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# Construction of Mathematical and Financial Concepts based on Realistic Mathematics Education

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## ABSTRACT

**Context:** Mathematics is present in people's daily lives and involves from simple and easy to complex ways of identifying financial issues, such as buying at the supermarket, exchanging or selling a good, dividing an account, planning a trip, or even long-term financing. However, the school hardly addresses the subject. **Objectives:** To describe the results of a study carried out with students from an initial teacher education course. **Design:** From the development of a didactic sequence, we sought a connection between the ideas of Financial Education anchored on Realistic Mathematics Education (RME). **Environment and participants:** The study was conducted with 11 students from a Pedagogy course at a Brazilian university. **Data collection and analysis:** The data were collected through written records of the students' activities, class observations, and audio and video recordings. The methodology of analysis was defined considering the procedures of Content Analysis, using the modality of thematic analysis that occurs in three phases: pre-analysis, exploration of the material and treatment of the results. **Results:** The discussions of the problem situations developed according to the principles of the RME aroused students' interest, curiosity, autonomy, cooperation, and reflection on financial situations, showing that this approach contributed significantly to the development of the didactic sequence. **Conclusions:** This study shows that students benefit from mathematics teaching based on everyday contexts, such as motivation and interest to learn mathematics. Students can also use their understanding of mathematical concepts to organise their financial life, and, in a conscious, responsible, and autonomous way, improve their quality of life.

**Keywords:** realistic mathematics education; financial education; didactic sequence; mathematisation; construction of mathematical concepts.

## Construção de Conceitos Matemáticos e Financeiros fundamentados pela Educação Matemática Realística

## RESUMO

**Contexto:** A matemática está presente no dia a dia das pessoas e envolve desde formas simples e fáceis até complexas de identificar questões financeiras, como comprar no supermercado, trocar

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ou vender um bem, dividir uma conta, planejar uma viagem ou até mesmo um financiamento de longo prazo. Porém, a escola dificilmente aborda o assunto. **Objetivos:** Descrever os resultados de um estudo realizado com acadêmicos de um curso de formação inicial de professores. **Design:** A partir do desenvolvimento de uma sequência didática, procurou-se estabelecer uma conexão entre as ideias da Educação Financeira ancoradas pela Educação Matemática Realística (EMR). **Ambiente e participantes**: O estudo foi realizado com 11 acadêmicos de um curso de Pedagogia de uma universidade brasileira. **Coleta e análise de dados**: Os dados foram coletados por meio dos registros escritos das atividades realizadas pelos acadêmicos, das observações das aulas, e das gravações em áudio e vídeo. A metodologia de análise foi definida considerando os procedimentos da Análise de Conteúdo, utilizando a modalidade da análise temática que ocorre em três fases: pré-análise, exploração do material e tratamento dos resultados. **Resultados**: As discussões das situações-problema desenvolvidas segundo os princípios da EMR, despertaram nos acadêmicos o interesse, a curiosidade, a autonomia, a cooperação e a reflexão sobre as situações financeiras, mostrando que essa abordagem contribuiu de forma significativa, com o desenvolvimento da sequência didática. **Conclusões**: Verifica-se com esse estudo que, com o ensino da matemática a partir de contextos que envolvem questões cotidianas, vários benefícios podem ser alcançados, como a motivação e o interesse para aprender matemática, e ao compreender os conceitos matemáticos, os estudantes poderão utilizá-los para a organização da sua vida financeira, e de forma consciente, responsável e autônoma, melhorar sua qualidade de vida.

**Palavras-chave**: educação matemática realística; educação financeira; sequência didática; matematização; construção de conceitos matemáticos.

## INTRODUCTION

This article presents partial results of research related to initial teacher education, in which the theme is the study of concepts of Financial Mathematics Education, with the Realistic Mathematics Education (RME) as a reference. The objective of the research was to understand how the promotion of learning situations based on the approach of Realistic Mathematics Education with academics from the Pedagogy Course could contribute in the conceptual and methodological aspects for the development of the teaching and learning process in the early years of elementary school.

The research focuses on this topic because, in the curricular guidelines, especially the BNCC, the contents related to the Financial Education are suggested as a way of contributing to the development of skills linked to the contents of Financial Mathematics. The theme is also important because mathematics is present in people's daily lives and involves from simple and easy to complex ways of identifying financial issues, such as buying at the supermarket, exchanging or selling a good, dividing an account, planning a trip, or even long-term financing. However, even with a variety of applications in people's routines, this subject is still a reason for concern among mathematical educators, because the school hardly addresses the subject. Thus, school mathematics is seen in a way too far from reality, and what students learn in class is not put into practice in their daily lives.

In this scenario, the relevance of working this theme and the importance of bringing the discussion of Financial Education into the classroom since the early years of Elementary School becomes evident. School, in this way, can provide students with financial formation, making them multipliers, who can help their families not only to do the math but also organise and plan their finances. For this, school and teachers must be prepared to insert this issue as part of the school culture.

It is essential to allow students to get in touch with the issues of Financial Education so that they learn how to plan and organise their financial life, acquire habits and mathematical knowledge, face financial situations and make decisions autonomously and consciously.

## THEORETICAL BACKGROUND

### a) The Realistic Mathematics Education

The Realistic Mathematics Education (RME) is an approach for Mathematics teaching that was proposed by the mathematician Hans Freudenthal (1905-1990). It aims to provide students with contact with problem situations in everyday life, awakening interest, creativity, and apprenticeship of mathematical contents.

The principles of RME are the contextualisation and the use of problem situations that must have a relationship with reality, in which the student can make a mental representation of something concrete or not, having the opportunity of experiencing Mathematics as a "human activity."

Mathematics as a human activity is an activity beyond problem-solving, searching for problems, but it is also an activity of organising a real matter of daily life or mathematical issues.

According to Freudenthal, people can reinvent Mathematics on their own or with help and some guidance, and else, there should always be a level within everyone's reach, making it available to all.

> This is a view at variance with that of prescribing to people a priori the mathematics they should learn. Learners should be allowed to find their own levels and explore the paths leading there with as much and as little guidance as each particular case requires. There are sound pedagogical arguments in favour of this policy. First knowledge and ability, when acquired by one's own activity, stick better and are more readily available than when imposed by others. Second discovery can be enjoyable and so learning by reinvention may be motivating. Third it fosters the attitude of experiencing mathematics as a human activity. (Freudenthal, 1991, p. 47).

For the author, Mathematics is an organising activity, in which the knowledge keeps building, structuring, schematising itself and modelling the world mathematically.

> What is mathematics? Of course, you know that mathematics is an activity because you are active mathematicians. It is an activity of solving problems, of looking

> for problems, but it is also an activity of organizing a subject matter. This can be a matter from reality which has to be organized according to mathematical patterns if problems from reality have to be solved. It can also be a mathematical matter, new or old results, of your own or of others, which have to be organized according to new ideas, to be better understood, in a broader context, or by an axiomatic approach. (Freudenthal, 1971, p. 413-414)

Thus, the RME is an approach focused on the students' activities for the construction of knowledge. With this approach, students can learn ideas and mathematical concepts from contextual problems related to the environment, daily life, that is, the students' reality.

