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Applying Ethnomodelling to Explore Glocal Mathematical Knowledge Systems

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ABSTRACT

Background: Ethnomodelling methods examine how members of distinct cultural groups have come to develop local mathematical knowledge. However, what may indeed be less evident is how mathematical thinking can be part of the way in which researchers and educators attempt to make sense of the underlying cultural frameworks within which mathematical ideas, procedures, and practices are embedded. **Objectives**: The main objective of this theoretical article is to present arguments that link mathematics and culture in order to develop an effective understanding of the development of dialogical mathematical knowledge. **Design**: The theoretical and methodological concepts of this qualitative study are supported by the assumptions of ethnomodelling that adds an important cultural perspective to the modelling process through the development of an extensive literature review on this topic. **Results**: We present arguments to show that the linking of mathematics and culture is appropriate and necessary for an effective understanding of the development of dialogical mathematical knowledge, which aims at providing a holistic understanding of human knowledge. This means that cognition is a process that is not only embodied and situated, as well as distributed because the members of distinct cultural groups create, process, accumulate, and diffuse mathematical information conjointly. **Conclusions**: We discuss the role of ethnomodelling in order to develop an understanding the connection between ethnomathematics and modelling. In this context, we present concepts related to the use of both local (emic), global (etic) approaches by applying the glocal (dialogical) approach found in ethnomodelling research.

Keywords*:* Ethnomodelling; Ethnomodels; Global approach; Glocal approach; Local approach.

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Aplicando Etnomodelagem para Explorar Sistemas de Conhecimento Matemático Glocal

RESUMO

Antecedentes: Os métodos de etnomodelagem examinam como os membros de grupos culturais distintos desenvolveram os conhecimentos matemáticos localis Contudo, o que pode ser menos evidente é como o pensamento matemático pode ser parte da maneira como os pesquisadores e educadores tentam dar sentido às estruturas culturais subjacentes nas por meio das quais as ideias, os procedimentos e as práticas matemáticas estão inseridas. **Objetivos**: O objetivo principal desse artigo teórico é apresentar argumentos que vinculam a matemática e a cultura, pois visa desenvolver uma compreensão efetiva do desenvolvimento do conhecimento matemático dialógico. **Design**: Os conceitos teóricos e metodológicos desse estudo qualitativo são sustentados pelos pressupostos da etnomodelagem que adicionam uma perspectiva cultural ao processo de modelagem por meio do desenvolvimento de uma extensa revisão da literatura sobre esse tema. **Resultados**: Apresentamos argumentos para mostrar que a articulação entre a matemática e a cultura é apropriada e necessária para uma compreensão efetiva do desenvolvimento do conhecimento matemático dialógico, que visa proporcionar uma compreensão holística do conhecimento humano. Isso significa que a cognição é um processo que não é apenas corporificado e situado, bem como distribuído porque os membros de grupos culturais distintos criam, processam, acumulam e difundem informações matemáticas conjuntamente. **Conclusões**: Discutimos o papel da etnomodelagem para desenvolver uma compreensão da conexão entre a etnomatemática e a modelagem. Nesse contexto, apresentamos conceitos relacionados à utilização das abordagens local (êmica) e global (ética), aplicando a abordagem glocal (dialógica) encontrada nas pesquisas em etnomodelagem.

Palavras-chave: Etnomodelagem; Etnomodelos; Abordagem Global; Abordagem Glocal; Abordagem Local.

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INITIAL CONSIDERATIONS

As researchers come to investigate the local mathematical knowledge of the members of distinct cultural groups, they may be able to find characteristics of mathematical ideas, procedures, and practices that we refer to as ethnomodelling (Rosa & Orey, 2010). However, an outsider's understanding of objectified *cultural traits* [1]is always an interpretation that may emphasize

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[1]According to Ting-Toomey and Chung (2005), cultural traits are systems of knowledge that consist of patterns, traditions, meanings, beliefs, values, actions,

inessential features and is in danger of misinterpreting the development of local mathematical ideas and procedures. The challenge that arises from this perspective is how to comprehend culturally-bound activities, often related to local mathematical ideas, procedures, and practices while reducing the possibility of contamination by researchers and educators (*outsiders*) cultural background. This interference or bias will with no doubt color the findings gleaned from the members of the cultural group (*insiders*) under study.

This happens when members of distinct cultural groups share their own perceptions of their culture (*emic*) opposed to outsiders' interpretation (*etic*). In this regard, emic constructs are descriptions and analyses acceptable by these members as meaningful and appropriate while etic constructs are categories and concepts used by the external observers who generate scientific and mathematical theories. In our point of view, research strategies prioritize the study of etic phenomena over the emic analysis of cultural traits. Yet, in any ongoing ethnomodelling research, there are two approaches to be considered in order to investigate and study mathematical ideas, procedures, and practices developed by these members:

1. *Global (Etic)*, which is defined as external or outsiders' view on beliefs, customs, and scientific and mathematical knowledge of the members of cultural groups (Rosa & Orey, 2019). These individuals are considered as *culturally universal* (Sue & Sue, 2003).

2. *Local (Emic)*, which is defined as insiders' view or perceptions of the members of distinct cultural groups about their own customs, beliefs, and scientific and mathematical knowledge (Rosa & Orey, 2019). These individuals are considered as *culturally specific* (Sue & Sue, 2003).

However, the use of emic or etic approaches depends upon the nature of the research itself because there are different conceptions about cultures and the ways they are studied (Cortes & Orey, 2020). For example, culture as being universal can be studied in every defined cultural group, but cultural differences can be studied only if there are at least two of these cultures. This perspective

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experiences, attitudes, hierarchies, religion, notions of time, norms, roles, spatial relations, concepts of the universe, artifacts, mentifacts, sociofacts, and symbols acquired by the members of distinct cultural groups, which are diffused and shared from generation to generation.

has implications for research since every culture should be identified by taking into account individual and diverse cultural understandings (D'Ambrosio, 2006). While the influences of cultures may testable from one culture alone, the effects of cultural differences are measurable only when members from distinct, different and diverse cultural groups interact (Li & Karakowsky, 2002).

