Understanding Teacher Education Within the Scope of the Model of Semantic Fields

João Pedro Antunes de Paulo[a]
Maria Aparecida Viggiani Bicudo[b]

[a] Instituto Federal Catarinense, câmpus Rio do Sul, Rio do Sul, SC, Brasil
[b] Universidade Estadual Paulista, câmpus Rio Claro, Programa de Pós-graduação em Educação Matemática, Rio Claro, SP, Brasil

ABSTRACT

Background: In Mathematics Education many works use the Model of the Semantic Fields (MSF) to investigate aspects of the teaching and learning Mathematics and teacher education. However, none of these works analyzes the development of this theoretical model or investigates the theories that support it. **Objectives**: present a meta-analysis of the results of the doctoral thesis (Paulo, 2020) which aimed to investigate the existence of a Mathematics teacher education project consistent with the MSF. **Design**: The study is part of a hermeneutics analysis based on the MSF's ways of proceeding. **Setting and Participants**: The research has a bibliographic nature and was developed from 31 works published between the years 1992 and 2012. The selection of works that constituted the research corpus took place by consulting all works published by the research group Sigma-t, being elected those dealing with teacher education. **Data collection and analysis**: From the corpus of the research, an analysis was carried out according to the MSF's characteristic way of proceeding, explained in the item "Methodology" of this text. This analysis resulted in the production of a report from which the authors articulated their conclusions. **Results**: The study shows the existence of a teacher training project being articulated over the years by the researchers who make up the research group investigated. In a complementary way, a systematization of "Plausible Reading" also resulted from our analyzes as a research procedure based on the MSF. **Conclusions**: We conclude this article by pointing out the potential and limitations of the research developed during the PhD, as well as referrals for the research developed.

Keywords: Model of Semantic Fields; Mathematics Education; Teacher education; Plausible reading; Qualitative research.

Compreendendo Formação de Professores no Âmbito do Modelo dos Campos Semânticos

RESUMO

Contexto: Na Educação Matemática muitos trabalhos se valem do Modelo dos Campos Semânticos (MCS) para investigar aspectos do ensino e aprendizagem da Matemática e da formação

Corresponding author: João Pedro Antunes de Paulo. Email: paulojpa@outlook.com

de professores. No entanto, nenhum desses trabalhos analisa o desenvolvimento deste modelo teórico, ou investigam as teorias que o sustentam. **Objetivos**: apresentar uma meta análise dos resultados da tese de doutoramento (Paulo, 2020) que teve por objetivo investigar a existência de um projeto de formação de professores de Matemática coerente com o MCS. **Design**: O estudo enquadra-se em uma análise hermenêutica fundamentada nos modos de fazer próprios do MCS. **Ambiente e participantes**: A pesquisa possui cunho bibliográfico e foi desenvolvida a partir de 31 trabalhos publicados entre os anos de 1992 e 2012. **Coleta e análise de dados**: A partir do *corpus* da pesquisa foi realizada análise de acordo com o modo de proceder característico do MCS, explicitado no item "Metodologia" deste texto. Esta análise resultou na produção de um relatório a partir do qual os autores articularam suas conclusões. **Resultados**: O estudo evidencia a existência de um projeto de formação de professores sendo articulado ao longo dos anos pelos pesquisadores que constituem o grupo de pesquisa investigado. De modo complementar resultou, também, de nossas análises uma sistematização acerca da "Leitura Plausível" enquanto um procedimento de pesquisa fundamentado no MCS. **Conclusões**: Concluímos este artigo apontando potencialidades e limitações da pesquisa desenvolvida no doutoramento, bem como, encaminhamentos para a pesquisa desenvolvida.

Palavras-chave: Modelo dos Campos Semânticos; Educação Matemática; Formação de professores; Leitura Plausível; Pesquisa qualitativa.

INTRODUCTION

In this article, we present studies carried out between the years 2017 and 2020. Our research question, "what premises support the understanding of teacher training from a philosophical perspective within the scope of the Model of Semantic Fields ?" set us on a quest to understand the presence of a teacher education project that could be understood within the production of the research and development group in Mathematics Education Sigma-t. This group was led by Romulo Campos Lins, who developed his activities within the Graduate Program in Mathematics Education at Unesp RC between the early 2000s and 2017. When Romulo started teaching for the institution, in 1992, he already supervised some researchers; however, the group had not yet become official.

The historical excerpt was established by analyzing the corpus constitution, covering 20 years of this group's productions.

Our results reveal the group's intention to propose new possibilities for the initial teacher education that take as guiding parameters the theoretical model proposed and developed within their research activities. We analyze these works and create a possible systematization for the principles of the teacher training proposal produced until then. We also look at the development of the theoretical model and investigate its constitution and the main theories with which it dialogues, highlighting how these are present in its structuring. We are also concerned with pointing out, in an introductory way, the possibility of the theoretical model supporting discussions within the Philosophy of Mathematics Education.

This article is organized in the following sections: "Theoretical reference," in which we present our theoretical contribution, emphasizing the notions that constitute the Model of Semantic Fields we consider the first to understand our analysis process;

"Methodology," in which we present our methodological path, discussing the notion of "Plausible Reading" that guided our research actions; "Results and analyses" in which we highlight the main results obtained in the research and how we articulate them, and "Conclusions," in which we present a comprehensive synthesis on what we consider relevant for the Mathematics Education inquiry region.

THEORETICAL REFERENCE

The genesis of the Model of Semantic Fields (MSF) is in the intention to read interaction processes in the classroom positively. Here, positive refers to always reading what is said and never what is not present, the absence, what is missing. In his doctoral thesis, Romulo C. Lins, the main author of this theoretical tool, suggested that "knowledge" is always produced in relation to a semantic field, a set of possible meanings in a specific context. This means that knowledge is not always the same; it is circumstantial, it always depends on who speaks, and from where they speak.

This theoretical model is born within Mathematics Education, and the empirical evidence that supports Lins's conclusions in the proposition and development of MSF is produced into the Mathematics classroom. Be it in the investigation of how children produce meaning for Algebra, or how Mathematics teachers talk about Mathematics.