Finally, two main points characterise the RME. The first one is that Math must be connected to real problems, i.e., situations must be part of the student's everyday life and so, promoting the learning of Math in a meaningful way, be meaningful to life, becoming something real for students. The second point is that Mathematics is a human activity, that is, situations must be organised mathematically and students, with the teacher's guidance, have the chance of organising and schematising the mathematical reality, which Freudenthal called mathematisation.

The RME is based on the six principles that highlight the use of context in problem-solutions to generate mathematical activities, the use of models for the representation of these contexts and situations, the importance of student's constructions for the teaching and learning process, the role of the teacher, and the integration and interrelation of mathematical concepts.

Gravemeijer and Terwel (2000), when defining mathematisation as "doing more mathematics," relate it with some characteristics of mathematics: generality, certainty, accuracy, and brevity. Among these characteristics, they observe the following strategies:

- for generality: generalising (looking for analogies, classify, structure);
- for certainty: reflecting, justifying, proving (using a systematic approach, creating and testing conjectures);
- for accuracy: modelling, symbolising, defining (limiting interpretations and validity);
- for brevity: symbolising and schematising (developing standard procedures and notations). (Gravemeijer & Terwel, 2000, p. 781).

Treffers differentiate this process of mathematisation idealised by Freudenthal in two phases, the horizontal mathematisation and the vertical mathematisation (Treffers, 1987). In the horizontal mathematisation, the author refers to horizontal "as an activity of making a subject accessible to the mathematical treatment" and vertical "as an activity to promote a more sophisticated mathematical processing.

De Lange (1999) explains that in the horizontal mathematisation, the "process of going from the real world to the mathematical world" occurs, and the vertical mathematisation "is working in the problem within the mathematical world" from the development of mathematical tools to solve the problem. Thus, we show in Figure 1 a scheme of how mathematisation can develop, based on the authors' propositions.

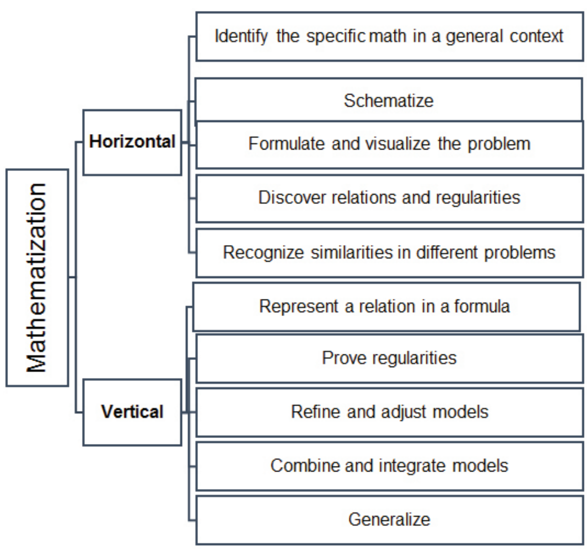
In the Realistic Mathematics Education, one must value the contexts and the mathematical connections in mathematics learning. It is essential to start by exploring and problematising issues from reality, close and familiar to the student, i.e., using practical situations from daily life so that they can be mathematised, instead of starting with purely mathematical problems or with formulas and ready-made definitions. Moreover, students must be guided by the teacher during this process to master the mathematical concepts.

Thus, the context problems, the representations, the informal strategies, and the formal mathematics intertwine in the RME ideals.

The informal models are the representations that students can make to solve the problems of context such as a scheme, a table, or drawings.

The context, followed by successive problems, models and the development of strategies constitute the tools for an individual's ultimate understanding of formal mathematics.

**Figure 1**
*Representative scheme* of the activities of Mathematisation




In this sense, when exploring daily experiences with real problems, students are autonomous to think, discuss and understand the reality mathematically, which allows them to develop ways to solve problems and acquire the necessary skills to face increasingly complex problems.

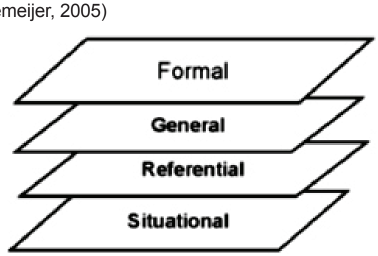
This idea of comprehension is intrinsically related to another RME important characteristic, called the principle of level. In the process of mathematisation, students go through different levels of understanding, from building the first informal solutions linked to context and reaching the levels of schematisation, until they can distinguish the general principles of a problem. This means that the organisation activities done informally, later, with reflection, become more formal.

That means, from the real situations or problems of context, students will build their models, and, after discussing and interacting in the classroom, they will use the mathematical models to solve other problems and obtain formal mathematic knowledge.

> The shift from model-of to model-for concurs with a shift in the students' thinking, from thinking about the modeled situation, to a focus on mathematical relations. In doing so, the students gradually build a framework of mathematical relations. Then, the model begins to derive its meaning from this emerging framework of mathematical relations, and the model becomes more important as a base for mathematical reasoning than as a way to symbolize mathematical activity in a particular setting. In this sense, the role of the model gradually changes as it takes on a life of its own. As a consequence, the model can become a referential base for more formal mathematical reasoning. (Gravemeijer, 2002, p. 2)

To distinguish between these two models, four types of mental and linguistic activities can be identified, which are denoted by Gravemeijer (1994, 1999, 2005) by levels of understanding. Although they are not a strictly ordered hierarchy, these levels are related to the use of strategies, models and languages from different cognitive categories, as shown in Figure 2:

**Figure 2**
*Levels of Understanding.* (Gravemeijer, 2005)




(1) **Situational Level:** the interpretations and tasks resolutions depend, basically, on understanding the context of the situation; informal strategies linked to the context

of particular situations, based on informal knowledge, common sense and experience, are used.

(2) **Referential Level:** the 'models of' refer to the activity in the situation described in the teaching tasks, i.e. they show graphical, tubular or notational models and the descriptions, concepts and procedures that describe the problem, but always referring to a particular situation.

(3) **General Level:** the 'models for' refer to mathematical relations and strategies that make it possible to act, discuss and reason; it is developed from the exploration, reflection and generalisation of what appeared at the previous level, but with the mathematical focus on strategies that go beyond the reference to the context.

(4) **Formal Level:** not dependent on model support for the mathematical activity, procedures and notations are conventional and formalised, that is, students are expected to analyse, understand, and solve problems with the use of formal mathematics.

