English and Taiwaneses upper secondary teachers' approaches to the use of GeoGebra

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RESUME

The idea of the integration of dynamic geometry and computer algebra and the implementation of open-source software in mathematics teaching underpins new approaches to studying teachers' thinking and technological artefacts in use. This study opens by reviewing the evolving design of dynamic geometry and computer algebra; teachers' conceptions and pioneering uses of GeoGebra; and early sketches of GeoGebra mainstream use in teaching practices. This research has investigated English and Taiwanese upper-secondary teachers' attitudes and practices regarding GeoGebra. More specifically, it has sought to gain an understanding of the teachers' conceptions of technology and how their pedagogies incorporate dynamic manipulation with GeoGebra into mathematical discourse.

Keywords: GeoGebra. Geometry. Algebra. Mathematic.

As abordagens do uso do GeoGebra por professores secundários ingleses e taiwaneses

RESUMO

A idéia de integração de geometria dinâmica e álgebra computacional e a implementação de software livres no ensino de matemática sustenta novas abordagens pedagógicas para estudar o pensamento e os artefatos tecnológicos em uso. Esse estudo abre uma revisão envolvendo o design da geometria dinâmica e a álgebra computacional; a concepção dos professores e os usos iniciais do GeoGebra; e exemplos do GeoGebra usados na prática. Essa pesquisa tem investigado as atitudes e práticas dos professores secundários Ingleses e Taiwaneses considerando o GeoGebra. Mais especificamente, tem procurado obter a compreensão das concepções dos professores sobre a tecnologia e como as suas pedagogias dinâmicas incorporam manipulação com o GeoGebra no discurso matemático.

Palavras-chave: GeoGebra. Geometria. Álgebra. Matemática.

INTRODUCTION

Algebra and geometry are two core strands of mathematics curricula throughout the world and are considered the 'two formal pillars' of mathematics (ATIYAH, 2001). It is therefore not surprising that they have been specifically targeted by the field of technology (SANGWIN, 2007). Many researchers consider mathematics education as one

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of the earlier education fields to introduce technology as an assistant tool in classrooms (PAPERT, 1980; NOSS; HOYLES, 1996).

The major application of technology in mathematics education is the integration of mathematical software in teaching practices. In respect of geometry, the most widely used computer applications, known as Dynamic Geometry Software (DGS) and include, Cabri-géomètre and Geometer's Sketchpad (GSP), etc. One important feature of DGS is the drag mode, encouraging interactions between teachers, students and mathematics (JONES, 2000). The drag mode can be used to explore and visualise geometrical properties by dragging objects and transforming figures in ways beyond the scope of traditional paper-and-pencil geometry (LABORDE, 2001; RUTHVEN, 2005). DGS also has options to visualise the paths of objects as they move. For algebra, the most widely used applications are known as Computer Algebra Systems (CAS) and include programmes such as Mathematica, Maple and Derive. Some graphical visualisation and symbolic representations of algebraic expressions are implemented in CAS. Using the metaphor of the two 'formal pillars' of mathematics, geometry and algebra are afforded prominent positions especially at the secondary level (HOHENWARTER; JONES, 2007). However, the connection between geometry and algebra, namely 'the beam connecting the two pillars', is apparently missing, as evident that in some countries geometry and algebra are entirely separate in their curricula (ibid).

Ruthven (2008) researches the specific examples of computer algebra and dynamic geometry, and highlights 'three important dimensions- interpretative flexibility, instrumental evolution and institutional adoption-of the incorporation of new technologies into educational practices'. Although research into current technology use of computer algebra and dynamic geometry in teaching practices separate each sphere into distinct areas for study; I argue against this separation as there are areas overlapping algebra and geometry such as functions and graphs (DUBINSKY; HAREL, 1992). Examining both together has great educational implications and the connections between the two should not be ignored (EDWARDS; JONES, 2006). However, there is a gap in the literature dealing with this linkage between both fields and the use of technology. Despite an awareness of the need for a combination of DGS and CAS (HOHENWARTER; FUCHS, 2004), software designers struggle to combine them as there are completely different constructs in software design. GeoGebra could be seen as pioneering software, although whether or not it is successful in linking DGS and CAS still needs research as the supporting evidence is limited at present.

LIKING GEOMETRY AND ALGEBRA

Since CAS and DGS are two completely different mathematical constructs, the 'beam' is weakly constructed within current mathematical software. Historically, CAS programmes have mainly provided algebraic and numerical computations while DGS have provided graphical and dynamic demonstrations. Hohenwarter and Jones (2007, p.127) point out that "[…] forms of CAS have begun to include graphing capabilities in order to help to visualise mathematics; likewise, DGS have begun to include elements of algebraic symbolisation in order to be useful for a wider range of mathematical problems". In recent years, the need to integrate CAS and DGS has become apparent as Schumann and Green (2000, p. 337) claim that "[…] [t]here is a need for further software development to provide a single package combining the desired features [of DGS and CAS]". The recently published software GeoGebra by Markus Hohenwater (2004) explicitly links the two (as evidenced by the name Geometry and alGebra). This integration aims to provide unprecedented opportunities for mathematics education (SANGWIN, 2007). GeoGebra affords a bidirectional combination of geometry and algebra that differs from earlier software forms. The bidirectional combination means that, for instance, by typing in an equation in the algebra window, the graph of the equation will be shown in the dynamic and graphic window. Similarly, by dragging the graph, the equation changes accordingly (HOHENWARTER; FUCHS, 2004). A closer connection between the visualisation capabilities of CAS and the dynamic changeability of DGS is therefore offered by GeoGebra (HOHENWARTER; FUCHS, 2004).