Between the 1990s and 2010, the MSF was structured based on the practices developed in master and doctoral research. This structure resulted mainly from the way of understanding what "knowledge" is and how it is produced; how the communicative process occurs; how knowledge is shared; and how these characteristics modify the formative processes, mainly in the initial Mathematics teacher education.

When assuming a posture that corroborates this theoretical model, our research takes as the main assumption that "knowledge" is statement-belief together with justification (Lins, 2012), believing that those who produce knowledge believe in something and enunciate that something, and have a justification for that belief. Enunciation (statement) is not restricted to speech. Gestures, scribbles, and the organization of things can also be understood as enunciation. It is usual, and, in this article, we will refer to "enunciative actions" as a way of covering all these possibilities.

Belief in this triad refers to the fact that whoever makes an enunciation makes it because it can be done. At least for the subject who enunciates something, that is possible. This subject of the enunciation acts in a coherent way with what is being enunciated. To explain this idea, Lins (2012) brings the example of someone who believes, being in a situation where he/she says it is not possible to see through the walls, not to search for a lost object in another room, looking "through" the walls. The fact is that if that someone believes what he/she says, then that someone acts coherently.

Justification, the third constituent element of "knowledge," refers to what the subject who enunciates believes authorizes him/her to enunciate. It is not an explanation; it need

not be. Justifications can be constituted from empirical experiences carried out by the subject who enunciates, as they can also be constituted from cultural transmission. It is, for example, a socially accepted justification, and passed on to generations, that some behaviors are sinful. Even if he/she did not experience them, or if he/she does not have an explanation, a subject makes statements that corroborate the knowledge of these sins.

There are also justifications taken as legitimate by the authority that is given to someone (Paulo, 2016). For example, the Mathematics teacher's authority may be justification for the belief that $2 + 2$ equals 4. Someone claims that there are infinities of different sizes, not because of an experience carried out in direct contact with the infinite, but because this someone feels authorized to say it, borrowing the legitimacy of the math teacher who told him/her.

Thus, in this theoretical stance, we emphasized that knowledge is always from someone because it is always produced from a belief, and justification is a constituent. This allows us to differentiate, for example, statements that appear to be the same. A person who justifies statement $2 + 2 = 4$ with authority borrowed from the Mathematics teacher, produces a different knowledge than someone who justifies statement $2 + 2 = 4$ based on Peano axioms.

This distinction is possible because it understands that justification is a constituent of knowledge. It is not a way of saying whether knowledge is true or not. It is part of the stated knowledge. Therefore, when taking different justifications, a subject can produce different knowledge. Also, for the same statement-belief, as in the previous example, they can be similar elements that constitute different knowledge.

From the theoretical point of view, this is important as it allows, in the classroom interaction process, that similar statements are not taken as equal. It is not by repeating what the teacher says that the student is producing the "same" knowledge as the teacher. It is always necessary for the teacher to explain and make the students also explain their justification for what they are saying. Which, they believe, authorizes them to state what they are stating.

In this theoretical stance, it is also a fact that knowledge cannot be transmitted. Even if different subjects express the same belief, the justification can be different, which will characterize different knowledge.

When paying attention to how knowledge is shared, it is necessary to distinguish between knowledge and what is left of it when the enunciation ends. We call "residue of enunciation" what remains when the enunciation ends. The sound of the words and the scribbles on a sheet of paper are all marks of a knowledge production process that has already occurred. An "enunciation residue" does not contain knowledge. It is just a culturally established mark of a knowledge production process.

We must open parenthesis for this explanation. We draw attention to the procedural character that knowledge assumes in this perspective. There is always a process of knowledge production. There are marks of that process. Even if the subject who enunciates turns his/her attention to what was enunciated, a true re-turn will no longer be possible

for him/her. He/she will always put himself/herself in a new process of producing knowledge from what has become the residue of his/her previous enunciation. Closing the parenthesis.

Because they are culturally instituted, the residues of enunciation are traces that bounds the knowledge production process. When a subject turns his/her attention to what is written on the blackboard in a Mathematics class, that attentive subject can produce any knowledge. For example, a child not completely internalized in a culture that speaks (produces knowledge) about Mathematics may see an alien language there (Lins, 2012), while an advanced calculus student, looking at the same residue, will produce knowledge about the techniques of integration and derivation rules about what his/her teacher talks.

The student of advanced calculus can also produce knowledge about an alien language, but, in the context of the Mathematics class, it is more plausible that his/her teacher is talking about techniques of integration and derivation rules than about an alien language. It is because it has been internalized[1] in different cultures that the subject of knowledge can produce different knowledge from the same "residues of enunciation". The greater his/her repertoire of ways of producing knowledge accepted as legitimate within cultures, the greater his/her ability to produce different knowledge from the same residue of enunciation.

One more example can be taken when imagining a polyglot subject. By being internalized in different cultures, with different languages that share the fact that they use similar alphabets - we refer to the ordered set of graphic signs -, the polyglot subject can produce knowledge taking any of the languages he/she knows as legitimate. He/she can read with accents or say that there is nothing written there that he/she can understand.

This characterization gives us clues to talk about how the communicative process is seen from this theoretical perspective. At the same time that knowledge is being produced, an enunciative action is being developed, and residues of that enunciation are being