> Os modelos que aparecem no nível situacional (modelos de situações particulares) são estendidos a outras situações e generalizando se com outra linguagem, tornando-se entidades em si, como ferramentas (modelos para) para resolver situações permitindo um raciocínio matemático mais formal[1]. (Bressan, Zolkower & Gallego, 2005, p. 88-89)

At the RME, the formal mathematics should be experienced the same way as the informal one, and from the development of didactic activities, grow and expand the mathematical reality of students. This way, students build the most formal mathematical reasoning when based on arguments identified in the new mathematical reality. "Initial modeling activities, done in problems of context related to students' reality, allow students to reach new realities, which in turn, can become objects of new modeling activities again." (Van den Heuvel-Panhuizen, 2003, p. 29)

Therefore, students must develop their own models, beginning with more familiar, more concrete situations. Gradually, those models will serve as support for new representations, becoming more mathematical with the process of formalisation and generalisation.

> These levels are dynamic, and a student can work on different levels of understanding for different contents or aspects of the same content. Instead of accurately describing what the learner can do in each of them, they serve to follow their overall learning processes. (Bressan, Gallego, Pérez, & Zolkower, 2016, p. 8)

[1] TN: Models that appear at the situational level (models of particular situations) are extended to other situations and generalise with another language, becoming entities in themselves, as tools (models for) to solve situations, allowing a more formal mathematical reasoning.

> We understand that models are given the role of bridging the informal understanding linked to the real and the imagined reality on the one hand, and on the other, the understanding of formal mathematics. (Van den Heuvel-Panhuizen, 2003)
>
> Models and collective reflection are the basic tools for changes of level. They are representations of situations in which they reflect essential aspects of mathematical concepts and relationships that are relevant to the given situation. The use of models in RME is far from the generalized concept of mathematical modeling, such as the translation of problematic situations into mathematical expressions that can function as models. In this view, the model is the result of the organization of an activity by the subject, holding a deep constitutive involvement between the model and the situation (Gravemeijer, 2002). At RME, we respect those models that emerge from the students themselves and others are inspired by informal strategies, whether they are used by students or presented in the history of mathematics (studied from didactic phenomenology). (Bressan, Zolkower, & Gallego, 2005, p. 88-89)

Summarising, this is the mathematical view as a human activity that refers to the role developed by the modelling activities. When students use the contexts and create a representation of those problem-situations, they go through various levels of mathematisation, and thus, develop the mathematical understanding.

### (b) The School Financial Education

In times of crisis, many people have or have already experienced financial problems. The possible causes are small unpredictable events or uncontrollable facts, such as those triggered by unemployment, high inflation, and outdated wages. Thus, many end up asking for credits to solve their financial problems. When well used, loans enable the consumption of necessary goods; however, the lack of planning and unconscious behaviours and attitudes make people extrapolate their financial limits, increasing their debts and problems. Thus, the topic of Financial Education becomes relevant as we consider the current problem that afflicts millions of Brazilian families.

The OECD (Organization for Economic Cooperation and Development) elaborated a Project in 2003 intending to educate the populations of its member countries financially. One of the guidelines is that Financial Education should start at school, for which teachers and prospective teachers must be trained to take that knowledge to the educational environment. Thus, the definition of Financial Education proposed by the OECD is:

> Financial education is the process by which financial consumers/investors improve their understanding of financial products and concepts and, through information, instruction and/or objective advice, develop the skills and confidence to become more aware of financial risks and opportunities, to make informed choices, to know where to go for help, and to take other effective actions to improve their financial well-being. (OECD, 2005, p. 26)

Following this document, to implement Financial Education, the Brazilian government passed Federal Decree No. 7.397/2010, creating the National Strategy for Financial Education (NSFE), a mobilisation instituted permanently, through the articulation of various government agencies and entities and civil society organisation that are part of the National Committee for Financial Education (NCFE). To ensure the gratuity of the initiatives they develop or support and their commercial impartiality, the NSFE aims to contribute to the strengthening of citizenship by providing and supporting actions that can help the population to make more autonomous and conscious financial decisions. Thus, the NCFE aims at making the Financial Education in Schools Program a public policy supported by curriculum references, especially the Common National Curriculum Base (BNCC), inserting this theme into the school culture.

According to Silva and Powel (2013, p. 10), the proposal for this program was built under the coordination of educators linked to the Instituto Unibanco and focused on high school. In opposition to the ideas of the OECD and NSFE, the authors have elaborated a curriculum proposal for School Financial Education to be developed throughout elementary school, as part of their mathematics education, which we use as guidance in our research, especially in the classroom work, with the application of the didactic sequence.

Believing that the focus of financial education is on the students and at the school level, the authors introduced the following concept for School Financial Education:

> School Financial Education is a set of information through which students are introduced to the universe of money and stimulated to produce an understanding of finance and economy; through a teaching process that enables them to analyze, make informed judgements, make decisions and take critical position on financial issues involving their personal, family and societal lives. (Silva & Powell, 2013, p.13)

In this proposal, the authors elaborated four guiding axes to be developed in the proposed curriculum, aiming to develop the students' financial thinking as part of their mathematics education, during their entire formation.

Based on these axes, the proposed curricular structure is based on the personal, family, and social dimensions, including relevant themes to draw students' attention, such as social issues related to money. The themes involve notions that are part of students' daily lives, which aim to stimulate the involvement in family issues, the participation in decision making, and to make Financial Education go beyond math classes, contributing to social and interdisciplinary work.

In this sense, it is crucial to bring the discussion of Financial Education into the classroom since the early years of elementary school, because, by providing students with financial training, they can also be multipliers, helping their families not only to do math, but also to help them organise and plan their finances. To reach that goal, the school and teachers must be prepared to insert this topic as school culture

For Villa, Silva, and Darroz (2018):

> [...] it is perceived that financial education plays a fundamental role in the development of skills that enable people to consume, save and invest responsibly and consciously, as well as helping people to realize their dreams that depend on their personal finances. It is also evidenced that the school can be the most appropriate place for learning subjects related to the financial education issues. (Villa, Silva & Darroz, 2018, p. 58)

Furthermore, for Campos (2012, p. 172), "Financial Education is rich in associated themes that can naturally emerge in the classroom when we approach financial decisions". In this sense, Almansa (2018), conceptualises School Financial Education:

> School Financial Education is a set of strategies and actions developed within the school environment, with the objective of inviting the student to reflect, from a mathematical perspective and in a multidisciplinary way, on financial and economic events that influence his life, the organization and budgetary planning of families and society in general. (Almansa, 2018, p. 112)

Hence, students must be given problem situations that involve these issues since their initial years of schooling. Thus, it is essential to work on the contents of the subjects in a contextualised way and according to the reality of the student.

In the National Curricular Parameters (NCP) of Elementary School Mathematics, Financial Education is approached in the transversal theme "Work and Consumption," in which related issues are presented in a way that they can become attractive contexts to be explored in the classroom. From this perspective, NCPs highlight the need for making connections with the contents of financial mathematics in the development of problem situations:

> In order to understand, evaluate and decide on some situations form daily life, such as the best way to pay some bill, of choosing a loan, etc, it is necessary to work problem situations concerning Commercial and Financial Mathematics, as calculating simple and compound interest, and divide in proportional parts as the contents needed to solve these situations are already integrated in blocks. (Brazil, 1998, p. 86)

In the BNCC (Brazil, 2017) wording, Financial Education is one of the themes contemplated in the skills of the curricular components, i.e., it is not a priority of the mathematics subject, and it is the schools' responsibility, according to their specificities, to treat it in a contextualised and interdisciplinary way.

