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Two Fish Moving in their Seas: How does the Body Language of Teachers Show itself who Teach Mathematical Equations?

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ABSTRACT

Background: "Culture hides much more than it reveals and, strangely enough, what it hides, it hides more effectively from its own participants" (Hall, 1959, p. 39). This quote corresponds well to a Persian proverb, also a well-known aphorism that has been widely cited in many ethnographic articles: "a fish will be the last to discover water." Being immersed in water, surrounded by it, makes it invisible and almost impossible to perceive. In other words, we often do not know our interactional behaviour as mathematics teachers when we perform it in our usual and localised professional practice. **Objective:** To discuss mathematics teacher's body language when teaching equations and thus perceive this language in terms of possible fruitful educational action when teaching equations in the classroom. **Design:** Qualitative methodology. **Data collection and analysis:** Based on theoretical references that deal with body language, corporeality, and perception, we analysed individually and comparatively the classes of two mathematics teachers who taught equations in Birmingham (United Kingdom) and Rolante (Brazil). Thus, particularly attentive to mathematical culture in the classroom and analysing the localised gestures in the teachers' teaching of equations and the non-verbal behaviour, we can understand mathematics teaching through body movement, which often goes unnoticed. **Results:** We understand from the results of this research that perceiving the body language of mathematics teachers, which is produced with speech, gives us indications of the materialisation of the meanings attributed to the equation and how this will possibly affect the very constitution of the student's mathematical knowledge, in terms of possible meanings attributed to each gesture. **Conclusions**: We consider that knowing the body language can favour the teacher's teaching, i.e., metaphorically, knowing the sea can favour the fish to swim.

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Keywords: Mathematics education. Teacher education. Embodied cognition. First degree equations.

Dois peixes movendo-se em seus mares: como se mostra a linguagem corpórea de professores que ensinam equações matemáticas?

RESUMO

Contexto: "A cultura esconde mais do que revela e, por mais estranho que pareça se esconder, esconde-se mais efetivamente de seus próprios participantes" (Hall, 1959, p. 39). Esta citação se enquadra muito bem com um provérbio persa, também, um aforismo bem conhecido que tem sido bastante citado em muitos artigos etnográficos, o qual se apresenta como "um peixe é a última criatura a descobrir a água". Estar imerso na água, cercado por ela, torna invisível e quase impossível percebê-la. Em outras palavras, muitas vezes desconhecemos nosso comportamento interacional, enquanto professores de matemática, quando o realizamos em nossa prática profissional habitual e localizada. **Objetivo:** Discutir como se mostra a linguagem corpórea do professor de matemática ao ensinar equações e, assim, perceber essa linguagem em termos de ação educacional profícua ao se ensinar equações em sala de aula, por exemplo. **Metodologia:** Metodologia qualitativa. **Coleta de dados e análise:** A partir de referenciais teóricos que tratam de linguagem corpórea, corporeidade e percepção, analisamos as aulas de dois professores de matemática que ensinavam equações, em Birmingham (Reino Unido) e em Rolante (Brasil), individualmente e comparativamente. Assim, prestando atenção especial à cultura matemática em sala de aula e analisando os gestos localmente situados no ensino de equações e o comportamento não-verbal dos professores, podemos compreender o ensino de matemática por meio do movimento do corpo, o qual muitas vezes passa despercebido. **Resultados:** Compreendemos com os resultados dessa pesquisa que perceber a linguagem corpórea dos professores de matemática, a qual é produzida-com-a-fala, nos dá indicativos da materialização do sentidos atribuídos à equação e como isso possivelmente afetará a própria constituição do conhecimento matemático do estudante, em termos de possíveis sentidos atribuídos a cada gesto. **Conclusões**: Consideramos que conhecer a linguagem corpórea pode favorecer o próprio ensinar do professor, ou seja, metaforicamente, conhecer o mar pode favorecer o peixe a nadar.

Palavras-chave: Educação matemática. Formação de professores. Cognição corporificada. Equações do 1ºgrau.

STARTING TO DIVE

We begin this study by "plunging into the sea," a dip into -perhaps deep- waters, but carefully planned and calculated. By focusing on mathematics teachers' body language, we first selected a standard mathematical topic, in our case, first degree equations. We did this because it is a relevant mathematical subject for the discovery of unknowns and the introduction of the idea of

variables, besides having historical (Ribeiro, 2008a, 2008b) and practical multimeanings in terms of the conceptions of mathematics teachers (Barbosa & Ribeiro, 2012). Empirically, we also observed that teachers, when teaching equations, do not realise how they gesticulate. They gesticulate without knowing later what gesture(s) they used during their classes. Hence, the Persian proverb, in our view, is a consistent metaphor with the situation apprehended: "A fish will be the last to discover water." Our teachers may be the last to realised their body language when teaching mathematics and how fruitful it can be in their teaching practice.

Nevertheless, the metaphor adopted evokes memories of a great international movie, winner of the Academy Award for Best Film in 2018, "The Form of Water." This film presents a plot in which Eliza, a woman who cannot communicate through speech, expresses herself through body and sign language, in England BSL – British Sign Language-, in Brazil, LIBRAS – Brazilian Sign Language. Eliza works as a janitor in an experimental laboratory in the United States that holds captive an amphibious man. Elisa interacts with the creature through non-verbal contact, and both end up establishing a linguistic, affective, and sexual relationship. In this case, would they be two fish? Could they be two humans? The identity issue opens up, establishing understandings beyond what is expressed in a Cartesian way (Rosa, 2008), as body language is a regulation mark for what one reads, expresses, understands, and feels about the other. It goes beyond the idea of uniqueness, the idea of human, hybrid, and what else could be labelled in the scenes that continued. Likewise, for us, the body can be considered a means of reading, expressing, understanding, and feeling, also opening possibilities to mathematics education, specifically teacher education. In teaching equations, we can connect mathematics teaching to the body as a means of communication. We recall that although the teachers we investigated are not fish, like in the film, they use their body language at all times and, as in the proverb, they do not notice its use.

