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From Work Tools to Didactic Resources for Mathematics Classes: Experiences in and for Technical Courses

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ABSTRACT

Background: High school technical vocational education (Educação Profissional Técnica de Nível Médio - EPTNM) is an educational modality provided by the LDB and responsible for the qualification of part of the workers at the national level. **Objective:** to articulate theoretical assumptions of mathematics education in pedagogical action with students of EPTNM, and discuss the role that work tools can play in mathematics education of students of technical courses integrated to high school. **Design:** qualitative research, approaching a bibliographic research. **Settings and participants:** 96 texts published in the national meetings of mathematics education proceedings from 2010 through 2019. **Data collection and analysis:** to collect the data, we downloaded the proceedings, identified the files containing specific descriptors, performed a reading located where those terms were used and then selected the articles that presented proposals or experiences related to the EPTNM. For data analysis, we started from gross units of information (excerpts from the articles) for the categories of information (defined by the concept of transparency, following the theoretical framework). **Results:** Based on the notion of transparency of didactic resources, we found that the visibility of the instruments gives tasks an interdisciplinary character, since students become familiar with working tools. On the other hand, the invisibility of resources is favourable to pedagogical action, mainly by promoting intellectual work from manual work. **Conclusions:** regardless of the level of transparency of the working instrument, its use as a didactic resource is recommended for mathematics education at EPTNM.

Keywords: Teaching resource; Work; Technical course; High school technical vocational education.

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De instrumentos do trabalho a recursos didáticos para aulas de Matemática: experiências em e para cursos técnicos

RESUMO

Contexto: A Educação Profissional Técnica de Nível Médio (EPTNM) é uma modalidade educacional prevista pela LDB e responsável pela formação de parte dos trabalhadores em âmbito nacional. **Objetivo:** articular pressupostos teóricos da Educação Matemática na ação pedagógica com estudantes da EPTNM, de modo a discutir o papel que os instrumentos do trabalho podem assumir na Educação Matemática de alunos de cursos técnicos integrados ao Ensino Médio. **Design:** pesquisa qualitativa, aproximando-se de uma pesquisa bibliográfica. **Ambiente e participantes:** 96 textos publicados nos anais dos Encontros Nacionais de Educação Matemática, realizados entre 2010 e 2019. **Coleta e análise dos dados:** para coleta, realizamos download dos anais, identificamos os arquivos que continham determinados descritores, realizamos uma leitura situada onde esses termos foram empregados e, então, selecionamos os artigos que apresentavam propostas ou experiências relativas à EPTNM. Para análise dos dados, partimos de unidades brutas de informação (excertos dos artigos) para as categorias agrupadas de informação (definidas pelo conceito de transparência, seguindo o referencial teórico). **Resultados:** A partir da noção de transparência dos recursos didáticos, verificamos que a visibilidade dos instrumentos confere às tarefas um caráter interdisciplinar, uma vez que os estudantes passam a se familiarizar com ferramentas do trabalho. Já invisibilidade dos recursos também é favorável à ação pedagógica, principalmente por promover o trabalho intelectual a partir do trabalho manual. **Conclusões:** independentemente do nível de transparência do instrumento do trabalho, sua utilização como recurso didático é recomendável para a Educação Matemática na EPTNM.

Palavras-chave: Recurso Didático; Trabalho; Curso Técnico; Educação Profissional Técnica de Nível Médio.

INTRODUCTION

As a Brazilian public policy, vocational education had its initial milestone in 1909, when Decree No. 7.566 of September 23 (Brasil, 1909) created nineteen apprentice schools. Since then, the institutions responsible for labour qualification in Brazil have had different configurations, adjusting to national demands and the interests of government officials. Currently, vocational and technological education (EPT) is a modality provided for in the Law of Guidelines and Bases of National Education – LDB (Brasil, 1996), aiming to prepare citizens for the exercise of professions, contributing to their being able to enter and act in the world of work and life in society.

Also according to the LDB, the EPT can be combined with high school, generating an intersection area called high school technical vocational education (Educação Profissional Técnica de Nível Médio - EPTNM). In this modality, the general preparation for work and professional qualification may be developed in an integrated, concomitant format or a subsequent form, within the high school institutions or in cooperation with institutions specialised in vocational education (Brasil, 1996). In the articulated model, the students can access basic education and professional qualification in the same institution (integrated offer) or in different institutions (concomitant offer).

According to the technical summary of the school census of the Ministry of Education, Vocational and Technological Education, the course "is composed predominantly of students under 30 years of age, who represent 78.8% of enrollment" (Brasil, 2020, p. 41). While the number of general high school enrollments decreased by 7.56% (from 8,076,150 to 7,465,891), the number of students attending technical courses integrated with high school increased by 28.31% (from 485,685 to 623,178). Considering both formats of technical courses, we have that approximately one in eight high school students is attending a vocational course, which reinforces the relevance of this field not only for research in mathematics education but for investigations in other areas of Brazilian education.