THE CASE OF GEOGEBRA: Geometry + Algebra= GeoGebra?

Hohenwarter (2004) developed GeoGebra with the intention of supporting secondary mathematics teaching by bridging students' understanding of the connection between geometry and algebra. GeoGebra is a multi-platform dynamic mathematical software with its window divided into two parts (Fig. 1, HOHENWARTET, 2006) – 'Algebra window' (left side) and 'Geometry and Graphics window' (right side).

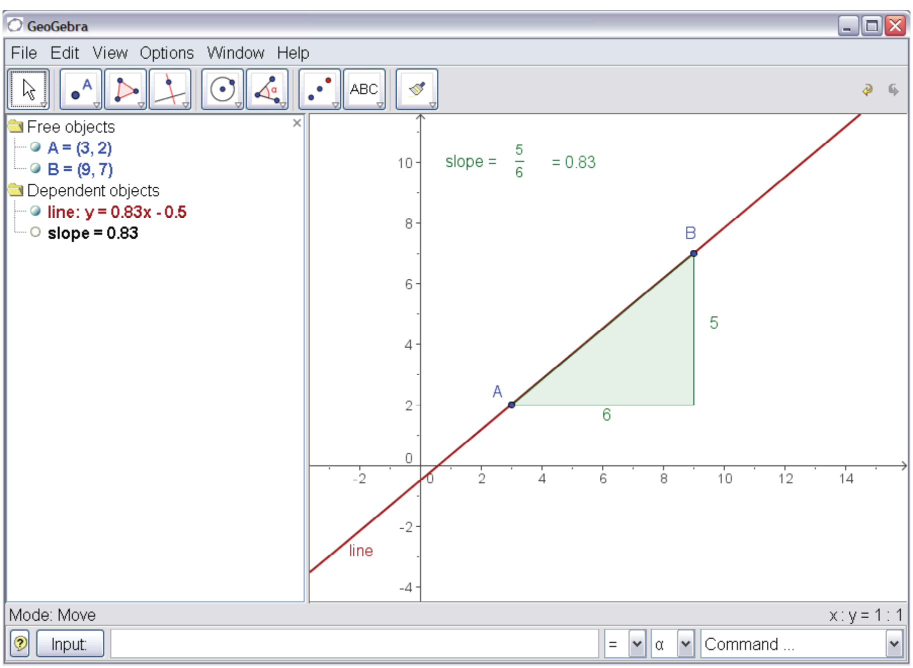

FIGURE 1 – GeoGebra window- Algebra window and Geometry and Graphic window.

On the one hand, GeoGebra is a dynamic geometry system, much like any other, which works with points, vectors, segments, lines, and conic sections. On the other hand, equations and coordinates can be entered directly into the grid at the bottom of the window (Fig. 1). It provides a bidirectional combination and a closer connection between visualisation capabilities of CAS and dynamic changeability of DGS.

Although most research attention on GeoGebra pertains to the teaching of geometry, GeoGebra has great potential in the teaching of algebra which lies mainly in functions and graphs. Functions can be defined algebraically and then changed dynamically afterwards (SANGWIN, 2007). For example, by entering the equation y=x2 the corresponding graph can be seen directly. The visualisation of two windows provides a connection between algebraic and geometric representations. It also works the other way around, by dragging the line or curve of the graph to change the equation. The change in the equation can be seen on the algebraic window. This encourages the investigation of the connection between variables in the equations and graphs in a bidirectional experimental way (HOHENWARTER; PREINER, 2007). This is particularly significant as it connects the crucial parts of multiple representations of mathematics, which are numerical, algebraic, geometrical and graphical; far beyond the reach of other DGS and CAS.

GeoGebra being open-source software may face criticism as it may be thought that free software lacks quality control compared to commercial software. Acknowledging that it would be insufficient to only provide free software without proper training and collegial support, the International GeoGebra Institute (IGI), therefore, is organised for supporting the collaboration between teachers and researchers and provides professional development for teachers (HOHENWARTER; LAVICZA, 2007). Since it is a non-profit organisation, funding has been sought mainly from Europe and the U.S. (HOHENWARTER et al., 2008). Teachers need a support system and professional development to improve their skills in teaching mathematics using GeoGebra (HOHENWARTER; PREINER, 2007). With this guidance and support from IGI, GeoGebra enhances teachers' willingness to integrate this new technology into their teaching practices. Despite its important ramifications, there has been little research into this area. It is hoped that this cross-cultural study will contribute to the IGI development of GeoGebra implementation in mathematic teaching in terms of pedagogical strategies and innovative ways of using GeoGebra in classroom practices.

One problem is that most mathematical software in mainstream use is commercial, which means the availability of software is subject to the school or student's finances. Therefore, some teachers or students who cannot afford to buy commercial software search for free software for their own purposes. There is positive potentiality and improvement offered by encouraging a collaborative community of open-source software users and voluntary software developers.