That said, we set out to study the gestures employed by two teachers when teaching equations, a woman and a man of different ages, in different periods, from different cultures and places, as we realised that body language is often ignored when teachers teach mathematical equations. We investigated what is hidden, i.e., our interactional behaviour as mathematics teachers. Thus, the guiding question of our study is:

"How does the mathematics teachers' body language show itself when teaching equations?"

In this perspective, we want to investigate this language in terms of fruitful educational action, i.e., to investigate the gestures produced in the classroom regarding the perception of their possible contribution to the teaching of equations. In doing so, reveal what the teachers themselves may not notice or reflect about, to highlight pedagogical aspects related to embodied cognition, which we believe may contribute to the learning of those teachers' students.

For this, we will highlight our conception of research, why we should do it, the participants' background, our methodological procedures to produce and analyse data and, therefore, discuss the theoretical aspects.

HOW DO WE PERCEIVE THE WATER AND WHAT IS IN IT? (RESEARCH METHODOLOGY)

To highlight the locus of our research, our surroundings and, in this case, the surroundings of the teachers investigated, we had to choose a research paradigm. A paradigm that would most help us analyse what we set out to do. That is, for the analysis of the mathematics teacher's body language when teaching equations, it would not be helpful to quantify identical or different movements; it would not be pertinent to quantify the repetitions, since we would not reach the perception of the "how" it happens, of the movement in terms of possible correlations with the equation solving method, for example. Thus, the qualitative study of the movements traced by teachers, their body language, in terms of procedural description of gestures each of the research participants performed, made us acknowledge that this methodology was appropriate.

According to Seidel and Rosa (2014, p.408), "Among the broad spectrum of possibilities of qualitative research, our commitment here is to bring ways of proceeding according to the worldview and phenomenological knowledge, in which '[...] the world is already 'there,' before reflection, as a presence' (Merleau-Ponty, 2006, p. 1-2)". What we investigate is not treated as an object external to our world-life. Thus, when we interrogate the phenomenon so that it can be known, we already conceive of the world-conscious, the world-with-the-subject, the world-with-the-things and everything that it does and is done with it. We are all situated in the "same" world of the phenomenon.

For us, there are different ways of seeing the quality of an object and of perceiving the phenomenon that encompasses that object (Seidel & Rosa, 2014, p.411). Thus, we set out to use the perception (Merleau-Ponty, 2006) of body language and, through this, to see, read, understand, feel, and express what

we perceived. Therefore, we developed an investigation with rigour criteria, which involved clarity and coherence of aspects underlying the worldview and knowledge assumed by us. We assume *the being-in-the-world-there-with* (Heiddeger, 2012) and, according to Rosa and Caldeira (2018, p. 1076),

> [...] when we refer to the body from a perspective of totality (Merleua-Ponty, 2011), it makes no sense to consider it an object or an alienable from the world, from its context, but to think of the body in terms of movement, perception, language, and experience-life, which refers to immediate contact with life and the understanding of it. The body is our vehicle of being in the world (Merleau-Ponty, 2011).

In other words, the body is not a recipient; it is not detached from the "being" nor the "world". Besides constituting the thought (McNeill, 1992), the body enables us to communicate what could be considered incommunicable. Thus, we set out to scale the research contexts, the surrounding world of our subjects, and describe the participants themselves and the procedures adopted.

The seas where they inhabit (the research context)

The data presented in this article come from two different nations, Brazil and the United Kingdom. We conducted an approach to examine communication (verbal - what is said, and visual - the gestures in space) in a British and a Brazilian school. The means of instruction were exclusively English in the British school and Portuguese in the Brazilian school, although there were students of various ethnicities. In the British school, there were first and second generations of students from Persian and Turkish heritage. Our sample was a class of 24 students aged 12.5 years on average. In the Brazilian federal school, an incoming class in 2019 consisted of 22 students, of which only 14 were attending courses during the data production period, five men and nine women. The institution classifies the class as out of the "standard" of the other classes in the PROEJA modality[1], since it is composed of many young students (around 20 years of age) and only two people over 50 years of age (which is the most common age in the other classes). Most of those students had given up attending high school at some point, causing a deviation in the

[1]" Proeja **was** initially created by Decree no. 5.478, of 06/24/2005, and referred to as Programa de Integração da Educação Profissional ao Ensino Médio na Modalidade Educação de Jovens e Adultos/Programme for the Integration of Vocational Education to High School Education in the modality of Youth and Adult Education ". (BRASIL, 2019).

age/grade expected for the regular education level. The arrival of adulthood made it difficult for them to enrol in conventional classes, leading them to seek this modality to complete basic education. Besides the different contexts in terms of the language used, geographical location, educational modality, and age group, the teachers participating in this research also have very dissimilar characteristics.

The "fish" (the participants)

Leo[2] is the British teacher that participated in the UK data production. From our perspective, he is a new teacher in terms of classroom experience. At the time of data production, Leo had three years of experience in mathematics teaching in a State school. He worked only in two different States schools in the UK, did not hold a university degree in mathematics, but took a two-year course to become a mathematics teacher, teaching young people aged 11 to 16.

On the other hand, in the Brazilian school, we have teacher Paula[3]. She had teaching experience in elementary and high schools in the State network, where she worked from 2013 to 2018. At the time of this research, she worked as a substitute teacher in this Federal school (since 2018), composing the collegiate of technical courses in agriculture, commerce (PROEJA) and computer science. In 2019, she taught the school subject Mathematics I - for students who had to repeat the subject (*dependência*) - students enrolled in the 2nd year of the courses mentioned above who had passed the other disciplines and failed only in mathematics. She also taught Mathematics I for students of the technical course in commerce, in the PROEJA modality; Mathematics III for students enrolled in the 3rd year of the technical courses in agriculture and livestock and computer science; and Applied Mathematics for students enrolled in the 1st semester of the concomitant/subsequent technical course in agriculture and livestock.