In pedagogical terms, the idea of *work as an educational principle* is the foundation for EPTNM, "having its integration with science, technology, and culture as the basis of the political-pedagogical proposal and curriculum development" (Brasil, 2012, Art. 6). This assumption is based on the construction of knowledge articulated to the idea of work as an intentional transformation of nature by human beings (Marx, 1996), so that theories, techniques, and tools can be seen as a synthesis of human action in the world[1].

Although the idea of work as an educational principle is in different guiding documents and course projects, mathematics teachers still seem to be unable to assign meaning to what is expected of their discipline (Silva & Oliveira, 2018). In this scenario, we conducted a doctoral research that aimed to articulate theoretical assumptions of mathematics education in pedagogical action with students of the EPTNM, with a view to a comprehensive education, aligned with the guiding principles of this modality (Sá, 2021). This article,

[1]At this point, it is worth commenting that the word "trabalho" ("work" in English) has its origin in the word "*Tripallium*" - name of a torture device formed by three pieces of wood (therefore, *tri-pallium*) (Albornoz, 2017). Hence, we realise an intimate relationship between work and the tools. We also emphasise that it was because of this origin that a correctional meaning was attributed to the work for a long time.

which stems from the several conclusions drawn from that doctoral dissertation, deals with the role that work instruments [2] can play in the mathematics education of students attending technical courses integrated with high school. This study takes up the main political-pedagogical assumptions of the EPTNM and relates them to mathematical tasks in the classroom. These activities were taken from texts published in the proceedings of the National Meeting of Mathematics Education (ENEM) held in the cities of Salvador (2010), Curitiba (2013), São Paulo (2016), and Cuiabá (2019).

To define the set of texts with which we would dialogue, we downloaded the 6,094 articles published in the proceedings, identified the files containing the words "técnico," "técnica" (technician) or "profissional" (professional) not only in the title and in the abstract, but in the body of the text, performed a reading located where those descriptors were used, and then selected the articles that presented proposals or experiences related to the EPTNM. With this protocol, we constituted a corpus with 96 publications, representing 1.58% of the total initial files.

To organise the experiences and dialogue with the theoretical frameworks, we developed an inductive analysis (Lincoln & Guba, 1985), based on gross units of information, represented by excerpts from the ENEM articles, for the clustered categories of information, defined by the concept of transparency of Meira (1998), Adler (1999; 2000), and Moyer (2001). Throughout the process, we sought to produce knowledge *from* the authors' narratives, trying to operate considering teachers as producers of knowledge, in a non-hierarchical view of the school know-how by the university know-how. From this perspective, it was up to us to systematise and socialise similar results in categories and discuss these tasks in the final section of this article.

Next, before presenting the experiences and our underlying reflections, we introduce the idea of work as an educational principle and show, still at the theoretical level, how the instruments for work can promote mathematics mediated teaching.

[2] We understand tools and instruments as synonyms that mean elements interposed between the worker and the environment, capable of enhancing the transformation of nature (Oliveira, 2009). Likewise, we will understand activities and tasks as synonyms that translate into teaching situations proposed by teachers and performed by students. We will toggle those uses only to avoid repetitions in the text.

THE WORK TOOLS: THEIR POSSIBILITIES FOR MEDIATED TEACHING AT THE EPTNM

In 2020, the plot of the samba school (escola de samba) Estácio de Sá, in Rio de Janeiro, Brazil, was "Pedra/Stone”. To justify her choice, carnivalesque Rosa Magalhães argued that “the stone is present in several parts of the history of our country: the first rock inscriptions, as guardians of prehistoric animals; as a source of wealth of the Portuguese kingdom; as a dwelling place for indigenous spirits and the unbridled exploitation of wealth on their lands” (Liesa, 2020, p. 9). At the beginning of the parade, the fifteen dancers of the front commission of Estácio de Sá showed how much human beings have changed themselves as they changed nature with the aid of the stone. Thus, they exemplified in the Marquês de Sapucaí that, throughout human existence, the transformation of nature was a requirement and a result of the transformation of the essence of the human being – an idea defended by Karl Marx in *Capital* (1996).

To understand better this relationship between Rosa Magalhães’ proposal and Marx’s principles, let us take the classic example *The Part Played by Labour in the Transition from Ape to Man*, by Engels (1876). According to the text, the ape’s hand, which became free when the animal started to use the upright position, ensured more flexibility, agility, and skill. From a Darwinian perspective, this characteristic was transmitted by genetic inheritance and improved from generation to generation. With this, the apes began to build roofs and wield clubs, increasing the specialisation of its hand. Thus, we concluded that by modifying the external nature, the ape modified its own nature; or, in other words, “the hand is not only the organ of labour; it is also the product of labour.” (Engels, 1876, p. 3).