In this context, Rosa and Orey (2017) state that the terms emic and etic are neologisms that were initially coined by an American Anthropologist Pike (1954) from phon***etic***, which refers to the sounds used in a particular language and phon***emic***, which provide the general aspects of vocal sounds and sound production in languages. For example, Agar (2005) affirms all the possible sounds individuals make constitute the phon***etics*** of the language. But, when people actually speak a particular language, they do not hear all its possible sounds because not all of them *make a difference*. The sounds that are locally significant, as modeled by linguists, are the phon***emics*** of that language. In the study of language' sound systems, there are two approaches that could be applied in the study of the members of distinct cultural groups, which provide the perspective of either the insiders or the outsiders.

Historically, the concepts of emic and etic began to be widely used both inside and outside of linguistics. For example, in the late 60s, Berry (1969) transferred Pike's cross-cultural psychology by using the term *etic* to analyze human behavior of members who focus on universals. Thus, *etic* behaviors are those that might be compared across cultures by using common definitions, categories, and metrics. An *emic* analysis of these behaviors would focus on behaviors unique to distinct cultures or on the diverse ways in which local activities are carried out in specific cultural settings. In this context, (Rosa & Orey, 2018) argue that cross-cultural researchers make a distinction between culture-specific (local/emic) and culture general (global/etic).

The local (emic) approach focuses on intrinsic cultural distinctions that are meaningful to the members of distinct cultural groups, whether the natural world is distinguished from the supernatural realm in the worldview of that specific culture. It attempts to describe particular knowledge by investigating, discovering, and elucidating mathematical ideas, procedures, practices locally developed. The primary judges of the validity of the descriptions regarding their own cultural, social, environmental, political, economic contexts (Rosa & Orey, 2012) is the people themselves. To reaffirm, an emic approach focuses on studying a construct from within a specific cultural group and tries to understand it from the context of the members themselves do from within. On the other hand, global (etic) approaches rely upon extrinsic concepts and categories that have meaning for external observers who are often the sole judges of the validity of these accounts. It involves developing an understanding of a construct by comparing it across cultures by using predetermined characteristics (Lett, 1996).

In this context, etic-oriented researchers examine phenomena from a cross-cultural perspective so that their observations are taken in accordance to externally derived criteria. This context allows for the comparison of multiple cultures where "both the objects and the standards of comparison must be equivalent across cultures" (Helfrich, 1999, p. 132). On the other hand, from a glocal (dialogical) approach, both emic and etic approaches provide a more complete understanding of mathematical knowledge and interests of the members of distinct cultural groups (Rosa & Orey, 2017). In this regard, we discuss the role of ethnomodelling in order to develop an understanding the connection between ethnomathematics and modelling related to the use of both local (emic), global (etic) approaches by applying the glocal (dialogical) approach found in educational research (Rosa & Orey, 2017).

The main objective of this article is to present arguments that demonstrate the practical potential for linking mathematics and culture. In so doing, we share our thoughts in relation to ideas and concepts that are necessary for creating an effective understanding of local mathematical knowledge, which aims at providing a more holistic comprehension of its development. We are particularly interested in looking at the theoretical and methodological concepts of this qualitative study supported by assumptions of ethnomodelling that add an important cultural perspective to modelling processes through the development of an extensive literature review on this topic. We also present concepts related to the use of both local (emic), global (etic) approaches by applying a glocal (dialogical) approach found in ethnomodelling research.

CULTURAL INFLUENCES ON MATHEMATICAL KNOWLEDGE

Cultural influences on mathematical knowledge are related to interactions between culture and mathematics. Society, cultures, communities, family values, and beliefs influence the formation of cultural systems, as well as the acquisition of scientific and mathematical knowledge (D'Ambrosio, 2006). Therefore, mathematical knowledge developed by the members of distinct cultural groups results from value systems enhanced in particular contexts that are developed over time as these members become socialized into a particular cultural group. Value systems include cultural elements that these members have in common within the cultural group to which they belong, as well as idiosyncratic behaviors unique to each member. Thus, culture influences the development of mathematical ideas and procedures by reinforcing the value of its manifestations in these practices (Rosa, 2010).

The development of mathematical ideas and procedures can serve as a vehicle to transfer meanings from culturally constituted environments to the school and/or academic contexts. This means that the interaction and communication of these ideas represent the influence of culture on the mathematical practices developed locally (Rosa, 2010). From an emic approach, culture may not be seen as a constructed apart from and causing the development of mathematical practices because it is not inseparable from the development of mathematical knowledge represented by the members of distinct cultural groups (Geertz, 1973).

Culture influences mathematics through its manifestations such as symbols (Hofstede, 1997), which represents a form in which mathematical knowledge can be stored and expressed. These manifestations may be considered as the ideas and procedures that are organized, evaluated, and constructed in order to help members of distinct cultures to assign meaning to (mathematical) phenomena that occur in the environment that surrounds them. Thus, any cultural group possesses different mathematical manifestations that encompass elements of their own culture (Rosa, 2010). In this regard, emic approaches focus on the meanings of symbols and cultural artifacts in the lives of the members of distinct cultural groups in order to explain how they organize and applies information to solve problems faced daily. We understand that scientific and mathematical knowledge embodies and expresses cultural principles. For example, symbols are a broad category of processes and cultural artifacts that carry meaning and are unique to a particular cultural group (Geertz, 1973).

Cultural artifacts are objects created by members of distinct cultural groups that provide clues and information about its creators and users. Mathematical ideas and procedures are cultural artifacts that are socioculturally situated as well as distributed among members from generation to generation (D’Ambrosio, 2006). This approach also includes embodied cognition in which cognitive activities use symbols as external resources that assist these members to develop "mental representation and manipulation of things that are not present, and this includes purely internal uses of sensorimotor representations in the form of mental simulations" (Wilson, 2002). However, mathematical symbols may not exist in all cultural groups and their meaning may be different from one specific culture to another (Rosa & Orey, 2008).

Since mathematical symbolism is generated at a sociocultural level, it is expressed through mathematical practices that become charged with cultural meanings. In this regard, members of distinct cultural groups are moved to use mathematical ideas that may be charged with symbolic meaning and values that have a central role amongst other manifestations of their culture (Hofstede, 1997). Hence, symbolic language as well as symbology generally expresses cultural values in through which language, mathematical ideas and procedures become part of the development of transmitted mathematical practices across generations.