Means for looking at the water (resources and methodological procedures)

To analyse the mathematics teacher's body language when teaching equations, we used two hours of interactive videos recorded in both the British

[2]We will use the name "Leo" as a pseudonym to safeguard the teacher's identity, according to an informed consent form he signed in the UK. The teacher, through this term, authorised the use of his images and speeches for research purposes, renouncing only the use of his first name.
[3]Teacher Paula, through an informed consent form, authorised the use of her images, speeches and her own name for the purposes of this research.

and Brazilian schools. The objective of the registers was to capture verbal and visual excerpts of language to investigate how those modes of communication play a role in the production of mathematical meaning. Although all video recordings were transcribed, only a few excerpts were of interest. Those parts often contained embedded gestures that produced, from our point of view, mathematical meanings. Therefore, when analysing the video recordings, we paid particular attention to gestures that referred to equation and verification of results by substitution. In other words, we emphasised to the detriment of others (Mason, 2002). Regulatory and disciplinary gestures such as "ok/good," "hold on," "go on," or "hush" were not included for analysis. Therefore, there were representational choices in data analysis regarding what to include and what not to include. Naturally, a researcher makes subjective choices in how the data is initially selected and recorded.

Table 1

Choices around recording and selecting interactional data (Farsani, 2015b)

	At the data recording site	Outside the data recording site
Researcher choices	What/when/who to record in the classroom?	What to select from the data set for further analysis? There are, of course, presentation choices and what to present in the transcript.

Table 1 illustrates the guiding questions of representational choices a researcher makes inside and outside research sites (Farsani, 2015b). Moreover, how transcription is presented determines how the data will be analysed. Therefore, transcription is, in fact, part of the analysis (Bezemer & Mavers, 2011). Subsequent transcriptions, along with sequential image recordings (since the video presenting the movement cannot be brought to the text) highlight and emphasise the movements of the body that accompany the term *equations* or *result checking*.

Also, there were other representational choices made regarding translation. Translating requires an interpretation — the data presented from the British context required translation into Portuguese. Naturally, the translator had to go back and forth in languages to find equivalent statements. From these aspects, we present the theoretical framework that supports our analysis, and later the data selected for this article, along with the analysis, in the light of this framework itself.

WHAT IS THE SHAPE OF THE WATER? (THEORETICAL FRAMEWORK)

To understand what is not familiar to us in daily life, i.e., to analyse a teacher's gestures when teaching mathematics to clarify what this analysis can help us in the task of being a teacher, we need an *a priori* referent. We need to highlight what has been scientifically done in terms of body language, body and perception and, after that, focus specifically on body language, the mathematics teachers' gestures when teaching equations.

In this perspective, the term "gesture" is used in a broad sense, as a physical movement of a body part (e.g., hands, arms, eyes, and face) (Kendon, 1983; Maschietto; Bussi, 2005; 2009) in communicative situations (Kendon, 1997; Streeck, 1988). However, it is not only this, a pure movement of a body as a visible object and the actions performed through it. It's not just a physical movement of a body part because,

> Once the loss of sensations is removed, a face, a signature, a conduct cease to be simple "visual data" of which we would need to seek, in our inner experience, the psychological significance, and the psyche of the other becomes an immediate object as a whole impregnated with an immanent signification. More generally, it is the very notion of the immediate that is transformed: henceforth, the immediate is no longer the impression, the object that is unique and the same as the subject, but the sense, the structure, the spontaneous arrangement of the parts. (Merleau-Ponty, 2006, p. 91)

Thus, the meaning given by the gesture is what we want to perceive. More than a physical classification, we assume the intentionality of the subject in motion as what can cause us to realise our posture as teachers and the relationship that the posture that of our bodies take in terms of mathematical cognition. Perhaps, we can perceive the "water" that surrounds us, our own time-space, the moment we realise how gestures, senses, structure occur in the actions of other mathematics teachers. We understand that,

> What allows us to reconnect the "physiological" and the "psychic" to each other is the fact that, reintegrated into existence, they are no longer distinguished as the order of being-in-itself and the order of being-for-itself, and that they are both oriented towards an intentional pole or a world. (Merleau-Ponty, 2006, p. 129)

What we defend is that we are already connected to the world, our own body moves and:

> Each determined movement occurs in a medium, on a background determined by the movement itself (...) We execute our movements in a space that is not 'empty' and unrelated to them, but which, on the contrary, is in a very determined relationship with them: movement and background are, in fact, only moments artificially separated from a single whole. (Merleau-Ponty, 2006, p. 192-193)

This artificiality causes theorists to separate the body into parts and seek a classical, objective, objectifying categorisation. For example, McNeill (1992, p. 11) used the term "gesture" to mean "[...] arm and hand movements [...] closely synchronised with the flow of the speech." Kendon (1972; 1980) refers to the spontaneous movements of the hands, produced while speaking, of "gesticulation." In similar lines, Sfard (2009, p. 194) defines the "gesture" as a "body movement that fulfils the communicational function," and this, for us, is not bad, nor negligible. In fact, a clear way to categorise and try to understand those movements and express them from parts of a whole that is not conceived. However, not treating the background in terms of the surrounding world makes the action limiting in terms of what phenomenology advocates.

In this article, then,

> Regarding spatiality, which is the only one that interests us at the moment, the body is the third term, always Implied, of the figure and background structure, and every figure stands out against the double horizon of the external and bodily space. Therefore, any analysis of bodily space that only considers figures and points should be rejected as abstract, since figures and points can neither be conceived nor be without horizons. (Merleau-Ponty, 2006, p. 147)

Therefore, we assume "gestures" as movements of hands or arms that are articulated while the gesticulators (teachers) intend to attribute educational meaning to what they say in the face of their context, the world, their surroundings, with that/those around them. Thus,

> The gesture is before me as a matter, it indicates some sensitive points in the world, invites me to find it there. Communication takes place when my conduct finds its own path on this path. I

> confirm the other, and the other confirms me. (Merleau-Ponty, 2006, p. 251-252)

In this sense, in the meantime, we included regulatory gestures, such as "be quiet/hush" or "sit down". However, "aesthetic gestures" such as those used to put hair on the back of the head/ear, or bite nails, particularly, are not considered significant in terms of educational meanings, although they are part of the whole.