From Engels (1876), we can observe that, in this mutual transformation between human being and nature, the instrument/tool brings with it the function for which it was created and used, thus representing the synthesis of human action in the world. Allegorically, the twenty-six wings (alas) of the samba school Estácio de Sá exemplified such ideas through the costumes that showed stones used in prehistoric and baroque arts, the treasuring of precious stones and the human exploitation in Carajás and Serra Pelada mines. In all those wings, Rosa Magalhães showed that the different ways of recognising the stone stem from a historical process underlying the human being’s performance in society.

In political-pedagogical terms, the proposal to adopt *labour as an educational principle* comes precisely from the need to value the role of the individual in society, building instruments and formulating theories:

> The individuals humanise the world and humanise themselves from the moment they act in this world by producing instruments and objects for their existence. The individuals *intentionally* humanise themselves and the world from the moment *they recognise themselves in this world as the creators of those forms and contents*, consciously determining their directions (Nascimento, 2014, p. 126–127, emphasis added).

From the psychological point of view, this process of humanisation is not straightforward, as the stimulus-response logic is. On the contrary, for some Soviet researchers, such as Vygostky, it is a relationship mediated by signs and instruments. In this context, the signs can be understood as elements that express other objects and situations - such as the word book, number five, and a traffic sign - and that are internally oriented, a way to direct psychological influence to the domain of the individuals themselves. On the other hand, the externally oriented instruments would be the elements interposed “between the worker and the object of his/her work, expanding the possibilities of transformation of nature.” (Oliveira, 2009, p. 29).

This Vigostkian relevance of the tools for human activity, the third theme indicated by Wertsch (1993), cited in Cole and Scribner (1991), comes from the psychologist’s connection to the historical-dialectical materialism of Marx and Engels:

> The Marxist theory of society (known as historical materialism) also played a key role in Vygotsky’s thinking. According to Marx, historical changes in society and material life produce changes in “human nature” (consciousness and behaviour). Although others had repeated this general proposal, Vygotsky was the first to try to correlate it with actual psychological issues. In this effort, he creatively elaborated Engels’ conceptions of human labour and the use of instruments as the means by which human beings transform nature and, in doing so, transform themselves (Cole & Scribner, 1991, p. 10–11).

Given the contribution of the instruments for the humanisation of the human being -and for their mediated learning-, some curriculum documents began to recommend their use in the school context of the EPTNM. For example, according to the LDB, education provided with technical and professional emphasis will consider "the inclusion of practical labour experiences in the productive sector or simulation environments, establishing partnerships and making use, when applicable, of instruments established by the legislation on professional learning." (Brasil, 1996, Art. 36, §6, item I). On the other hand, the National Curriculum Guidelines for the EPTNM proclaim that the structuring of the High School Technical Vocational Education courses, guided by the conception of a technological axis, implies considering "the technological matrix, covering methods, techniques, tools, and other elements of technologies related to the courses" (Brasil, 2012, Art. 13, item I).

In 2018, the National Council for the Federal Network of Vocational, Scientific, and Technological Education Institutions (Conif) published a document entitled "Diretrizes Indutoras para a oferta de cursos técnicos integrados ao Ensino Médio na Rede Federal de Educação Profissional, Científica e Tecnológica" (inductive guidelines for the provision of technical courses integrated with high school in the Federal network of vocational, scientific and technological education), ideated by the Education Leaders Forum. One of the 23 inducing guidelines proposes to

> Ensure the realisation of vocational practices that enable the students to contact the world of work and ensure the theoretical-practical formation intrinsic to the technical training profile, through professional activities, intervention projects, experiments, and activities in special environments, such as laboratories, workshops, pedagogical companies, ateliers, among others (Conif, 2018, Diretriz 7, p. 16).

In short, our research recognises the work instruments as a synthesis of the human beings' action in the world, seeing them as an essential element for the humanisation of individuals and their learning. Thus, considering the recommendation to incorporate the instruments in the school context of the EPTNM, we will now reflect on their impact on mathematical learning.

THE TEACHING MATERIALS IN THE CONTEXT OF MATHEMATICS EDUCATION

The teacher's daily life is permeated by resources such as textbooks, videos, educational software, and laboratory scripts to assist the teaching and/or learning process. However, the nature of the course and the teachers' education and beliefs involved influence the adoption of a given didactic resource. For example, in a longitudinal study conducted with U.S. mathematics teachers, Moyer (2001) identified two major categories for the use of teaching materials in the classroom: "fun math" and "'real math." While in the first group, many of the teachers explained that teaching mathematics with resources (such as games) made learning more fun, in "real mathematics" classes, traditional materials such as books, paper, and pencils gained prominence, since the activities focused on rules, procedures, and algorithms.

In our research, we understand didactic material as "any useful instrument to the teaching-learning process" (Lorenzato, 2006, p. 18). For the author, however, the didactic materials make up only one of the parties responsible for student learning. In his words, "the DM [Didactic Material] never goes beyond the category of auxiliary means of teaching, the methodological alternative available to the teacher and the student and, as such, is no guarantee of good teaching or significant learning and does not replace the teacher" (*ibid*, p. 18). In mathematics, we can say that "manipulative materials are objects designed to represent explicitly and concretely mathematical ideas that are abstract" (Moyer, 2001, p. 176).

