear Equation made by subject S1 was the same as by subject S2; namely, they made a Linear Equation by classifying time, the number of clothes, and the salary for each employee pair in Queen Taylor and King Taylor. For example, the number of clothes sewn by the employees Amin and Awis at Queen Taylor was symbolised by $a + b = 31$. In contrast to subject S3, she made a Linear Equation by grouping the work results of employees based on the number of clothes per day and the salary per clothing per day. The subject determined the number of clothes per day that each employee sewed by dividing the number of clothes over time. For example, the number of clothes per day sewn by Queen Taylor's employees Amin and Awis was symbolised by $x + y = \frac{31}{8} = 3{,}9$. As for the salary per clothing per day, subject S3 made a Linear Equation with the formula $\frac{\text{Quantity of clothes}}{\text{number of days}} x \frac{\text{salary}}{\text{number of clothes}}$. For example, the salary per day for Amin and Awis at Queen Taylor was $\left(\frac{31}{8} x \frac{1{,}285{,}000\ \text{IDR}}{31} = Rp.161{,}658\right)$. Thus, the Linear Equation for per day salary was denoted by $x + y = 161{,}658$. Based on the Linear Equation, the subject created a visual form of the mediator in the form of a Linear Equation System, as shown in Figure 2 and Figure 3.

Figure 2

The answer of S1 representing the quantity of sewn clothes

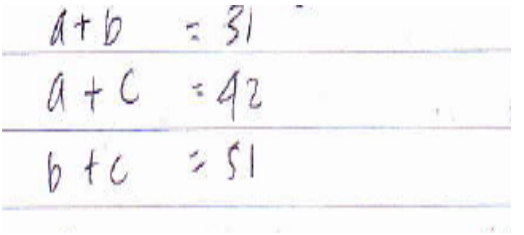
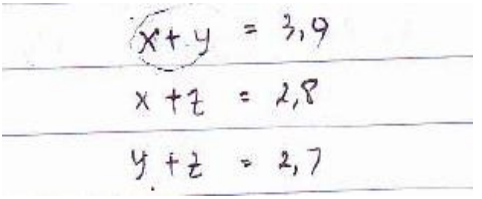


Figure 2 shows the visual mediator in the form of a Linear Equation System made by S1 to represent the number of clothes sewn by the employees at Queen Taylor. The subject represented the number of clothes sewn according to the working time of each pair of employees. For example, the number of clothes that Amin and Awis sewed was 31 clothes in eight days.

Figure 3

The answer of S3 representing the quantity of sewn clothes



Meanwhile, Figure 3 shows the visual mediator in the form of a Linear Equation System to represent the number of clothes per day sewn by employee pairs at Queen Taylor. S3 thought that the number of clothes sewn by the employee pair was counted per day. For example, the number of clothes that Amin and Awis sewed was 3.9 clothes in one day. The Linear Equation System created by all subjects was a symbolic visual mediator because they expressed the elements and values in the table in the form of a system of linear equation. S1 and S2 produced a system of linear equations according to the values in the table. However, S3 did not directly use the values in the table, but she modified the values in the table and then expressed them in the form of a linear equation system.

The narrative form appeared when the subject revealed the reasons for using the Linear Equation System. The subject revealed that the linear equation system was used to determine the work result of each employee. To solve the linear equation system, the subject used elimination and substitution methods. Thus, the subject could determine which employee was the most successful at their job.

The narratives expressed by the subjects were substantiation narrative because there were reasons used by the subjects related to the use of the linear equation system. In this context, the reason why they created a linear equation system was because the solution of the linear equation system could determine the value of each variable. In addition, the subjects also used memorisation

narratives because the subjects solved linear equation system according to the existing rules in the linear equation system, including making linear equation containing variables and constants, combining several linear equations so that they became a linear equation system, using substitution and elimination methods to solve it. S3 also conducted a memorisation narrative in the form of determining the formula to obtain the number of clothes per day and the wages per dress per day for the pair of employees. For example, S3 made a formula for the number of clothes per day for each pair of employees in the form of $\frac{\text{Quantity of clothes}}{\text{the number of days}}$, while the formula for wages for pair of employees per cloth in one day was $\frac{\text{Quantity of clothes}}{\text{number of days}} x \frac{\text{salary}}{\text{number of clothes}}$". For example, the number of clothes per day for Budi and Tono was obtained from $\frac{33}{8} = 4{,}1$, and the wage for Budi and Tono was obtained from $\frac{1{,}372{,}000}{33} = 41{,}575$. Thus, the wage per cloth in one day that Budi and Tono received was $4{,}1\ x\ 41{,}575 = IDR\ 170{,}475$.