---

[1] For Lins (2012, p. 29, our translation), "'to be internalized' means, precisely, 'to be owned'." For this author, internalization is the process in which cultures internalize subjects. He says, "it is better to say so than to use the usual convention and say that 'the subject has internalized x'. […] With the new formulation, we can talk about legitimacies and legitimate ways of producing meaning without having to talk about groups of people" (Lins, 2012, p. 13, our translation). From this perspective, the subject is internalized in cultures as he/she produces knowledge, seeking to speak in interlocution directions that he/she believes will be taken as legitimate by that culture.
[In] this note, we consider it essential to bring other conceptions of internalization circulating in the Human Sciences, notably in Sociology and Psychology, as a counterpoint to Lins's.
Internalization: in sociology and other social sciences, many authors, like George Herbert Mead (1969), work to explain this concept. It refers to the acceptance of attitudes, of a set of norms and values accepted and practiced in the social environment through the socialization process. Through this process, well explained by Bronfenbrenner (1971), which includes unconscious learning, without critical reflection, these attitudes, norms, and values become part of the person's way of being. Bronfenbrenner (1971) brings a study on the internalization process through relationship practices that lead to obedience and this to self-discipline, internalizing the values implicit in those practices. Bernstein (1974) brings studies on the sociology of language, explaining the internalization process when dealing with themes such as social class, language codes, and grammatical elements, for example. Lev Vygotsky provides an alternative definition for internalization, through the internal reconstruction of an external operation. The concept worked in psychoanalysis is introjection. It refers to a formation process of the superego through the absorption of uses and customs prevailing in the social structure, practiced and assumed by parents and authorities to whom the individual owes obedience and respect. In these conceptions, what comes from the outside - understood as culture, and in a more defined way, as society- plays a predominant role in the formation of the person.

produced. At the same time that knowledge is produced, traces of legitimacy are instituted so that, based on the residue left from this enunciative action, certain knowledge is taken as legitimate. That is, any knowledge can be produced from that residue, but some, not all, will be taken as legitimate. What establishes the legitimacy of the latter, and not of the former, is the culture within which the residue of enunciation was produced.

The fact that "knowledge," from this perspective, is always circumstantial stands out. It is because it is produced within this culture, that is, in these circumstances, space and temporally situated, that some knowledge is legitimate in detriment of others.

From this perspective, we have the subject who enunciates a knowledge and the residue of enunciation that is enacted when enunciation finishes. There is also a third element, characterized as "interlocution direction." As we stated earlier, the subject who enunciates does not enunciate any knowledge, but one who, he/she believes, is legitimate in the context in which he/she is producing that enunciation, just as with the advanced calculus student in the previous example. We call "interlocution direction" this anticipation of the legitimacy of the speech of the subject of knowledge. He/she always enunciates in a direction that he/she believes to be legitimate, i.e., in a direction where what he says will be accepted.

Let us resume the example. If he/she is in calculus class, the student will produce, based on the residue of enunciation in the book, knowledge in terms of techniques integration and derivation rules because, anticipating the legitimacy of his/her speech, he/she believes that his/her teacher would also say that. Here, the interlocution direction is one that this student believes he/she shares with his/her teacher. At home, when talking to a child who reads his/her calculus book, that same student can produce knowledge by speaking in a direction that he/she believes would be shared by a child. He/she will not talk about techniques integration and derivation rules. Perhaps he/she speaks in terms of an alien language or some other direction that he/she believes that child would also speak.

Thus, communication consists of three elements: the subject who enunciates knowledge, the residue of his/her enunciation, and the interlocution direction. The interlocution direction is constituent because it establishes that knowledge will be legitimate. It is the demarcation of the culture that establishes what knowledge can be produced from the enunciation residue.

It is in the process of internalizing interlocution directions taken as legitimate in a specific culture that the subject of knowledge is being internalized by that culture (Paulo, 2016). In other words, it is because it seeks to speak in directions that a given culture takes as legitimate that whoever produces knowledge becomes part of that culture. He/she plausibly produces interlocution directions, anticipating the legitimacy of what he/she says. This plausibility is what constitutes the dynamic character of cultures. It is not a question of producing the "same" knowledge, but knowledge that is plausibly shared among the subjects of a culture. See, for example, Lins and Gimenez (1997), who characterize " Mathematics on the streets" and "Mathematics in the school."

By setting communication in motion, we establish what, from this perspective, we call the "communicative process." The subject of some knowledge produces an enunciation that becomes residue in a direction that he/she considers legitimate. Another subject produces knowledge from the residue of enunciation of the other subject, and produces a new enunciation. Thus, the knowledge produced by a subject becomes residue from which the other subject produces knowledge. Once the process is set in motion, the roles shift, and the communicative process takes place.

Insofar as these subjects who produce knowledge share legitimacy, i.e., they have been internalized by the same culture, these knowledge productions converge in interlocution directions plausibly shared. It is to the extent that these directions are shared that the sensation of knowledge transmission occurs. It is to the extent that one produces an enunciation with the justification that the other accepts it as legitimate, that they feel that they speak of the same thing (Lins, 2012).

Dantas, Ferreira and Paulo (2016) distinguish two processes of interaction. For these authors, a "productive interaction" is characterized by the sharing of interlocution directions. Sharing means that a subject of knowledge produces statements taking as legitimate the rationale that another subject of knowledge would also take as legitimate. For these authors, there are also situations of interaction in which enunciations are produced, but justifications are not shared. In this way, the subjects in this interaction do not share, even if plausibly, a direction of interlocution. This interaction, for the authors, is not a productive interaction. This second type of interaction corresponds to situations in which it seems that the subjects are talking about the same thing, but, in fact, they are not.

In this way of understanding the communicative process, we see the possibility of both synchronous and asynchronous communication. In the second case, the subject of a knowledge that produces enunciations from a residue, will not have the possibility of interaction with the subject of knowledge that produced those residues. At this point, an issue that deserves to be highlighted arises.

When assuming the possibility of asynchronous communication, the MSF highlights that it is not to the other subject of knowledge, to the person in front of him/her, whom whoever produces an enunciation is addressing. Once again, affirming the idea that there is no transmission of knowledge. Whoever produces an enunciation always enunciates in an interlocution direction. Even if it is in front of a person, it is not to that person that an enunciation is addressed, but to an interlocution direction that would plausibly adopt the same legitimacy as the subject who enunciates.

In the case of asynchronous communication, the subject who produces knowledge does so in an interlocution direction that he/she takes as legitimate for, plausibly, sharing the same culture as the author of that residue. This does not guarantee at all that there is such an author for the residue from which a subject of knowledge is producing knowledge. It just guarantees that this subject that produces knowledge is able to establish coherence to that residue and produce knowledge from it in some direction, i.e., about some culture of which he/she, the subject that produces knowledge, is part. See as an example Luchetta's work (2017).