According to Lorenzato (2006), the didactic materials can perform various functions according to the objective to which they are meant. Therefore, the teacher must ask him/herself what he/she wants from this resource. In other words, the researcher argues that the way to use each didactic material depends strongly on the teacher's conception of mathematics and its teaching. Internationally, Moyer (2001) pointed out factors that contributed to the popularity of manipulable materials for teaching mathematics. In the national territory, Fiorentini (1995) described some ways of seeing and conceiving mathematics teaching in Brazil, and highlighted the role of teaching materials in the student-teacher-mathematical know-how triad. In the following paragraphs, we will summarise the ways of conceiving mathematics teaching systematised by Fiorentini (1995), citing the Classical Formalist, Empirical-Activist, Modern Formalist, Technicist, Constructivist, and Socio-cultural trends.

Until the late 1950s, mathematics teaching was mostly guided by a *Classical Formalist* trend, characterised by an emphasis on the ideas and forms of Classical Mathematics, following a Euclidean model and a Platonic perspective (Fiorentini, 1995). Under this conception, the teacher was considered the knowledge transmitter, whereas the student assumed a receiver's passive position. The didactic materials were summed up in books and instructional manuals to *repeat* and *memorise* the reasoning dictated by the teachers.

To overcome the *Classical Formalist* trend and include the nature of the developing child, the *Empirical-Activist* trend emerged, based on the active pedagogy (Fiorentini, 1995). Here, from a knowledge transmitter, the teacher shifts to being a mentor or mediator, since, from this new perspective, students take an active position in the didactic process. Still, this trend maintains the Platonic conception, which makes the main function of teaching materials lead the student to *access* or *discover* the pre-existing knowledge.

As the name suggests, the *Modern Formalist* trend resumed the *Classical Formalist* trend, especially regarding formal language and mathematical rigour. The title of "modern" was guaranteed by the replacement of the Euclidean model by the formal language of contemporary mathematics (Fiorentini, 1995). Associated with the context of the Second World War and the participation of Brazilians in the international movement of reformulation and modernisation of the mathematics curriculum (Movement of Modern Mathematics), teaching, according to this trend, was again centred on the teacher.

Pedagogical technicism was a U.S. trend of the 1960s and 1970s that sought to optimise school results, hoping to make it "more efficient." To this end, the *Technistic* trend was characterised by the implementation of "special teaching and school administration techniques" (Fiorentini, 1995, p. 15). With influences from Behaviorism, teaching happened through the so-called programmed instruction, with the computer as the primary didactic material. In a crossover with the *Modern Formalist* trend, technicism gave rise to a *formalist technicism*, perceived in manuals by Sangiorgi, Scipione, and Castrucci, with definitions followed by exercises of the type "solve [the problem] according to the model". Then, in search of an even more pragmatic character, the *mechanistic technicism* abandoned the theoretical understanding and focused only on rules and algorithms. Therefore, it made use of teaching materials such as games and other memorisation activities.

The *Constructivist* trend for the teaching of mathematics arose from Piagetian genetic epistemology, denying the rationalist theory that sustained formalism and the empiricist theory that determined the existence of knowledge exclusively from experiences and senses. In didactic terms, "it replaces the mechanical, mnemonic, and associationist practice in arithmetic with a pedagogical practice that aims, with the aid of actual materials, at the *construction* of the structures of logical-mathematical thought" (Fiorentini, 1995, p. 19, emphasis added). In this case, we highlight that it was the actual materials that gained strength as didactic materials in this trend.

Finally, the *socio-cultural theory*, initiated in the last decade of the 20th century, reveals the contradictions between school mathematics and that mobilised by individuals in contexts related to life and work. In this case, the teaching is based on the discussion of problems that relate to the reality of the students, and the didactic materials can come from games, handicrafts, experiences between indigenous and landless people, activities of the civil construction, experiences at fairs etc.

From the above, we observed the influences of beliefs on mathematics and teaching in conducting activities with didactic resources. In an approach close to the socio-cultural theory of mathematics, Adler (2000) proposes a hybridised practice for teaching, coordinating topics of academic mathematics, and contextualised contents in life and living in society. Thus, the researcher argues that teaching materials can also come from academic and daily mathematical practices. That is, here we identify another reinforcing argument for the use of work instruments for classes at EPTNM. With this, we can include in the set of didactic resources not only chalk, a film, and a geometric solid, but also manuals and equipment adopted in professional activities.