Finally, the subject carried out a routine in the form of regularity in using word use, visual mediator and narrative. S1 and S2 performed the same routine when representing data when they constructed tables into a linear equation system. There were three forms of the linear equation system they made for each tailor. In this case, the three linear equation systems were relevant to many variables in the table: sewing time, sewing number of clothes, and wages. They selected the values in the table to be substituted into the linear equation system. The routine carried out by S1 and S2 was an exploratory routine because the two subjects could select relevant values and use the main elements in the table to be made into a Linear Equation System. Meanwhile, S3 carried out a ritualised routine because she could create and complete a linear equation system, but the form of the linear equation system she made did not contain the values in the table. However, S3 modified the variables and values in the table to be expressed in the form of a linear equation system. For example, the variable number of clothes was changed to the form of the number of clothes per day, and the wage was converted into the form of wage per cloth per day.

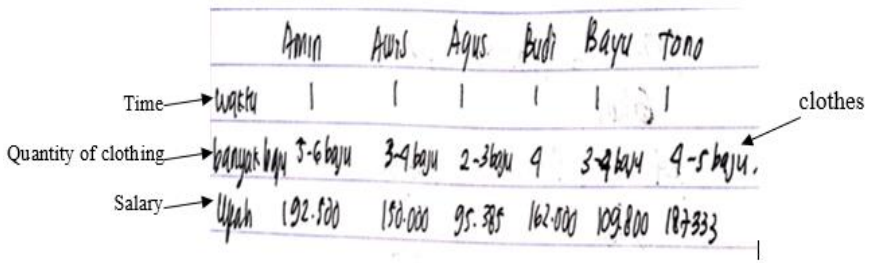
In the data analysis and interpretation, subjects compared and made decisions in choosing one of the six employees who were the most successful in their work. The subjects made decisions based on the results of representing data. Word use utilised by the subjects in analysing and interpreting data appeared when the subjects used the words "greater than" and "more than" in determining the most successful employees in their jobs. These words were a

colloquial word use because they had everyday meanings and also had a mathematical meaning in determining employees who were successful in their work. For example, S1 chose Amin as the most successful employee because Amin could sew more clothes and earn more than other employees. In this case, the words "more" and "bigger" were the keywords in which these keywords were a colloquial word used to select one of the successful employees.

Based on the results of the completion of the linear equation system (Table 2), one of the subjects, namely S1, made a visual mediator in the form of an iconic mediator (in the form of values in the table). In this case, the values in the table were used to express the number of clothes sewn and the employee's wage in one day, as shown in Figure 4. Based on the values in the table, S1 easily determined who was the most successful employee in his job. This was expressed by S1 when interviewed as follows: "*I change the time to sew clothes in one day to make it easier to compare the number of clothes and the wage of each employee at the same time*".

Figure 4

Iconic mediator made by S1



Subjects' substantiation narrative appeared when describing the reasons for deciding to select the most successful employee. Each subject made different decisions in determining the most successful employees. S1 chose Amin, S2 chose Agus, and S3 chose Tono as the most successful employee. S1 narrated that the most successful employee depended on sewing the most and earning the most, i.e., a successful employee would be the one who sewed the most and consequently would get the highest pay.

S1: *If an employee sews more clothes than the target, the employee will definitely get a higher salary, and otherwise, an employee sewing fewer clothes will gain a lower salary.*

To determine the clothes sewn by each employee and the wages in one day, as shown in Figure 4, S1 used a memorisation narrative by using the formula $\frac{\text{Quantity of clothes}}{\text{time}}$ to determine the number of clothes per day, and using the formula $\frac{\text{salary}}{\text{time}}$ to determine the employee's wage in one day. For example, the number of clothes Amin sewed in one day was $\frac{11}{2} = 5{,}5 \approx 5 - 6$ clothes. In this case, S1 revealed that it was impossible for many clothes to contain a decimal number, so he changed 5.5 clothes to 5-6 clothes. This means that Amin could sew 5 to 6 clothes in one day. The S1 expression is shown in the interview transcript as follows

S1: There are no clothes in the form of decimals, if I round it, Amin sews 5 clothes in one day, in 2 days there will be 10 clothes. However, in the available data, Amin sews 11 clothes in 2 days, so it can be said that on the first day Amin finishes 5 clothes and on the second day Amin finishes 6 clothes, so that the total is of 11 clothes.

Furthermore, Amin's wage in one day was $\frac{385{,}000\ \text{IDR}}{2} = 192{,}500\ \text{IDR}$. Based on this consideration, S1 chose Amin as the most successful employee because Amin could gain more wages than other employees in one day.