Gesture and speech form a unified communicative system and "[...] present a unique cognitive representation" (McNeill, 1985, p. 353). Gestures not only help to "constitute thought" (McNeill, 1992, p. 245), but they also help the speaker to have access to a wide range of possible resources to make sense of what is said (Kendon, 1980). A combination of gesture and educational discourse balance, i.e., without being restricted to verbal language, can support students to build meanings (Alibali, 1999), help students cognitively build broad communicative flows to expand the way of thinking (Kendon, 1997), and interpreting information (Kendon, 1995). Therefore,

> [...] thought and expression are constituted simultaneously, when our cultural acquisition is mobilised in the service of this unknown law, just as our body suddenly performs a new gesture to acquire the habit. Speech is a true gesture and contains its meaning, just as the gesture does, too. That is what makes communication possible. (Merleau-Ponty, 2006, p. 249)

In addition, we believe that the silent and non-verbal messages that are communicated through teachers' gestures are of great value in education and educational research, not only in terms of teaching but also in assessing students' understandings (Farsani, 2015b). Teachers' gestures, in our view, can help convey pertinent mathematical information (Farsani, 2015b; Farsani, 2016). Since

> In the gesture of the hand that rises toward an object is included a reference to the object not as represented object, but as this well-determined thing toward which we project ourselves, near which we are by anticipation, that we attend. Consciousness is the being for the thing through the body. A movement is learned when the body has understood it, i.e., when it has incorporated it into its "world," and moving your body is to aim things through it, is to let it correspond to its request, which is

> exercised on it without any representation. (Merleau-Ponty, 2006, p. 193)

In mathematical terms, we understand that, when gesticulating, the teachers allow themselves to perform gestures that accompany their verbal message and gestures that are performed independently, without the verbal messages that accompany them when teaching. Those independent gestures are also called emblems, gestures that have a specific cultural meaning. For example, a positive gesture, indicated with a thumbs-up, although it means "ok" in most of the world, also means the number "one" for Europeans, "five" for Japanese, and "sixty" for Iranians. Thus, emblems are consciously sent and consciously received with a particular meaning that is identified by a specific cultural group (Barakat, 1973; Farsani, 2015a). So, we understand that:

> [...] it seems impossible to give words, as well as gestures, an immanent meaning, because the gesture is limited to indicating a specific relationship between man and the sensitive world, because this world is given to the viewer by natural perception, and because thus the intentional object is offered to the witness at the same time as the gesture itself. Verbal gesticulation, on the contrary, aims at a mental landscape that, in the first place, is not given to everyone and whose function is precisely to communicate. But here, what nature does not give, culture provides. (Merleau-Ponty, 2006, p. 193)

Thus, for McNeill (1992), some gestures accompany his verbal messages in four different perspectives. 1) "Beat gestures" are rhythmic gestures that musicians and politicians often use. The beating gestures do not express any verbal semantic content but are intrinsically aligned with the prosody of speech. 2) "Iconic gestures," unlike the beating gestures, directly portray the semantic content through a form or physical property. For example, when describing a person, the gesticulator may raise a hand, palm down, to indicate the "height" of the person being described. This height is a physical property of the person; therefore, it is a complementary visual description. 3) "Metaphoric gestures" portray the semantic content via metaphor. For example, a gesticulator may perform exactly the same gesture by raising one of the hands over the head with the palm down to indicate "high intelligence." Therefore, in many situations, the trajectories of shape and movement of iconic and metaphoric gestures are precisely the same. In many cases, what tells the difference between an iconic and a metaphoric gesture, according to the author, depends mainly on the verbal message that accompanies it. That is, whether the

accompanying message conveys something tangible/concrete or reflects a metaphoric notion. 4) "Deictic gestures," pointing at objects or locations. These gestures can be performed with the index finger extended and with other fingers or with the whole hand. However, although categorised, we do not take those gesture perspectives as a way of objectifying the movement, since

> Plato also granted the empiricist the power to point at things, but in fact, even the silent gesture is impossible if what he designates has not already been torn from his instantaneous existence and monadic existence, treated as the representative of his previous appearances in me and of his simultaneous appearances in another one, i.e., subsumed to a category and elevated to the concept. (Merleau-Ponty, 2006, p. 171)

This means that we do not understand gestures disconnected from our own bodies and, consequently, from the world. The being-in-the-world (Heiddeger, 2012) emphasises the being that is in the world and being the world with it, because, "Being-in-the-world does not claim being within the world, but fundamentally being world, and this in the experience of being in being, of existing in the infinite dimension of being, i.e., of existing in the opening of the-being" (Schuback, 2012, p.27). From this perspective, we assume that we live with the world and with every apparatus that is in it, without dichotomising, in the sense of not conceiving the existence of a being who thinks "about" the world, but the existence of a being who thinks, acts, and lives "with" the world. Therefore,

> I signal through the world, I signal there where my friend is; the distance that separates me from him, his consent or his refusal are read immediately in my gesture, there is no perception followed by a movement, perception and movement form a system that changes as a whole. If, for example, I realise that they do not want to obey me, and consequently, I modify my gesture, there are not two distinct acts of conscience there, but I see the ill will of my partner, and my impatience gesture is born from this situation, without any thought interposed. If I now perform "the same" movement, but without targeting any present or even imaginary partner and as "a sequence of movements in itself", i.e., if I perform a "flexion" of the forearm on the arm with "supination" of the arm and "flexion" of the fingers, my body, which was just the vehicle of movement, becomes its goal; its motor design does not target

> anyone else in the world, it targets my forearm, my arm and my fingers, and it targets them while they are able to break their insertion in the given world and draw around me a fictitious situation, or even while, without any fictitious partner, I consider this strange machine of meaning with curiosity and make it work for fun. (Merleau-Ponty, 2006, p. 160)

This fact, then, leads us to refer to the intentionality of the being that is there and that moves there in the world. There is an intentionality that characterises a thinking loaded with nuances of the context, of the world in which one inhabits. In this sense, there are deictic gestures that are performed differently in different countries and cultural groups. Culture, then, establishes the difference. For example, pointing at lips (Enfield, 2001), pointing at the eyes (Wilkins, 1999), and, for us, pointing at the nose, eyebrow, and neck and chin set can be considered common practices worldwide. However, those so-called "practices" are not watertight. They are not isolated actions that only draw our attention. They are not pure movements performed by a body considered as an object. But they are expressions of what was perceived, they are interrelations of what is experienced with the body.