Because of this possibility, the MSF does not make a value distinction between the knowledge produced by a gypsy woman who reads tea leaves at the bottom of a cup and the chemist who reads the report of an experiment carried out in a laboratory. It is not the fact that here there is a person who intentionally produced a specific residue, called a scientific article, and there the arrangement of tea leaves is attributed to fate, which determines that one is a communicative process, and the other is not. However, from the MSF perspective, the existence of a subject capable of producing knowledge from those residues determines the existence of a communicative process. In other words, a communicative process is established by the existence of a culture, from which justifications can be taken as legitimate, which makes the production of knowledge plausible in some interlocution directions.

In the gypsy's case, what makes the knowledge produced from reading the residue of enunciation that forms at the teacup plausible is not the existence of an author who put them there, but the existence of a culture in which people read the shapes and configurations of the leaves. Legitimacy is in the culture, and not in the author. It is the culture that lends authority, and therefore legitimacy, to an author to say what he says.

Once more. It is because it is part of a culture that the subject of some knowledge is capable of producing the enunciations he/she produces.

In the case of reading a scientific article, it is by sharing a culture that establishes the language, that field as science and that method as scientific, that a subject can produce from that residue knowledge in a specific interlocution direction. A scientific field is constituted because there is a group of subjects capable of anticipating the legitimacy of specific ways of producing knowledge. And it is within this group, or about it, that specific knowledge is taken for granted. Some ways of producing knowledge are instituted as legitimate, and some regimes of truth are established. These institutions are not stable and are related to the dynamics of the culture, as we mentioned earlier.

By assuming a posture that corroborates this theoretical perspective, our research advances precisely in the understanding of reading residue of enunciation and producing knowledge aiming at an interaction that will not occur. In other words, we produce enunciations according to the legitimacy that we have when studying residues of enunciation. From the theoretical perspective of the MSF, this process is characterized as a "Plausible Reading" of these residues. Plausible because it is acceptable, it seems to be possible (Lins, 2012). In other words, we produce statements and constitute interlocution directions that are consistent in their own terms. In the next section, we present this notion and discuss its implications for doing research.

METHODOLOGY

The "Plausible Reading" is closely related to the Model of Semantic Fields (MSF) production trigger, that is reading situations of interaction in the classroom, and intervening while they happen. We say that it is related to, not that it is, the way of reading these

situations, because we understand that, as a process, Plausible Reading also encompasses the cases of asynchronous communication that we mentioned earlier, enunciations that are directed to an interaction that will never happen.

The way we understand the process of producing a Plausible Reading was summarized by Paulo (2020). For this author, reading plausibly involves the following elements: (*i*) constituting an author; that is, when faced with an residue of enunciation, we must inquire about the legitimacy that mobilized the production of that residue. It is not a question of imagining a *persona* who enunciated that residue but constituting legitimacies that make that residue coherent in itself. (*ii*) Constituting an object. From the theoretical perspective of the MSF, whenever knowledge is produced, an object is produced, i.e., something about what something is said (Lins, 2012). In the process of reading a residue plausibly, we also constitute an object. The concern here is for this object to be consistent with the legitimacy that we have constituted for an author. (*iii*) Constituting a kernel. The notion of kernel, from the MSF perspective, has to do with the procedural character of knowledge production. As stated, knowledge is produced within semantic fields, and a semantic field is formed around a kernel. Similar statements-beliefs produced related to different kernels constitute different knowledge. (*iv*) making a statement. The process of reading plausibly is characterized by the production of an enunciation, under the terms of the MSF, from a residue. Lins (2012) states that reading is production, reading is saying "what is here is..." (*v*) constituting an interlocution direction, and (*vi*) anticipating legitimacy. Items *v* and *vi* refer to the communication process as we characterized it in the previous item of this text. Since Plausible Reading is a process of producing knowledge from some residue, then that knowledge is produced in an interlocution direction, and we are concerned with anticipating its legitimacy, i.e., we are concerned with plausibly mobilizing the justifications that would be considered legitimate in some cultural context. Rather than being delimited by the legitimacies that we have already mobilized before, this cultural context is delimited by an author that we had constituted at the beginning of the process. This happens because, as a method that seeks to produce knowledge from a specific residue of enunciation, Plausible Reading seeks to operate from the legitimacies of that subject of knowledge that we have instituted as an author of the residues that we read. In other words, reading plausibly is an attempt to see the world through the eyes of others (Lins, 2012).

The Plausible Reading process is characterized by an attempt to operate according to the legitimacies of others, failing to accept as legitimate, or unique, the legitimacies that the reader can mobilize. To read plausibly is to seek to produce new legitimacies and operate with them, producing statements that would be accepted by an author that we believe produced the residues of enunciation from which we produce knowledge.

In the process, we establish a set of legitimacies and produce statements that are consistent with this set established. The concern is to operate and produce knowledge concerning this group and not to something outside it. There are no value judgments about those legitimacies established. According to Lins (2012), the concern is to establish a context in which it is acceptable; it seems to be so, the knowledge we produce makes sense.

We draw attention to the fact that the establishment of such a context is possible only insofar as we share, to some extent, a culture from which residue may have been produced. Moreover, it is in these terms that plausible is employed. From this perspective, we understand that the reading we produce is always circumstantial, and, like the knowledge explored in the previous item, it is the culture that establishes the existence of the reading we produce.

We do not claim that we cannot produce knowledge from the residues of enunciation produced from a culture that has not internalized us. Take, for example, a book written in Mandarin. A reader who does not know this language shares very few legitimacies with a culture that writes in that language. Besides the fact that he/she recognizes that it is a book and, if there is any iconography, he/she can affirm, based on the images, what the book is about, he/she cannot produce knowledge in the same interlocution directions as that person who can read Mandarin will produce when reading the book. This reader will not be able to take as legitimate the justifications mobilized by that someone that reads Mandarin, in the process of producing knowledge from that residue, because he/she does not share with him/her a culture that reads and writes in Mandarin. On the other hand, from that residue this reader can produce knowledge by mobilizing legitimacies of the very culture. These legitimacies would not possibly be accepted by someone who reads Mandarin.

