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Mathematical Investigation Contributions for the Financial Education and Economics Teaching

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ABSTRACT

Background: The proposal of financial education in classrooms is born in the expectation of changing an irresponsible consumption scenario that society is going through and should be included in all student educational contexts, from elementary to higher education, aiming to provide consumers-individuals with basic notions of economics and consumption. **Objective:** Analyse how investigative tasks can contribute to the financial education and economics teaching. **Design:** Methodology of mathematical investigation. **Setting and participants:** The research was carried out with twelve students in the discipline of Financial Mathematics, in a mathematics program at a university in the state of Rio Grande do Sul (Brazil). **Data collection and analysis:** This article is characterised as qualitative research and the data produced (tasks, recordings, questionnaire, forum) were analysed through discursive textual analysis. **Results:** Possibilities of conjectures and resolving strategies; investigative activity importance, in group, for the creativity and autonomy learning and development; critical thinking development in the decision making process; relations between the task and daily life situations; and analysis of difficulties during task execution. **Conclusions:** Through this study, we conclude that the investigative tasks have contributed for the teaching and learning processes, when the small groups made discoveries and followed different paths during each problem situation solving, they demonstrated enthusiasm, creativity, and autonomy. When the results were compared, at the moment of interaction with the bigger group, the groups realised they could have chosen other paths, or any important factor, which could have been part of the analysis, had not been considered, what yielded several discussions in the classroom as well as in the virtual forum, strengthening a critical and collaborative thinking development.

Keywords: mathematical investigation; financial education; economics; teaching; learning.

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Contribuições da Investigação Matemática no Ensino de Educação Financeira e Economia

RESUMO

Contexto: A proposta da educação financeira em sala de aula nasce da expectativa de mudar o cenário de consumo irresponsável que a sociedade atravessa e deve ser inserida em todos os contextos educacionais dos alunos, do ensino fundamental ao superior, visando proporcionar aos consumidores-indivíduos noções básicas de economia. e consumo. **Objetivo:** Analisar como tarefas investigativas podem contribuir no ensino de educação financeira e economia. **Design:** Metodologia da investigação matemática. **Ambiente e participantes:** A pesquisa foi desenvolvida com doze estudantes da disciplina de matemática financeira, do curso de licenciatura em matemática de uma universidade no estado do Rio Grande do Sul (Brasil). **Coleta e análise de dados:** Este artigo caracteriza-se como pesquisa qualitativa e os dados produzidos (tarefas, gravações, questionário, fórum) foram analisados mediante a análise textual discursiva. **Resultados:** Possibilidades de conjecturas e estratégias de resolução; importância da tarefa investigativa, em grupo, para o aprendizado e desenvolvimento da criatividade e autonomia; desenvolvimento do espírito crítico no processo de tomada de decisão; relações da tarefa investigativa com situações do cotidiano; e dificuldades durante a resolução da tarefa. **Conclusões:** Por meio deste estudo conclui-se que as tarefas investigativas contribuíram nos processos de ensino e de aprendizagem, os pequenos grupos ao fazerem descobertas e percorrerem caminhos diferentes durante a resolução de cada situação-problema demonstraram entusiasmo, criatividade e autonomia. Quando compararam seus resultados, no momento da socialização para o grande grupo, perceberam que poderiam ter escolhido outros caminhos ou que algum fator importante, que poderia ter feito parte da análise, não tinha sido considerado, o que rendeu várias discussões tanto em sala de aula como no fórum do ambiente virtual, fortalecendo o aprendizado e desenvolvimento do espírito crítico e colaborativo.

Palavras-chave: investigação matemática; educação financeira; economia; ensino; aprendizagem.

INTRODUCTION

The proposal of financial education in classrooms, according to Santos, Menezes, and Rodrigues (2016), is born in the expectation of changing an irresponsible consumption scenario that society is going through. Kistemann Jr. (2011) emphasises that financial education should be included in all student educational contexts, from elementary to higher education, aiming to provide consumers-individuals with basic notions of economics and consumption. According to Bauman (2008), the ceaseless pursuit of happiness associated with the purchase of new objects has, in a way, become a problem for a significant number of the population, that ended up becoming defaulters. Moreover, according to Franzoni, Del Pino, and Oliveira (2018), little knowledge about economics can impair social well-being growth, as an informed and knowledgeable society can make better choices.

Therefore, this qualitative approach study brings an analysis of how mathematical research tasks can contribute to the teaching of financial education and economics in a mathematics teaching degree course. Mathematical research is a teaching and learning methodology, where the formulation of conjectures is the main characteristic (Ponte, Brocardo, & Oliveira, 2015).

The data collection instruments used were the recordings of the discussions of the classroom investigative tasks (in small groups) and recordings with the filming of the talks (in the large group), recording of mathematical investigation tasks, individual questionnaires and discussions in the virtual environment forum. *Moodle* platform was used as a virtual learning environment that aims to contribute to learning and encourage students to interact through the discussion forums. The objective of including the forum in this study is so that debates on the socialisation of investigative tasks do not end in the classroom, but continue online, to strengthen learning and develop students' critical, creative, and collaborative attitude. Finally, we emphasise that data analysis was based on the Discursive Textual Analysis (Moraes & Galiazzi, 2016).

The first section of our work brings the introduction, and the second section presents the theoretical framework. The third section addresses the methodology used. The fourth section refers to data analysis and presents the main results. The last section explains the conclusions of this study.

THEORETICAL FRAMEWORK

Financial education is among the topical themes present in the National Common Curricular Base (BNCC). This is the set of knowledge understood as essential for strengthening citizenship and aimed at helping the population to make more autonomous and conscious financial decisions (Brazil, 2016). The definition of financial education, according to the Central Bank of Brazil:

> It is the process by which individuals and societies improve their understanding of financial concepts and products. With clear information, education and guidance, people acquire the necessary values and skills to become aware of the opportunities and risks associated with them, and then make well-founded choices, know where to look for help, and take other actions that improve their well-being. Thus, financial education is a process that consciously contributes to the formation of responsible individuals and societies committed to the future (Banco Central do Brasil, 2019, p. 1).