Similarly, D'Ambrosio (2006) affirms that researchers and educators should be respectfully encouraged to acknowledge and recognize that members of distinct cultural groups possess valid scientifically mathematical knowledge. Thus, we acknowledge that mathematical thinking, procedures, and practices are developed and used in specific sociocultural contexts with specific needs and ways of life so that members of distinct cultural groups are able to survive and transcend. Thus, it is important, relevant, and necessary to analyze the relation between culture and mathematics, questioning the predominant view that mainstream mathematics is culture-neutral.

RESEARCH IN ETHNOMODELLING

Research in ethnomodelling is related to the mathematical practices developed by members of distinct cultural groups that tend to privilege organization and presentation in order to facilitate communication and transmission through generations (emic). The representational idea of this local mathematical knowledge through scientific methods help researchers and educators to construct and understand the world (etic) by using small units of information called ethnomodels that compose its entire representation.

Ethnomodels help to link the development of mathematical practices to the cultural heritage of the members of distinct cultural groups, who detain necessary information to solve problems and situations described in systems taken from their own reality (Rosa & Orey, 2018). The emphasis of ethnomodelling research considers the processes that help the construction and development of local mathematical knowledge systems. These include collectivity, creativity, and inventivity (Ascher, 2002).

According to this approach, it is impossible to imprison mathematical ideas, procedures, and practices in registers of univocal designation of reality because there are distinct systems that provide unambiguous representations of reality as well as universal explanations (Craig, 1998). This means that mathematics cannot necessarily be conceived as a universal language because its principles are not always the same everywhere around the world (Rosa & Orey, 2007). The production process of mathematical ideas, procedures, and practices operates within the register of interpretative singularities regarding possibilities for symbolic construction of local mathematical knowledge (Rosa & Orey, 2013a).

Ethnomodelling studies local mathematical processes developed by the members of distinct sociocultural groups. Many interesting ethnomodels have been formulated by using data obtained from studies related to ethnomathematics, and which propose a rediscovery of knowledge systems adopted by the members of diverse groups (Babbitt, Liles & Eglash, 2012; Rosa & Orey, 2011). When this knowledge applies mathematical ideas and procedures through the elaboration of ethnomodels, we are able to understand the origin of mathematical practices more efficiently.

In ethnomodelling research, emic constructs represent the accounts, descriptions, and analyses of mathematical ideas, procedures, and practices expressed in terms of conceptual schemes and categories that are regarded as meaningful and appropriate by members of the cultural group und. This means that emic constructs are in accordance with the perceptions and understandings deemed appropriate by the insider's culture (Lett, 1996). The validation of these constructs comes with a matter of consensus from those who do the mathematics under study, that is local people who must agree that emic constructs match shared perceptions that portray characteristics of their culture.

Emic mathematical knowledge can be obtained through elicitation and observation because observers infer local perceptions. In emic approaches, researchers and educators put aside their own bias, prior theories, and assumptions in order to let those who do the activity under study to explain, and allow for understanding mathematical themes, patterns, and concepts that emerge locally (Rosa & Orey, 2015). Some of its strength lies in its appreciation of the uniqueness of the context being studied in its respect for local viewpoints, and potential to uncover unexpected findings.

On the other hand, the "etic constructs are accounts, descriptions, and analyses of mathematical ideas, procedures, and practices expressed in terms of conceptual schemes and categories that are regarded as meaningful and appropriate by the community of scientific observers" (Lett, 1990, p. 130). An etic approach uses as its starting point theories, hypothesis, perspectives, and concepts from outside of the cultural setting being studied, which are developed by researchers and educators. Etic constructs are precise, logical, comprehensive, replicable, and observer-researcher independent (Rosa & Orey, 2017).

The validation of etic knowledge becomes a matter of logical and empirical analysis, in particular, the logical analysis of whether the construct meets the standards of comprehensiveness and logical consistency concepts. It is important to emphasize that the particular research technique that is used in the acquisition of scientific and mathematical knowledge have no bearing on the nature of that knowledge. Etic knowledge may be obtained at times through elicitation as well as observation. One of the strengths of the etic approach is that it allows for comparison across contexts and populations, and the development of more general cross-cultural concepts (Morris, Leung, Ames, & Lickel, 1999).

Ethnomodelling emphasizes the organization and presentation of mathematical ideas and procedures developed by the members of distinct cultural groups in order to facilitate its communication and transmission across generations, which adds cultural aspects to the modelling process. In this regard, these members construct ethnomodels of mathematical practices found in sociocultural systems, which link cultural heritage with the development of mathematical practices (Rosa & Orey, 2017). It is our understanding that this approach helps the organization of pedagogical action in classrooms by using emic and etic aspects of mathematical knowledge through the development and elaboration of ethnomodels.

EMBODIED AND SITUATED COGNITION IN ETHNOMODELLING RESEARCH

Culture is a lens that shapes reality, as well as it is blueprint that specifies a plan of action or expectations. At the same time, since there are aspects of a culture that are unique to the members of distinct cultural groups, who together have grown, learned, and act daily in diverse contexts, such as economic, social, cultural, political, and environmental in which they live (Rosa & Orey, 2015).

In this perspective, language is often the ultimate cognitive cultural artifact because it is seen as playing a far more central role in shaping the social interactions that occur in distinct cultural groups. As language communities, members of distinct cultural groups develop their own forms of communication through negotiations that occur as part of this process. This approach creates opportunities for students to learn and develop knowledge and meanings that comprise the school curricula (Shuell, 2001).

These environments allow these members to extend their cognitive processes beyond the brain, the body, and their immediate environments. This means that cognition is not only embodied and situated, but also distributed because they are able to create, process, accumulate, and diffuse information conjointly, including mathematical knowledge. As alluded here, cognition is embodied and situated in environments and distributed among agents, symbols, cultural artifacts, and external structures through generations (Rosa & Orey, 2015).