Any reading can be done, just as any knowledge can be produced, but culture, shared to some extent by the subjects that produce knowledge, demarcates legitimate ways of producing knowledge from an residue of enunciation.

We assume it as plausible and consider it acceptable, because we do not have access to the culture of an author that we have instituted for the enunciation residue from which we produce knowledge. As we discussed in the previous section, we cannot be sure that such author exists. In fact, for the MSF, it is not important that an author exists, because, as we discussed, it is the culture that lends legitimacy to the knowledge produced. In other words, plausible readings are only produced from or for cultures that we can make exist[2].

An example of plausible reading can be taken from Luchetta's work (2017).

> […] In this paragraph, Euler is constituting the object "series," so, for him, the "***infinite series** arises from the expansion of the finite expression*;" and "*both must have the same value*." And he tells us that every series has a finite expression that originated it. Thus, we can produce meaning for the *sum of a series* by saying that adding an infinite series means returning the series given to the finite expression that generated it. […] Furthermore, the *generality of algebra principle* assured Euler that this sum was valid for any value of *a*. This principle consisted of the

---

[2] The "make exist" here is very important. Cohen's Monster Theory helps us explain why. Every monster is dual in its corporeality, at the same time repel and attract. It is because while it is strange, it is also familiar. So it is with the cultures. Even if we create a new one, a subversion of all the previous ones, it keeps something similar to its precedents.

following assumption: "If an analytical formula was derived by the rules of algebra, then it was thought valid in general." (Ferraro, 2008, p. 209-210, our translation). (Luchetta, 2017, p. 87-88, highlight in the original text).

When producing a plausible reading from Leonhard Euler's "*Elements of Algebra*," the author sought to understand the "series" object produced by author Euler in his work. In her thesis, the author establishes the difference between the way of producing knowledge plausibly mobilized by Euler, and what is accepted as legitimate today. For her, many statements-belief enunciated by Euler are still accepted today, but not the ways of producing knowledge; statements-belief can be justified in different semantic fields.

Paulo (2020) assumes the Plausible Reading as presented, and analyses a set composed of 22 scientific articles, a master's dissertation, 5 doctoral theses, a *livre-docente*[3] thesis, a memorial, and an interview granted to us. Except for the interview, this material was produced between 1992 and 2012 by Romulo Campos Lins and researchers under his supervision during their master's and doctoral courses.

The selection of this material was based on a previous survey of the researcher's and his advisors' productions. Survey carried out with the Lattes platform. Of these works, we highlight those that deal with "teacher education." This criterion was established in line with the research question carried out (Paulo, 2020) about the teacher education produced throughout the group's work.

In this research, 31 works were selected to constitute the corpus, analyzed according to the MSF's way of proceeding, as previously presented. In this process, we highlight how the group investigated understood the "teacher education" object, how the theoretical model was constituted, and we understood how necessary it was to turn our attention to the theories with which the authors dialogued in the MSF production process.

Thus, after analyzing the group's productions, we proceeded with the study of the works that were in the references of the works we analyzed. The selection criteria for these works was based on our understanding that they were configured as references that contributed to the development of the theoretical model, excluding those authored by the group itself that had already been analyzed.

This selection resulted in works by A. R. Luria, "Cognitive Development: its Cultural and Social Foundations", L. S. Vygotsky, "Thought and Language" and "Mind in Society"*,* Nelson Goodman, "Of mind and other matters" and "Ways of Worldmaking", Tomas Silva, "A produção social da identidade e da diferença" (The social production of identity and difference) and "Monstros, ciborgues e clones" (Monsters, cyborgs and clones), and Jeffrey Cohen, "*Monster theory* (*seven theses*)."

---

[3] TN: title Brazilian universities award someone who applies for a specific university public call, to certify the person's superior capacity in teaching and research. In the process, aimed only at people with a doctorate degree, a board of experts and PhDs evaluates and ranks that person's academic trajectory.

The analysis of these works took place to show which objects were being constituted by each author. We sought to understand each of them in their own terms and expose the legitimacies constituted in the reading of each author. In a second step, we turned our attention to the MSF as we had constituted it in the study that we carried out, and analyzed how the legitimacies that we constituted in the study of the authors mentioned in the previous paragraph were depicted in the MSF and productions of the group analyzed.

Our conclusions are presented in Paulo (2020), in which we systematize our understanding of the teacher education project evidenced from the study of the research *corpus* in the form of a "Didactic Model of Semantic Fields." So we named it, because we understood the group's effort to systematize possible ways through which any teacher could implement the constituted theoretical model in the classroom.

RESULTS AND ANALYSES

The analysis carried out, as presented in the previous item, allowed us to enter an aspect that until then had not been explored by the Brazilian Mathematics Education. The Model of Semantic Fields (MSF), throughout more than 20 years of history, has been mobilized in several research works. Such works are committed with presenting the notions that constitute the theoretical model and their articulation with the problem in question, but none of them investigated the architecture and development of the theory.

In producing this analysis, Paulo (2020) does not exhaust this aspect, but inaugurates it, problematizing some aspects of the MSF proposal and development. In our research, we present possibilities for future works that also propose to follow this path.

Regarding the development of the MSF, in a general way, the formalization of the theoretical model can be seen in the following works: in articles published in the years 1993 and 1994, the theorization begins to be called the "Theoretical Model of the Semantic Fields". They bring the notions of "knowledge." "meaning," "kernel," and "semantic field" to the center. In 1996, the notion of "communication" is presented in an article published in the Conference Day, from the British Society for Research into Learning Mathematics. In 1998, at the sixth National Meeting of Mathematics Education, the notion of "communication" is added to the notion of "communicative space." In the following year, 1999, in a book that brought together works by professors from the Graduate Program in Mathematics Education at UNESP/RC, the distinction between "biological subject" and "cognitive subject" was established.