My rationale behind carrying out this inquiry into GeoGebra is not only due to its being open-source software with freely available support and online materials (SUZUKI, 2006), but also due to its unique capacity to integrate geometry and algebra. The significance of this research is not only the investigation of how GeoGebra usage can be incorporated into the teaching of either geometry or algebra alone, but more importantly, how the teaching of geometry and algebra can be linked using GeoGebra, thus contributing to a better understanding for students of their interrelationships. Studies such as this one will contribute to knowledge of GeoGebra-mediated teaching and the future pedagogical development. Nevertheless, one might ask the question: 'does GeoGebra offer sufficient linkage between geometry and algebra?; does it provide both functionalities of DGS and CAS?' I, therefore, aim to explore whether GeoGebra offers linkage between geometry and algebra in teaching practices.

COMPARATIVE STUDY

Recent research has indicated that culture influences the ways that teachers behave and inter-culture differences appears to be stronger than intra-culture differences (SCHMIDT et al., 1996; GIVVIN et al., 2005; ANDREWS, 2007). In particular, comparing eastern and western traditions with their respective Confucian and Socratic underpinnings can be enlightening as there are great differences in teacher beliefs and practices (LEUNG, 1995; TWEED; LEHMAN, 2002; ANDREWS, 2007). There is little comparative research of technology use in mathematics education, especially between Eastern Asian and Western countries (GRAF; LEUNG, 2001). Consequently, seeing how culture influences technology-mediated mathematics teaching is a pertinent issue.

There are large-scale quantitative studies such as TIMSS and PISA and small-scale qualitative studies. These studies highlight both similarities and differences between mathematics education in different cultural contexts in depth and in breadth. Large scale surveys are limited, however, by the fact that they often compare students' academic achievements without taking cultural and social factors into consideration (PRAIS, 2007). Quantitative studies such as TIMSS have also been reproached for their uncritical evaluation and for promoting globalisation over curricular and cultural diversity (ANDREWS, 2007). In contrast, small qualitative studies acknowledge cultural differences without attempts for generalisation. Particularly, when comparing East Asian and Western traditions with their respective Confucian and Socratic underpinnings, there is a significant difference between what are classically designed with the educational traditions (LEUNG, 1995; KAISER et al., 2005; TWEED; LEHMAN, 2002). In particular, Kaiser et al. (2005) proposed a framework analysing East Asian and West European cultural traditions in mathematics education. The framework by Kaiser et al. (2005) is adapted partially in terms of teaching styles as I undertake a small-scale qualitative study in countries that exemplify East and West with a focus on teachers' perspective and their use of technology in mathematics teaching. The Eastern country chosen is Taiwan since it is viewed as 'the one most often cited admiringly by educators in the West for the level of its students' educational achievements (BROADFOOT et al., 2000) and a high mathematics performing country in international comparative studies such as TIMSS and PISA (MULLIS, 2003; OECD, 2004; 2007). The Western country chosen for the study is England due to its contrasting educational system (BROADFOOT et al., 2000). A cross-cultural study between Taiwan and England will help obtain a sense of the commonalities and discrepancies of teachers' conceptions and practices in relation to GeoGebra use. I have chosen to research at the upper-secondary level (students aged 15-18) as this level is less researched but is a crucial step for bridging students' secondary mathematics learning and higher education. Therefore, the overarching research questions are: (1) What are the upper-secondary mathematics teachers' conceptions of technology in relation to GeoGebra in England and Taiwan? (2) In what manner is GeoGebra used for the teaching of geometry and algebra by Taiwanese and English teachers? (3)How are the teachers' conceptions of technology and GeoGebra related to their teaching practices in both countries?

METHODOLOGY

Since there is little research into GeoGebra usage to date, this study is exploratory (MARSHALL; ROSSMAN, 2006; CRESWELL, 2007). In brief, exploratory and multiple-case studies are my chosen methodology as the research focuses on this particular mathematical software, requiring specific teachers who utilise GeoGebra to teach upper-secondary level mathematics. Comparing and contrasting cases of teachers with interest in using GeoGebra from Taiwan and England provide a comprehensive understanding of how GeoGebra can be used in two very different cultural traditions, pedagogies and curricula.

I define mathematics teaching with the use of GeoGebra in Taiwan and England as the two main units of analysis. These have embedded cases of teachers who use this software. Moreover, within the units, four cases of English and Taiwanese teachers are studied to obtain evidence of their views on GeoGebra teaching practices. To achieve the comparability between cases and units, pre-determined themes: *teacher background, views on technology and GeoGebra, software comparisons* and *ways of using GeoGebra* have been set for research design and data. A complete set of data was collected from four school visits. All of the interviews were audio and video-recorded, lasted for approximately an hour each and took place in classrooms using either a laptop or a computer connected to an interactive whiteboard. Through observations during the interviews the teachers demonstrated ways they utilised the software. The interview data were collated and summarised for each of the four cases.

THE CASES

Jay

Jay has been teaching mathematics for twelve years in two senior high schools in Taiwan (students aged 15 -18). Jay's views about the incorporation of technology into teaching practices are generally more negative than positive. He inferred that both students and teachers viewed computers as a tool for entertainment rather than a learning or teaching tool. On the contrary, he held positive attitudes only with regard to GeoGebra. He claimed GeoGebra to be a convenient tool, which can be used for demonstrations, checking and visualisation as well as research. He mentioned that GeoGebra provides powerful capabilities that other software packages cannot offer: 'It is actually very good, especially when you want to do addition and subtraction in the grid coordinate system.' In general, Jay was discouraged by the current educational environment regarding technology and both students' and teachers' attitudes toward mathematical software in Taiwan. He also asserted that support from mathematical software was limited as human brains do the logical deduction. However, he emphasised that GeoGebra provides quality functionalities that encouraged his use of this software in his teaching practice. The salient categories emerged from the data are listed as follows:

Tool use	Graphing, calculations, visualisation, demonstration, dragging, checking, test and verify, teaching and research
Mathematics topics	Cartesian coordinate systems, both algebra and geometry
Teaching style	Textbook-oriented
Infrastructure	Laptop demonstration in the classroom

Li

Li has thirteen years of teaching experience at the upper-secondary level (Year 10-12 equivalence) in Taiwan. Since his first degree was in applied mathematics, he gained an interest in IT during his undergraduate study. He was enthusiastic about new technologies and volunteered to translate the Traditional Chinese version of GeoGebra. Moreover, he had been creative in using different software packages, free software in particular, and trying to use a combination of different open-source software to make teaching materials. He has written some journal articles comparing new, free software packages detailing how they might be incorporated into mathematics teaching for Taiwanese teachers. He maintains the school mathematics website which includes GeoGebra related teaching and problem-solving materials. In addition, he proposed and conducted GeoGebra training courses and workshops in senior high schools in Taipei. He had also set up his website and uploaded his up-to-date GeoGebra materials and step-by-step tutorial materials for students or teachers. Li had a similar opinion to Jay on students' and teachers' attitudes towards the use of computers. However, he was positive that exploiting GeoGebra can change students' attitude towards mathematics. Some of his designed teaching materials and tutoring examples of using GeoGebra in solving examination problems were displayed on the websites. He also encouraged students to use the websites for reference and discussion. His ideal teaching environment would incorporate technology and GeoGebra. The salient categories emerged from the data are listed as follows:

Tool use	Graphing, calculations, demonstration, problem-solving, revision, investigation, and interaction
Mathematics topics	Geometrical topics and algebraic calculations
Teaching style	Curriculum-based, textbook-oriented and exam-driven, self-developed teaching materials and website with GeoGebra
Infrastructure	Home, IT room or computer and projector in classroom

Richard

Richard has taught secondary and A-level mathematics for twelve years in England. He is skilled in computer programming and is in charge of the school mathematics website where a combination of GeoGebra, Yacas and JavaScript are used for developing online mathematics materials and tests. He designed a piece of DGS and used it to teach before starting to use GeoGebra. Previously, he was working as a software developer and cooperated with the NCETM GeoGebra project. Richard has an ambivalent view of GeoGebra. He expressed that he was not convinced that GeoGebra links geometry and algebra but then stated that: 'it does the connection between algebra and geometry much better than other programmes – anywhere you can enter a number you can also enter a formula'. He asserted that GeoGebra had changed the way he taught as he had been taking students to IT rooms more often and some students liked the revision with GeoGebra as it sped up some processes of preparation for examinations and for accuracy. He stressed 'the fact that you can animate any variable by turning it into a **slider** is a very powerful feature'. The salient categories emerged from the data are listed as follows:

Tool use	Graphing, calculations, demonstration, revision, student activities, investigation with the slider
Mathematics topics	Mainly geometrical topics, gradients of a curve and transformations
Teaching style	Activity-based, a combination system of paper-and-pencil and computer environments
Infrastructure	Home, IT room or computer and projector in classroom

Tyler

Tyler has taught mathematics to 11-16 year olds in a college for twelve years. He has also acted as an AST[1] supporting schools and as a part-time school consultant, cooperated with the NCETM GeoGebra project and hosted a GeoGebra training workshop at his college. Tyler's utterances reflectehd a view of GeoGebra as an environment for exploring dynamic geometry rather than algebra. He viewed GeoGebra as a replacement to Cabri, which he used before GeoGebra. However, he mentioned that his experience with GeoGebra was approximately half a year, which meant that there were areas of using GeoGebra that were under-explored and underdeveloped, such as using GeoGebra in teaching algebra.

Some criticisms about current usage of technology in schools were brought up in terms of the IT rooms and school websites. He described his intention to change the way his pupils work from being passive to actively involve in learning through software. Moreover, he did not expect that students would not undertake much thinking in the IT room. In addition, some school mathematics websites have mathematics tests for pupils to log on to at home with their personal passwords which, in his view, allowed no room for discussion and interaction. He pointed out that GeoGebra is interactive and intuitive so he could set up diagrams and activities for students to interact with easily: 'This is

[1] Advanced Skills Teacher.

different. This is maths by interacting; this is maths by trying things out, by conjecturing, by having a go.' He emphasised that GeoGebra could not only be used as a presentation tool by teachers but also as an investigation tool for pupils. An enthusiasm for GeoGebra was apparent in Tyler's strategies of using GeoGebra in mathematics teaching.

Overall, Tyler was reflective and explorative about different practices with GeoGebra, and eager to find out possible areas where GeoGebra could be useful in mathematics teaching. He also drew a distinction between '*knowing how'* to use it and '*getting used to'* using it in relation with GeoGebra. This inferred that he acknowledged the differences between *using* GeoGebra and *teaching* with the use of GeoGebra. The salient categories emerged from the data are listed as follows:

Tool use	Demonstration, interaction, investigation, exploration, testing hypothesis, creation, projection capability and the slider
Mathematics topics	Mainly geometrical topics
Teaching style	A whole-class teaching activity
Infrastructure	Home, IT room or computer and projector in classroom

CROSS-CASE ANALYSIS

Emerging categories

Some extracted findings from each case were collected in the within-case analysis. By following the constant comparative method (GLASER; STRAUSS, 1967), several categories emerged from the data when comparing incidents applicable to each category. The classification involved subdividing the data as well as assigning the data into as many categories as possible that fitted an existing category. For example, the category of teachers' conceptions and uses of GeoGebra as an 'educational tool' emerged quickly from comparisons of the teachers' responses to the ways in which they viewed and used GeoGebra as a tool for a variety of purposes.