The Organisation for Economic Cooperation and Development (OECD), the World Bank, and the Ministry of Education justify the financial education teaching by the increasingly early involvement of young people in financial decisions, the growth in financial products the banks offer today, the complexity of choosing the best investment, financing, payment method, and type of pension, etc. Besides, the World Bank and the OECD have shown concern about the degree of household indebtedness. Excessive consumerism has led many families to debt, which increased the intervention of the Brazilian Central Bank both in the economy, to reduce consumption, and in schools, to teach financial education.

Lusardi and Mitchell (2014) highlight that most people around the world are considered financially illiterate and confirm the importance of knowledge of economics in financial education teaching so that students can make better financial choices. Therefore, financial education encompasses not only the knowledge of financial mathematics, but also of economics, expecting that students can make better choices and analyse more deeply the financial problems of everyday life. According to Kistemann Jr. (2011), students must be given strategies that can help them make decisions and conduct everyday situations, to stand out as critical individuals. Therefore, we chose mathematical research to teach financial education and economics to help students develop critical thinking in their decision-making process.

Ponte, Brocardo, and Oliveira (2015, p. 23) define mathematical research as a "teaching-learning activity" that involves four main moments. The first moment refers to the initial knowledge, involving the verification, analysis, and elaboration of questions about the problem situation. The second moment consists of the development of ideas and findings, which is based on assumptions drawn from a situation. These hypotheses are called conjectures. The third moment implies testing conjectures, when students will confirm or not their assumptions. In the fourth moment, students demonstrate and assess the situation, following with an argument that justifies their reasoning.

Mathematical research is related, according to Ponte, Brocardo, and Oliveira (2015), to the formulation of conjectures that we want to test and prove. The students are invited to act as mathematicians, presenting the results and debating with their peers and teacher. In this way, it is possible to prepare individuals for the world, who can make better choices, be aware of their actions and consequences, standing out critically in the face of some theme and/or problem. In the area of mathematics, the citizen of this century needs to "use strategies, concepts, and mathematical procedures to interpret situations in various contexts, whether daily activities, facts of the natural sciences and humanities, or even economic issues" [...] (Brasil, 2017, p. 523). This competence in the mathematics area, among others, contributes to the formation of critical and reflective citizens, in which students begin to investigate the challenges of the contemporary world and interpret economic situations to make better choices in the face of a problem.

Also, according to Brazil (2017), students' skills include: solving and elaborating everyday problems, financial mathematics, and other areas of knowledge; interpreting rates and indexes of a socioeconomic nature, such as human development index, interest rates, inflation, and exchange, investigating the processes of calculation of these numbers; preparing spreadsheets for the control of family budgets; solving and elaborating problems involving percentages in various contexts and compound interest, highlighting their exponential growth. However, Teixeira (2015) emphasises that the financial mathematics contents (simple and compound interest, amortisation systems, etc.) are being transmitted to students in a decontextualised way, excessively concerned with teaching through formulas and tables, without referring to everyday life, which hinders learning. According to the author, it is necessary to bring theory and practice together, aiming to connect this discipline with financial education.

METHODOLOGY

To achieve the objective proposed for this study, i.e., to analyse how mathematical research can contribute to the teaching of financial education and economics, we applied and explored ten mathematical activities in Financial Mathematics with twelve students attending the 6th semester of a mathematics teaching degree course of a university in the state of Rio Grande do Sul. The problem-situations elaborated are intended to meet the BNCC (Brasil, 2017) and the objective and expected skills for the discipline of Financial Mathematics, contributing to the initial education of the undergraduates by establishing relationships between financial mathematics and financial education and economics through investigative tasks.

The primary purpose of each financial education issue proposed is to analyse consumer behaviour, students' thinking in the decision-making process of everyday-life financial problems. It also intends to explain: the implications of compound interest over time; the difference between term and spot value; the importance of doing market research and comparing prices to minimise cost and/or maximise satisfaction; how much the understanding of percentage, exchange rates, currency conversion, pre- and post-fixed interest rates can facilitate the process of choice; which forms of investment, pension plans, capitalisation schemes, ways of financing (real estate, leasing, consortium, direct consumer credit) are more advantageous, according to the scenario of the current economy and its possible forecasts. Yet, how the understanding of economics and financial mathematics influences consumer decision-making and is essential to achieve financial education. Thus, we believe that prospective teachers may feel more prepared to teach financial education.

Due to the size of the class, four groups were formed. The small groups were assembled based on each student's choice of a word: *Research*, *Mathematics*, *Finance*, and *Teaching*. For ethical reasons, we did not disclose the real names of the participants, who are identified here as A1 through A12.

All investigative tasks were designed for high school, according to the skills provided for in the BNCC (Brasil, 2017) for the discipline of Mathematics, but can be adapted for elementary school. This article will analyse only two of those tasks, as shown in Figures 1 and 2.