This approach implies that the many diverse traditions, actions, and patterns of behavior, shared by cultures have social and cultural meanings that require background knowledge or *know how* in certain situations, such as the development of mathematical ideas, procedures, and practices (D'Ambrosio, 2011). In this case, situatedness "refers to having one's behaviour strongly affected by the environment (…). Embodiment is a type of situatedness; it refers to having a physical body [every material object] and thus interacting with the environment through the constraints of that body" (Matarić, 2001, p. 82). Therefore, it is important to emphasize that situated cognition:

> (…) takes place in the context of task-relevant inputs and outputs. That is, while a cognitive process is carried out, perceptual information continues to come in that affects processing, and motor activity is executed that affects the environment in task-relevant ways. (Wilson, 2002, p. 626)

In this regard, “embodiment in the field of cognitive science refers to understanding the role of an agent’s own body in its everyday, situated activity” (Gibbs, 2006, p. 1). Generally speaking, embodiment can be viewed as the mechanism that makes it possible to be situated, and consequently it can be argued that embodiment might be a *stronger* position than situatedness (Lindblom, 2007), which implies that both concepts are of crucial importance in the alternative *embodied*, *situated*, and *distributed* approaches to the study of ethnomodelling.

Consequently, situatedness and embodiment are also considered as approaches that may be applied in the conduction of an ethnomodelling research because:

1. *Cognition is situated*. It takes place in the context of a real-world environment, involving perceptions and actions (Wilson, 2002) of the members of distinct cultural groups, which are performed in order to solve problems faced daily. In this regard, situatedness is related to mathematical ideas, procedures, and practices that are situated in the sociocultural contexts of these members. In other words, cognition is contextualized according to the relevance of the inputs and outputs of activities developed locally (Lindblom, 2007), such as mathematical practices.
2. *Cognition is embodied.* Embodiment refers to the “experiences that arise from the living body in its interactions with a material/physical as well as a social and cultural world” (Lindblom, 2007, p. 14). It is related to mathematical ideas and procedures that are embodied in the cultural artifacts and practices developed by the members of distinct cultural groups. These embodied activities possess cognitive and/or epistemic meaning because they may be considered as part of problem solving processes that function as external scaffolds for the development of higher level cognition (Anderson, 2003) developed by these members.

Cognition should be considered an activity structured by the bodies and its situatedness in its environment as embodied actions. The embodied view of cognitive science stresses physical, temporal, and functional situatedness, and enforces interaction between the members of distinct cultural groups and their environments. Such a holistic view prevents inappropriate simplifications and unrealistic assumptions of mathematical ideas, procedures, and practices because it enforces dealing with unexpected contingencies, provides specificity, and incorporates energetic and resource considerations (Matarić, 2001), which allows members to develop direct cultural interaction that is undeniably crucial for ethnomodelling research.

We characterize the role and relevance of embodiment in social interactions and cognition in order to develop a thorough and integrated understanding of mathematical knowledge developed by the members of distinct cultural groups through ethnomodelling, which supports and explains the relationships that actually exist, which originate from the embodied cognition. In this context, embodiment and situatedness relate to ethnomodelling research because of their possibilities for actions that allow the interaction between the activities, such as mathematical practices that the members of distinct cultural groups perform in accordance to their surrounding environments.

Embodied actions are situated in sociocultural contexts in which these members apply them in their social interaction as ways of facilitating and coordinating different social, cultural, and cognitive processes (Adolph & Berger, 2006). Thus, the nature of social interaction is relational because meanings and intentions are emergent phenomena in this environment. Embodiment may provide concrete evidences of the development of mathematical knowledge when members of distinct cultural groups solve problems faced in their daily life, which is situated in distinct environments, such as physical, political, economic, social, environmental, and cultural.

According to this context, embodied practices developed in distinct contexts provide sufficient scaffolding for the understanding of these members as a way of acting and creating meaning for those practices (Gallagher, 2007). In this regard, we argue here that embodiment is the part and parcel of social interaction and cognition in the most general and specific ways, and in which dynamically embodied actions themselves have meaning and agency for the construction of mathematical knowledge through the conduction of ethnomodelling research and curricular activities in the classrooms.

CULTURALLY-UNIVERSAL (ETICS) VERSUS CULTURALLY-SPECIFIC (ETIC) APPROACHES IN ETHNOMODELLING

The discussion in relation to the current understanding of universality of mathematics is complex. Often, mathematics is regarded as a neutral and culturally free research area that is not connected to social and cultural values. Traditional mathematics as taught in schools is often thought of as a culturally free discipline that involves learning supposedly accepted universal ideas, procedures, concepts, practices, and contents (Rosa, 2010). Ethnomodelling research has been developed to confront the taboos that mathematics is a field of study that is universal and acculturated (Rosa & Orey, 2017).

We believe that this approach avoids the pervasive view of mathematics as Eurocentric and value-free misrepresents the evolution of modern mathematics (Joseph, 2000). This perception is also reinforced by student experiences of the way mathematics is taught in schools. In this context, educators' view of mathematics is transmitted to their students and helps to shape their views about the nature of mathematics (Brown, Cooney, & Jones, 1990).

Even though the universality of mathematical truths is not in question, it is in the last four decades that the view of mathematics as culture free has been challenged (Rosa & Orey, 2006). This means that "there is no sense in regarding mathematics learning as abstract and culture free" (Bishop, Hart, Lerman, & Nunes, 1993, p. 1) because the learning process cannot be abstract and context free since it cannot be free of societal and cultural influences. In this context, it is worth noting that the contextualization of mathematics has been described as the identification of mathematical practices developed by the members of distinct cultural groups in diverse contexts (Nasir & Cobb, 2007).

If mathematics can be considered as a cultural construct, then it is equally a product of cultural development (Rosa & Orey, 2017). This assertion contradicts the claims that modern mathematics is universal, objective, and culturally neutral. Since mathematical knowledge results from social interactions in which relevant ideas, facts, concepts, principles, and skills are acquired as a result of the influence of cultural contexts, then mathematics is not a universal formal domain of knowledge (Dossey, 1992).

On the other hand, there are at very least six universal mathematics activities such as counting, measuring, designing, locating, explaining, and playing (gambling, guessing, and explaining) that are thought to be practiced by the members of distinct cultural groups. These activities are also widely developed across cultures in order to provide the fundamental facets used to probe traditional daily living and scientific and mathematical practices and are inseparably intertwined with other aspects and activity of any culture (Bishop, 1991).

In this regard, Rosa and Orey (2007) argue that there are some cultural differences found within these six universal mathematical activities. Even though they may be considered universal, it is important to recognize that they are merely universal to those members who share the same cultural features, historical perspectives, and linguistic backgrounds. This means that school/academic mathematics may look the same in many cultures because there is a competitive social, economic, environmental, and political ethics that demands a competitive mathematical development.