Categories appeared when comparing the interview data across the cases. In relation to environments within which teachers use GeoGebra, infrastructural change of IT facilities and settings seemed to be one of the major concerns. With regard to teachers' behavioural change, thwo aspects, teachers' mathematical and IT background and the transition that they experienced through using GeoGebra, were scrutinised. The third category is the way they viewed GeoGebra as an educational tool. The fourth main category – mathematical topics had been targeted for different levels of mathematics. Out of those categories, some sub-categories emerged, which will be discussed in the following analysis.

After splitting categories into sub-categories, I followed Dey's (1993: 139) strategy for splicing categories: 'when we splice categories, we join them by interweaving the different strands in our analysis'. This is for the purpose of integrating categories and their properties. Following the sequence of splitting, splicing categories and linking the data, a framework for analysing cross-cases was then identified. In the final stage, there are four main categories (Appendix XI) in relation to the use of GeoGebra integrated:

(a) the ways in which teachers view and use GeoGebra as an educational tool;

(b) the transition that teachers experience when they go through different stages from learning GeoGebra to teaching with the use of GeoGebra;

(c) the mathematical topics that teachers choose for teaching aided by GeoGebra;

(d) the infrastructural change of technology environment under which teachers work in relation to their practices of GeoGebra.

These four dimensions are used to examine the differences and similarities among these four cases in the following.

Educational tool

The case studies show that, besides differences in teachers' views on and methods of using GeoGebra, they all referred to GeoGebra as an educational tool. Two possibilities of GeoGebra as an educational tool are that teachers might view it as a tool or use it as a tool in their classroom practices. As a consequence, this dimension overlaps two themes – views on and uses of GeoGebra. Applying comparative analysis cross the cases and themes, GeoGebra can be identified as an educational tool for:

- research and analysis;
- immediate feedback and reflective checking;
- creating teaching materials and online materials;
- demonstration, presentation and visualisation;
- problem-solving, computation and calculation;
- classroom activities, tasks- investigation, experimentation and conjecture;
- geometrical proof of theorems;
- revision for examinations.

Jay viewed GeoGebra as a tool for research, checking, calculation, teaching and demonstration and used it mainly for presentation in the classroom. He mentioned that GeoGebra was a 'resurrection' tool that activated and visualised some mathematical concepts in textbooks. He also stressed his position of viewing GeoGebra merely as a tool which was useful and convenient. After one year of using GeoGebra, he had not changed the way he viewed it as an additional tool for speeding up teaching processes. He did not give students guidance to learn or to engage with GeoGebra. Jay's limited ways of using GeoGebra could be the result of his conceptions that its effectiveness was low and that not many teachers would use it or students find it a useful tool.

Li considered GeoGebra as a tool for a broader range of affordances, such as making teaching materials, editing online tutoring worksheets for problem-solving, conjectures, geometrical proof of theorems, students' reference after school and revision for examinations. This is likely to reflect his high level of enthusiasm and confidence in GeoGebra. Moreover, his extensive production of GeoGebra applications could be inferred from his profound mathematics content knowledge. However, a lack of pedagogy in teaching with GeoGebra seemed apparent. During observation of a lesson in the IT room, he used GeoGebra as a revision tool. Students followed his pre-prepared worksheet step-by-step to observe how graphs change when different functions were typed in. It seemed that students simply acted according to the required task and did not engage in actively thinking about the task. Therefore, this is understandable that students might unlikely to be inspired or motivated by learning through GeoGebra. This view of missing appropriate pedagogy was also indicated in his aspiration to raise students' motivation to learn by using GeoGebra- he uttered: 'I hope to use GeoGebra to move students' hearts and grasp them back'. Although Li's self belief that his design work with GeoGebra might persuade students to engage more fully with mathematics, the unappreciative reaction of his students to his efforts indicate otherwise. This has prompted him to improve the situation.

Richard regarded GeoGebra as a tool for a variety of practices, even for different subject areas such as physics. He asserted that 'you really can do anything' with GeoGebra, such as designing tests or tasks on school websites. Nevertheless, his main use for it was as a presentation device in the classroom and a tool for revision for examinations in the IT room. His enjoyment of mathematics was derived from combining different software packages for producing online tests for students to practice at home. One limitation of his use of GeoGebra stemmed from the fact that most of the material he designed only required 'yes' or 'no' answer. Additional explanation or help was not offered if students answered questions incorrectly. His intention was to help students learn through these online tests, IT room activities, classroom tasks and demonstration. Arguably, these activities might assist students with procedural understandings rather than conceptual ones.

Tyler did not appear to consider GeoGebra as a tool but rather as an environment for exploring mathematics. However, he stated that he would use GeoGebra as a presentation tool in the classrooms but preferred students to use it as a tool for working on tasks, investigation and testing conjectures. He was aware students simply observing teachers present work with the software hinders their interactive participation and is different from doing the work themselves. Therefore, he claimed that GeoGebra is most useful when students actually experiment and investigate with it. He viewed GeoGebra as an educational tool, not only for teachers but also for students.