Besides reflecting on the academic-social approach to mathematics in the classroom, Meira (1998), Adler (2000), and Moyer (2001) explore the concept of *transparency* that, for us, is enlightening for classroom practices, and particularly in relation to resources and their use:

> [...] Resources in school mathematics practice need to be seen to be used (visible) and seen through to illuminate mathematics (invisible). Transparency is not an inherent feature of the resource, but rather a function of its use in practice, in context. As resources are harnessed to support and enable learning in a hybrid practice like school mathematics, their transparency becomes more complex. [...] (Adler, 2000, p. 214)

According to Adler (1999; 2000) and Meira (1998), transparency is not an inherent feature of the resource, but a function of its use in practice. In this sense, using equipment in a professionalising discipline can constitute an invisible resource and the same material, in a mathematics class, becomes visible. Moreover, in the same activity, a given material may be visible to a student who has never manipulated it yet, while invisible to another who has already done it.

SOME EXPERIENCES WITH STUDENTS FROM TECHNICAL COURSES

From Lorenzato's (2006) understanding that didactic materials can be any tool considered useful for the teaching-learning process, let us include in that group work tools such as multimeters, temperature sensors, storage silos, regulatory standards, and software for architectural projects. In this section, we will bring experiences that have adopted those work tools as visible (that must be noticed to be used) and invisible resources (that function as a mediating element for the construction of mathematical thinking).

As mentioned before, we recognise that teachers are producers of knowledge and, at first, we sought only to systematise and socialise similar results in the form of categories. Our reflections, emerging from these tasks, will appear in the final section of this article, where we will bring new notes for the pedagogical practice of mathematics at the EPTNM. For categorisation purposes, we considered the teacher's intentionality, trying to observe in the activity report whether the tool had a prominent role (visible resource) or was a means to teach the content proposed (invisible resource).

Work tools as invisible resources

This subsection will address experiences in which work tools were presented as invisible resources for the activity, functioning as a mediating element for constructing mathematical thinking. However, we emphasise that some of those resources may have been visible to a student who had not come in contact with it before. In this case, to organise the text, we considered the teacher's intentionality to make the tool invisible, treating it only as a means to teach the content.

The first experiences observed in this movement are related to the teaching of geometry, associated with the discipline of Technical Design, using hand-made projects and software such as Autocad and Sketchup. The reports used computer graphics with the argument of bringing the pedagogical proposal closer to the professional reality of the students of the Technical Course in Constructions of the Federal Institute of Minas Gerais (Gomes & Lopes, 2016) and the Technical Course in Electrotechnical Technology of the Federal Institute of Espírito Santo (Marques & Chisté, 2016).

Gomes and Lopes (2016) analyse the recent changes in the teaching of Technical Design, determined by new technologies. In the article, the authors reflect on traditional methods to build knowledge of the graphic expression and present considerations about experiences with various didactic resources. In the end, Gomes and Lopes (2016) show how the socialisation of experiences with the gradual insertion of technologies in the classes of the technical course of Buildings in federal institutions has helped teachers overcome dilemmas and obstacles.

Marques and Chisté (2016) report an experience with students of the Technical Course in Electrotechnical Technology Integrated with High School, members of the Research Group on Technical Design. The researchers began their activity with the poem "O Desenho," by Cecilia Meireles. They then screened Charles Chaplin's film Modern Times to foster a critical view of technicism, and a part of a Walt Disney Production cartoon, "Donald in Mathmagic Land," to associate the content proposed with different areas of knowledge. Finally, the students projected a golden image in AutoCad and, inspired by Antoni Gaudi's works, they produced a mosaic. In the following activity, Marques and Chisté (2016) built a digital project of Villa Savoye (Le Corbusier), in three dimensions, with the aid of Sketchup software. The work was the same as the students had studied in a previous meeting, when they manually prepared a sketch of the Villa.

In view of the three articles synthesised, we perceive that Gomes and Lopes (2016) and Marques and Chisté (2016), in different ways, sought to build a teaching proposal for the Technical Design that should be interdisciplinary, historically contextualised, and that favoured the appropriation of systematised knowledge – aligned with Marx's (1996) and Engels's (1876) proposals:

> Given this historical review, we realise that it is important that the student not only learns techniques, as has been occurring in some technical courses, but also studies the context in which a particular technique emerged, its purpose, what it represented, and how important it is today and in its reality. (Marques & Chisté, 2016, p. 4)

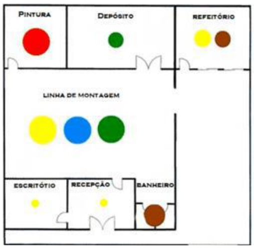
In the field of quantities and measures, Abadi, Reis, and Sad (2019) present an experience developed at the Federal Institute of Espírito Santo with students of the second period of the Technical Course in Occupational Safety, offered by the National Program for the Integration of Vocational Education with Basic Education for Youth and Adults (Proeja). In this experience, the instruments used were the Labour Regulatory Standards (Normas

Regulamentadoras do Trabalho - NR), which are requirements and procedures related to Occupational Safety and Health, of mandatory compliance by private and public companies and government agencies that have employees governed by the Consolidation of Labour Laws (CLT) and promulgated by the Ministry of Labour and Employment, through Ordinance No. 3214/78 (Brasil, 1978).