It is in this unique context and assemblage of culturally constructed symbolisms that enables the manipulation of the representations of mathematical knowledge because members of distinct cultural groups develop procedures in their cognitive systems, which also presents a process that occurs in the context of socially constructed activities. Thus, Rosa and Orey (2018) state that mathematical skills that students learn in schools are not logically constructed based on abstract cognitive structures, but rather forged out of a combination of previously acquired *mathematical tacit knowledge*[2], skills, and new cultural inputs.

Therefore, mathematics arose out of the needs of organized communities, which cannot be divorced from the activities and practices developed locally and globally in a *glocalized society*[3] (Rosa & Orey, 2018).

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[2]Tacit mathematical knowledge is related to the ways in which students use mathematical concepts by relating them to their own experiences, beliefs, and cultural values. The main components of tacit knowledge are mental symbolism, mathematical language, methods, symbolic operations, strategies, procedures and techniques locally developed, which are often applicable in solving contextualized problems. (Ernest, 1998).

[3]Glocalized societies enable the development of active, interactional, and dialogical processes in which requires an ongoing negotiation between the local and the global mathematical, scientific, technological, and engineering knowledge through a cultural dynamism. The complexities of a glocalized society require members of distinct cultural groups to be equipped with a new set of core knowledge and abilities that

According to this context, research in mathematics education, more specifically, ethnomodelling can be done from three basic viewpoints, which are phrased in terms of the *Culturally-Universal)*, *Culturally-Specific* and *Dialogical* perspectives.

1. *Culturally-Universal Perspective (Global//Etic/Outsiders)* refers to mathematical phenomena that is constant throughout the world, which does not vary across cultures. Some mathematical concepts are generalizable across cultural groups and the general idea of mathematical practices is considered a universal phenomenon (Kline, 1953; Goldman, 1988). In this perspective, an etic approach understands the mathematical phenomenon cross culturally rather than cultural specific meanings (Rosa & Orey, 2012). However, it is naive to state that the members of distinct cultural groups do not share universal mathematics characteristics. For example, Bishop (1991) states that many of the everyday activities of members of cultural groups involve a substantial amount of mathematical application.

2. *Culturally-Specific Perspective (Local/Emic/Insiders)* focuses on studying the sociocultural aspects of mathematical phenomena from within a specific cultural context in order to understanding it as their members comprehend it (Gudykunst, 1997). In keeping with the emic approach, a demand for local or culturally specific approaches has emerged besides the Western and Euro-American approaches. Examples of local or culturally specific topics and research instruments encouraged some researchers to support the valorization of emic perspective. Many theories and methods seem to be susceptible to cultural differences and to demand culturally contextualizations (Rosa & Orey, 2010).

3. *Dialogical Perspective (Glocal/Cultural Dynamism/Encounters Dynamics)* uses both emic and etic knowledge in order to understand processes of dialogue and

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enables them to solve problems as well to gather and evaluate evidence that empowers them to make sense of information gained and accumulated from diverse media sources in order to develop decision making processes (Rosa & Orey, 2016).

> dynamics of the encounters. While the traditional concepts of emic and etic are important points of view for understanding and comprehending cultural influences on ethnomathematics and mathematical modelling research, this third perspective is important for the development of ethnomodelling investigations. In this perspective, etic claims that the knowledge of the members of any given cultural group have no necessary priority over its competing emic claims because there is a relation of interdependency between these two approaches. This perspective stresses the development of investigations through a mutual exchange between the emic and etic, which could be considered as a qualitative transformation in the research process (Rosa & Orey, 2018).

One of the primary issues raised in mathematics education concerns itself with the position of researchers and educators in relation to different types of universals such as the etic approach (culturally universal and global), the emic approach (culturally specific and local), and dialogical approach (cultural dynamism and glocal) perspectives into the mathematics curriculum. Most of these professionals may operate from the etic position because they believe that mathematical ideas, concepts, procedures, and practices occur in the same way in every culture. Thus, they base their beliefs on Western ideas in which every cultural group construct, develop, acquire, accumulate, transmit, and diffuse the same kind of scientific and mathematical knowledge (Rosa & Orey, 2016).

Researchers and educators consciously or unconsciously transmit views, values, beliefs, attitudes, feelings, and scientific and mathematical ideas of their own culture or the society being studied into universal principles by assuming that all cultures are homogeneous (D'Ambrosio, 2006). For example, the results of the study conducted by Rosa (2010) in relation to the perceptions of school leaders and their English Language Learners (ELL) students revealed that 17 (65.4%) out of 26 school leaders are limited by their own cultural orientations in relation to school issues, which may not contribute to the academic success of these students who are guided by another cultural orientation.

Although diversity among and within the members of distinct cultural groups have been identified, due to the lack of cultural specific theories in local cultures, Western born-made theories are overemphasized in mathematical classrooms (Rosa, 2010). In the educational context, minimal modifications in the pedagogical practices of mathematics are required because scientific and mathematical knowledge are considered universal and equally applicable across cultures. Thus, if the assumption that the origin, process, and manifestation of scientific and mathematical knowledge are similar across cultures, then universal guidelines and strategies for the pedagogical work would appear to be appropriate in application to the members of all cultural groups.

It is necessary that researchers and educators consider that lifestyles, cultural values, and worldviews influence the development of mathematical knowledge because its development arises from distinct cultural contexts (D'Ambrosio, 2011). This is one of the most important issues currently confronting these professionals. For example, many researchers and educators who believe that cultural background and life experiences of the students influence their development of scientific and mathematical knowledge propose the use of culturally specific strategies in the pedagogical work of teaching and learning mathematics. Thus, such professionals are pointing out that worldwide current guidelines and standards for mathematical instruction are culturally bound (Rosa, 2010).

Considering the mathematics education research field, should researchers and educators be based on culturally universal or culturally specific approaches? Some of these professionals believe in cultural universality, which focus on similarities and minimize cultural factors while others take on techniques and beliefs of cultural specificity, which focus on cultural differences. According to this perspective, researchers and educators do not agree on the nature of mathematics (Rosa, 2010). One of the primary issues is whether mathematics is *external* or *internal* to individuals. This argument is pertinent to the relation of culture and mathematics in that internalists perceive connections between mathematics and culture while externalists see mathematics as culture free (Dossey, 1992). On the other hand, many scholars believe mathematics activity is highly cultural (Eglash, 1997; Rosa & Orey, 2008).