Figure 1

Investigative Task - Flight Ticket/ Baggage Cost - Purchase Decision

Let's assume a friend got a job in Natal (Rio Grande do Norte/ Brazil), needs to move from Porto Alegre (Rio Grande do Sul/ Brazil) in the next few hours and decided to go by plane. Soon, he will need to choose between economy or business class tickets and the number of suitcases he will take, which can vary from two to four, according to his forecasts. We know that if the passenger exceeds the limit established for each suitcase checked (weight: 23kg; size: 158cm linear) he must pay an additional value. The passenger can carry only one carry-on bag on the aircraft, provided that it does not exceed 8 kg and 115 cm linear. In this activity, the airlines allow each passenger to dispatch up to 3 bags. According to a survey conducted on the internet, your friend needs to choose from the following airlines:

AIRLINE "A"

R$1,230.00 without checked bag
R$1,430.00 entitled to checked bag
R$60.00 = 1 checked bag
R$100.00 = 2 checked bags (total value)
R$120.00 = 3 checked bags (total value)
R$80.00 = overweight allowed (up to 32 kilos); $23 < P \leq 32$; per bag
R$160.00 = overweight allowed (up to 45 kilos); $32 < P \leq 45$; per bag
+ R$110.00 = over 45 kilos (per bag)
R$110.00 = above the allowed size (per bag); P = Weight

AIRLINE "B"

R$1,267.00 without checked baggage
R$1,270.00 entitled to checked baggage
R$60.00 = 1 checked bag
R$100.00 = 2 checked bags (total value)
R$120.00 = 3 checked bags (total value)
R$80.00 = overweight allowed (up to 32 kilos); $23 < P \leq 32$; per bag
R$160.00 = overweight allowed (up to 45 kilos); $32 < P \leq 45$; per bag
+ R$100.00 = over 45 kilos (per bag); P = Weight
R$100.00 = above the allowed size (per bag)

AIRLINE "C"

R$972.00 without checked baggage
R$1,496.00 entitled to checked baggage
R$80.00 = 1 checked bag
R$110.00 = 2 checked bags (total value)
R$200.00 = 3 checked bags (total value)
R$12.00 per kilo exceeding allowed weight (per bag)
R$130.00 = above the allowed size (per bag)

AIRLINE "D"

R$1,000.00 without checked baggage
R$1,400.00 entitled to checked baggage
R$60.00 = 1 checked bag
R$100.00 = 2 checked bags (total value)
R$130.00 = 3 checked bags (total value)
R$130.00 = overweight (per bag)
R$130.00 = above the allowed size (per bag)

a) Is it better to purchase the economy class ticket without the right to checked baggage and pay for it separately or buy business class ticket with the right to check a suitcase? Explain.
b) Which airline is the most advantageous? Demonstrate your reasoning mathematically.
c) If your friend decides to take more than one checked bag, does the airline's choice remain? Explain.
d) From the calculations made, what are the possible formalisations to be applied to any situation of the same nature?
e) Are there other factors that could have been part of the analysis in the decision-making process and that were not considered in the calculations performed? Explain.

Figure 2

Investigative Task - International Travel Cost – Purchase Decision

A family of four stayed 15 days in Moscow (Russia) to watch the World Cup. The average expenditure per person in the city is equal to 4,107.25 roubles per day, and this family had to choose between buying roubles in Brazil at the airport + IOF of 1.1%, dollar or euro at foreign exchange brokers in Brazil + IOF of 1.1% to take for travel. For credit card purchases outside Brazil, the cost of IOF increases to 6.38%.

Assuming that the family has reached the average expenditure per person, in roubles, in the 15 days and knowing that:

IOF = Financial Transaction Tax

1 dollar ⟷ 63.21 roubles
1 euro ⟷ 73.70 roubles

1 real → 0.26 dollars
1 real → 0.22 euros
1 real → 16.25 roubles

a) How many reais does it take to buy a unit of foreign currency? Find the exchange rates.
b) Set exchange rate and formalise your thinking.
c) Is it worth buying dollars or euros in Brazil and then exchanging the currency in Russia? Explain.
d) If our currency devalues more against the dollar than the euro the next day, is it still worth buying the dollar? Explain.
e) We know that in airport exchange brokers in Brazil 1 real = 14.164 roubles + 1.1% IOF. How many reais would be spent if the family decided to exchange all reais for roubles in Brazil? What was the reasoning?
f) What would be the result if the family decided to spend only on the credit card? Don't forget the 6.38% IOF. How many conversions will be made?
g) Formalise your reasoning considering the calculations made (items a, b, c, d, e, f) so that it can be applied to any situation of the same nature.
h) Compare the results and choose the most advantageous situation for the family, justifying your answer.

After each task, which lasted around 3 hours, the students went to the computer lab to answer the questionnaire about both the task and what they had learned. The resolution for each mathematical investigation activity per group was recorded in a notebook and posted in the forum of the virtual environment to encourage discussions outside the classroom. Thus, students were free to analyse in more detail the resolutions of the activities of their peers and cooperate, which fostered students to interact more in the class through remarks to enrich learning, and which helped them develop autonomy and critical thinking.

The data were collected through recordings of the discussions of investigative tasks in the classroom (small groups) and recordings with the filming of the discussions (large group); record in the notebook of mathematical investigation tasks, posted in the virtual environment (per group); questionnaires applied and posted (individual); and discussions in the forum of the virtual environment.

Given the above and based on the objective proposed, this study is characterised as qualitative research. According to Gerhardt and Silveira (2009, p. 31-32), this "is not concerned with numerical representativeness, but with the understanding of a social group, with aspects of reality that cannot be quantified, focusing on the understanding and explanation of the dynamics of social relations." In this context, to understand how investigative activities can contribute to the teaching of financial education and economics in a mathematics teaching degree course, a qualitative approach was chosen to help us understand how the teaching and learning processes happen, without taking into account quantifiable aspects.

In qualitative research, according to Bogdan and Biklen (1994), the way something happens is the relevant aspect. Minayo (2010, p. 21) adds that "qualitative research works with the universe of meanings, motives, aspirations, beliefs, values, and attitudes." Therefore, the researcher is not only interested in the final product, but in the process, in the meaning of people's thoughts and ways of being.

Thus, we analysed the emerging data through applying the DTA (Discursive Textual Analysis), which, according to Moraes and Galiazzi (2016), is configured as a comprehensive step methodology, requiring the researcher to pay attention and be rigorous at each step of the process. At first, the DTA aims to disassemble the texts and their examination in the smallest detail. Subsequently, relationships between each unit are established, seeking the identity between them to, then, capture what emerges from the totality of the text towards a new understanding of this whole. According to Moraes and Galiazzi (2016), the DTA is composed of three stages, the first of which is the unitisation process, in which the text is deconstructed, fragmenting it into units of meaning. The unitisation process is, therefore, the essential stage in the development of the DTA, since this unit contains the most significant messages of the texts analysed. The second corresponds to the organisation of categories, which can be regrouped continuously. At last, in the third, a metatext is produced with the new understandings obtained.