> The common experience finds a convenience and a sense relationship between the gesture, the smile, the accent of someone who speaks. But this relationship of reciprocal expression, which makes the human body appear as the manifestation, on the outside, of a certain way of being in the world. (Merleau-Ponty, 2006, p. 160)

Thus, when we mention the body from a perspective of totality (Rosa & Caldeira, 2018), as previously announced, it makes no sense to consider it an object or an alienable from the world, from its context, but to think of the body in terms of movement, perception, language, and lived experience, which refers to immediate contact with life and the understanding of it. "The body is our vehicle of being in the world" (Rosa & Caldeira, 2018. p. 1076). In other words,

> What we call body schema is precisely this system of equivalences, that invariant immediately given by which the different motor tasks are instantly transposable. This means that it is not only an experience of my body, but also an experience of my body in the world, and that is what gives a motor sense to verbal orders. (Merleau-Ponty, 2006, p. 196)

Thus, although we classify gestures when we take the perspective of cognitive psychology to "read" the body language of mathematics teachers, we are not making this theoretical movement as if objectifying our subjects' body. We know that the body, for a long time, could not escape the same determinations that made the object an object, the characterisations without which the body would have no place in the system of experience. Expressly, those predicates of value that our reflective judgment confers should be sustained in the "being" through a first layer of clearly characterised physicochemical properties (Merleau-Ponty, 2006). However, this has already been overcome, since,

> [...] it is, therefore, necessary that even "automatic" movements are announced to consciousness, i.e., that there are never movements in themselves in our body. And if all objective space exists only for intellectual consciousness, we must find the categorical attitude even in the movement of catching. Just like physiological causality, awareness cannot begin anywhere. And one must either renounce physiological explanation or admit that it is total — or deny consciousness or admit that it is total; one cannot refer some movements to body mechanics and others to consciousness, the body and consciousness are not limited to each other, they can only be parallel. (Merleau-Ponty, 2006, p. 174)

The movement, then, occurs considering the body, because it is about how we perceive the movement itself, inhabiting space-time. Not only from a mathematical perspective but beyond, i.e., reveals itself in a totality. The movement, from this perspective, does not submit to space and time; it **actively assumes** them (Merleau-Ponty, 2006). Thus,

> It is through my body that I understand the other, just as it is through my body that I perceive "things." Thus "understood," the meaning of the gesture is not behind it, it is confused with the structure of the world that the gesture draws and that on my account I resume, it is exposed in the gesture itself — just as, in the perceptual experience, the meaning of the chimney is not beyond the sensitive spectacle and the chimney itself, just as my looks and my movements find it in the world. (Merleau-Ponty, 2006, p. 253)

Our own body is a means of perception because "[...] through our body we are connected to the world, often launching ourselves towards this world"

(Rosa & Caldeira, 2018, p. 1077), and this enables us to seek to understand the body language of mathematics teachers who teach equations. We then bring the data produced when filming two teachers who taught equations and analyse those data, seeking to answer our research question.

HOW DO FISH SWIM? (DATA ANALYSIS)

We began our analysis by bringing sequential images of the teachers' movements along with their respective "speeches" during classes. Concomitantly, we bring their gestures articulated with those speeches, perceived in the same recording period. In the case of the British teacher, we will also bring his speeches in his mother tongue (English) for verification and a better understanding of our translation. In both cases (the British and the Brazilian teacher), we will identify the excerpts chosen per date of the meeting, the start and end time of each excerpt chosen, and through a title that, for us, summarises the central idea of what was analysed. Also, each sequence of figures will be identified through letters underlying the figure number, composed of the set of images. We continue with the first excerpt from teacher Leo's class.

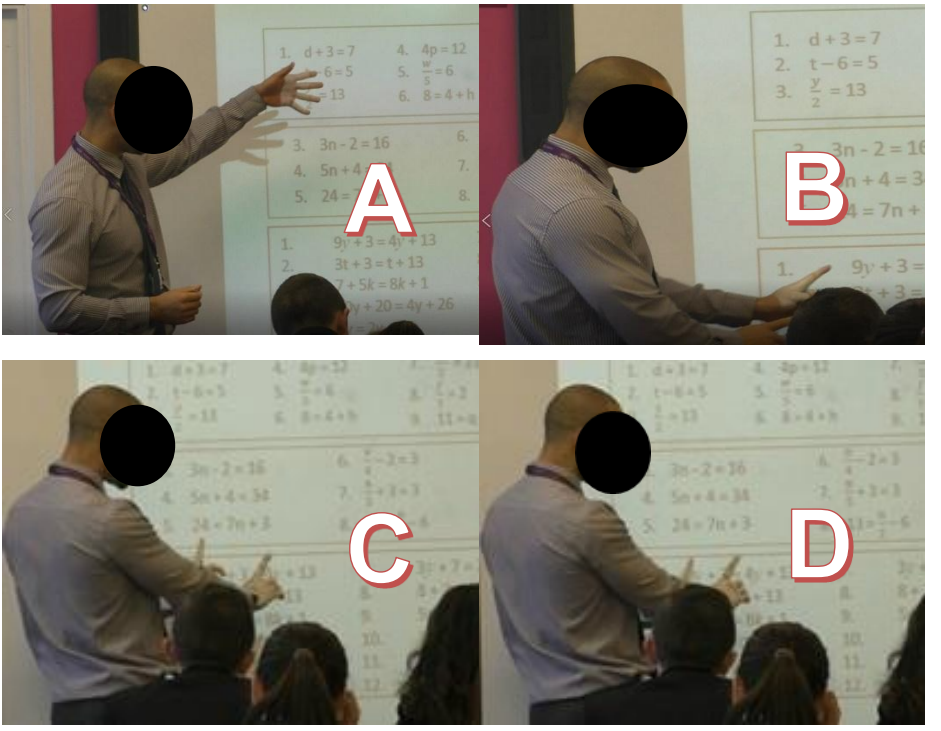
Thus, in the excerpt we present, teacher Leo discusses the idea of checking answers regarding a list of 1st degree equation solving exercises that he had given to his students. In the meantime, he explains how to solve the equations and resumes the explanation in correcting each question. The excerpt then begins with a speech that presents the possibility of the value remaining unknown if his students do not verify the answer.