Internalists such as Bishop (1988) and D'Ambrosio (1985) understand that mathematics is a cultural product that is developed as a result of the development of various mathematical activities, such as counting, locating, measuring, designing, and playing. Other mathematicians, such as Kline (1953) are externalists because they believe that mathematics activity is culture free. Thus, they do not believe in the connection between mathematics and culture.

For example, the results of the study conducted by Rosa (2010) revealed that 16 (61.5%) out of 26 school leaders in 9 (nine) high schools in a school district in Sacramento, California, possess an externalist view of mathematics, which means that they perceive it as being *culture-free*.

Researchers and educators must also be aware of their own worldviews, and the danger of it influencing their findings. As they become more mindful of how their worldviews and values shape their perceptions, then they can become more open to apply aspects of ethnomodelling in their pedagogical practices. This may lead them to a clear decision between these two approaches. These professionals may also use one paradigm or multiple paradigms as a dialogical perspective (Rosa & Orey, 2018) in order to best fit their worldview. Different paradigms give rise to contradictory ideas and contested arguments (Greene & Caracelli, 2003). These "contradictions, tensions, and oppositions reflect different ways of knowing about and valuing the social world" (Creswell & Plano Clark, 2007, p. 27).

From the non-universalistic viewpoint, distinctions can be made specifying a hypothetical construct as culturally-specific and local, which can be distinguished from the culturally-universal and global (Lonner & Berry, 1986). In multi-cultural comparisons, the etic refers to a mathematical phenomenon that has a common meaning across cultures, often referenced as core mathematical meanings. Conversely, emic refers to different phenomena across cultures, where each emic aspect is related to the shared local knowledge (Berry, 1969; Rosa & Orey, 2017). In this context, the etic approach may be defined as "the collective programming of the mind which distinguishes the members of one group or category of people from another" (Hofstede, 1997, p. 5). The focus of this definition is related to the comparison of one cultural group with another.

Researchers and educators who follow an etic approach in research generally look for universal or culture-free scientific and mathematical concepts and theories (Rosa, 2010). They may be searching for variables and constructs common to all cultures that can be directly compared in order to discover how scientific and mathematical knowledge of the members of those cultural groups are different from or similar to each other. Thus, the emic mathematical practices may be defined through the use of a:

> (…) lens through which all phenomena are seen. It determines how these phenomena are apprehended and assimilated.

> Second, culture is the blueprint of human activity. It determines the coordinates of social action and productive activity, specifying the behaviors and objects that issue from both. (McCracken, 1988, p. 73)

Emic approaches do not intend to directly compare mathematical knowledge developed by members of distinct cultural groups, but promotes a complete understanding of mathematical ideas and procedures practiced by these members through thick description (Geertz, 1973). The methods used in conducting emic research do not provide culture-free measures that can be directly compared; instead, they provide culture-rich information about these practices. In this approach**,** information and observations are constructed to reflect the studied culture's own scientific and mathematical knowledge, language, and belief systems (Rosa & Orey, 2017).

The emic approach contrasts with the etic in that it refers to information collected in terms of the conceptual system and categories of the researchers, educators, and other outsiders. To collect emic data, it is usually necessary to use the local language, dialect, or jargons, and mathematical ideas and procedures in order to gather information in a very open-ended and nondirective way. In the etic approach, the observations and data are constructed in the researchers' system of categories, metrics, and definitions (Rosa & Orey, 2017).

By studying members of distinct cultural groups according to pre-established etic procedures may impede the discovery of cultural diversity, whereas an emic analysis broadens this view (Headland, Pike & Harris, 1990). For example, Rosa and Orey (2018) state that emic approach seeks to understand particular mathematical phenomena from the point of view of its adherents while the etic approach does the same, but by means of analytical tools and concepts drawn from outside.

The choice of emic versus etic approaches depends on several important factors that includes the nature of the research question itself, and is influenced by the researchers' resources and training, and the purpose of the study. Luna (2001) states it is possible to conclude that both etic and emic approaches refer to similar constructs, but from different points of view related to mathematical knowledge developed between-cultures versus the ones developed within-cultures.

In our opinion, this dialogical approach offers ways of connecting emic and etic features of the sociocultural world because it is the point in which both the local and the global knowledge systems intertwine. This pedagogical action entails ways in which students are encouraged to construct an understanding of the nature of mathematics, thereby, connecting it and communicating it with other knowledge fields in an interdisciplinary fashion.

CULTURAL REPRESENTATIONS OF ETHNOMODELS

Traditional mathematical models, more often than not, do not fully take into account the implications of the varied cultural aspects of human social systems. The cultural component in this process is critical because its accounts "emphasize the unity of culture, viewing culture as a coherent whole, a bundle of practices and values" (Pollak & Watkins, 1993, p. 490) that are incompatible with the rationality of the elaboration of traditional mathematical models. In the context of mathematical knowledge, what is meant by the cultural component varies widely and ranges from viewing mathematical practices as socially learned and transmitted to the members of distinct cultural groups to mathematical practices viewed as made up of abstract symbolic systems with an internal logic that gives a symbolic system its mathematical structure (Read, 2004).

If the former is considered, then it is the process by which transmission takes place from one member to another, which is central to elucidating the role of culture in the development of mathematical knowledge. If the latter is considered, then culture plays a far reaching and constructive role with respect to mathematical practices that cannot be induced simply through observation of these practices (Read, 2004). If mathematical knowledge developed by the members of distinct cultural groups consists of abstract symbol systems whose form is the consequence of an internal logic; then students may learn specific instances of the usage of that symbology, as well as derive from those instances a cognitive based understanding of the internal logic of the mathematical symbolic system (Rosa & Orey, 2017).