S2 decided to choose Agus as the most successful employee because Agus had the highest salary (1,240,000 IDR in Table 2) compared to other employees. S2 revealed that the most successful employee was based on salary alone; if the employee got the highest pay, the employee was categorised as the most successful employee. Meanwhile, subject S3 revealed the reasons for choosing the most successful employee based on the salary per clothing per day. For this reason, S3 chose Tono as the most successful employee.

Although S1 and S2 had used the same visual mediator representing the data, the narrative expressed by S1 and S2 was different in making the decision to choose the most successful employee. In this case, S1 involved all aspects or variables in making decisions, such as time, how many clothes were sewn, and the salary, while S2 involved only one aspect, i.e., choosing the most successful employee based on salary. Likewise, with S2 and S3, although the

symbolic mediator they used was different when representing data, they used substantiation narrative when they made the decision to choose one of the most successful employees, in which only involved the wage aspect. Snippets of interview transcripts of all subjects in choosing a successful employee were as follows:

S1: The most successful employee in the job is Amin because Amin sews more clothes, and the wages per day he receives are greater than other employees.

S2: The most successful employee is Agus because Agus gets more wages than other employees.

S3: The most successful employee is Tono because the wages per garment per day he receives is higher than other employees.

All subjects carried out an applicability routine by completing the linear equation system by substitution and elimination. The results of the subject's answers from solving the Linear Equation System can be seen in Table 2.

Table 2

The summary of the subject's answer for each employee

Taylor	Employee	Subject's Answer	
		S1 = S2	S3
Queen Taylor	Amin	Duration : 2 days Quantity of clothing:11 clothes Salary : 385,000 IDR	Clothing quantity per day: 2 clothes The salary per clothing per day: 78,349 IDR
	Awis	Duration : 6 days Quantity of clothing : 20 clothes Salary : 900,000 IDR	Clothing quantity per day: 1,9 clothes The salary per clothing per day: 83,309 IDR
	Agus	Duration : 13 days Quantity of clothing : 31 clothes Salary : 1,240,000 IDR	Clothing quantity per day: 0,8 clothes The salary per clothing per day: 29,983 IDR
King Taylor	Budi	Duration : 5 days Quantity of clothing : 20 clothes Salary : 810,000 IDR	Clothing quantity per day: 2,05 clothes The salary per clothing per day: 84,886 IDR
	Bayu	Duratin : 10 days Quantity of clothing : 32 clothes Salary : 1,098,000 IDR	Clothing quantity per day: 1,45 clothes The salary per clothing per day: 43,537 IDR
	Tono	Duration : 3 days Quantity of clothing : 13 clothes Salary : 562,000 IDR	Clothing quantity per day: 2,05 clothes The salary per clothing per day: 85,571 IDR

CONCLUSIONS, DISCUSSIONS, AND SUGGESTIONS

We introduced a commognitive framework that could be used as a tool when students carried out a statistical reasoning process in making complex decisions. A complex decision in this study happened when the students chose one of the six employees who was the most successful in the work. In this case, the data displayed were in the form of a table containing much information such as time, the number of clothes, and wages, which was not displayed for every employee but for some employees' pairs. In this case, students were required to carry out a statistical reasoning process, such as representing, analysing and interpreting data. In the process of representing data, the commognitive framework was a tool for students to display data from tables in other forms. Meanwhile, in analysing and interpreting data, the commognitive framework was a tool for students to make decisions and choose one of the most successful employees in their work.

Based on the results of this study, four important things needed to be done when students did statistical reasoning using a commognitive framework. First, a visual mediator matching the data display was necessary when students represented data. Someone created a visual mediator depending on how he/she made sense of the data displayed. The ability to read data displays becomes the basis for students to begin making a visual mediator. For example, data display in tabular form was then expressed by students in the form of a linear equation system so that the students' way of reasoning when reading tables would impact the way they created visual mediator. We suggested that the visual mediator created was adjusted to the values or features in the data display. This was based on the visual mediator produced when representing data was the foundation for being able to analyse and interpret data. Sfard (2008) suggests that the visual mediator is critical in building effective communication. Furthermore, Tabach and Nachlieli (2011) argue that the visual mediator used in communication often influences a person's ideas about what is being discussed and the discursive action chosen. This is different from Viirman (2014), who states that routine plays a major role in describing discourse. We assume that routine results from the regulation of word use, visual mediator, and narrative. Thus, if the wrong visual mediator is used, it will result in the ritualised routine. In this case, the procedures or steps used by students in completing the linear equation system were correct, but because the visual mediator made was not per the data displayed so that the result of the completion of the linear equation system could not be used as a mediator to make logical decision. This is supported by Ripardo (2017), who states that the visual mediator is a tool, while routine is a process, and endorsed narrative is the result of a discourse. Therefore, the role

of the visual mediator is very crucial in discourse (Sfard, 2008), including discourse in decision making.