Comparing the four teachers' behaviours with GeoGebra, Richard and Li approached GeoGebra in a similar fashion although they are from different countries. They both had a combination system of working with GeoGebra, creating their own teaching materials and websites as well as providing revision section for students' examination preparation. However, Jay and Tyler both approached GeoGebra differently. Jay was more demonstration-oriented which indicated that his teaching practice was consistent with his conceptions of GeoGebra being software for visualising mathematics. Tyler's practice was student interaction-based which might be in relation to his conceptions of GeoGebra being interactive.

Teacher transition

Teachers might experience changes in their manipulation of GeoGebra providing more time and exploration. According to the interview with Markus, he thought that teachers seem to go through phases and changes from starting to learn GeoGebra to teaching utilising GeoGebra. These four stages are:

(a) Preparation- teachers begin with basic constructs, such as making triangles, circles and graphs of equations. They create diagrams for preparation of arranged lessons and generate printed worksheets or test sheets.

(b) Presentation – teachers start using GeoGebra in the classroom for demonstration, either displaying pre-prepared files or constructing graphs step-by-step in front of students.

(c) Interaction – teachers design whole class activities and encourage interactions between students and GeoGebra.

(d) Investigation – teachers ask open questions and students work in pairs to investigate their mathematical ideas, conjectures with GeoGebra.

Given this framework for examining teacher transition, I determined that Jay was the only one who stayed at the presentation stage; Li and Richard seemed to move on to the interaction stage whilst Tyler had proceeded to the investigation stage as a result of his personal expertise as an AST. However, I could argue that these teachers are not teachers who are new to using technology: some might have experiences using other software in the past, particularly similar DGS packages. Therefore, they did not necessarily need to go through the first stage. For instance, Jay had experience using GSP during the past twenty years and he started GeoGebra straight into the second stage without changing for years. His perception of the uses of GeoGebra was limited and possibly so were his intentions of exploring different uses of mathematical software. Consequently, there are probably teachers who stay at one stage, never moving forward.

The data suggested that teachers can be categorised into three types: unskilled teachers who have never used technology in teaching, technology-skilled teachers and GeoGebra advanced skills teachers. Some teachers who are not used to technology can download GeoGebra online materials or worksheets for their classroom practices. They could be at the pre-stage phase where they might simply want to use it for demonstrations and are unwilling to learn more advanced mechanisms of the software. Teachers who are skilful using technology are possibly the ones who progress from stage to stage. Advanced skills teachers use GeoGebra across all stages as a network. They change their plans adapting to different topics or student abilities and employ GeoGebra for preparation of lessons to encourage interaction with students, preparation for presentation on particular topics or preparation of activities for student investigation. Given more time and experiences of teaching with GeoGebra, a combination of all stages is exploited.

Mathematical scope

The mathematical topics that GeoGebra supports in terms of my research focus can be categorised as algebraic topics, geometric topics or both algebraic and geometric related topics. The differences and commonalities of the four teachers' choices of mathematical topics using GeoGebra are discussed.

Jay taught with GeoGebra mainly on topics related to coordinate systems, which is a possible subcategory of both algebra and geometry. Li listed all topics related to a wide range of mathematics areas apart from 3D topics which can also be set to both algebra and geometry. Both Taiwanese teachers viewed algebra and geometry as two sides of a coin that should not be separate. Li pointed out that there were no particular separation in the curriculum, therefore, they both sometimes taught algebra and geometry at the same time which seems to be the reason why they used GeoGebra to teach both algebraic and geometric related topics.

In contrast, Tyler and Richard used GeoGebra mainly for geometric topics possibly in consequence of their perception that GeoGebra is a DGS. They expressed that they would not want to use GeoGebra for all topics as there are certain topics that are not appropriate for incorporating technology. Therefore, they preferred to find out what topics GeoGebra is appropriate for then use it for those specific topics. For example, Tyler had shorter period of time exploiting GeoGebra and was interested in exploring GeoGebra for topics related to algebra. According to the interviews, both Richard and Tyler chose to use different software for algebraic topics as they did not seem to be convinced by the algebraic capability of GeoGebra. The difference between Taiwanese and English teachers' choices of topics might be due to discrepancies in the structure of mathematics curricula and their perceptions of GeoGebra.

According to the teacher evaluation of GeoGebra and other software, most of them regarded GeoGebra as a replacement to GSP and Cabri. However, GeoGebra could not work with particular topics such as 3D topics for Taiwanese teachers. Jay chose to use Archimedes 3D whilst Li chose SketchUp for the 3D related topics. Richard designed online materials with Yacas and JavaScript for algebraic topics whilst Tyler used Autograph for teaching topics related to functions. When the weaknesses of GeoGebra capability were discovered all teachers were proficient in embracing other software packages for their chosen topics.

Infrastructural change

The infrastructure of the educational environment is closely related to the ways GeoGebra can be used. Since GeoGebra is open-source software, one advantage offered by it is that both teachers and students have options to use it at school and at home. Teachers can use GeoGebra at home for either research or preparation for mathematics teaching materials whilst students can do coursework with it at home. According to the interviews, some of the teachers encouraged students to go on GeoGebra-related websites to practice mathematics exercises at home. Most teachers used GeoGebra to demonstrate mathematical objects or visualise mathematics in their classrooms using a laptop or a computer connected to a projector. Some of them brought students to the IT room and a few prepared laptops in the classroom for students to investigate. There are therefore three different environments that teachers used GeoGebra – at home for research or preparation, in classrooms for demonstration or student interaction and in IT rooms for activities, revision or student investigation.