> **Scene 1: January 2017 – Video S2220020 (04:36-05:05) – Leo – This is the metaphoric gesture of an ancient weighing scale**
>
> **Leo:** *Or the value is ever unknown. So, for example, check the three steps, so, [name of student], what you need to do with yours. Ok. What was your answer for this one (pointing at the equation 9y + 3 = 4y + 13 on the whiteboard)? Y = 2, so nine times two is eighteen plus a three is twenty-one. That side it twenty-one, this side also needs to be twenty-one. Ok, four times two is eight plus thirteen is twenty-one. So it balances right (gesture for balance: He establishes this gesture in a four-stroke process: first, he closes all his fingers in both hands with only the index fingers on both hands pointing upwards, second, he places the right hand pointing upwards in a slightly upper position than the left hand, third, he places the left hand pointing upwards in a slightly higher position than his right hand, and fourth, he finally*

positions the tips of his both index fingers pointing to the same level or imaginary line indicating that now they are level). So that is how you are gonna check it (he is making another rotating circular gesture which appears to be in the format of substitution, that is checking through substitution).

Figure 1

Leo showing the weighing scale



When analysing Leo's class, we extracted the excerpt (Scene 1) that matches moments (*prints*) of the filming made at that time (Figure 1). Thus, we began to indicate the mathematics teacher's body language when teaching equations. From this perspective, the perception of Leo's body as he communicates in class, also with verbal language, makes us re-signify that body language is a participant in the act of teaching, since it is not only a prerogative of the subject him/herself but an action that is potentially significant in the constitution of students' knowledge. We affirm this as, beforehand, when Leo states, "*Or the value is ever unknown, whatever it is"*, he, at the same time,

moves his hand as if "drawing" circles in the air (Figure 1-A). Repeatedly, this circular movement, in our view, carries the idea of some space, which one must think about, one must question: what is it, the unknown? An amount? A number? A variable? An incognito? We understand that at this moment, in performing this gesture, Léo not only brings a movement, a circle, a possible "hole," but also re-signifies the "unknown." His gesture ceases to be simple "visual data", but a re-signification of what is "unknown", given by the very notion of the immediate that is transformed through his body language, since that movement without any vocal announcement could immediately have another meaning. Henceforth, the immediate is no longer the impression, but the sense, the structure, the spontaneous arrangement (Merleau-Ponty, 2006) of what Leo attributes as the unknown in the equation.

Nevertheless, when Léo asks "*What was your answer for this one?"*, at the same time, he points to equation *9y + 3 = 4y + 13* on the board (Figure 1-B), which shows that Léo performed a deictic gesture that indicates objects or locations (McNeill, 1992). This gesture was performed with the index finger extended, which means that in the gesture of the hand pointing toward an object (the equation on the board), a reference to the equation is included, not as represented equation, but as an answer, as the discovery of a value that should be checked. That is, this well-determined "thing" toward which we project ourselves (Merleau-Ponty, 2006), i.e., the mathematical sense for that expression in mathematical language. Consciousness is the being for the thing through the body. A movement is learned when the body has understood it, i.e., when it has incorporated it into its' world, 'and moving your body [in this case, pointing to the equation] is to aim things through it, it is to let it correspond to its request, which is exercised on it without any representation" (Merleau-Ponty, 2006, p.193). In the sequence, Leo uses "Metaphoric gestures" (McNeill, 1992) (Figures 1-C and 1-D) because he portrays the semantic content via metaphor. Which one? He puts his left and right index fingers in parallel so as to highlight the two members of the equation as if it were a scale. By stating "*So it balances right"* demonstrates that his gesture reflects a metaphoric notion, since it is about balance. In the meantime, by announcing that "*Y = 2, so nine times two is eighteen plus a three is twenty-one. That side it twenty-one, this side also needs to be twenty-one. Ok, four times two is eight plus thirteen is twenty-one.* (Figure 1-D)", Leo articulates the idea of equilibrium, of a balance, i.e., he refers to the conception of equation, which, according to Pérez & Marin (1928, p.15 *apud* Miguel, Fiorentini, & Miorin, 1992, p.47)" [...], is any equality that expresses a relationship between the known and unknown quantities of a problem, being the quantities known the

data of the problem or equation, and the unknown quantities, the incognitos", emphasising equality when he says that "*this side also needs to be twenty-one*," equal to the first member of the equation. Metaphorically, when placing the two index fingers at the same height (Figure 1-D), Léo establishes a reference to equality, to balance, between the members of the equation. When gesturing, he indicates a relationship between his fingers and the sensitive world, given by the equality sign, because the members of the equation have the same value, by equality itself, by the balance of heights that in Figure 1-C above was disproportionate, unequal. Because of this, we realised that Leo, with his body language, naturally, launches himself to the resolution of the equation pointed. We understand that the act of equating finger height is done because the world is given to the viewer by perception, and because thus the intentional object (teaching the resolution of equation $9y + 3 = 4y + 13$) can be revealed to the witnesses (their students) at the same time as the very equilibrium gesture (Merleau-Ponty, 2006).