Second, a narrative needs to be accompanied by students' understanding of mathematics and statistics when representing data. This is because when the narrative represents the data, it has an effect on the student's narrative in analysing and interpreting the data. For example, S3 made memorisation narrative in the form of a formula wage per day for employee pair in the form $\frac{\text{Quantity of clothes}}{\text{number of days}} x \frac{\text{salary}}{\text{number of clothes}}$. We considered that the formula is the same as $\frac{\text{salary}}{\text{number of days}}$. This means that the wages per clothing per day are similar to the wages in one day that each employee pair receives. So, initially, S3 assumed that the pay of each employee pair involved a lot of clothes being sewn, but we considered that the involvement of many clothes was not visible when S3 determined the employee's wages. In this case, we argue that these students did not fully understand mathematics and statistics, which caused them to make a memorisation narrative that was irrelevant to the data display. Next, the S3 made the variable number of clothes sewn per day displayed in decimal form. Meanwhile, data in decimal numbers were a continuous type of data which were not suitable to represent many clothes. Understanding the type of data is paramount to representing them. Mishra et al. (2018) state that the type of data is a significant part of collecting, analysing, and presenting data. One type of data is quantitative data containing discrete data and continuous data. In this case, discrete data is data in the form of integers (such as the number of clothes, people etc.), while continuous data is data in the form of decimal numbers (such as salary, time). Memorisation narrative that was not accompanied by an understanding of mathematical or statistical concepts resulted in the appearance of ritualised routine. In this way, students' decision became confusing.

Third, the commognitive framework was used by students when representing data functions as an intermediary or liaison to make decisions. The decisions taken depend on how students use word use, visual mediator, narrative, and routine based on the results of representing the data.

Fourth, substantiation narrative needed to be accompanied by narrative memorisation when analysing and interpreting data to choose one of the most successful employees. Decision-makers needed to use substantiation narrative by involving all aspects/variables in the data displayed so that it did not only affect one factor when they made a decision. For example, decision-makers did not look only at high wages without involving other aspects in determining the

most successful employees, but they also involved time and number of clothes sewn, which owes to the risks arising from the high pay received by the employee, such as the risk of taking a long time to complete the job. Viktorovna et al. (2018) state that decision making is always associated with risk. Thus, decision-makers could choose the smallest risk arising from the selected decisions. In addition, they needed to use the memorisation narrative by using the rules related to mathematical or statistical concepts in making a decision. For example, when S1 determined the each employee's daily payment, it did not mean that the employee received the same amount every day. In this context, the employee's wages were the average received per day. The narrative could emerge from the literate word use in students' minds in the form of the word "average". However, none of the students used "average" word when narrating the reasons for deciding on choosing a successful employee. Meanwhile, Jones et al. (2004) provide a solution for using proportional reasoning when comparing the aspects arising in decision-making.

In this study, the authors introduced the importance of using a commognitive framework in the statistical reasoning process. This is because many students could not read the data displayed in a table to impact decisions made based on the information obtained from the table. Thus, this study could contribute to students or decision-makers using a commognitive framework when faced with decision-making problems.

Thus, further research can be carried out to examine in depth how students use the commognitive framework at each level of statistical reasoning. For example, the narrative of students who are at the idiosyncratic, transitional, quantitative, and analytical levels. In this study, students' narratives that could not involve various aspects of decision making. Jones et al. (2004) argue that this type of student statistical reasoning is included in the quantitative level, in which students can make quantitative comparisons. Yet, they experience difficulty connecting various ideas, especially connecting different aspects of decision making. Therefore, there needs to be further research on how to use a commognitive framework at the level of statistical reasoning.

This study was limited to data containing cases of comparing the value between two variables. For instance, one employee sewed more clothes than another employee, and one employee earned more than other employees. Therefore, it is necessary to have several questions displaying inversely proportional data. For example, cases with data in which employees sewed fewer clothes, and the wages they received were high. Thus, students must do more challenging statistical reasoning using a commognitive framework.

AUTHORS' CONTRIBUTIONS STATEMENTS

DR and TN had the research ideas and developed theories in this study. DR also processed and analysed data. INP had provided advice on research methodology. Meanwhile, HS developed instruments and collected data. All authors actively participated in the discussion of the results, reviewed and approved the final version of the work.

DATA AVAILABILITY STATEMENT

All data in this study are available from the first author (DR).

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