Jay and Li conceived of the infrastructure of GeoGebra usage for conventional presentation of mathematical work in classrooms. Before teaching, both of them used GeoGebra at home, however, Jay would use it for research and making teaching portfolios whereas Li used it for making GeoGebra worksheets, online tutorial materials and examination sheets as well as teaching preparation. During teaching, Jay used GeoGebra solely for demonstration in classrooms where he brought a laptop and showed graph works to the class whenever needed. Li would not use GeoGebra in classrooms but in IT rooms for demonstration and revision.

Richard and Tyler worked with GeoGebra in different ways. Richard used it at home for designing mathematical tasks and tests on the school website, whereas Tyler set up activities for presentation at home and student investigation in IT rooms. Richard mainly used it for demonstration in classrooms and revision in IT rooms. Tyler used it for activities in classrooms and tasks for students to investigate in IT rooms where they work in pairs, making conjectures and testing their mathematical ideas out. Comparing these four cases, the English teachers taught both in classrooms and IT rooms whilst the Taiwanese teachers chose one environment instead of switching between classrooms and IT rooms.

Most teachers expressed that there was a certain degree of difficulty in approaching appropriate IT facilities as the time spent on setting up laptops and projectors or getting students in IT rooms and logging on to the computers could take up to 20 minutes in one lesson. In addition, there were distractions when computers were available as students occasionally attempted to check emails, surf the web, or listen to music. These factors could contribute to their frustrations towards implementing GeoGebra.

Compared with English teachers, Taiwanese teachers held more negative attitudes with respect to infrastructure of technology and therefore it influenced their ways of using GeoGebra. This is not only because it is more time-consuming but also due to students' passive response to technology. For example, Jay stated that, 'I don't dare to say that it enhances students' motivation in learning. In fact, I don't even put the idea in my head. Because learning in the field of mathematics, is considered a hard subject, students need very strong motivation if they are willing to learn or they are interested in logical thinking.'

CROSS-CULTURAL EXPLORATION

There are several areas with respect to the use of GeoGebra in Taiwan which are different from England. However, ascertaining the commonalities and differences of the use of GeoGebra between Taiwan and England is not particularly easy as cultural influence is a complex issue. In addition, the presentation of four cases cannot offer a broad understanding or generalisation of what is happening in both countries. What this study offers is an exploration into teachers' commonalities and discrepancies in using GeoGebra in England as compared to their Taiwanese counterparts according to their personal characteristics, conceptions and practices.

By adopting Kaiser et al. (2005)'s framework for analysing mathematics education in Eastern and Western traditions, teachers' conceptions of mathematics and their practices in relation with GeoGebra and cultural influences are chosen for cross-cultural comparison. In an attempt to identify 'what is universal' and 'what is context bound' (OSBORN et al., 2000), this study would help understand the role played by cultural context and the ways in which teachers use GeoGebra with different forms of pedagogy as Taiwan and England have contrasting values.

Responding to 'what is context bound', there are three aspects generated from the data that could be seen as significantly different between the cultures in England and Taiwan. Firstly, teachers' attitudes towards technology in both countries varied. The participating Taiwanese teachers held negative conceptions of technology use for teaching practices, whereas the English teachers were positive about it not only because they were confident and comfortable about using technology but also because students seemed to have a higher level of acceptance. Secondly, the Taiwanese teachers experienced greater difficulties pertaining to infrastructure as the classroom settings were not particularly designed for technology use in Taiwan whilst the English classroom settings implemented interactive whiteboards and projectors which offered convenience for teachers.

Finally, in terms of pedagogy, the Taiwanese teachers tended to follow a curriculum based teaching strategy and mostly related GeoGebra exercises to textbooks; therefore, GeoGebra was used specifically for assistance of visualisation of textbooks examples. Again, the English teachers appeared to be more creative and flexible in choosing their teaching methods. As the Taiwanese educational system has an examination-driven culture, there are several areas being used extensively such as problem solving for university entrance examinations and proof of theorems as well as revision for examination preparation. In contrast with Taiwan, the English educational system has a focus on individual learning, therefore, there seemed to be an emphasis on students' individual investigation and interaction with GeoGebra. Identifying 'what is universal' cross cases, one noticeable commonality is that all teachers conceived GeoGebra as a useful tool for mathematics teaching practices.

FINDINGS

Teachers' practical elaboration of GeoGebra can be seen as interrelated within the four dimensions. The infrastructure of technology has a great impact on the ways in which teachers regard GeoGebra as an educational tool since if technology facilities are not available or advanced, it would definitely influence the way teachers use the software.

Given technology provision, teachers' mathematical content knowledge and conceptions may affect their mathematical scope utilising GeoGebra. Certainly, provided there is sufficient support for the use of GeoGebra, teachers might start experiencing changes in their behaviour with GeoGebra. This teacher transition will move them from beginners to advanced users of GeoGebra as well as help them develop their pedagogical practices in teaching practices.

In spite of these common dimensions between Taiwan and England, there are substantial discrepancies in technological artefacts and adaptation of curricular resources which underpin English and Taiwanese teachers' decisions and practices with GeoGebra applications. These significant differences could be explained by the two opposing Eastern and Western cultural traditions.