In the initial works, Lins was concerned with establishing the theoretical model and explaining how human cognition occurs from this perspective. It was necessary to think about those concepts to lay a solid foundation in learning. The concept of cognition Lins takes from the Soviet psychology, Vygotsky Circle, not as a complete adoption of these theorists' ideas, but as an approximation to their way of thinking. This approach and concern with cognition reflect the strong presence of the Soviet theorists in the works during this period.

Gradually, the productions of the group formed by Lins and the researchers under his supervision moved from cognition to the learning of Mathematics in Basic Education, in Higher Education, to the teacher educationand, finally, to culture in a general way. All phases are centered on Mathematics Education as a space for action and a region of inquiry.

This first period of production runs from 1992 to 1999. A second period can be characterized between 1999 and 2004, when the group's interests moved from the production of the theoretical model, focusing on the Mathematics teacher education. The productions revolve, at first, in thinking about the mathematical education in undergraduate courses. This moment is marked by a criticism to the traditional way of education, expressed in the works as a "*três mais um*" (three plus one) model, and also marked by the concern with the proposition of what is now called education through Mathematics, focusing on the transformation of Mathematics subject matters into Mathematics Education subject matters.

The third period, from 2004 to 2012, which ends with the article in a book published for the twentieth anniversary of the theoretical model, is marked by the central interest in the relationship between culture and teacher education. Patrícia Linardi's doctoral dissertation (Linardi 2006) is representative of the direction of the group's productions towards teacher education in undergraduate courses in Mathematics. The author investigates what she calls traces of mathematical training, of subjects of mathematical content, in the practice of Mathematics teachers. These traces are understood as what remained from the Mathematics subject matters in the Mathematics teacher's practice, and can be observed from what the teacher says about his/her practice.

Looking at this aspect of teacher educationand at Lins' productions, the group began to discuss the Mathematics teacher education. During this period, the second book was published at PPGEM/RC (2004). In this issue, there is an article by Lins entitled "Matemática, Monstros, significados e Educação Matemática" (Mathematics, Monsters, meanings, and Mathematics Education), in which topics of the period appears. It is his concern to discuss culture.

At the end of this period, Lins produces an "axiomatization" of the theoretical model. In his book published in 2012, Lins writes a chapter entitled "Modelo dos Campos Semânticos: estabelecimentos e notas de teorizações" (Model of the Semantic Fields: establishments and theorizing notes). The chapter is a synthesis of the MSF notions that had been published until then. Researchers who were under his supervision were also included, bringing, in the form of chapters, their doctoral theses.

A fourth period can be characterized from the year 2012 until the current publications. In this last period, the group's concerns were focused on thinking about teacher education centered on the Categories of everyday life. Atomizing the publications that mobilize the MSF is also a characteristic of this period. These are no longer centered on the research group investigated in Paulo (2020) and are scattered among other groups constituted of researchers who were under Lins's supervision. Paulo (2020) does not approach any analysis of those productions. However, we understand that an investigation

of the productions would reveal the paths that are being followed, showing how the MSF is being worked.

When we turn our attention to "teacher education," the object of study of the research carried out by Paulo (2020), we characterize three distinct periods in the group's productions. These periods are distinguished by the approach that was launched on the topic of teacher education within the group investigated. They are intertwined in the periods of development of the theory and are delimited by the years of proposition of the research projects of the group investigated.

In the first period, delimited by the years 1999 and 2004, the group's attention turns to teacher education for the first time. As highlighted by Silva and Paulo (2020), this focus on teacher education is loaded with experiences produced in doing research on teaching and learning Algebra that occurred in the previous period. During this period, the group was concerned with thinking about a new curriculum for Mathematics initial teacher education courses.

Corroborating discussions at the national and international levels, the group criticizes the "three plus one" format that is characteristic of undergraduate degree courses and, using triggers as the results of Silva's (2003) and Oliveira's (2002) research, proposes a new organization for these courses. Both Silva and Oliveira evidenced in their conclusions that Mathematics teachers, when putting themselves into the activity of producing meaning for texts in mathematical discourse, mobilized categories that the researchers called non-mathematics. The researchers also pointed out that although they took courses to provide fundamental knowledge for teachers, these students did not mobilize this knowledge when they were talking about mathematical objects.

Based on these results, the group proposed that instead of courses focused on the categories of Mathematics, such as Algebra, Calculus, and Geometry, initial training courses should be centered on notions that can be problematized from different perspectives, such as the notion "Space." The group understood that this notion is familiar to both students and teachers. The student can talk about "space" without mobilizing mathematical notions. When starting to talk about it, the teacher can share with this student other ways of talking about the notion, and among these new ways will be the categories of Mathematics. In other words, the notion "Space" is focused from common sense, from Calculus, from Geometry, from Algebra, and so on. The focus is on what can be said from each of these perspectives and what differs between them.

> [...] this is exactly where the strength of the approach we developed was revealed. Upon reaching this point, you can say to the student: "I am not asking you to give up this (this object); I'm just realizing that we are, in Linear Algebra, talking about something else." The objective is not that the student would learn Mathematics more, but that he/she would learn to recognize the existence of this extremely common process in the production of meanings and, thus, become more capable of recognizing it when it occurs in his/her classrooms: this is teacher education BY mathematics (Lins, 2002, p. 74, highlights from the original).

Also in this period, based on the characterization Lins and Gimenéz made in 1997, the group proposed a distinction between what is usually called Mathematics. When referring to the Mathematics of the mathematician, the Mathematics of the Mathematics teacher, and Mathematics on the streets, the group was talking about different ways of producing knowledge that mobilizes different legitimacies. This is important for the group, as it allows them to produce a reading of school Mathematics that is not deficient concerning the Mathematics produced by the mathematician. It is not a simplification, but a different way of producing knowledge.

In the following period, 2004 to 2012, the group's works dealing with teacher education were concerned with analyzing possibilities for the implementation of this education proposal designed in the previous period. In this movement of thinking about modes of operation, the notion of "Categories of everyday life" becomes evident. This nomenclature means those ways of producing knowledge, distinct from those taken as legitimate by the Mathematics of the mathematician and mobilized by the subjects in the process of interaction. In other words, the Categories of everyday life include those ways of producing knowledge that the subject that produces knowledge mobilizes in his/her daily lives, that is, it is about ways that are, for him/her, usual.