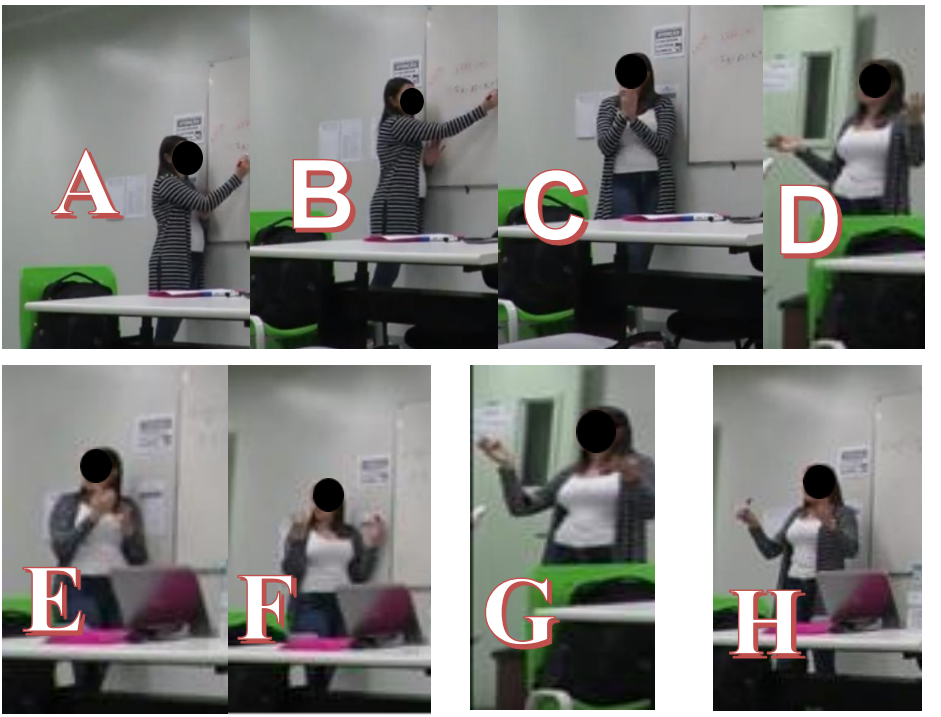
Similarly, we analysed the body language of Paula, a Brazilian teacher, based on evidence of the relationships between the gestures and movements performed by her and what she expressed in terms of solving a first degree equation by assigning meaning to it verbally. In this perspective, the excerpt chosen (Scene 2) presents, as in Scene 1, the teacher correcting equation solving exercises. Paula corrects the exercises while resuming the explanation of the process of solving a first degree equation. In the following excerpt, she discusses the resolution of equation $7x - 10 = x + 50$ and subsequently confers the result.

> **Scene 2: January 2019 – Video 1 - MVI_0249 (00:00-05:48) – Paula - Solve both this and this side of equality, the two answers will be the same**
>
> ***Paula:*** *So look, our goal is to find out what numbers we're going to put here in place of the x that's going to solve this part of the equation, this part of the equation and the two of them are going to give the same answer, ok? That's always the goal. So, we start isolating x, remember how we did last week, right? So, it's going to be 7 x minus x equals 50 plus 10, is it ok so far? 7 x minus x is 6 x, 50 plus 10, 60, x is going to be equal to 60 divided by 6, x equals 10, right? What does it mean? It means that if I put the number 10 here in place of x and solve both this and this side of equality, the two answers will be the same, it must be the same. So, if I carry out 7 times 10, minus 10, which is this side, it must give exactly the same answer as if I add 10 plus 50. Here if I do*

it, it's going to be 70 minus 10, here it's going to be 60, 60 equals 60. This means that the calculation worked, that the number I have to put here in place of x is the number 10. Alright?

Figure 2

Paula makes gestures for the weighing scale analogy



As we followed teacher Paula's class, we noticed that her gestures and movements to explain how to solve a first degree equation were analogous to what she said. We can state this because, when she says, "*So look, our goal is to find out what numbers we're going to put here* [indicating with the whiteboard marker the 1st member of the equation, *7x – 10*, Figure 2-A] *in place of the x that's going to solve this part of the equation,"* and after that, she says "*this part of the equation* [indicating the other part, that is, the 2nd member of the equation, *x + 50*, Figure 2-B]," she uses "deictic gestures" (McNeill, 1992), which, in this case, aims to locate the member of the equation she is mentioning, and then locate the other part of the equation in terms of space, without properly defining it. Thus, in Scene 2, the teacher made the gesture

with the marker, but her body, her hand, moved as if using her index finger straight. In this sense, we do not understand as "[...] abstract any analysis of bodily space that only takes into account figures and points, since figures and points can neither be conceived nor be without horizons" (Merleau-Ponty, 2006, p. 147). Putting it another way, Paula uses her body not as part of the system, not as a figure pointing to nothingness, as if stretching her arm in the act of reaching something (although this act also implies a horizon), but as a whole. She intentionally launches herself to what she expresses, points to the first member of the equation and then the second member, to give meaning to what she is synthesising: "*and the two of them will give the same answer, ok? This is always the goal*" [Paula joins both hands when she says "*the same answer*," Figure 2-C], indicating the idea of equality, in which juxtaposed hands (Figure 2-C) equate both sides, the two members of the equation, in this case, indicated by both hands. From this perspective, we consider that "Speech is a true gesture and contains its meaning, just as the gesture does, too. That is what makes communication possible" (Merleau-Ponty, 2006, p. 249), since when she says "*the two will give the same answer*," i.e., the two parts (members) of the equation have the same value, she expresses the sense of equality and, at the same time, equals the hands, joining them in a similar position, at the same height, or even in the same position, if we consider both hands as a reference. In our view, this favours students' understanding in terms of equality.

In addition, after explaining the resolution of the equation in a way that isolates the unknown in one of the members of the equation, the teacher states "*and solve both this* [opening her arms, Figure 2 -D, immediately taking her right hand on the left, Figure 2- E, so that her students perceive each side of the equality and then the side of the equality to which she referred] *and this side of the equality, the two answers will be the same, they must be the same*," which allows us to consider that "[...] it seems impossible to give words, as well as gestures, an immanent meaning, because the gesture is limited to indicating a specific relationship between a person and the sensitive world (Merleau-Ponty, 2006, p. 253), as well as words. So, even though she could indicate the side of the equality on the board, as she had done previously, using the marker, the teacher did not point, but made use of other gestures, with her body (overlapping her hands). We understand this action as body schema because, "[...] it is precisely this system of equivalences, that invariant immediately given by which the different motor tasks are instantly transposable" (Merleau-Ponty, 2006, p. 196).