The activity aimed to promote the learning of mathematical contents of ratio, proportion, and scale based on the NR-5, which deals with the Internal Commission for Accident Prevention, and NR-18, which establishes the Working Conditions and Environment in the Construction Industry. The practice consisted of constructing a risk map (Figure 1) and a model of a lift for the school space. From those pedagogical actions, the students perceived mathematics and its importance in a professional practice.

Figure 1

Risk map using drawing techniques. (Abadi, Reis & Sad, 2019, p. 6)



Tipos de Agentes	Cor	Proporção do Risco		
		Elevado	Médio	Pequeno
Físico	Verde	●	●	●
Químico	Vermelho	●	●	●
Biológico	Marrom	●	●	●
Ergonômico	Amarelo	●	●	●
Acidente	Azul	●	●	●

As the Regulatory Standards are mostly a textual tool, Abadi, Reis, and Sad's (2019) experience allows us to reflect on the role that language can play in this type of experience. Taking Frighetto and Zorzi (2016) as a reference, as we will detail later, we have that school mathematics and mathematics practised at work, despite preserving conceptual similarities, may show differences in the field of language. In this sense, recognising those differences and incorporating professional practices in a hybridisation process (Adler, 2000) may represent different ways of producing mathematics knowledge in the school context. Therefore, it is important that teachers of a specific formation participate in the tasks that incorporate work tools because, in this way, the terminologies and internal meanings of professional practice in the context of mathematics classes can be preserved.

In algebra, in another experience, Antonello, Santarosa, and Ferrão (2019) presented, in the Mathematics class, a task to introduce the concept of function from an experimental procedure of the context of the Electrotechnical course using the Dias Blanco panel[3] and the multimeter[4]. The participants were first-year students of the Electrotechnical Course Integrated with High School of the Industrial Technical College of Santa Maria. The planning and validation of the proposal included the teachers who taught Eletrotécnica I (electrotechnical technology) and Mathematics, respectively.

The intervention was performed in three moments. First, the activity was presented to the students, and they calculated the electrical resistances (R) for conductive wires, varying the lengths (L) and the areas of the cross-sections (A), resuming the relationship $R = \rho.\frac{L}{A}$, where ρ is the resistivity that depends on the material of the wire. After that, the students were sent to the electronics laboratory where, in groups, they determined the value of the resistances experimentally, according to the parameters of the first task (Figure 2). Finally, in the computer laboratory, each group tabulated the empirical data in Excel and constructed the graphs according to the variations proposed, resulting in linear and rational functions.

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[3] Didactic material to study Ohm's law, the Wheatston Bridge, electrical resistance, and resistivity. Although the Dias Blanco panel was not originally a work tool, we maintained this experience due to the use combined with the multimeter.

[4] Tool that estimates electrical quantities. In a single device, the multimeter integrates several measuring instruments, such as voltmeter, ammeter, and ohmmeter, and may also include a capacimeter, frequency meter, thermometer, among other optional items.

Figure 2

Student manipulating Dias Blanco panel and multimeter. (Antonello, Santarosa & Ferrão, 2019, p. 6).

Despite the participation of the electrotechnical technology teacher, the experience shared by Antonello, Santarosa, and Ferrão (2019) points to a transparency of the tools used, especially when they show only the motivating aspect of the manipulation of the didactic materials:

> The experimental procedure, such as resistance measurements using the Dias Blanco Panel and the multimeter, was found to stimulate students' curiosity and motivated them in the development of activities. (Antonello, Santarosa & Ferrão, 2019, p. 6)

At the end of the report, the teachers-researchers conclude that the activity allowed students to formalise and adequately represent the concepts worked and establish some relationships between the disciplines of

Mathematics and Electrotechnical technologies, showing that learning was significant.

Work tools as visible resources

Based on Adler's (1999; 2000) notion of transparency, we will present in this subsection the work instruments that were used as visible resources for mathematics class in technical courses. In other words, we shared experiences in which the tools assumed a prominent role in the class and needed to be noticed for teaching to happen.

The first experience we bring is Matté's (2019), who, to address the content of functions, developed a situation of mathematical modelling in the Technical Course in Mechatronics of the National Service of Industrial Learning of Rio Grande do Sul (Senai/RS). In terms of instruments, the activity was based on experimental data collected by resistance temperature sensors, which can be classified into Negative Temperature Coefficient (NTC) and Positive Temperature Coefficient (PTC), according to how temperature variations react. In the first type, the resistance is inversely proportional to the temperature, and its value drops exponentially as temperature increases. The second presents a coefficient of variation of electrical resistance as a function of positive temperature, which indicates that its resistance increases exponentially as the temperature rises (Matté, 2019).

In the classroom, Matté (2019) prepared an experiment with a glass of hot water in the centre of a pot with ice. To analyse the temperature, the researcher and his students submerged the sensor and the mercury thermometer in the hot water of the cup. The sensor, at that moment, was connected to the probes of the multimeter, as shown in figure 3.

