

Development of Mathematical Thinking Skill from the Formulation and Resolution of Verbal Arithmetic Problems

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ABSTRACT

Background: mathematics teachers interested in improving student performance in the face of the low academic results presented, we seek, **Objective:** articulate the skills of mathematical thinking with the formulation and resolution of verbal statement arithmetic problems (PAVE). **Design:** the methodology was focused on action research from the design and application of a didactic sequence developed from three categories of analysis: thinking skills, formulation, and solving of arithmetic problems. **Setting and participants:** basic education students starting high school. **Data collection and analysis:** we created and implemented a didactic sequence that includes two directions: one for the formulation and the other the resolution of PAVE. Each one was monitored from three activities: opening, development and closing. **Results:** difficulties in formulating and solving verbal statement arithmetic problems were evidenced in those students. **Conclusions:** after applying the intervention, changes were evidenced in the formulation and resolution of verbal statement arithmetic problems in the group of students. Some difficulties detected in the students are related to the length of the statement, the order of presentation of data, the situation of the question, the size of the numbers used, elements that affect the syntactic and mathematical structures of the PAVE

Keywords: Formulation and resolution of arithmetic problems; Mathematical thinking; Thinking skills.

Desarrollo de Habilidades del Pensamiento Matemático desde la Formulación y Resolución de Problemas de Enunciado verbal

RESUMEN

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Contexto: los profesores de matemáticas interesados en mejorar los desempeños de los estudiantes ante los bajos resultados académicos presentados, buscamos, **Objetivo:** Articular las habilidades del pensamiento matemático con la Formulación y Resolución de Problemas de Enunciado verbal (PAEV). **Diseño:** La metodología se enfocó en la Investigación–Acción desde el diseño y aplicación de una secuencia didáctica desarrollada desde tres categorías de análisis: habilidades de pensamiento, formulación y resolución de problemas aritméticos. **Escenario y participantes:** estudiantes de educación básica que inician la secundaria. **Colección y análisis de datos:** creamos e implementamos una secuencia didáctica que contemplo dos direcciones: una para la formulación y la otra para la resolución de PAEV. Cada una se monitoreo desde tres actividades: de apertura, de desarrollo y de cierre. **Resultados:** Se evidenció en los estudiantes que inician la secundaria, dificultades para formular y resolver problemas aritméticos de enunciado verbal. **Conclusiones:** luego de aplicar la intervención, se evidenció cambios en la Formulación y Resolución de problemas aritméticos de enunciado verbal en los estudiantes. Algunas dificultades detectadas en los alumnos están relacionadas con la longitud del enunciado, el orden de presentación de los datos, la situación de la pregunta, el tamaño de los números utilizados, elementos que afectan las estructuras sintáctica, semántica y matemática de los PAEV.

Palabras clave: Formulación y resolución de problemas aritméticos; Pensamiento matemático; Habilidades de pensamiento.

Desenvolvimento de Habilidades de Pensamento Matemático a partir da Formulação e Resolução de Problemas de Enunciado Verbal

RESUMO

Contexto: professores de matemática interessados em melhorar o desempenho dos alunos frente aos baixos resultados acadêmicos apresentados, buscamos, **Objetivo:** Articular as habilidades de pensamento matemático com a Formulação e Resolução de Problemas de Enunciado Verbal (PAEV). **Design:** A metodologia centrou-se na Pesquisa-Ação a partir do desenho e aplicação de uma sequência didática desenvolvida a partir de três categorias de análise: habilidades de pensamento, formulação e solução de problemas aritméticos. **Ambiente e participantes:** alunos do ensino fundamental iniciando o ensino médio. **Coleta e análise de dados:** criamos e implementamos uma sequência didática que inclui duas direções: uma para a formulação e outra para a resolução do PAEV. Cada um é monitorado a partir de três atividades: abertura, desenvolvimento e encerramento. **Resultados:** Dificuldades na formulação e solução de problemas aritméticos de enunciado verbal foram evidenciadas em alunos que ingressaram no ensino médio. **Conclusões:** após a aplicação da intervenção, evidenciaram-se mudanças na Formulação e Resolução de problemas aritméticos de enunciado verbal dos alunos. Algumas dificuldades detectadas nos alunos estão relacionadas com o comprimento do

enunciado, a ordem de apresentação dos dados, a situação da pergunta, o tamanho dos números utilizados, elementos que afetam as estruturas sintáticas, semânticas e matemáticas do PAEV.

Palavras-Chave: Formulação e resolução de problemas aritméticos; Pensamento matemático; Habilidades de pensamento.