The education project proposed in the previous period began to be considered in relation to the Categories of everyday life, and the education itself began to be understood as to how to enable the teacher to move between everyday life and the mathematician (here we can say Categories of the Mathematics of the mathematician) and how to enable the teacher to develop the skill of reading his/her students, knowing what daily life is for them, also assuming the posture of thinking their daily lives as a starting point.

This characterization of everyday life and its mobilization in the teacher education project, as thought by the group, was made possible by the inclusion of cultural studies in the references with which they dialogue. At the beginning of this period, the group acknowledges that the education process can be characterized as the sharing of cultures, a process of expanding the repertoire of ways of producing knowledge. As proposed in the works of that period, education is a sharing that does not aim at replacing ways of producing knowledge already mobilized by the subject in training. Thus, Mathematics is seen as another way of talking about things, rather than the science that unveils their true meaning.

> […] In front of the student - here representing the normal, ordinary citizen - a cultural body appears (which cannot be denied), in the form of a scribble, a few words. The other speaks about this thing, thus creating a demand for the student also to speak about it, that *he/she produces meaning for it*. But he/she *cannot*: what can the student say when the teacher affirms - and "*demonstrates*"; - that the cardinality of real numbers is greater than the cardinality of rational numbers? One infinite greater than the other? This is truly monstrous for the student and for the teacher - the Mathematical representative of the mathematician. Although this "fact" is recognized as peculiar, it is nothing more than a pet monster: so it is,

although we recognize the distance between this and the "the common life" (Lins, 2004, p. 116, highlight in the original).

This cultural aspect becomes nuclear in the following period, characterized in Paulo (2020) as started in 2012 and continuing until the present day. The Categories of everyday life became triggers of the education process. The works seek to think of situations that are driven by the processes of "defamiliarization " and "decentralization."

These notions had already appeared in the previous period and are used to characterize the encounter between different cultures. The process of defamiliarization occurs in the encounter with what is unusual, what cannot be said. In terms of the MSF, a residue of enunciation   from which the subject of a knowledge is unable to establish coherence by mobilizing only the legitimacy that he/she has already internalized. The decentralization process, a possible stance taken in the face of defamiliarization, is characterized by the attempt to produce legitimacies for what is strange. It is the movement to search for legitimacy that makes the stranger coherent, that he/she finds another place where it is possible to say something that in this place cannot be said. It is not a naturalization of the strange, it is not to make it normal, but to understand that the strange is produced from another way of producing knowledge.

In this research, we show the difference between these periods in terms of the productions that characterize each of them. When analyzing these productions, we highlight the concern with teacher education and the search for the systematization of a proposal for teacher education. Despite the several studies analyzing different perspectives of this proposal and the implementation of some of its principles carried out in continuing education courses, there is no systematization in the group's production for the project being considered. In the subsequent periods, there is no systematic proposition for the Mathematics initial teacher education.

In the preparation of this article, we have highlighted that the proposal for the Mathematics teacher education does not differ between the characteristic periods of the group's productions. It is always resumed and articulated with the references that are adopted, but, in short, it aims at a curricular restructuring of the training courses. It becomes clear to us authors that, when proposing a curriculum that does not take the categories of the Mathematics of the mathematician as a structuring element, the idea of initial education provided in a degree is disrupted. For the curriculum designed by the group and exposed in some of its aspects, as we stated, there is no complete explanation, it is not necessary, nor recommended, to distinguish between initial and continuing education. We understand that the education proposed by the group takes place in an amalgamation with the teaching profession.

In Paulo (2020), there are intermittences that, as we understand them, structure a education project designed by the group in the period we investigated. This project was named "Didactic Model of Semantic Fields," corroborating ideas presented by Bathelt

(2018). Here, we presented more systematically these intermittences that underlie a possible teacher education project that assumes the MSF as a theoretical perspective.

1. Didactic action. These are acts that intentionally create conditions to reach the epistemological limit of a specific way of producing knowledge, opening the possibility of an invitation to visit new places. Under the terms of the MSF, a didactic action aims to foster interactions that explain the rationales, bringing the difference to the enunciative plane in which the cognitive places of students and teachers can be problematized.

2. Mathematical maturity. It is constituted from a repertoire of experiences that makes the teacher feel capable of producing knowledge, operating with legitimacies of the Mathematics of the mathematician.

3. Curriculum. Thought in a way that the formation can no longer be dissociated from the professional practice of the Mathematics teacher. As currently thought, the disciplines give way to categories from which a notion can be focused on different ways of producing knowledge. Professional performance is part of this process, and the space to think about and propose interventions is central.

4. Through Mathematics. Mathematics, as seen in Western culture, is considered another way of producing knowledge that can be taken as legitimate. The "through" here refers to assuming the ways of producing legitimate knowledge within the Mathematics of the mathematician to produce new knowledge with them for everyday situations. The key is to make a difference. By assuming different legitimacies, different knowledge can be produced.

5. Practice. The practice is understood, in this perspective, as a formation imbricated with the exercise of the teaching profession. It is not an extension of the workload of practical components in the undergraduate curriculum. It is to dissolve initial training in professional practice.

6. Working groups. The formation is the constitution of a repertoire of legitimate ways of producing knowledge, legitimate ways of being in the world, in a specific culture. As a possibility of education, working groups are spaces for carrying out formation movements in which teachers, at different moments in their careers, can share legitimacies, constituting other interlocution directions and, collaboratively, instituting ways of being teachers.

7. Positive reading. Reading positively is always looking for identities that allow interaction to be maintained. Interacting is, for this conception of education, the condition for sharing something.

These 7 bullet points synthesize a possible structure for education thought by the research group Sigma-t, according to the study we conducted. They lay the foundations for a education that is not centered on a single way of producing knowledge, that of Mathematics of the mathematician, and is structured as an amalgamation of training with teacher practice. What is central to this proposition is the understanding of knowledge as from the theoretical perspective assumed by the group.