So, in the first stage, the data, which had been transcribed from the recordings and the filming of class discussions of each investigative task, were organised into a chart, where each column related to a group of students and the corresponding keywords were placed. Afterwards, the same procedure was done for the notebook records per group for each investigative task. Ultimately, the keywords of the individual data related to the posts in the discussion forum (*Moodle*) were included in another chart, and, then, the questionnaires about each investigative task were applied. In the second stage, we established the categories to produce metatexts by category in the last stage of the DTA, connecting the theoretical framework of this study to the students' responses.

ANALYSIS AND RESULTS

The students had the opportunity to experience in practice, in solving each task, the four moments of the mathematical investigation described above. The two investigative tasks analysed in this article targeted to understand the students' thinking in the decision-making process and explain the importance of doing market research and comparing prices to minimise cost and/or maximise satisfaction, demonstrating how understanding exchange rates and currency conversion can facilitate the process of choice. In short, how the understanding of economics and financial mathematics influences consumer decision-making and is important in mathematics teaching to achieve financial education, as highlighted by the National Common Curricular Base. Thus, the data collected were grouped into five categories, namely: a) Possibilities of conjectures and resolution strategies; b) Importance of the investigative task, in groups, for the learning and development of creativity and autonomy; c) Development of the critical spirit in the decision-making process; d) Relationships between the investigative task and everyday situations (theoretical and practical connection); e) Difficulties during the resolution of the task. The following are the categories that emerged from the students' statements, and the discussions raised and their connections with some authors:

a) Possibilities of conjectures and resolution strategies

Regarding the first task (Figure 1) on cost of plane ticket and baggage, all groups found that the choice of an airline depends on the number of checked baggage and problems of overweight and/or size, changing the order of preferences. The best option to minimise cost is to buy the ticket in economy class and buy the luggage. The most advantageous airline is "C" up to two checked baggage without problems of overweight and/or size. Still, if the number of checked bags is three, the lowest-cost airline will be "D." However, if the consumer prefers to travel only by company "B," it is better for him to purchase the ticket in business class, paying R$3.00 more (R$1,270.00 – R$1,267.00), due to having the right to check baggage; if he decides to take two or three bags, he will pay less than in economy class.

Among the possibilities tested by the groups are: 1) size problem in all luggage, the best option is economy class, airline C is preferable up to two bags and company D preferable for three checked bags; 2) overweight problem in all luggage, the best option is airline C, up to three checked bags, for weight greater than 23kg and less than or equal to 32kg. Airline D is preferable to up to three checked bags, for a weight greater than 32kg; 3) weight and size problem in all luggage, airline C is the most viable for up to three checked bags (weighing more than 23kg and less than or equal to 32kg + size problems). Airline D is preferable for weight greater than 32kg + size problem in up to three bags. The prices of airline B (business class) are interesting if we compare with other airlines (economy class), in various situations (number of checked baggage, problems of overweight and/or size). The decision will depend on consumer preferences, willingness to pay, and analysis, for example, of the quality of on-board service, the number of connections and stopovers, the history of flight delays, food, size, model and condition of the aircraft, among others. The paths taken by the groups were different to reach those conclusions, among others.

Two groups (A and B) solved the problem situation using tables, highlighting several possibilities of combinations of luggage for the following situations: no weight and size problems in luggage; weight problem and no irregularities in size; no overweight and size problems; weight and size problem in luggage. However, group C constructed a single table to find the conjectures, and group D was creative and efficient. As shown in Figure 3 below, group D placed the information of the task statement in Matrices [1X4] of one row and four columns to find the generalisations and answer the letter *"d"*: What formalisations are possible, from the calculations performed, to be applied to any situation of the same nature?

Figure 3

Investigative Task 1 - Flight Ticket and Baggage Cost (Research Data: Group D)

$p \Rightarrow$ weight (each suitcase) $P_x = \sum p, \forall p > 23kg \Rightarrow$ summation of weights above 23kg

$P_1 = 23 < p \leq 32$; $P_2 = 32 < p \leq 45$; $P_3 = > 45$

X = economic cost without luggage; Y = executive cost with luggage;

M1 = 1 bag; M_2 = 2 bags; M_3 = 3 bags; D_L = dimension > 158cm

Matrix Data: A_{11} = Airline A; A_{12} = Airline B; A_{13} = Airline C; A_{14} = Airline D

$m_{p0} \Rightarrow$ number of bags such that $p \leq 23kg$ $m_{p1} \Rightarrow$ number of bags such that $23kg < p \leq 32kg$

$m_{p2} \Rightarrow$ number of bags such that $32kg < p \leq 45kg$ $m_{p3} \Rightarrow$ number of bags such that $p > 45kg$

$m_{px} = m_{p1} + m_{p2} + m_{p3} \Rightarrow$ number of bags with excess weight $m = m_{p0} + m_{px} \Rightarrow$ total number of bags

$d \Rightarrow$ linear dimension (each bag) $m_{d0} \Rightarrow$ number of bags with $d \leq 158cm$

$m_d \Rightarrow$ number of bags with $d > 158cm$ $m_{d0} + m_d = m$

According to the caption above and task statement information, it is possible to define the following matrices:

$X = [1230 \quad 1267 \quad 972 \quad 1000]$ $Y = [1430 \quad 1270 \quad 1496 \quad 1400]$ $M_1 = [60 \quad 60 \quad 80 \quad 60]$

$M_2 = [100\ 100\ 110\ 100]$ $M_3 = [120\ 120\ 200\ 130]$ $P1 = [80\ \ 80\ \ 12 \Rightarrow (P_x \div m_x - 23)\ 130]$