INTRODUCTION

The Curricular Guidelines and Standards in the Colombian context propose five processes for the mathematical activity that must be developed when students go from elementary to high school. One of these processes is related to the formulation and resolution of problems known as *Problemas Aritméticos de Enunciado Verbal* - PAEV (verbal statement arithmetic problems) that appear at the end of elementary education and beginning of high school.

The children's group observed children entered high school (sixth grade in Colombia). We could observe a) low level of reading understanding, b) not understanding how to formulate and/or solve arithmetic problems, c) problems in understanding and oral expression, and d) disconnection between concepts or formation of conceptual networks of different disciplines. These weaknesses hinder the students' learning process from achieving the basic skills required, according to the parameters of the educational policies established by the Ministry of Education -MEN. Proof of that is the high failure rates in each school period (internal institutional tests), as well as in the results of external tests (Saber 5° tests, applied by the State).

In the school environments of the Colombian context, there are curricular guidelines for different areas of knowledge. Especially for mathematics, they propose five ways of thinking: 1) numerical thinking and numeral systems, 2) spatial thinking and geometric systems, 3) metric thinking and measurement systems, 4) random thinking and data systems. 5) variational thinking, algebraic, and analytical systems (MEN, 2006). Within these five types of thought proposed, it is possible to integrate the formulation and resolution of problems. For this, this work seeks to make contributions to the relationship between the formulation and resolution of arithmetic problems with mathematical thinking in populations that finish elementary school education and begin high school.

The sample for this research was focused on students making the transition from basic primary to secondary education (sixth grade), in a state

public institution, located in the south-west of the city of Bogotá and that offers its services to children who live in subnormal neighbourhoods in the area. We seek to identify students' changes in the formulation and resolution of arithmetic problems based on the design and implementation of a didactic sequence that sought to develop elementary mathematical thinking skills, from Márquez's (2014) proposal.

Considering that problem-solving learning begins almost at the same time as reading, we find that children's elementary verbal arithmetic problems are expressed through statements that are not intended to clarify the understanding of the problem but are part of the task that must be faced for its resolution. These aspects generate difficulties for students to understand and relate the length of the statement, the order of presentation of the data, the context of the question, and the size of the numbers to solve the problem. The target population of this study shows to have little understanding of these elements when they create a mathematical problem that has a solution. In general terms, we find pretentiousness in the use and handling of the syntactic, semantic, and mathematical structures to formulate and solve the PAEV.

BACKGROUND

Gaulín, (2001) indicates that in the field of mathematics education, a privileged space has been granted to the resolution of mathematical problems since the mid-20th century, Pólya being one of its precursors. Since then, problem-solving has had three educational perspectives: 1) teaching for problem-solving; 2) teaching about problem-solving; and 3) teaching through problem-solving (Gaulín, 2001). Since then, problem-solving has been incorporated into school curricula and, therefore, it is a skill that must be developed in students throughout their school lives.

Lara (2012) identifies three general currents that were interested in thinking skills since the late 1980s: the acquisition of intellectual skills through academic programs, the development of critical thinking, and thinking processes. Lucero (2009), in turn, confirms the importance of working on thinking skills to support the integral development of the Being. Based on the results of the research carried out by the INDAGAR group¹ of Universidad

¹The research was called "Caracterización de las habilidades básicas de pensamiento que aplican, en la solución de problemas de la cotidianidad, los estudiantes del grado séptimo de la institución educativa municipal en San Juan de Pasto." (Characterisation

Mariana (Mariana University, Colombia) in the period 2004-2009, the author created a model to develop the thinking skills expressed in two books: *Desarrollo de habilidades de pensamiento (1991)* (Development of thinking skills) and *Aprende a pensar (1993)* (*Learn to think*).

Araya (2014) studies how to present and enhance four thinking skills (observation, induction, hypothetical-deductive reasoning and abstraction in problem-solving), with fifth-grade school children, through a comparison of pre-test and post-test between the control group and the experimental group, after applying an “intelligent didactic institutional plan in mathematics” (p. 9) for six months.

Hernández (1996) focuses on the resolution of *verbal arithmetic problems* in students between the ages of 8 and 11, who are taught a system of visual-geometric representation to address this type of problems, focused on the development of cognitive, heuristic, and metacognitive skills. There, students had to solve both additive and multiplicative problems. It highlights the importance of the semantic meanings of the statements, since, in the problems the students created, they used few words that indicate additions or subtractions.