There is a distinction between Financial Mathematics, which is linked to the application of mathematical concepts, and Financial Education, which is related to the formation of people's behaviour and attitudes concerning finances. In this sense, following the BNCC guidelines, this work aimed to approach the concepts related to Financial Mathematics by using the Financial Education approach, as this topic is suggested as a context for the development of skills linked to the Financial Mathematics contents, such as percentage and interest.

Regarding the skills to be developed in the early years of Elementary School, they should focus on activities that involve personal strategies, mental calculations, and the use of a calculator. Also, the school should implement activities related to money-management situations such as buying, exchanging, and selling products, forms of payment, using terms related to Financial Mathematics, besides emphasising the ethical, conscious, and responsible product consumption.

Below, we present the methodological aspects of this research.

## METHODOLOGY

To understand how the promotion of learning situations based on the Realistic Mathematics Education can contribute to the development of the teaching and learning process at school and realise how this happens, we chose the qualitative research approach. Thus, this study had as its research phenomenon a group of students from the Pedagogy Course, observed by the application of a didactic sequence containing a problem situation approaching the School Financial Education, following the ideas and principles of the Realistic Mathematics Education.

### Environment and participants

Participants in this research were undergraduate students from the Pedagogy Course at the Universidade Regional Integrada do Alto Uruguai e das Missões – Uri, in the municipality of Santiago/RS. During the second semester of 2018, we proposed to the students to create a group of studies and research to work on several topics on Financial Education and Financial Mathematics contents of Elementary School. Of the 52 academics enrolled in the Pedagogy Course, 13 (attending the fourth, sixth and eighth semesters) participated in the meetings of the group entitled "Study and Research Group on School Financial Education." To constitute the unit of analysis of this study, the undergraduates who participated in the activities with more than 70% of meetings attendance were selected, totalling 11 students.

This study was carried out by the ethical procedures of the Franciscan University, and its proposal was approved by the Research Ethics Committee (REC) on September 4, 2018, with the CAAE registration: 96950318.2.0000.5306.

### Research procedures

The activities of the didactic sequence were developed as follows. In the first moment, the students, in small groups, carried out the activities the researcher (author S.M.F.) proposed, answering questions related to Financial Education. To search for solutions, they were encouraged to use representations, symbols or mathematical models related to the problem given. Besides assisting and helping those who had difficulties, they were also incited to discuss with their colleagues not only the mathematical strategies, but also their opinions, and make a critical analysis of the issues approached. Finally, each group presented their solutions and reflections to the class, which the other colleagues could compare, correct, and incorporate their answers.

In the moments described above, as proposed by Realistic Mathematics Education, the students were the protagonists of their learning, and the researcher adopted the role of participating observer, initially conducting the activities as a guide and experiencing the classroom and the direct contact with the students, participating in the moments of discussion through questions or making remarks when necessary.

### Data collection instruments

The data were compiled using the researcher's field diary, class observations, written records of the participant's activities and their reflections, written at the end of each meeting. Also, the classes were photographed and recorded in audio and video.

The field diary was used by the researcher to record the qualitative observations about the students' behaviour in the activities developed during the classes. The audio-visual materials were used to capture what was not perceived during the researcher's observation and to record the activities developed.

The documents produced by the students served as instruments for the researcher to monitor and verify the process of individual and collective learning, the advances in the levels of skills, the understanding of concepts of Financial Mathematics and the reflection of Financial Education topics, addressed during the activities proposed in the didactic sequence.

### Data analysis

According to Moraes (2003), qualitative research has increasingly used textual analysis, intending to deepen the understanding of the phenomena investigated from rigorous and careful analysis. Thus, to understand the information, we analysed the records in the researcher's field diary about the class observations, the reports and records of the students' activities, and the video and audio of the meetings.

For the systematisation, codification, understanding, and reflection on the information obtained, an interpretative analysis of the phenomenon investigated was done, based on the characteristics and principles that guide the Realistic Mathematics Education and on the conception and experiences lived by the researcher.

The research analysis methodology was the Content Analysis proposed by Bardin (2009). This methodology represents a set of communication analysis procedures, which uses systematic and objective methods to describe the content of the messages. The thematic analysis was the modality of the Content Analysis that, according to Minayo (2007, p. 316), happens in three phases: pre-analysis: organisation of what will be analysed; material exploration: coding of the material from which the categories arise; results treatment: data interpretation and the highlight of the information obtained.

Initially, to delineate the understanding concerning the problem investigated in this work from the written resolution of the students' activities, the focuses of observation were outlined for data analysis, according to the Realistic Mathematics Education.

Along with the development of the research, in front of the set of data obtained after the application of the didactic sequence, the categories of analysis were again outlined and rebuilt according to the research objectives, and the work of decoding and analysing followed the three categories described in Table 1:

**Table 1**

*Categories of analysis according to the objectives and instruments of the research*

| Research Objectives | Categories of Analysis | Instruments |
|---|---|---|
| - to analyse strategies and processes students use in learning situations, from the formulation and resolution of realistic problem-situations involving school financial education;<br><br>- Identifying, distinguishing, and analysing the relationship between the students and the process of mathematisation, reinvention, levels of understanding and with progressive mathematisation, anchored in the principles from Realistic Mathematics Education, from the development of the didactic sequence | a) The mathematisation phases, the occurrence of the phases and the transition from horizontal to vertical mathematisation;<br><br>b) The levels of understanding reached during the development of the activities;<br><br>c) The strategies that identify the characteristics related to mathematics: generality, certainty, accuracy, and brevity. | - Written records of the students' resolutions during the development of the activities;<br><br>- Audio-visual Material: speech and dialogue transcriptions of the students during the activities application;<br><br>- Written observations on the researcher's field diary;<br><br>- Written records of the students' reflections at the end of each meeting. |

We present below how the decoding and analysis of the data was done, according to each category:

### (a) The mathematisation phases, the occurrence of the phases and the transition from horizontal to vertical mathematisation

Table 2 presents the indicators, called Comprehension, Representation, Discussion, Abstraction, and the description of their respective descriptors, defined according to De Lange (1999, 2006), to identify the evidence of the mathematisation phases performed by the students during the activity. Therefore, for each indicator, its descriptors were defined, and from these, we verified the evidence for each phase.