$P2 = [160\ 160\ 12 \cdot (P_x \div m_x - 23)\ 130]$ $P3 = [270\ 260\ 12 \cdot (P_x \div m_x - 23)\ 130]$ $D_L = [110\ \ 100\ \ 130\ \ 130]$

Thus, the two equations that generalise the various possibilities found are:

For economy class without baggage included: $X + M_m + \sum_{n=1}^{3}(M_{pn} . P_n) + M_d . D_L$

For business class with baggage included: $X + M_{m-1} + \sum_{n=1}^{3}(M_{pn} . P_n) + M_d . D_L$

In his report, student A3 was surprised because one of the groups resolved the activity without a table. He evaluates that he would find it difficult to solve it without tables, but at the same time, he notes that the way the group thought about the resolution is interesting:

> I found it quite intriguing the way they thought when they carried out the task, as they did it without the use of a table, I would have gotten lost in the calculations and the reasoning. I would have had a hard time doing this activity without a table. Congratulations to the group (A3).

Student A1 emphasises that there are several ways to solve a problem, and that, although the groups took different paths, they managed to understand the activity:

> I believe that how conclusions are reached are only in different ways, but we expect to succeed. In this activity, as in the others, all groups who took different ways were able to complete the task and, this, that is cool to learn. Her group did not make a table but managed to understand the activity (A1).

At the same time, student A5 highlights that organising the data in several tables makes the analysis more accessible, and students reach a greater understanding of the problem situation. Also, the table makes the most advantageous option more evident:

> The tables are extremely organised and easy to understand. Separate tables make it easier to see the best airline. And the answers are clear, it is possible to have a better understanding of the question (A5).

We can see that mathematical research provides different thoughts of resolution between groups of students, in which it is necessary to analyse all possibilities before concluding, as emphasised by student A9:

> Very well organised, the formalisations of your group, easy to understand, exploring the various possibilities of the case (A9).

Ponte, Brocardo, and Oliveira (2015) point out that when we solve a problem, we have a path of discovery, and this process can become more significant than its solution. This can be seen in the dialogue between students A1 and A3, when they show that there is a set of possibilities to solve the problem, and that the answer found is not always correct or unique, because there are other factors that were not discovered during the development of the activity:

> I saw that they made separate tables of economic and business class, it was well organised. The interesting thing about these activities is how much they show that we can reach the same conclusion in different ways, respecting the reasoning of each one of us. Perfect, your speech, A3, when everyone takes a path and finds the same result, it shows that mathematics has not only a single but several methods to reach an outcome. I agree with what you said, A1, because if we think about how we are often assessed both in primary and higher education, sometimes they do not take into account the different means we used to reach the same conclusion or

> that the solution found may not have been perfect, because it still depends on other factors that have gone unnoticed (Dialogue between A1 and A3).

According to Abrantes (1999, p. 155), "a contribution of the practice of activities that involve students in open problems is that it deals with fundamental processes of activity and mathematical thinking, such as formulating problems and doing and demonstrating conjectures." In this context, this methodology corroborates the National Curricular Parameters (Brasil, 1998) and the National Common Curricular Base (Brasil, 2017).

> In interaction with their colleagues and teachers, students must be able to investigate, explain, and justify the problems solved, with emphasis on the processes of mathematical argumentation [...]. The identification of regularities and standards requires, besides reasoning, representation and communication to express the generalisations, as well as the construction of a consistent argument to justify the reasoning used. [...] Regarding the competence to argue, its development also presupposes the formulation and testing of conjectures, with the presentation of justifications (Brasil, 2017, p. 519).

Regarding the investigative task international travel cost (Figure 2), it is noteworthy that none of the students knew what exchange rate was. After working with currency conversions (real, rouble, euro, and dollar), they managed to define the concept. According to Pinho and Vasconcellos (2006), the exchange rate is nothing more than the measure of the conversion of the national currency into the currency of other countries, which meets the responses of the groups:

> Price of one currency against the other (Group A).
> Ratio between local currency and a given foreign currency (Group B).
> Value of the currency of one country relative to that of another country (Group C).

Relationship between monetary values of two countries (Group D).

Therefore, exchange rate is the price (cost) of the foreign currency in terms of the national currency, that is, the price (cost) in reais (R$) of each foreign currency. It is noteworthy that the groups reached the same conclusions in almost all questions, but by taking different paths. Two groups (A and B) were concerned with finding, first, the total cost of travel (CT = average expenditure per person x No. of people x No. of days), in roubles and working with the exchange rates found in letter *"a,"* to answer the questions from letters *"c"* to *"h"* by a rule of three or by multiplying the total cost (CT) by the corresponding exchange rate (T). Group C solved the task with the rates found in letter *"a"* but did not calculate the cost of travel to answer the questions of letters *"c"* and *"d"*; it only found the generalisation. Group D and group C worked with the rates provided in the task statement. They did not calculate the cost of travel, only found the generalisations (Figure 4) that could be applied to any situation of the same nature by testing, validating the conjectures, and finding the solution to the problem. For the conversion of a value *"a"* of a currency A to a value *"b"* of a currency B, group D used the following operation: bB = aA. Exchange $Rate_{B\text{-}A}$; while group C solved the question by a similar equation: $CR_{Mn} = CR_{Me} \cdot T_{Mn\text{-}Me}$[1].

Figure 4

Investigative Task 2 - International Travel Cost (Research Data: Group D)

c) Is it worth buying dollars or euros in Brazil and then exchanging the currency in Russia? Explain.