Bosch (2012) addresses aspects of mathematical thinking, citing the diversity of meanings for the term “thinking”. Bosch (2012), citing Cantoral et al., characterises it as the combination of mathematical topics and a series of “thinking” processes such as abstraction, justification, and visualisation. This allows us to refer to mathematical thinking as the relationship between intuitive thinking and analytical thinking because there must be a connection between them to solve a mathematical problem.

Saido, Siraj, Bin Nordin, and Al_Amedy (2018) categorise mental processes based on Bloom’s taxonomy seen from two elements: *Lower Order Thinking Skills - LOTS* and *Higher Order Thinking Skills - HOTS*. LOTS includes to remember, understand, and apply; HOTS includes to analyse, evaluate, and create. Based on these contributions, Márquez (2014) suggests mental processes designed to encourage the development of thinking in any area of knowledge, proposing an organisation by levels and types of knowledge.

of the basic thinking skills that students of the seventh grade of the municipal educational institution in San Juan de Pasto apply in solving everyday problems.)

THEORETICAL FRAMEWORK

Two conceptual axes were approached: 1) Arithmetic problems, their classification and formulation, from the position of Puig and Cerdán (1988). 2) Thinking skills, from Márquez's (2014) proposal.

Arithmetic problems

Also known as verbal statement arithmetic problems (PAEV, in the Spanish acronym), Puig and Cerdán (1988), cited in Espinoza et al. (2015), propose:

In the statement, the information provided is quantitative, since the data are usually quantities; the condition expresses relationships of a quantitative type, and the question refers to the determination of one or more quantities, or relationships between quantities. Problem-solving, or what needs to be done to answer the problem question, essentially seems to consist of performing one or more arithmetic operations.

From this theoretical position of the PAEV, it is possible to analyse three structures with which the richness of a problem proposed by the students in formulation tasks can be evidenced: *Semantic structure*, divided into additive and multiplicative, each with its own subdivisions. *Syntactic structure* indicates the linguistic complexity with which a written statement is posed; considering the length of the text, the interrogative proposition and the type of numbers used. *Mathematical structure*, where the number of stages and the number of steps to achieve a solution reside. In Table 1, we present an adaptation of the PAEV classification considering their semantic, syntactic, and mathematical structure guided by the contributions of Espinoza et al. (2015).

Table 1

PAEV classification (Espinoza et al., 2015, p.21)

Type of structure	PAEV	Sub classification	Characterisation	Example
		Combinati on	There is a part - part-whole relationship between sets and the request is to	There are 223 hens and 168 ducks on a farm. How many birds are there in all?

Semantic structure	Additive		find the cardinal of one of these elements.	
		Change	There is an initial quantity which is subject to changes in a temporal sequence.	There are 264 boys in the school, followed by another 264. How many boys are there now?
		Comparison	There are a reference quantity, a compared quantity, and the difference between them.	Real Madrid has scored 89 goals. Barcelona has scored 22 more goals. How many goals has Barcelona scored?
	Multiplicative	Isomorphism of measure	There is a relationship of simple direct proportionality where the correspondence rule is expressed.	The school is going to buy 150 notebooks; each one costs 125 pesos. How much will all the notebooks cost?
		multiplicative comparison	There is a comparison between two quantities of the same type of magnitude through a scalar.	240 children can fit in the schoolyard. The 3rd-grade class can accommodate 30 children. How many times more children can be accommodated in the playground than in the 3rd-grade class?
		Measurement product	There exists a Cartesian multiplication between two sets or space of measures. Area or volume problems apply, among others.	I have 6 consonants and 5 vowels. How many different syllables that start with a consonant can I spell?

Syntactic structure	Linguistic complexity, understood as the type of language used, natural, analytical, symbolic. Viewed from three complexities: structural, cognitive, and developmental.
Mathematical structure	The information is presented exclusively through verbal language, and to solve them it is necessary to apply one or more of the four elementary operations. The data are offered in the form of quantities, either verbally or numerically, and quantitative relationships are established between them. The questions prompt us to determine one or more of these quantities. It is expected that there will be clarity on the number of stages and steps to follow.

Uses of the term “complexity” in linguistics

Pallotti (2015) remarks the polysemy of the term in the linguistic bibliography and classifies the different meanings of complexity into three large blocks: *Structural complexity*: formal property of texts and linguistic systems that has to do with the number of their elements and their relational patterns. *Cognitive complexity*: it has to do with the processing cost associated with linguistic structures. *Developmental complexity*: considers the order in which linguistic structures emerge and are learned in the acquisition of second (and possibly first) languages. However, the concept of complexity has been interpreted in different ways in linguistic studies, allowing different types of complexity to be distinguished depending on the type of analysis to be carried out. Thus, Dahl (2004) recommends distinguishing the complexity of the system from structural complexity. The *complexity of the system* refers to the properties of a language; measures the number of distinctions within a category and calculates the content of the speaker’s proficiency. *Structural complexity* calculates the quantity of structure in a linguistic object and analyses the structure of expressions.