**Table 2**
*Relation of indicators and descriptors of the Mathematisation phases*

| | Indicators | Descriptors |
|---|---|---|
| **Mathematisation Phases** | **Comprehension** | C1 – Demonstrates the understanding of the problem situation; |
| | | C2 – Relates the context data/real world to the experience or mathematical knowledge; |
| | | C3 – Organises the data from the problem situation; |
| | | C4 – Identifies the relevant mathematics in the problem situation; |
| | | C5 – Understands reality mathematically. |
| | **Representation** | R1 – Represents the problem situation as a model; |
| | | R2 – Represents symbology, notation and appropriate formulas in the resolution; |
| | | R3 – Shows ways of representation (schemes, tables, diagrams, ...) in the resolution; |
| | | R4 – Shows mathematical skills in the resolution; |
| | | R5 – Provides informal solutions related to the context of the problem situation; |
| | | R6 – Develops models or mathematical concepts. |
| | **Discussion** | D1 – Analyses the resolution strategies; |
| | | D2 – Compares, discusses or relates ideas/concepts with classmates and/or teacher; |
| | | D3 – Argues and interacts with classmates. |
| | **Abstraction** | A1 – Reflects on the resolutions/learnings and discoveries; |
| | | A2 – Summarises solutions; |
| | | A3 – Shows formal mathematical reasoning; |
| | | A4 – Represents the solution to the mathematical problem formally. |

According to Gravemeijer (1994, 1999, 2005) and Van den Heuvel-Panhuizen (1996), in the indicator Representation (R), for instance, the emphasis is given to the informal models. That is, to the types of representations, strategies and notations carried out in the development of the activities that students can make to solve the problems of context, as a scheme, a table or drawings. In this sense, for the descriptor R6 we remember the Gravemeijer (2002) definition, that a model is a result of the organisation of an activity by the student, ensuring a deep constructive involvement between the model and the situation.

Table 3 presents the individual evidence observed in the students (named: A1, A2, ..., A10) by the descriptors during each activity. The evidence was decoded through the symbols (+, -, 0), and respectively mean totally, partially, or no evidence identified in the descriptor.

**Table 3**

*Results of the individual evidence in relation to the descriptors (model)*

| Indicators | Comprehension (C) | | | | | Representation (R) | | | | | | Discussion (D) | | | Abstraction (A) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Academics | C1 | C2 | C3 | C4 | C5 | R1 | R2 | R3 | R4 | R5 | R6 | D1 | D2 | D3 | A1 | A2 | A3 | A4 |
| A1 | + | + | + | + | + | + | + | + | + | + | 0 | + | + | + | + | + | + | + |
| A2 | + | + | + | + | + | + | + | + | - | + | - | + | + | + | + | - | + | 0 |
| A3 | + | + | + | + | + | + | + | + | + | + | 0 | + | + | + | + | + | + | + |
| A4 | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| A5 | + | + | + | + | + | + | + | + | + | + | 0 | + | + | + | + | + | + | + |
| A6 | + | + | + | + | + | + | + | + | + | + | 0 | + | + | + | + | + | + | + |
| A7 | + | + | + | + | + | - | + | + | - | + | + | + | + | + | - | - | - | 0 |
| A8 | + | + | + | + | + | + | + | + | + | + | + | + | + | + | - | + | + | + |
| A9 | + | + | + | + | + | - | + | + | + | + | - | + | + | + | + | + | + | + |
| A10 | + | - | 0 | + | + | - | + | 0 | - | - | 0 | - | + | - | 0 | 0 | + | 0 |
| A11 | + | + | + | + | + | + | + | + | + | + | 0 | + | + | + | + | + | + | + |

Based on the average of the results obtained in the Comprehension, Representation and Abstraction indicators, the analysis was carried out to verify the occurrence of the phases and the transition from horizontal to vertical mathematisation, defined by Treffers (1987), identified in the resolution of the students during the performance of each activity.

### (b) The levels of understanding reached during the development f the activities

In RME mathematical learning, students from several levels of understanding develop the ability to solve a contextual problem and, through schematisation. They obtain basic knowledge until they can find the formal solution of a mathematical problem. In this category, we try to identify, through the resolutions presented by them, the four types of mental and linguistic activities, denoted by Gravemeijer (1994, 1999, 2005) by levels of comprehension. The levels are related to the use of strategies, models, and languages in different cognitive categories, and are represented by four levels: situational, referential, general, and formal.

**(c) The strategies that identify the characteristics related to mathematics: generality, certainty, accuracy, and brevity.**

Gravemeijer and Terwel (2000) related mathematisation with some characteristics from mathematics: the generality, certainty, accuracy, and brevity. Authors also noticed different strategies to observe these characteristics during "doing mathematics."

Therefore, Table 4 was developed to identify this category, that is, to understand these characteristics in the resolutions made by students.

**Table 4**
*Characteristics of Mathematics*

| Mathematising involves the following features: | |
|---|---|
| **Generality** | Generalise – activity of organisation, mathematisation |
| | Analogy (similarity, proportion) |
| | Classify (select, quantify, order) |
| | Structure (organise, plan, construct) |
| **Certainty** | Reflect: express, think, reveal, convey |
| | Justify: prove, substantiate, demonstrate |
| | Prove (systematic approach): order, organise |
| | Elaborate and test conjectures: deduct, suppose, infer |
| **Accuracy** | Model |
| | Symbolise |
| | Define (limit interpretations and validate): interpret, explain, determine, clarify, confirm |
| **Brevity** | Symbolise |
| | Schematise (standard procedures and notations) |

Next section presents two (2) problem situations that were developed with the academics, as well as the results of the analysis from the categories previously described.

## RESULTS AND ANALYSIS

This section presents the analysis and discussion of the activities proposed in one of the meetings done with the research group. Two problem situations were proposed to be solved by the group, one after the other had been completed.

To extend the knowledge of Financial Education from the daily context, the objectives of the activity are: to allow the undergraduate students to understand and reflect on financial planning, interest rates, the value of money in time, credit modalities, offers and opportunities, risks and pitfalls with the use of money in a consumer society, discussing and analysing mathematically the conscious use of credit, based on the thematic axes proposed by Silva and Power (2013).

The studies from Muniz (2010) corroborate in this perspective on the Financial Education approach, for beyond the math lessons:

> We understand that educating a citizen financially goes beyond teaching financial mathematics. Although it is the central and, therefore, necessary and indispensable subject, it is not enough. Educating financially is a much broader action, which includes: learning math to understand financial situations; understanding the behavior of money in time; consciously organizing (future) personal finances; mathematically discussing the conscious use of credit; understanding economic issues such as GDP, inflation and its different indexes, IOF, IR among others; [...]. These issues should certainly be part of the financial education of students who will make up the economically active population of a country. (Muniz, 2010, p. 2)

**• Problem Situation 1: calculation of real estate financing instalments**

The context of this problem situation is the calculation of the instalments in real estate financing. In Figure 3, we present the Problem Situation 1, with information on how the instalments of the financing are paid. From the Reading, students should identify and interpret those data to determine the values of the first, second and tenth instalments. At the end of the activity, they were also asked to present a model to represent the calculation of an instalment for the payment of the financing.

The process of mathematisation intended in this activity consisted in interpreting the context data to determine the balance due in the financing mathematically from the number of instalments already paid, and the amount charged in interest on the financing for each instalment in relation to the balance due and then determining the value of each instalment in the period requested. Then, they had to check how much the balance due was so that they could calculate the interest on this amount and then determine the value of each instalment.