If we want to exchange x reais for roubles, using dollar as intermediary:

$$x\ BRL \cdot TUSD/BRL = x\ BRL \cdot \frac{0{,}26\ USD}{1\ BRL} = 0.26\ x\ USD$$

$$0.26\ x\ USD \cdot TRUB/USD = 0.26\ x\ USD \cdot \frac{63{,}21\ RUB}{1\ USD} = 16.43\ x\ RUB$$

If we want to exchange x reais for roubles, using the euro as intermediary:

$$x\ BRL \cdot TEUR/BRL = x\ BRL \cdot \frac{0{,}22\ EUR}{1\ BRL} = 0.22\ x\ EUR$$

$$0.22\ x\ EUR \cdot TRUB/EUR = 0.22\ x\ EUR \cdot \frac{73{,}70\ RUB}{1\ EUR} = 16.21\ x\ RUB$$

Therefore, it is worth buying a dollar in Brazil and then making the exchange in Russia.

In the generalisations found, two groups did not add the IOF (Financial Transaction Tax), another group did not notice, in the resolution of the letter *"f,"* that it would have to convert the expense in roubles into dollars. The total cost of the credit card trip in reais (R$) would depend on the dollar quotation on the day of purchase or at the closing of the invoice (choice of the cardholder)[2]. Another group did not realise that if the family wanted to buy roubles at the airport, the quotation was not so advantageous.

Two groups, too, did not find the generalisations for the letter *"d,"* they only tested the hypotheses, stipulating different percentage values of the devaluation of the euro and the dollar currencies. Based on those assumptions of changes in exchange rates, they found the solution through the exchange rate equation itself (ratio between domestic and foreign currency prices) or rule of three. Group D found a generalisation for this situation (Figure 5), concluding that the choice between dollars or euros

[1] CR = Cost/Revenue; Mn = Domestic Currency; Me = Foreign Currency; T = Exchange Rate.

[2] Note: The obligation of credit card operators to use only the quotation of the dollar on the day of purchase for conversion of the value into real entered into force on 03/01/2020, by determination of the Central Bank (Circular 3918), (Agência Brasil, 2020).

will depend on the percentage of the devaluation of the real in relation to these two currencies.

Figure 5

Investigative Task 2 - International Travel Cost (Research Data: Group D)

d) If our currency devalues more against the dollar than the euro the next day, is it still worth buying the dollar? Explain.

Considering the devaluation of the real against the dollar and the euro, the two new exchange rates can be represented as: $_{TUSD/BRL}$ = 0.26 - d; $T_{EUR/BRL}$ = 0.22 - e

As the currency depreciated more against the dollar than the euro, then d = e + a

Therefore, you can rewrite:

$T_{USD/BRL} = 0.26 - (e + a) = 0.26 - e - a = (0.22 - e) + 0.04 - a = T_{EUR/BRL} + 0.04 - a$

Considering that there has been no change in the exchange rates of the rouble against the dollar and the euro, we can say that for x reais we will have the following expressions in roubles:

$63.21 \cdot (T_{EUR/BRL} + 0.04 - a) \cdot x \Rightarrow$ rouble value after conversion to dollars

$73.7 \cdot T_{EUR/BRL} \cdot x \Rightarrow$ value in roubles after conversion to euro

$(63.21 \cdot T_{EUR/BRL} + 2.5284 - 63.21a) \cdot x$

$(63.21 \cdot T_{EUR/BRL} + 10.49 \Rightarrow \text{TEUR/BRL}) \cdot x$

After rearranging, they noticed that, for the conversion into dollar to remain worth it, one must have the following:

$2.5284 - 63.21a > 10.49 \cdot T_{EUR/BRL}$

$2.5284 - 63.21a > 10.49 \cdot (0.22 - e)$

$2.5284 - 63.21a > 2.3078 - 10.49e>$

$a < 16.59\%\ e + 0.0034899$

Thus, the choice of the dollar will only be worth it if our currency does not devalue against the dollar more than 16.59% of the devaluation value against the euro, plus 0.0034899.

Ponte, Brocardo, and Oliveira (2015, p. 23) emphasise that in mathematical research "the student is called to act as a mathematician not only in the formulation of questions and conjectures and the performance of tests and refutations but also in the representation of results and the discussion and argumentation with his/her colleagues and the teacher." Student A9 highlights how clear and organised the representation of results (conversions) was with the use of tables:

> I liked the organisation in tables with a clear view of conversions, in our group we ended up choosing to always solve from equations (A9).

Student A6 emphasises that it was possible to evaluate the reasoning and understand the activity in general at the time the findings were socialised and discussed by the class:

> We started the task using the rule of three in our calculations, and from that we began to formalise our thinking; by socialising with the peers, we could evaluate our

> reasoning and understand what factors we failed to consider during the resolution. From simple calculations, the student gets to more complex formulas (A6).

Student A7, when completing the four stages of the mathematical investigation, highlights that he understood the activity, there was learning by successfully validating the conjectures, and he was sure he could apply it in other situations of the same nature:

> By finding, testing, and validating the generalisations, we are learning and understanding what happens, and we can apply those formulas to other events of the same nature (A7).

Therefore, investigating concerns making discoveries, exploring problems and hypotheses, constructing arguments and justifications that support the idea. Ponte (2003) points out that investigating is discovering relationships, patterns, trying to identify and prove the properties raised by the investigator. For an activity to be considered as an investigation, according to Oliveira, Segurado, and Ponte (1998), it is essential that the situation is motivating and challenging, and the resolution process and the solution or solutions of the issue are not immediately accessible to the student.

b) Importance of the investigative task, in groups, for the learning and development of creativity and autonomy

Student A1 emphasises how colleagues in the other group were creative in solving the activity without using a table. According to Ponte, Brocardo, and Oliveira (2015), mathematical research activities encourage students to develop creativity and autonomy, defining objectives and conducting research, formulating strategies, testing their conjectures and critically analysing the results obtained from the groups.

> I found the way they were able to solve the problem challenging, since they did not develop a table. I really liked the way they made the generalisations because ours, we didn't know what name to give to the parameters that emerged (laughs), but you were very creative. By understanding the choice process, it is possible to understand which airline is cheaper to fly. Congratulations to group (A1).