CONCLUSIONS

The analysis of the production of the Model of Semantic Fields allowed us to show that this theoretical model emerges as a search for enabling teachers to read the interactions that happen during the class, in order to make it possible, in the dynamics of the process, to maintain the interaction with their students.

From the perspective of the MSF, it is in the interaction that the possibility of sharing ways of knowledge production resides, this sharing being the objective of the educational processes. It is necessary to highlight that, from this theoretical perspective, sharing means expanding the repertoire of ways of knowledge production. The substitution of one mode for another serves a political project and, from this perspective, is not part of the nature of the educational process.

In this way, the teacher education project that we highlight works through Mathematics, using it as a means of promoting the expansion of this repertoire, as this area of knowledge takes as legitimate ways of producing knowledge that are not mobilized in everyday life. In other words, understood as Western science, Mathematics assumes, from this perspective, the role of putting under questioning the naturalized ways of producing knowledge, and, at the same time, what is commonly said about (from) mathematical objects is also questioned. A duality metaphorized by Cohen's Monster Theory. This is because when using the legitimacy of everyday life to talk about mathematical objects, the characteristics that separate the ways of producing knowledge considered legitimate in Mathematics are eliminated.

Operating an educational project, seen here as a proposal for the education of Mathematics teachers, from the perspective of the MSF, requires a deep change in didactic actions because knowledge is understood from here as being of the order of enunciation, rather than of the announced. We warn that our educational system and schools are organized based on the idea that knowledge is transmitted and can be accumulated.

AUTHORS' CONTRIBUTION STATEMENT

MAVB supervised the development of JPAP's doctoral research project, the results of which are presented in this article. MAVB collaborated in understanding the analysis carried out and in conducting the articulations that bring together ideas, specifying the theory studied. JPAP was responsible for formatting this article. Both authors discussed and contributed to the final version of this article.

DATA AVAILABILITY STATEMENT

The data supporting the results can be consulted in the institutional repository through the link <http://hdl.handle.net/11449/191665>. Supplementary data may be provided by the author JPAP upon reasonable request.

REFERENCES

Bathelt, R. E. (2018). *Ensaios para um modo de ler modelos didático-teóricos em educação matemática: um estudo sobre a ótica do Modelo dos Campos Semânticos* (219 f.). Tese de doutorado, Educação Matemática, Instituto de Geociências e Ciências Exatas, Universidade Estadual Paulista, Rio Claro.

Bernstein, B. (1974). *Class, codes, and control: theoretical studies towards a sociology of language*. Lonon: Routlege.

Bronfrenbrenner, U. (1971). *The ecology of human development: experiments by nature and design.* Cambridge: Havard University press.

Dantas, S. C.; Ferreira, G. F.; Paulo, J. P. A. (2016). Uma noção de interação colaborativa elaborada à luz do Modelo dos Campos Semânticos e da Teoria da Atividade. *Revista Paranaense de Educação Matemática*, 8, 213-236.

Linardi, P. R. (2006). *Rastros da formação matemática na prática profissional do professor de matemática* (291 f.). Tese de doutorado, Educação Matemática, Instituto de Geociências e Ciências Exatas, Universidade Estadual Paulista, Rio Claro.

Lins, R. C.; GIMENEZ, J. (1997) *Perspectivas em aritmética e álgebra para o século XXI*. Campinas: Papirus.

Lins, R. C. (2002). *Análise sistemática e crítica da produção acadêmica e da trajetória profissional*. Tese de Livre Docência. Programa de Pós-Graduação em Educação Matemática, UNESP/Rio Claro.

Lins, R. C. (2004) Matemática, monstros, significados e educação matemática. In Bicudo, M. A. V.; Borba, M. C. *Educação Matemática*: pesquisa e movimento (pp. 92-120). São Paulo: Cortez.

Lins, R. C. (2012) O Modelo dos Campos Semânticos: estabelecimentos e notas de teorizações. In Angelo, C. L. *et al.* (Org.). *Modelo dos campos semânticos e educação matemática: 20 anos de história* (pp. 11-30). São Paulo: Midiograf.

Luchetta, V. J. O. (2017). *Uma possível produção de significados para as séries no livro Elementos de Álgebra de Leonard Euler* (244 f.). Tese de doutorado, Educação Matemática, Instituto de Geociências e Ciências Exatas, Universidade Estadual Paulista, Rio Claro.

Mead, G. H. (1969). *The social psychology*. Chicago: Anselm Strauss (ed). The university of Chicago Press.

Oliveira, V. C. A. (2002) *Sobre a produção de significados para a noção de transformação linear em Álgebra Linear* (187 f.). Dissertação de mestrado, Educação Matemática, Instituto de Geociências e Ciências Exatas, Universidade Estadual Paulista, Rio Claro.

Paulo, J. P. A. (2016). *Contando uma história: ficcionando uma dissertação sobre a relação entre professor e aluno* (136 f.). Dissertação de mestrado, Educação Matemática, Instituto de Geociências e Ciências Exatas, Universidade Estadual Paulista, Rio Claro.

Paulo, J. P. A. (2020). *Compreendendo formação de professores no âmbito do Modelo dos Campos Semânticos* (294 f.). Tese de doutorado, Educação Matemática, Instituto de Geociências e Ciências Exatas, Universidade Estadual Paulista, Rio Claro.

Silva, A. M. (2003) *Sobre a dinâmica da produção de significado para a Matemática* (244 f.). Tese de doutorado, Educação Matemática, Instituto de Geociências e Ciências Exatas, Universidade Estadual Paulista, Rio Claro.

Silva, A. M.; Paulo, J. P. A. (2020) Processos de convergência a um modelo teórico. In. Paulo, J. P. A. (2020). *Compreendendo formação de professores no âmbito do Modelo dos Campos Semânticos* (294 f.). (pp. 260-284) Tese de doutorado, Educação Matemática, Instituto de Geociências e Ciências Exatas, Universidade Estadual Paulista, Rio Claro.