**Figure 3**

*Problem Situation 1: Calculation of real estate financing instalments*

1) A couple does a real estate financing of a R$ 180,000.00, to be paid in 360 monthly instalments, with an effective interest rate of 1% month. The first instalment is paid one month after the release of the funds, and the monthly instalment is R$ 500.00 plus the interest of 1% on the outstanding balance (amount due before payment). Note that at each payment the balance due is reduced by R$500.00 and consider that there are no instalments in arrears.

a) If payment is made in this way, what is the amount in reais to be paid to the bank in the first instalment?

b) What is the amount, in reais, to be paid to the bank in the second instalment?

c) How much, in reais, is to be paid to the bank in the tenth instalment?

**- Results analysis and discussion**

We present the analysis of the resolutions of the undergraduates in this problem situation, according to the categories of analysis outlined in the methodology. As the activity developed was done from the resolution of two problem situations, only the aspects related to the phases of mathematisation will be presented now, and later, the levels of understanding and the characteristics of mathematics identified in the strategies of the academics during all the activity of resolution will be shown.

**a) The mathematisation phases**

In Table 5, we present the result of the analysis done to verify the evidence in the phases of mathematisation realised by the students in Problem Situation 1.

**Table 5**

*Results of the evidence of mathematisation in Problem Situation 1*

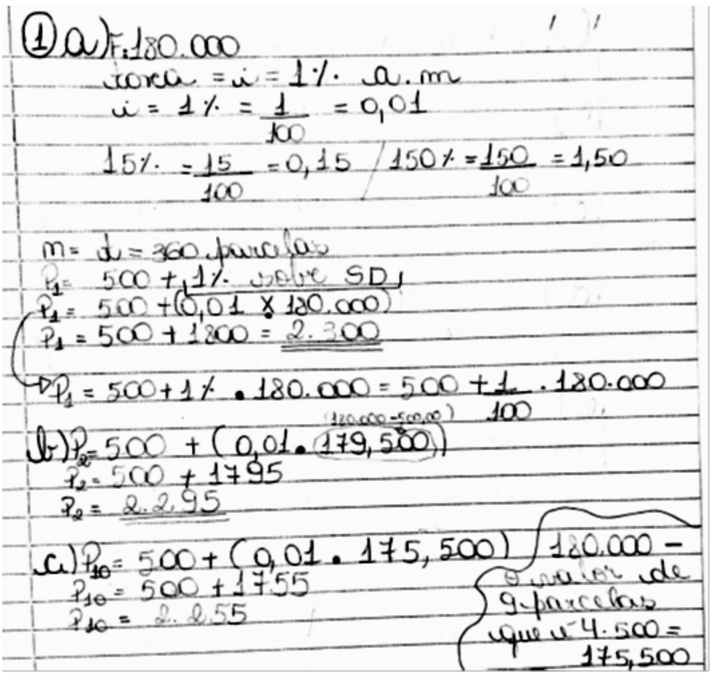
| Indicators | Comprehension (C) | | | | | Representation (R) | | | | | | Discussion (D) | | | Abstraction (A) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Undergraduate Students | C1 | C2 | C3 | C4 | C5 | R1 | R2 | R3 | R4 | R5 | R6 | D1 | D2 | D3 | A1 | A2 | A3 | A4 |
| A1 | + | - | + | + | - | + | + | 0 | + | + | 0 | - | + | + | + | 0 | + | 0 |
| A2 | + | + | + | + | + | + | + | 0 | + | + | - | + | + | + | + | - | + | + |
| A3 | + | + | + | + | + | + | + | - | + | + | + | + | - | - | + | + | + | + |
| A4 | + | - | + | + | - | + | + | - | + | + | + | + | + | + | + | + | + | + |
| A5 | + | + | + | + | + | + | + | - | + | + | + | + | - | + | + | + | + | + |
| A6 | + | - | - | + | - | + | + | - | + | + | - | + | - | - | + | + | + | + |
| A7 | + | 0 | + | + | 0 | + | + | - | + | + | + | - | - | - | + | - | 0 | 0 |
| A8 | + | + | + | + | + | + | + | - | + | + | + | + | + | + | + | + | + | + |
| A9 | + | + | + | + | + | + | + | - | + | + | - | + | + | + | + | - | + | + |
| A10 | + | - | + | + | - | + | + | - | + | + | - | - | + | + | + | - | + | + |
| A11 | + | - | - | + | - | + | + | - | + | + | 0 | - | - | - | + | 0 | + | 0 |

It is possible to verify in the previous table that the students are more successful in the problem Comprehension (C), followed by the Representation (R). They show much difficulty in all phases of mathematisation, from the initial interpretation of the problem situation, to find the meaning of the data, to the solution itself.

In the **horizontal mathematisation**, the students identified the value to be financed and represented the interest rate of 1% in decimal form. Then, they checked the financing terms, i.e., the number of instalments that should be paid. In the **vertical mathematisation**, they schematised the value to be paid in the first instalment as *p = 500 + 1% on the outstanding balance*, and then, reducing the reality, they found the expression *p = 500 + 0,01 x (180.000)*, obtaining the value for the first instalment of the financing. Later, using the same reasoning, they calculated the value of the debit balance and made the calculations for the value of the second instalment of the financing. To calculate the value of the tenth instalment, they immediately noticed the relation and understood that they needed to know what the amortised value was after 9 instalments paid to obtain the outstanding balance, discounting from the initial debt the value of 9x500, thus, ending the phase of the Representation (R).

In Figure 4, we present student A8's resolution, as described above.

**Figure 4**
*Resolution by student A8*




During the Discussion (D), RME characteristics were highlighted when academics related the problem situation to the student finance, as the majority of students participating in the study use the FIES to pay for their undergraduate course, that is, they related and interpreted the situation to other daily contexts involving the mathematical thinking.

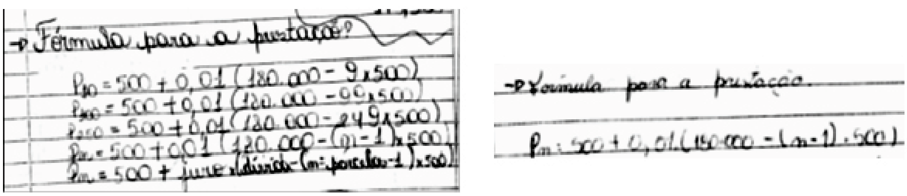
> An important aspect of this approach is that it creates opportunities for the students to develop their mathematical knowledge based on daily experiences. More importantly, it opens these connections. What, in turn, allows students evolve to more concrete levels of understanding, whether they solve problems. So, this approach allows students to work at different levels. (Gravemeijer, 2005, p. 104)

Concerning the results obtained in the resolutions, the undergraduates noticed that the amortised value paid each month was little in comparison to the value of the instalment paid. It helped them prove mathematically that this kind of financing was not fair, as the value of the paid instalment reduced very little the value of the debt, making a critical reflection on the theme. In this sense, we observed the intuitive use of mathematics to base the consumption decisions, recognising the interest paid on loan.