Student A4 emphasises that the interaction and collaboration provided by mathematical research help to understand the activities, contributing to learning in a differentiated way:

> The investigative activity generated differentiated learning because we learned among ourselves and overcame the difficulties through dialogue. Our group is very

> collaborative, interactive, and participatory, facilitating calculation and questions understanding (A4).

Based on this assumption, mathematical research contributes to integration and socialisation, as it provides an encouraging and creative environment, where students have the freedom to expose their thoughts and resolutions to colleagues and teachers.

Student A5 agrees with A4, finding that mathematical research is a differentiated form of learning, in which the student needs to reflect and seek their own knowledge. Ponte, Brocardo, and Oliveira (2015, p. 23) emphasise that the "[...] active involvement of the student is the fundamental condition of learning."

> The form of differentiated learning, first proposing a problem-situation and making the group think and question can be a positive point, as it encourages students to seek their own knowledge (A5).

Therefore, it is necessary to develop the student's ability to engage with their own learning, create their own strategies, and enable them to engage in activities that require exploration. Student A1 emphasises how group investigative activity contributes to learning. Teaching is no longer rigid, bringing students closer to the teacher and colleagues:

> Bringing mathematical research activities of a financial nature to students is interesting and challenging, since opinions may differ on certain occasions and teaching is no longer rigid. It brings the teacher closer to the student and makes the students really learn about the content inserted there (A1).
>
> The activity was great, since I didn't know what the exchange rate was, I was able to formalise and define a concept that I had never seen before. This shows that building knowledge with students is the best way to teach. I like it when I learn things I didn't know (A1).

According to the National Common Curricular Base (Brasil, 2017, p. 223), it is essential that students:

> Interact with their peers cooperatively, working collectively in the planning and development of research to answer questions and in the search for solutions to problems, to identify consensual aspects or not in the discussion of a particular issue, respecting the way of thinking of colleagues and learning from it.

The cooperation and interaction that small group studies provide are fundamental for students to gain confidence, know how to face their difficulties, discuss the problem with colleagues and learn from them. Student A2 highlights the importance of the activity being carried out in a group. Student A7 also highlights how cooperation, teacher encouragement and discussions were crucial for learning:

> We did every task together, and in a very cooperative way, I also thank the highlights of the teacher and colleagues during the class that made a lot of difference (A2).
>
> The cooperation that the research activities provided and the teacher instigating us favoured the learning of the content and encouraged the class to participate in the discussions, providing greater reflection and understanding about each situation, I am very satisfied with the classes, the teaching should be this way (A7).

The activities in small groups allow students to dialogue about the problems experienced, thus building spaces for reframing their reality and learning. According to Anastasiou and Alves (2005, p. 75-76), "working in a group is different from being part of a group of people, being fundamental the interaction, sharing, respect for the singularity, the ability to deal with the other in its entirety, including their emotions." The teacher should act as a mediator, according to Ponte, Brocardo, and Oliveira (2015), instigate the investigation and discuss the situations proposed. It is of paramount importance to develop challenging moments in a group, in which each student feels motivated to get involved throughout the activity.

c) Developing critical thinking in the decision-making process

According to the reports of students A1, A3 and A6, we can see that mathematical research activities contributed to the development of the critical spirit when they were requested to analyse the costs and benefits of all possible possibilities in a decision-making process:

> Very useful activity for our lives to make us more critical about situations of the same nature (A3).
>
> We have learned to be more critical concerning the analysis of airline ticket prices and take into account the possible unforeseen events that may occur. Also, if we make an international trip, we can calculate the money needed for the trip and what is the most advantageous and economical option (A6).
>
> It is important to emphasise decision making, because, when an individual makes a wrong decision, he/she ends up having a financial loss. In this case, when the individual travels and knows how to choose a quality and cheaper

> airline, he/she has positive finances for leisure after the trip. Understanding the process of choice becomes essential for the individual not to be financially impaired (A1).

"The teacher needs to be in constant formation, always seeking contributions and methodological strategies aimed at learning and training active and critical citizens, because after all, forming citizens is one of the teacher's functions" (Moreira *et al.*, 2017, p. 8). Based on the mathematical research activities, the undergraduates could analyse each situation better, reflect on the problem, becoming more critical, before accepting a single result as true.

Goldenberg (1999, p. 37) adds that investigative activities "motivate students, and develop skills that contribute to a broader knowledge of concepts and facilitate learning," as students A3 and A5 emphasise:

> I learned a lot from this activity because I had no idea what exchange rate was when they spoke on television; it seemed something from another world, something that was out of my understanding. Now I understand about currency devaluation and having logical thinking (A3).
>
> It is essential to carry out this type of activity that instigates us, makes us learn, helps us think and develop critical thinking (A5).

In this context, mathematical research activities on financial education and economics were elaborated, so that future teachers can learn to teach their students and from critical and reflective thinking act more actively in society, contributing to the economic and sustainable development of their country.

d) Relationships between the investigative task and everyday situations (connection theory and practice)

The National Curriculum Parameters - PCNs (Brasil, 1998) already signalled that, in mathematics, students should be able to question reality, formulate problems and solve them, using logical thinking, creativity, intuition and the capacity for critical analysis. Moreira *et al.* (2017, p. 8) complement: "in the discipline of mathematics it is necessary to take into account problems that involve the students' daily lives, that lead them to reflect, investigate, look for solutions and participate critically in the process of teaching and learning [...]"

From this perspective and according to the reports of students A1, A3 and A5, mathematical research activities become more attractive to the extent it is possible to establish relationships with problem situations of everyday life:

> The activity fostered the organisation of spreadsheets/tables, which facilitated understanding the problem of everyday issues, of air travel spending. I find this interesting, because when we make a trip in the future, we will know how to organise and choose an airline we can afford (A1).
>
> This activity was very interesting, because they are situations that can occur in our lives and we never stop [our daily activities] to analyse them (A3).
>
> An investigative activity can be a valid method to teach financial education, especially when it is related to our life, to our reality. This generates interest in the student through a group challenge, which motivates them to solve the problem (A5).