Thinking skills

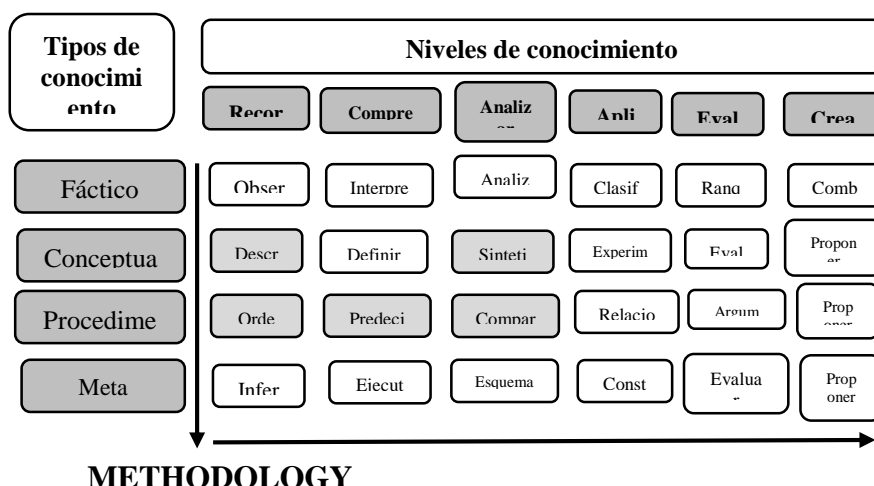
Márquez (2014), who points out that the development of thinking skills “fosters lasting, meaningful learning with greater applicability in decision-making and problem-solving” (p. 33), resumes research related to thinking and formulates a proposal that develops twenty-four skills. These are organised in a matrix that decomposes knowledge in two directions. On one side, it presents

six levels, and on the other, it establishes four types, each one in ascending order, as shown in Figure 1.

Figure 1 shows our adaptation for the taxonomy for thinking skills following the proposal of Márquez (2014).

Figure 1

Taxonomy, thinking skills. (Márquez, 2014, p. 66)



The action research was implemented with a group of students who made the transition from elementary school to high school in a state-owned public institution, located in the south-west of the city of Bogotá, which serves children living in subnormal neighbourhoods of this zone. This context prevents them from managing basic resources for an acceptable school performance.

The research was carried out in three phases. 1) *Planning*: identification of the problem, background, supporting theoretical framework that led us to design and apply a diagnostic test, from two instruments: one to identify the formulation of PAEV that the students made (the invention of problems). The other, to monitor the resolution of these types of problems. For both instruments, the semantic, syntactic, and mathematical structures were considered seen from the conceptual and procedural aspects, with nine items

that seek to discern the complexity in the formulation as seen from the students' reasoning in the process of solving the problem situations posed by them. We analysed and triangulated the results obtained, identifying flaws in the three structures, focused on three levels of knowledge that we considered to be of greater relevance (*remember, understand, and analyse-synthesise*), indicated in light-grey colour in Figure 1. This allowed us to plan and design a didactic sequence (DS) of intervention focused on the formulation and resolution of the PAEV.

For the design of the DS, the basic characteristics proposed by Díaz-Barriga (2013) were considered from three moments: *Opening activities*: those that allow creating an appropriate environment for learning, in which the students participate in discussions, express questions, and usually show their previous knowledge regarding the concept considered. *Development activities*: they allow the students to generate interaction between their previous knowledge and new information to give it meaning. *Closing activities*: they allow to synthesise all the tasks carried out and look at the learning developed.

Phase 2) *Execution and observation*: the DS was applied during an academic period, for approximately nine weeks. The information collected was observed, systematised, and triangulated in the diagnosis and application of the DS. It was systematised according to the descriptors proposed in Table 2 (shown in section 4 “results” of this manuscript). The answers were organised following three classification possibilities: (Sí - Yes) when a student complies with the descriptor, (No) when he/she does not comply with it, and NA (No se aplica - Does not apply) when an item is omitted or what is written in it has no relation to the descriptor. Phase 3) *Evaluation*: an exit test was carried out that evaluated the intervention of the DS, and it was contrasted with the results of the diagnosis, to verify if there was an evolution in the mathematical processes of the students. The information was collected using the Google forms tool, where it is possible to create dynamic tables and filters by categories.