To finish the activity, to make the vertical mathematisation more sophisticated, the researcher asked students to present a formula for the calculation of the instalment. As some of them did not understand, they were asked to represent the expression for the calculation of the 10th, 100th and the 250th instalment and, from the similarities, they realised the regularities and could present a model for the loan instalments, as presented by students A8 and A5, in Figure 5.

**Figure 5**

*Resolutions by students A8 and A5*




The models shown above reveal that the situation encouraged students to present their productions and to use the rule of three. The researchers, in directing the discussion to the representation of the instalment, without imposing, but in a guided way, intended to provide students with ways to present their reasoning and answers, characteristics recommended by the guided reinvention.

- **Problem Situation 2: calculation of the value of a debt in a loan**

The context of the problem situation is the calculation of a debt after a year, considering three modalities of credit. In Figure 6, we present the problem situation 2 involving the acquisition of a loan, with the information about the credit modalities and the respective interest rates asked in the years 2016 and 2017.

**Figure 6**

*Problem Situation 2: calculation of the value of a debt in a loan*

2) In recent years, Brazil has been going through a generalised crisis in the economy. Faced this situation, the various modalities of credit have reacted differently. The table below shows the annual (accrued) and monthly interest rates of some loan and financing modalities for May 2017, compared to the same period in 2016.

| Modalidades de crédito | Taxas de juros | | | |
|---|---|---|---|---|
| | (% ao ano) | | (% ao mês) | |
| | mai-17 | mai-16 | mai-17 | mai-16 |
| Cartão de crédito | 159,96% | 148,99% | 8,29% | 7,90% |
| Cheque especial | 325,07% | 311,48% | 12,82% | 12,51% |
| Crédito pessoal não-consignado | 132,62% | 129,76% | 7,29% | 7,18% |
| Crédito pessoal consignado - setor privado | 42,28% | 43,55% | 2,98% | 3,06% |
| Crédito pessoal consignado - setor público | 25,97% | 27,76% | 1,94% | 2,06% |
| Financiamento veículos | 24,25% | 26,33% | 1,83% | 1,97% |
| Crédito pessoal consignado - INSS | 27,83% | 30,66% | 2,07% | 2,25% |

Adaptado de: <http://minhaseconomias.com.br/blog/dividas/cheque-especial-e-credito-rotativo-cartao>. Acesso em 02 jul. 2017.

Suppose three friends, Abel, Bernardo, and Cláudio each took a loan of R$ 50,000.00 in May 2016, to be paid back after one year. Abel used the credit card, Bernardo the special check, and Cláudio made a personal consigned credit at the public sector.

What is the interest rate used in Abel's loan?

What is the interest rate used in Bernardo's loan?

What is the interest rate used in Cláudio's loan?

Which friend made the best deal?

Considering that the three friends have not made any payment since the date of the acquisition of the loan, in May 2017, in approximate values, what is the amount of debt of each one?

What is the percentage of Bernardo's debt when compared to Claudio's?

From the reading and interpretation of the data, the process of mathematisation consisted of the academics organising information, relating the credit modality that each person used to make the loan with the interest rate charged for each modality. After checking the rate, they should observe and analyse which of the friends made the best deal, calculate how much the debt would be after a year of the acquisition of the loan and, finally, compare the value of this debt between two friends.

**- Results analysis and discussion**

**(a) Phases of mathematisation**

In Table 6, we present the result of the analysis done to prove the evidence in the phases of mathematisation performed by the students in problem situation 2.

**Table 6**

*Results of the evidence in the phases of mathematisation in problem situation 2*

| Indicators | Comprehension (C) | | | | | Representation (R) | | | | | | Discussion (D) | | | Abstraction (A) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Students** | **C1** | **C2** | **C3** | **C4** | **C5** | **R1** | **R2** | **R3** | **R4** | **R5** | **R6** | **D1** | **D2** | **D3** | **A1** | **A2** | **A3** | **A4** |
| **A1** | + | + | + | + | - | + | + | 0 | - | + | - | + | + | + | 0 | + | - | - |
| **A2** | + | + | + | + | + | + | + | 0 | + | + | - | - | + | + | + | + | - | - |
| **A3** | + | + | + | + | + | + | + | 0 | + | + | - | + | + | + | + | + | + | - |
| **A4** | + | + | + | + | + | + | + | 0 | - | + | - | - | + | + | + | + | - | - |
| **A5** | + | + | + | + | + | + | + | 0 | + | + | - | + | + | + | + | + | + | - |
| **A6** | + | + | + | + | + | + | + | 0 | + | + | - | + | + | + | + | + | + | - |
| **A7** | + | - | - | - | - | + | + | 0 | - | + | - | - | - | + | - | - | - | - |
| **A8** | + | + | + | + | + | + | + | 0 | + | + | - | + | + | + | + | + | + | - |
| **A9** | + | + | + | + | + | + | + | 0 | + | + | - | + | + | + | + | + | + | - |
| **A10** | + | + | - | - | - | + | + | 0 | - | + | - | 0 | - | - | - | 0 | - | - |
| **A11** | + | + | + | + | - | + | + | 0 | - | + | - | - | 0 | 0 | - | 0 | - | - |

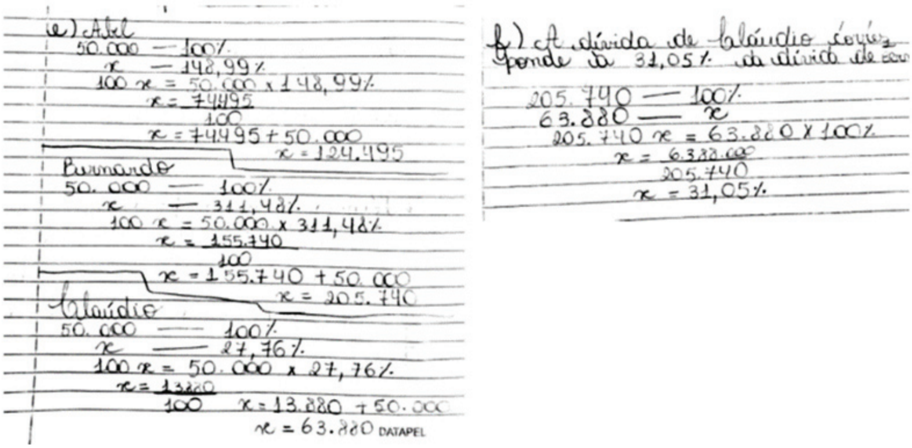
In Table 6, we can observe that the academics were more successful in the problem Comprehension (C), followed by the Discussion (D). in this activity they did not present as much difficulty as they did in the previous one, but the lack of concentration disturbed a little the resolution work.