Figure 3

Experiment for data collection. (Matté, 2019, p. 5).

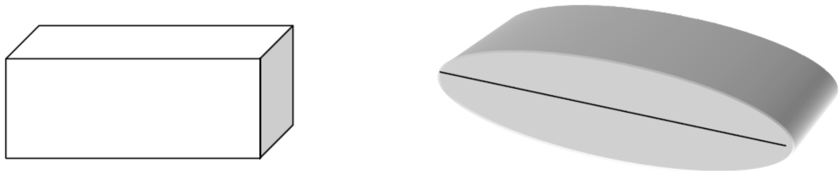
The students measured the resistance values during temperature variation from 80°C to 0°C, at intervals of 5 degrees. Then, they organised those data into tables, constructed graphs, and formulated a functional model that related the resistance values as a function of water temperature. According to the tables prepared, "the students realised that even though they were referring to sensors with the same nominal resistance, there were large differences between the resistance values collected and concluded that this owes to the percentage of tolerance existing between the sensors" (Matté, 2019, p. 5). With the model obtained, the author presents the community with another possibility of addressing the content of functions.

As noted above, the activity shared by Matté (2019) and Antonello, Santarosa, and Ferrão (2019) on the Dias Blanco panel and the multimeter has an interdisciplinary potential because it involves electrotechnical, physical, and mathematical content. However, what is the difference between the experience with the multimeter from the one that used sensors, to the point that in the first, the feature is classified as invisible, and in the second, the opposite happens? The answer to this question lies in the purpose of the tool. In Matté (2019, p. 4), the intention was to show "that there are no sensors presenting equal measures of resistance," which justifies the existence of tolerance percentages. In Antonello, Santarosa, and Ferrão (2019), the Dias Blanco panel and the bench multimeter were used only to collect experimental values. In other words, in Matté (2019) case, besides mathematical learning, knowledge about the manipulated instrument was also produced, which ensured its visibility in the task.

In another experiment, this time in the field of geometry, Nascimento and Nascimento (2010) developed a task that consisted of calculating how many trucks would be needed to fill a livestock silo (Figure 4). This situation was posed by one of the students of the first year of the Technical Course in Agriculture and Livestock Integrated with High School of the Federal Institute of Goiás, who was concerned with similar activities developed by his own family.

Figure 4

Form of livestock truck and silo. (Nascimento & Nascimento, 2010, p. 6).



Questioned by one of the students, Nascimento and Nascimento (2010) narrate that the class went to the mechanisation sector of the institution together with two mathematics teachers and a teacher from the specific area of the technical course. There, while the truck measurements were easily taken, thanks to its parallelepiped shape, the silo was a challenge to the group of students, because the curved region was not a semicircle and the tool was not polyhedral in shape either. After discussions, the future technicians chose to measure the central height and then another three measurements on each side, always equidistantly. From there, the average of the heights was calculated, transforming this structure into another parallelepiped.

Based on this experience, Nascimento and Nascimento (2010) defended interdisciplinarity and highlighted the importance of mathematical know-how in the daily lives of agricultural technicians, justifying with the numerous situations of weights, values, and measures.

> The work we carried out sought to point to a path that does not lead to the resolution but can, at least, minimise the problems in the professionals' daily lives. Thus, we believe that the activities of thinking mathematically in vocational education can become more pleasant and more productive if conducted through a methodology based on mathematical interpretation and interdisciplinarity. (Nascimento & Nascimento, 2010, p. 9)

In a proposal similar to that of Nascimento and Nascimento (2010), Frighetto and Zorzi (2016) investigated the possibilities of teaching geometry with students from the area of agriculture and livestock. The project "Geometry in poultry planning" was designed because the authors realised that the students mobilised to learn the mathematics presented in the discipline of Poultry Production and, in contrast, showed no interest in mathematical content, although both knowledges were close.

According to Frighetto and Zorzi (2016), to carry out the project, the students were organised into groups that should produce the two-dimensional and three-dimensional geometric representation of equipment and develop the structural and economic planning for each type of aviary. Through a seminar, the students compared the plans and analysed the specificities of the machinery. During the presentations, the students discussed the feasibility of each type of poultry farm and the economic, cultural, and structural possibilities of poultry farmers, in addition to the cost and benefit relationships of the prospective construction. The adequacy of the aviary to the standards provided for by the legislation and good working conditions were also discussed throughout the project.

From Frighetto and Zorzi's experience (2016), the visibility of the instruments used lies in the fact that they served as a model for mathematical calculations, replacing traditional geometric solids.

> Academic, school and field mathematics, in the case of the study, the mathematics practised in agriculture and livestock, more specifically, poultry farming, represent different language games that have similarities of families. The recognition of those differences and the inclusion of non-school cultural practices, here strongly represented by the technical discipline teacher, may represent other ways of producing mathematics knowledge in the school context. (Frighetto & Zorzi, 2016, p. 3).