Skovsmose (2000) points out that mathematics education is not reduced to the technique of teaching mathematics but is configured as an action of mathematically educating for life. Ulhôa *et al*. (2008, p. 2) point out that "the citizen of this century cannot have the same profile of skills as the last century. It can no longer ignore what is happening in the world, he/she needs to be properly inserted in the social milieu." Therefore, the inclusion of contents of financial mathematics, financial education, and economics in the discipline of Mathematics broadens the knowledge acquired. Through them, the student can establish relationships between theory and practice, perceive the connection of the content and its application in everyday life, in their lives, as highlighted by students A5 and A7, which meets the thinking of Skovsmose (2000), Ulhôa *et al.* (2008), Moreira *et al.* (2017), and D'Ambrósio (2000):

> The activity elaborated can be a way to teach financial education, as it empowers the student to be aware of real situations (A5).
>
> The problem situation is very realistic, which incentivises us to solve it and apply it in our daily lives (A7).

D'Ambrosio (2000) adds that students need to have mathematics knowledge to face everyday problems, know how to analyse them critically so that they can make better choices. Thus, according to the authors above, the teacher should relate the contents developed to students' everyday life, stimulating their autonomy and critical thinking and, thus, contribute to building citizenship, as stated in the BNCC (National Common Curricular Base). In this context, students must learn financial education not only to solve their financial problems but also to understand the economic, social and environmental problems experienced in their daily lives, showing concern about solving them, fighting for their rights, and fulfilling their duties, contributing to increase their quality of life and the economic and sustainable development of their country and the world as a whole.

e) Difficulties during task resolution

Student A2 highlights that the group had difficulty understanding the activity and finding the equations, which are used to using the form:

> We were in doubt about the question where we had exchange dollars into roubles, but we managed to conclude [...], we still had doubts regarding the generalisation of some formulas, in fact we are not used to finding formulas, we always use the forms (A2).

Student A1 reports that the group was not visualising the logic of converting national currency into foreign currency and that they could not find a generalisation for a question that could be used for any similar situation:

> We found it hard to interpret the letter "c" question because we were not seeing the logic of using the real currency to multiply the total value of the dollar/euro that it would be needed to buy the roubles currency. And another situation, we cannot find a formalisation for the question of the letter "d." Of course, later, we understand those difficulties from a simpler example we elaborated (A1).

Ponte, Brocardo, and Oliveira (2015) comment that investigating does not mean working with too difficult problems but with situations that at first seem to be confusing, but one tries to clarify. It is precisely because of their open structure that investigative activities present a high degree of difficulty. Student A8 emphasises the importance of investigative activity to develop critical thinking and establish relationships between theory and practice:

> Some information such as exchange rate, we have never heard of, and in this case, it was the whole class. The exciting thing is that from the mathematical investigation activity, everyone was able to define what it was, through critical thinking, through formalising the calculations, and linking theory with practice (A8).

Mathematical research is considered an activity that drives thought and occupies a central role in the teaching and learning process, according to Ponte, Brocardo, and Oliveira (2015). Therefore, investigative tasks can encourage students to identify what they know about the theme of each problem-situation and what the best strategies to be used to reach some conclusion are, helping students develop autonomy and critical sense. According to the students' report, we observed that the mathematical investigation tasks proposed (open problem-situations) boosted their thinking in the search for strategies to solve the problem, and helped them to assess the situations to detect possible errors made, making evident their awareness of the degree of difficulty presented in each task and what the causes would be.

CONCLUSION

This qualitative study aimed to analyse how mathematical research activities can contribute to the teaching of financial education and economics in a mathematics teaching degree course. Data analysis enabled us to achieve the objective proposed. Thus, the data collected were grouped into five categories, namely: a) Possibilities of conjectures and resolution strategies; b) Importance of the investigative task, in groups, for the learning and development of creativity and autonomy; c) Development of the critical spirit in the decision-making process; d) Relationships between the investigative task and everyday situations (theoretical and practical connection); e) Difficulties during task resolution.

The representative data of the categories above demonstrate that from the resolutions of mathematical investigation tasks, small-group reflections (potentialities and difficulties during the execution of tasks), and discussions during the socialisation of activities in the classroom and virtual environment forum, students could develop argumentation and critical thinking. The activity also stimulated their creativity, autonomy, and collaborative spirit, and contributed to the teaching and learning processes of financial education and economics.

The small groups took different paths and made important discoveries during the resolution of each problem when they compared the results during the socialisation for the large group. Then, they realised that they could have chosen other paths, or that they could have neglected some important factor that was part of the analysis and influenced the decision-making process. Students showed enthusiastic about reading each activity, questioning, raising hypotheses, finding generalisations for the first time (the formulas were always given), testing and validating the conjectures, realising, from the class socialisation of each activity, that there are several paths and several factors that are part of the analysis to reach an outcome, and that the answer found is not always the correct one.

Therefore, the potential of this methodology in the learning and development of critical thinking is undeniable. The undergraduates were so excited about the problem situations that were so close to their daily lives that the discussions were not exhausted in the classroom. The forum allowed each student to expand their knowledge further, going on with reflections and debates and strengthening learning. Finally, it is crucial to problematise the contents of financial education and economics and relate them to financial mathematics in 'undergraduates' initial education in a contextualised and innovative way, where everyone feels pleasure in learning, knows how to argue, and develops critical sense. In this sense, we chose the mathematical research methodology to teach financial education and economics, aiming to link theory with practice (everyday problem-situations), developing the critical thinking and skills of the basic education and/or higher education prospective teachers.

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AUTHORSHIP CONTRIBUTION STATEMENT

Authors PF and MQ discussed the methodology and theoretical foundation. The first author, PF, collected and analysed the data. Authors PF and MQ discussed the results and contributed to the final version of the article.

DATA AVAILABILITY STATEMENT

The data that support the results of this study will be made available by the corresponding author, PF, upon reasonable request.

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