In other words, emic knowledge is acquired in accordance to the *insiders'* point of view. Thus, *emic* ethnomodels are grounded in mathematical ideas, procedures, and practices that matter to members of distinct cultural groups whose mathematical practices are being modeled. On the other hand, many ethnomodels are *etic* because they are built on an outsider's view of these members' work. In this regard, etic ethnomodels represent how the modeler thinks the world of these members works while emic mathematical models represent how members who live in these cultural groups think their world really is. Yet, it is necessary to highlight this while emic knowledge plays an important role in ethnomodelling research; etic knowledge should be taken into consideration when conducting ethnomodelling research (Rosa & Orey, 2018).

In contrast, if mathematical knowledge consists of a set of socially learned and transmitted mathematical ideas, procedures, and practices, then the cognitive aspects of its development plays an important role when constructing ethnomodels of mathematical practices of sociocultural systems. The cognitive aspect needed in this framework is also a decision process by which the members of distinct cultural groups either accept or reject an ethnomodel as part of their own repertoire of mathematical knowledge (Rosa & Orey, 2015). We understand that the conjunction of these two scenarios appears to be adequate to the depth needed to encompass the full range of cultural phenomena because mathematical practices are diffused to these members through generations.

Ethnomodels are described as cultural artifacts that are pedagogical tools used to enable the understanding of systems taken from the reality of the members of distinct cultural groups (Rosa & Orey, 2013b). In this regard, ethnomodels may be considered as external representations that are precise and consistent with the scientific and mathematical knowledge that is socially constructed, developed, and shared by members of specific cultural groups. The main objective for the elaboration of ethnomodels is to *translate* emic constructs such as mathematical ideas, procedures, and practices in order to establish relations between local conceptual knowledge and the mathematics embedded in these constructs (Eglash et al, 2006).

According to Read (2004), there are two ways in which we make sense of mathematical phenomena. First, there is a level of cognition that the members of distinct cultural groups share, to varying degrees, with the members of their own group, which include cognitive models that they may elaborate at a non-conscious level that provide an internal organization of external mathematical phenomena as well as the basis upon which mathematical procedures and practices take place. Second, there are culturally constructed representations of external mathematical phenomena that provide its internal organization.

However, the “form of the representation arises through formulating an abstract, conceptual structure that provides form and organization for external phenomena in a manner that need not be consistent with the form and patterning of those phenomena as external phenomena (p. 167), which relates to ethnomodelling research. The implications for ethnomodelling are that ethnomodels are considered as representations of symbol systems organized by an internal logic of the members of distinct cultural groups.

Thus, Rosa and Orey (2012) state that ethnomodels are considered cultural constructs because one of the main objectives of its elaboration is to comprehend the way of thinking of these members, as well as to understand how they organize and model their mathematical ideas and procedures from their own point of view in order to mathematize their own reality. On the other hand, a model built without a first-hand sense for the world being modeled should be viewed with suspicion Researchers and educators, if not hindered by their prior ideology, should come out with an informed sense of the distinctions that make a difference from the point of view of the mathematical knowledge of the people being modeled. In so doing, they should be able to inform outsiders (etic/global) what matters to insiders (emic/local).

Dialogical Ethnomodels

Currently, the emic-etic debate continues to be one of the most intriguing research questions in ethnomodelling. Many researchers elaborate ethnomodels that contain questions such as:

1. *Are there mathematical patterns that are identifiable and similar across cultures?*
2. *Is it better to focus on these patterns particularly arising from the culture under investigation?*

Usually, an emic ethnomodel will focus on a single culture and employs descriptive and qualitative methods to study mathematical ideas, procedures, and practices. They focus on the study within a given cultural group context in which researchers and educators develop research criteria relative to internal characteristics or logic features of cultural systems. In this regard, meaning is gained relative to the context and therefore not transferable to other contextual settings (Rosa & Orey, 2013a).

For example, emic ethnomodels are not intended to compare the observed mathematical patterns in distinct contexts. On the other hand, etic ethnomodels are more comparative because they exam different cultures by using standardized methods. It is important here to state that comparisons are no evaluative, from a: *this is better or less than*, but comparative from the point of view: *I see that you do this that way*. These ethnomodels develop mathematical-theoretical ideas that are assumed to apply in all cultural groups while an emic construct is one that applies only to a specific culture (Rosa & Orey, 2016).

The rationale behind the emic-etic (dialogical) dilemma is the argument that mathematical phenomena in their full complexity can only be understood within the context of the culture in which they occur. The emic approach tries to investigate the mathematical phenomena and their interrelationships and structures through the eyes of the people native to a particular cultural group. Thus, in the dialogical approach, the etic perspective claims to acknowledge any given cultural group have no necessary priority over its competing emic claims.

According to this point of view, Eglash et al (2006) stated that there is a necessity to depend "on acts of *translation* between emic and etic perspectives" (p. 347). In this regard, cultural specificity may be better understood with the background of communality and the universality of theories and methods and vice versa. In our point of view, it is necessary that the insights that have been acquired through subjective and culturally contextualized methods be verified with methods independent of the subjectivity of the observer and researcher in order to achieve a scientific character.

Mathematization of the Gable as an Example of a Dialogical Ethnomodel

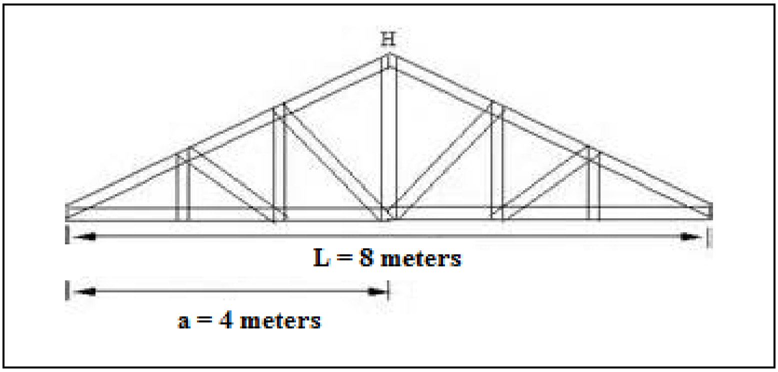
Results of the study conducted by Rosa and Orey (2017) show that the gable is one of the most popular roof designs due to its attractive symmetrical shape and efficiency at shedding water and snow. A cultural group member defined as a roofing contractor, can easily describe the practices acquired for the construction of a roof gable, which is the most commonly used type of pitched roof construction. After they choose the type of the material, such as red roofing tiles or shingles, in order to begin the construction of the roof, it is necessary that roofing contractors calculate the slopes of the beams that form the triangles in the gable.