Thus, Frighetto and Zorzi (2016) reflect that thinking about the pedagogical process from an interdisciplinary perspective requires dialogue and partnership between teachers, so that they recognise the different forms of manifestation of language games practised in their disciplines.

AS CONCLUSIONS: EMERGING REFLECTIONS AND CLASSROOM IMPLICATIONS

How and why teachers use teaching resources are complex issues. We agree with Moyer (2001) that the teachers' use of resources is "[...] intertwined with questions of knowledge of mathematical content, teachers' awareness of the conceptual structure of mathematics and the ability to teach this content to students [...]". In this sense, we restrict our gazes to the experience reports produced and published in the proceedings of the National Meetings of Mathematics Education (ENEM).

From what has been exposed throughout the article, the ethnocultural social theory, characterised by Fiorentini (1995), is very promising for our discussion, especially when it aligns with our Freirian ideology of valuing popular knowledge brought by students or professionals. Moreover, it dialogues with Adler's hybridisation proposal (1999), since the author proposes an articulation between academic and popular mathematics topics in the classroom. The experiences synthesised in this article also reinforce Lorenzato's (2006) hypothesis: it is not enough for a teacher to have a laboratory with didactic materials (that is, a mathematics teaching laboratory). For the researcher, when planning the class, the mathematics teacher must ask him/herself: "*Why a didactic material?" "When should I use it?"* and "*How should this material be used?"*

In the case of the technical courses, the *why* and *when* are based on the argument that the use of the work tools as mathematics didactic materials resumes the discussion about technological education, which, according to Frigotto, Ciavatta, and Ramos (2005), is present in the field of the technical vocational education when "it addresses knowledge associated with the technologies used in the production processes and, thus, can train people for the social and professional management of these technologies to occupy a specific space in the social and technical division of labour" (Frigotto, Ciavatta & Ramos, 2015, p. 41). Thus, bringing the work tool to class can imply an appreciation of the role of the human being in society and, therefore, walk towards assuming work as an educational principle.

The discussion on *how* a specific material will be used is anchored in the teacher's objectives and the notion of transparency of didactic resources developed by Adler (1999; 2000). In general, the visibility of a resource can be seen, at first, as something negative for the mathematics education of apprentices. Let us take, for example, a first situation of the use of the geoplane to discuss the properties of convex polygons. The novelty of the geoplane resource, *per se*, will make the material visible, delaying the start of mathematical discussions and sometimes diverting the focus of students. For this reason, we recommend that the teacher should allow a moment of manipulation and familiarisation of the resource before each task, to reduce its visibility and direct students' eyes to the elements that are intended to be discussed.

When we operate within the scope of EPTNM, the visibility of didactic resources, initially considered harmful because they distract the students, becomes potential when the intervention is with students from technical courses and uses work tools. In this case, the visibility of resources may be favourable to interdisciplinarity and professional qualification, mainly because, through manipulation, students become familiar with work tools. On the other hand, the invisibility of resources is also welcome, especially if we consider that it promotes intellectual work from manual work, ensuring the "overcoming of the bourgeois educational conception that is guided by the dichotomy between manual and intellectual work and between vocational instruction and general instruction" (Frigotto, Ciavatta & Ramos, 2015, p. 41). Therefore, we conclude that, regardless of the level of transparency of the work tool, its use is recommended for mathematics education in professional and technological education.

To conclude, we must consider that the didactic materials adopted in mathematics classes were, *a priori*, instruments for work. Thus, because they are unconventional objects, authors such as Lorenzato (2006) and Adler (2000) highlight the difficulty underlying those resources, as they are not self-explanatory objects, i.e., mathematics is not shining through them. This makes the dialogue between mathematics teachers and the specific training of the technical course even more important. In this aspect, it is still necessary to consider that the tools must be handled under the supervision of a specialist, otherwise, they could endanger the safety of students and teachers involved in the activity.

We reinforce that, to make the discussion more objective, we chose not to present all the experiences identified in the preliminary survey. Therefore, we strongly recommend that readers seek in the proceedings of the ENEM other teaching proposals, expanding the possibilities of approach that we present in this production. We also invite readers to know the other doctoral research results that gave origin to this article. In the doctoral dissertation, we also discussed three ways of seeing and conceiving the role of mathematics in the integrated curriculum of professional education, and began a debate on the mathematical formation of young people and adults who are inserted in a society marked by labour and social security reforms, increased unemployment, uberisation processes, and growth of informality.

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AUTHORSHIP CONTRIBUTION STATEMENT

L.C.S. conceived the idea of this article and collected and analysed the data presented. L.C.S., V.G. and A.J. discussed and collaborated in the structure of the analyses and the final formulation of this article.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study are only available for consultation at REMEP – Repository of Mathematics Education Experiences in Professional Education through the link http://www.sbembrasil.org.br/remep/remep.html. The authors make their data available to the public free of charge under a Creative Commons BY-NC-ND license, that is, as long as they credit the repository, but without being able to change it in any way or use it for commercial purposes.

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