When they began to identify the data informed in the problem situation, they soon questioned what interest rate should be used for the calculations, whether it was 2016, date of the acquisition of the loan, or 2017, year in which the current value of the debt was verified. The researcher then advised that they could analyse the issue, choose one of the rates and justify the choice. Of the 11 academics present, six answered that they had chosen the rates corresponding to 2016, and most justified that it was the date on which the loan was taken out. Student A5 exemplified that the interest rate for the student financing (FIES) he takes does not change, it is the same since the acquisition, and, based on that experience, he justified his choice.

From this justification, we can see that, by trying to solve the problem, the student made an analogy between the problem situation and his reality, a fundamental characteristic in the process of **horizontal mathematisation**, in which mathematics served to organise a real situation, because it is connected to the student reality.

In sequence, we present the calculations made by the student A8 to determine the current value of the debt, as shown in Figure 7.

**Figure 7**
*Resolutions by student A8*




We can see that the students did not make the calculations from the expressions, as in the previous activity, or another model to represent the situation, but they used the rule of three to find the solution. It is important to highlight that in this meeting, the students showed more difficulties in comprehension and interpretation of the questions compared to the previous meetings. From the observations done during the performance of the activity in the classroom, we can conclude that these difficulties were about making the calculations involving the interest rates.

However, these discussions were significant and contributed to the approach of Financial Education in the classroom, as presented by Muniz (2016):

> [...] Financial Education should contribute for reflection and mathematical formation (including) of students, from different lenses, encouraging them to think about their actions in relation to consumption, savings, financing and investments. It should also help raise awareness of the advantages and benefits that can come from the practice of financial planning, setting goals, identifying how one spends and with what one spends. As well as bringing reflections on how individual decisions are related to the collective, that is, that their personal decisions affect family life and more broadly, society. (Muniz, 2016, p. 4)

It is noticeable that by working with everyday contexts, academics can reflect on financial issues, enabling group discussion and the development of more critical thinking, contributing to analysis to more conscious decision making, and making the learning of mathematics more interesting.

**(b) The Levels of Understanding**

From the strategies presented by the undergraduate students, the models showed are arithmetic and algebraic, associated with the determination of values always related to the problem situation, and the resolutions presented can be in **situational and general levels.**

In Problem Situation 1, based on the interpretation of data related to the situation context, the students determined the values of the real estate financing instalments using arithmetic operations, involving percentages and operations with decimal numbers and, subsequently, represented the calculation of the instalment of the financing by a mathematical expression, generalising the calculations presented in the beginning. In Problem Situation 2, based on the experience, they defined the interest rate and calculated the loan values using the rule of three as a model and the operations with decimal numbers. Also, comparing the debt values according to the percentage rates, they developed concepts of proportionality and interest, realising the meaning of the interest rates charged on loan.

**(c) The characteristics of Mathematics**

During the activity development, we identified many characteristics while the 'doing mathematics' happened. At first, the students found it very difficult to represent the problem situation 1 in a mathematical language. In situation 2, they found less difficulties, as they were requested to interpret data presented in the Table, due to the comparison of the interest rates charged.

The use of contexts related to the need of asking for financing and loans helped students **understand** better their realities. When they made **analogies**, they **perceived** the use of mathematics, the importance and the need to know the interest rates charged in an instalment plan and be aware and resist to traps such as the special checks, credit cards, or other services with exorbitant interest rates.

The use of real contexts also fostered much **discussion** and **reflection** among the students, which contributed to changes concerning the point of view, opinions, and beliefs, as denoted by the following reflections:

> A5: *"But I thought the special check was a good thing, but it's special just for the bank, so."*
>
> A3: *"Wonderful class, I solved real problems, because it was **very significant to know** the interest rate, where it is in all the purchases we make."*
>
> A10: *"In this meeting, we **learned** about interest, it was very **significant** because we are constantly witnessing this issue in our daily lives, when buying products in stores and when we hire some financing."*

Therefore, the benefits of using daily financial contexts contributed to increasing the students' motivation and interest, as for them, they were given something new, but that it is completed related to their realities. The results of this activity show that, at first, even the students with mathematical difficulties got actively involved in the process of mathematisation, being able to connect themselves to the problem situations, as they corresponded to the reality that they have already experienced or could have imagined. Moreover, during the process, they demonstrated autonomy and confidence in their skills, when giving meaning to the problem and relating the application of mathematics in different contexts.

## CONCLUSIONS

The objective of this article was to describe the results of a doctoral research work that aimed to understand how promoting learning situations based on the approach of Realistic Mathematics Education can contribute in the conceptual and methodological aspects to the development of the process of teaching and learning at school in the early years of Elementary School. Therefore, this study had as investigation phenomenon a group of students from the Pedagogy course, observed by the application of a didactic sequence containing problem situations approaching the School Financial Education, following the ideas and principles of the Realistic Mathematics Education.

As for the results, we highlight that the RME approach contributed significantly to the development of a didactic sequence, emphasising as positive points the students' role as protagonists of their learning, and the teacher's role as a guide, intervening in this process through questions and remarks to the students during the activities.

We found that the use of real contexts in problem situations encouraged students to participate actively in the activities, producing an environment of interaction, exchange of ideas among colleagues, knowledge-sharing, and discussion among peers. In the RME, the students are responsible for their own learning. The application of the didactic sequence, the interaction, the discussions in small groups, the sharing of explanations, questions, and exchange of ideas encouraged students to reflect, advance levels of mathematics understanding, and develop autonomy. This opportunity lived by the students is a characteristic of the Guided Reinvention.

The search for solutions for the problem situations allowed students to make connections between the mathematical contents and everyday problems. The discussions aimed at problem interpreting and employment of social and mathematical thinking aroused students' interest, curiosity, autonomy cooperation, and reflection on financial situations, i.e., the prerequisites that contribute to the construction of mathematical knowledge, the confrontation of new problems and the making of financial decisions.

Based on the above conclusions, we consider that mathematisation should be considered as a means to favour the construction of teacher-oriented mathematical knowledge at all levels of education. Thus, provide learning environments aiming to develop concepts from real situations and consider the need to reflect on what has been accomplished, adapting it to the real world, are differentiated actions that should be encouraged in mathematics teaching.

Concerning the perspectives of continuity of the research, we emphasise the need for more studies on how to approach Financial Education in the school environment. Concomitantly, teachers' training for the development of these activities in the classroom must be addressed.

Through mathematics teaching based on contexts involving financial and everyday issues, several benefits can be reached: the contexts can increase students' motivation and interest in learning mathematics, making them use mathematical concepts to plan and organise their financial life and, in a conscious, responsible and autonomous manner, improve their quality of life.

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## AUTHORSHIP CONTRIBUTION STATEMENT

Author V.B. supervised the development of the S.M.F. doctoral research project, in which the data analysed in this article was constituted. Author S.M.F. was responsible for elaborating the format of this article and developing data organisation and analysis. Both authors discussed and contributed to the final version of this article.

## DATA AVAILABILITY DECLARATION

The data on which this study is based will be made available by the correspondence author S.M.F., upon prior request.

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