Gabled roofs often possess a ridge near or at the center and more often than not, slopes in two directions. It is simple and common in design, economical to construct, and can be used on any type of structure, and in any type of climate. Roofing contractors use triangles because they are stable, rigid and have no mobility. The main objective of the roof is to provide protection from climate because they must be strong enough to withstand high winds and shed moisture and often snow and ice quickly. Roof slope and rigidness are for shedding water and any excess weight provided by snow and ice and can bear extra additional weight (Rosa & Orey, 2017).

In the case of many roofs in Brasil, roofers calculate the slope of the roof by applying a ratio between the height and the length of the gable, which is expressed as a percentage. For example, the percentage of the slope (trim) for the roof to the tiles is at least 30% so that rainwater (snow and ice not being a problem in Brasil) can quickly drain. According to this approach, for each meter (100 cm) that runs horizontally, there is a vertical rise of 30 cm. Thus, if the length of the gable is L= 8 meters, roofing contractors mentally perform the percentage calculation by using a = 4 meters, which is half of that measure. Then, they multiply it by the percentage of the slope of the roof. For example,30% of 4 meters corresponds to the height of 1.20m. Figure 1 shows the scheme of a gable used in roof constructions.

Figure 1

Scheme of a gable used in roof constructions (Rosa & Orey, 2017, p. 158)



Conversely, researchers and educators have described this mathematical practice (emic, local) by using the Pythagorean Theorem (etic, global). However, it is important to understand the dialogical (glocal) relationship between these two approaches. For example, the informal calculation (emic knowledge) of the height (trim, flow) of the gable does not preclude the use of the Pythagorean Theorem (etic knowledge) by these professionals. In other words, they strive to compare, interpret, and explain this mathematical knowledge they observe and experience.

This dialogical approach is concerned with the stability of relations between these two different cultural approaches. In our point of view, both approaches are essential to developing a clearer understanding of the social and cultural behaviors that shape mathematical ideas, procedures, and practices. The embodied and situated cognition enforced interaction between members of this particular and distinct cultural group with their own environment. Such holistic views allow for realistic assumptions of mathematical ideas, procedures, and practices because they provide specificity and incorporate energetic resource considerations that allow for direct cultural interaction, cooperation, and collaboration. This approach provides an explanation for developing empowering pedagogical visions for educations as a transformative endeavor. The idea of transformation entails structural shifts in the thinking process and actions, thereby promoting a critical, creative, and holistic approach in all aspects and levels of teacher development in mathematics education.

Final Considerations

Local knowledge, like language, can be used to make a living and solve problems in a particular community or environment. It evolves informally, in situ and is dynamic and creative, and it is constantly growing and adapting to meet new conditions. The expression local knowledge refers to the knowledge of any group of people who have lived in a particular and limited area or small region for a long period of time, which allowed them to develop a body of scientific and mathematical knowledge through generations of living in close contact with their own social, cultural, and natural environment (Johnson, 1992). It is embedded in a specific and dynamic system in which spirituality; kinship, local politics, scientific ideas, mathematical practices are tied together and influence one another.

In this context, ethnomodelling establishes relations between the local conceptual framework and mathematics embedded in relation to local designs. It is easier to use strictly and explicit numeric systems such as counting than to look at embedded mathematics such as architecture and crafts because it requires ethnomodelling. Hence, the mathematics knowledge can be seen as arising from emic rather than etic origins. In some cases, the translation to Western mathematics is direct and simple such as counting systems and calendars. In other cases, the mathematics is embedded in a process such as iteration in bead work, and in Eulerian paths in sand drawings (Eglash et al., 2006). In this regard, this act of translation is named ethnomodelling (Rosa & Orey, 2019).

It is crucial that ongoing research on ethnomodelling shows sophisticated scientific and mathematical practices, not just trivial examples by directly challenging the epistemological stereotypes most damaging to minority groups (D'Ambrosio, 1985). Thus, ethnomodelling research often uses the term *translation* to describe the process of modelling local cultural systems with a Western academic mathematical representation (Eglash et al., 2006; Rosa & Orey, 2017). However, as with all translation, the success is always partial. Intentionality is one of the areas in which the process is particularly problematic. Thus, it is important to analyze insights acquired through a variety of subjective and culturally contextualized methods.

The rationale behind the emic-etic (dialogical/glocal) dilemma is the argument that mathematical phenomena in their full complexity can only be understood within the context of the culture in which they occur. Defined in that manner, the usefulness of emic (local) and etic (global) approaches is evident through cultural dynamism (glocalization). We emphasize that ethnomodelling offers a broader view of mathematics and modelling, which embraces the diversity of ideas, notions, procedures, processes, methods, and practices rooted in distinct cultural environments. This aspect leads to increased evidence of cognitive processes, learning capabilities, and attitudes that direct learning processes occurring in classrooms.

In addition, by reflecting on social and political dimensions of mathematics, another important aspect of ethnomodelling is the possibility for the development of innovative pedagogical action for a dynamic and glocalized society. In this regard, glocalization means the acceleration and intensification of interaction and integration among members of distinct cultural groups. It also recognizes that they develop unique techniques, methods, and explanations that allow them for an alternative understanding, comprehension, new actions, and a transformation of societal norms. In this context, answers to the most fundamental research questions including the origins of humanity, the characteristics of human nature, and the form and function of human social systems is part of the worldview of every culture.

Hence, researchers and educators have been enculturated to some particular cultural worldview, and they therefore need a means of distinguishing between the answers they derive as enculturated individuals and the answers they derive as anthropological observers. Therefore, defining emics and etics in epistemological terms provides a reliable means of making that distinction. Yet, from an ethnomodelling perspective, both emic and etic approaches are considered as two sides of the same coin that help researchers and educators to gain a more complete understanding of the mathematical knowledge developed by the members of distinct cultural groups through its dialogical approach.

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Both MR and DCO conceived the ideas presented in this article and developed the conceptualization of the theoretical basis of ethnomodelling. They both jointly produced the final version of this article in all phases of its elaboration.

DATA AVAILABILITY STATEMENT

The theoretical data collected and analyzed during the development of this article are available from the corresponding author MR on readers' reasonable request.

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