Teacher Paula continues: *"So, if I add 7 times 10, minus 10, which is this side, it must give exactly the same answer as if I add 10 plus 50* [placing

her hands side by side, parallel to her body, Figure 2-F]. *Here, if I do it, it's going to be 70 minus 10* [raising her right hand above the level of her left hand, indicating one side, Figure 2-G], *here it's going to be 60, 60 equals 60"* [to equate the height of her hands again, parallel to her body, Figure 2-H) . Thus, we realised that, when placing her hands side by side, Paula materialises what it would be to "*give exactly the same answer*," which reinforces the replacement of the unknown of the expression by the number ten in both members. Thus, Paula attributes new gestures to the idea of equality. Thus, following Merleau-Ponty (2006) statements, Paula's thought and her bodily expression are constituted simultaneously, because her cultural acquisition of the meaning given to equality is mobilised in the service of this sense conceived by her, just as her body suddenly performs a new gesture in the acquisition of what comprises, perhaps, balance, height similarity of the sides of her body (right and left hands) in line with the members of the equation. Also, in this sense, there is a confirmation of this equivalence of the sides, of the limbs, since, when multipying the first member, attributing the value of the unknown 10, she states, "*Here, if I do it, it's going to be 70 minus 10"*, raising her right hand (Figure 2-G), to represent the "here", the first member, which was on the same side as her right hand to those who saw Paula and the whiteboard on their back. That is, Paula's experience in the classroom, which is common to her, finds a relationship of convenience and meaning between the gesture made, her position in front of the whiteboard, the member of the equation to which she referred, because, agreeing with Merleau-Ponty (2006), this relationship of reciprocal expression is what makes the human body appear as the manifestation; in this case, Paula's body confirms a specific way of being in the world. In other words, she, as a maths teacher, to be situated as someone who teaches equations.

Nevertheless, Paula resumes the balance position of her hands (Figure 2-H), a balanced position that demonstrates that "*here, it's going to be 60, 60 equals 60*." Just like Leo, Paula, through her body, brings to consciousness the idea of balance, equality. She makes a movement learned by her body that understood it, i.e., when she incorporated the idea of equality into her "world," and moved her body to put her hands at the same height, again, this movement becomes the act of aiming the equation through it, it is to let it correspond to its request, which is exercised on it without any representation (Merleau-Ponty, 2006).

Thus, we attribute our perception of the gestures, Leo's and Paula's movements, our results to understand the Persian proverb "the fish will be the last to discover water." In this case, both Paula and Leo showed little evidence

of being aware and executing the gestures reflexively, in the sense of purposefully performing such gestures. However, we understand that it is up to each teacher to perceive their gestures and movements while teaching, since they can contribute to mathematics teaching, materialising through the body what is expressed verbally. This, then, may mean another possible way of communicating, of favouring the meaning given to what is taught, of constituting mathematical knowledge on the part of the student. In this article, it was up to us to make that clear. Therefore, we started to see the sea, the "water" that surrounds them.

THE SEA (FINAL CONSIDERATIONS)

This article aimed to investigate the mathematics teacher's body language when teaching equations. In this investigative aspect, we intended this movement because we consider that while classical psychology considers that the body can be understood as an opening to the perspectives of the world (SEIDEL, 2013), it also understands it under a separation from consciousness, since it places the observer before the observed object, in an objective space. In this sense, "[...] as for my body, I do not observe myself; to be able to do so, it would be necessary to have a second body [...]" (Merleau-Ponty, 2006, p. 135). In this perspective, it would be difficult for teachers to understand how much their body participates in their practice. Thus, it is important to perceive the bodily movement of those teachers when teaching equations, so that they and others who will read this study understand how much their gestures, their movements are shown, and will possibly give other meaning(s) to the constitution of their students' mathematical knowledge.

Thus, we assume that the body language of the mathematics teacher who teaches equations is another participant in the process of constitution of knowledge of equation solving. The body language of mathematics teachers, which is produced in speech, suggests that the conception of equality becomes a gesture of balance, with hands at the same height, for example. That a member of the equation, indicated as one side of the body, is taken and calculated, in general terms, with the same procedures on the other side. In general, equality, the balance expressed by the teachers' body, by their hands, either a man (Leo) or a woman (Paula), with little didactic experience (Leo) or a good experience (Paula), in England (Leo) or in Brazil (Paula), in 2017 (Leo) or 2019 (Paula), are significant in terms of the teaching practice of each one and are also made a possible way for the constitution of the mathematical knowledge. In this sense, we do not establish a single bodily posture and do not fail to consider the

subjectivities of each one. However, in the same mathematics class format, we perceive relevant bodily movements that enable the materialisation of speech by the body and the meaning that the teacher attributes to each moment of equation solving. Thus, gestures are similar, since they are impregnated with the meaning given to the equation, which, although having multimeanings (Ribeiro, 2008a, 2008b), assumes a conductive line that, according to Pérez and Marin (1928, *apud* Miguel, Fiorentini, & Miorin, 1992), is conceived as equality that expresses a relationship between quantities.

Hence, based on our theoretical framework, we understand the body as our "vehicle of being in the world" (Merleau-Ponty, 2006), which allows the meanings given to the equation and its forms of resolution to show themselves, to manifest themselves in the act of teaching. We assume the importance of analysing the teachers' gestures and understanding how they present themselves in mathematics class, aiming at opening to perception and embodied cognition, at improving our understanding as teachers of the mathematical ideas we express through our body and the possible meanings students may give them. From Merleau-Ponty's (2006) perspective and according to our understanding, perception can transform experiences with the world into shared movement. As teachers, we can understand and intentionally launch ourselves to the idea of a class whose mathematical meanings are fostered by attention to our gestures and movements during our classes.

AUTHORS' CONTRIBUTIONS STATEMENTS

M.R. participated in all stages of the study. In theoretical terms, he mainly discussed the ideas coming from philosophy. Concerning the data production stage, he was the only one responsible for the data production carried out in Brazil. D. F. also participated in all stages of the study, however, in theoretical terms, mainly discussed the ideas coming from psychology. Regarding the data production stage, he was the sole responsible for the production of data in the UK.

DATA AVAILABILITY STATEMENT

The authors agree to make their data available at the reasonable request of a reader. It is up to the authors to determine whether a request is reasonable or not. Thus, the data supporting the results of this study referring to the Brazilian teacher will be made available by the corresponding author, MR, upon reasonable request, while the data corresponding to the British teacher will be made available by author, DF, upon reasonable request.

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