Analysing the data thematically across the case studies revealed four salient dimensions in relation to GeoGebra-assisted teaching: **educational tools, teacher transition, mathematical scope and infrastructural change**. The findings are introduced in the following, which indicate that understanding the linkage between teachers' conceptions and practices is crucial. Firstly, the teachers' conceptions of GeoGebra seemed to be strongly rooted in their conceptions of the effectiveness and infrastructure of technology. The English teachers imbued a more positive attitude towards technology than their Taiwanese counterparts. However, teachers in both countries expressed favourable opinions regarding GeoGebra's agreeable contribution to their teaching. Secondly, GeoGebra was commonly used as a tool for visualisation, demonstration and interaction of mathematical topics, whereas for algebraic topics it was rarely utilised in England. It appeared that the English teachers associated GeoGebra primarily with geometric topics. Conversely, Taiwanese teachers worked with GeoGebra on both geometric and algebraic topics as they did not consider algebra and geometry to be necessarily separate; possibly as a result of the structure of Taiwanese curriculum and textbook-oriented culture. Thirdly, there were three different environments where teachers engaged with GeoGebra: – preparation of teaching materials at home, presentation and interaction in classrooms and activities for pupil investigation in IT rooms. Teacher transitions evolved from and were influenced by the infrastructure as they moved from preparation to presentation, incorporating interaction with pupils and finally encouraging investigation.

In effect, GeoGebra can be implemented in upper-secondary mathematics teaching as a network of preparation, presentation, interaction and investigation whereby teachers mediate their practices with flexibility. Based on the findings above, I present the general schema of this thesis (Fig.1). Arguably, there is a conceptual change in accordance with infrastructural change when technology is introduced in mathematics teaching. Teachers are the first to encounter this re-conceptualisation of pedagogical practices. They not only experience changes in their conceptions but also modification of their practices when they experience the transition. This transition would possibly alter teachers' choices of the mathematical scope and their uses of GeoGebra as an educational tool in light of their new pedagogical practices.

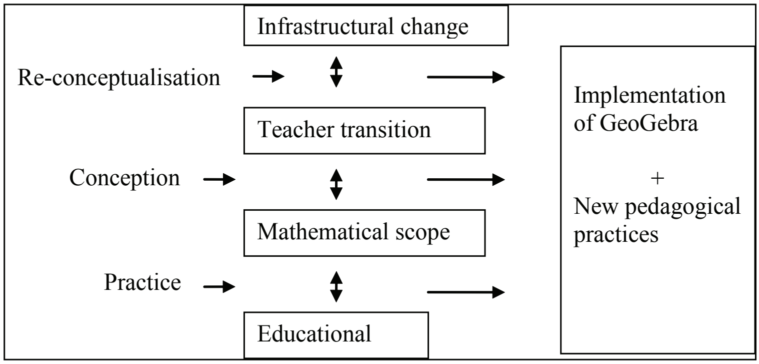

FIGURE 2 – The general schema of teachers' conceptions and practices of GeoGebra.

CONCLUSION

There are several areas with respect to the use of GeoGebra in Taiwan which are different from England. However, ascertaining the commonalities and differences of the use of GeoGebra between Taiwan and England is not particular easy as cultural influence is a complex issue. In addition, the presentation of four cases cannot offer a broad understanding or generalisation of what is happening in both countries. What this study offers is an exploration into teachers' use GeoGebra in England differently from their Taiwanese counterparts according to their personal characteristic, conceptions and practices.

There are three aspects generated from the data that could be seen significantly different between the cultures in England and Taiwan. Firstly, teachers' attitudes towards technology in both countries varied. The participated Taiwanese teachers held negative conceptions of technology use for teaching practices, whereas the English teachers were positive about it not only because they were confident and comfortable about using ICT but also students seemed to have higher level of acceptance. Secondly, the Taiwanese teachers experienced greater difficulties pertaining to infrastructure as the classroom settings were not particularly designed for technology use in Taiwan whilst the English classroom settings implemented interactive whiteboards and projectors which offered convenience for teachers. Finally, in terms of pedagogy, the Taiwanese teachers tended to follow a curriculum based teaching strategy and mostly related GeoGebra exercises to textbooks; therefore, GeoGebra was used specifically for assistance of visualisation of textbooks examples. Again, the English teachers appeared to be more creative and flexible in choosing their teaching methods. As the Taiwanese educational system has an examination-driven culture, there are several areas being used extensively such as problem solving for university entrance examinations and proof of theorems as well as revision for examination preparation. In contrast with Taiwan, the English educational system has a focus on individual learning, therefore, there seemed to be a stress on students' individual investigation and interaction with GeoGebra.

Teachers' practical elaboration of GeoGebra can be seen interrelated within the four dimensions. The infrastructure of ICT has a great impact on the ways in which teachers regard GeoGebra as an educational tool since if ICT facilities are not available or advanced, it would definitely influence the way teachers use it. Given ICT provision, teachers' mathematical content knowledge and conceptions may affect their choices of mathematical topics utilising GeoGebra. Certainly, providing sufficient support for the use of GeoGebra, teachers might start experiencing changes in their behaviour with GeoGebra. This teacher transition will move them from beginners to advanced users of GeoGebra as well as help them develop their pedagogical practices in teaching practices. In spite of these common dimensions between Taiwan and England, there are substantial discrepancies in technological artefacts and adaptation of curricular resources which underpin English and Taiwanese teachers' decisions and practices with GeoGebra applications. These significant differences could be explained by the two opposed Eastern and Western cultural traditions.

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