RESULTS

In Table 2, we show the triangulation between skills of mathematical thinking, formulation, and resolution of the PAEV, to identify in which structure there are difficulties. From this instrument, we designed the diagnostic test that showed weaknesses in the four types of knowledge proposed in Márquez (2014) and the detection of serious flaws in the syntactic, semantic, and, thus, in the mathematical structure. Due to the lack of space, we will focus

only on the types: conceptual and procedural approached from three levels, remembering (analysed from describing, ordering); understanding, (analysed from predicting); and analysing (from synthesising and comparing) considering the type of structure involved.

Table 2

Triangulation between skills of mathematical thinking, formulation, and resolution of PAEV from each structure

Thinking skill:	Skill indicator	Category	Subcategory	Code	Descriptor	
Describe	The student relates characteristic elements of objects or situations	Formulation	Syntactic structure	1	The problem statement has three or more propositions	
				2	The problem statement contains at least two quantities	
				3	The problem has an additive structure of change (there is a variation on an initial quantity)	
				4	The problem has an additive combination structure (there are two or more isolated quantities without variations and they are part of a whole)	
			Semantic structure	5	The problem has an additive comparison structure (there are two or more quantities and the difference between them must be calculated)	
				6	The problem has a multiplicative structure of isomorphism of measure (there is a simple direct proportionality relationship)	
				7	The problem has a multiplicative comparison structure (in the situation a scalar intervenes on a variable as a method of comparison)	
				8	The problem has a product of measure structure (it poses a Cartesian product of measures)	
		Resolution	Analysis phase	9	Identify the relevant information of the problem	
				10	Identify which question the interrogative statement of the problem asks	
				11	The mathematical structure of the problem is mixed, (it combines additive structure and multiplicative structure)	
			Formulation	Mathematical structure	12	The mathematical structure of the problem is additive (+, -)
					13	The mathematical structure of the problem is multiplicative (\cdot , \div)

Order	The student organises information according to an indicated or invented pattern	Resolution	Exploration phase	14	The mathematical structure of the problem is more than one stage
				15	Proposes a logical sequence of steps that allow the problem-solving
				16	Shows signs of diagrams, tables, or schematics to help understand the problem
				17	Shows signs of successfully developing additions
				18	Shows signs of successfully performing subtractions
			Execution Phase	19	Shows signs of multiplications developed correctly
				20	Shows signs of correctly developing divisions
				21	Follows all the steps designed to solve the problem
				22	Verifies that his problem has a solution
				23	The interrogative statement can be answered with the numerical information of the problem
Predict	The student predicts possible causes or consequences of a situation	Formulation	Syntactic structure	24	The interrogative proposition has a solution that is coherent with the type of numbers used and the relationships between variables involved in the statement
				25	The interrogative proposition is one of assignment (ask for the quantity of a variable)
				26	The interrogative statement is relational (a scalar is present for comparison)
				27	The interrogative statement is conditional (dependence between two variables measured by an explicit quantity)
				28	Shows signs of numerical strategies that allow him to reach the solution of the problem
			Resolution	29	The problem presents the numerical information and the interrogative statement related to it
				30	The problem involves more than one type of number (natural, decimal, etc.)
				31	Writes a logical and correct answer to the interrogative proposition of the problem
				32	Writes a numerically correct but partial answer to the interrogative proposition
				33	Formulates problems that require different arithmetic processes for their solution
Synthesize	The student integrates a whole from the parts	Resolution	Verification phase	34	Shows signs that he reviews the answer to a problem
				34	Shows signs that he reviews the answer to a problem
	The student identifies	Formulation	Mathematical structure	33	Formulates problems that require different arithmetic processes for their solution
				34	Shows signs that he reviews the answer to a problem

Compare	similarities and differences between objects or situations	Resolution	Verification phase	35	Compares his solution with another and considers his to be the correct one
				36	Identifies similarities between solving processes of the same problem
				37	Identifies differences between processes for solving the same problem

Figure 2 shows some images, from a series of them, that we offered to the students in the diagnostic phase, inviting them to produce a mathematical problem, allowing them to mix information between images or with data that they could place. We identified weaknesses in the syntactic and semantic structures because children created problem-situations by just writing sentences that involved quantities, followed by questions that sometimes had no solution; others, the information they provided was insufficient to answer the question asked.

Figure 2

Images given to students (Source: see footnote 4, 5, and 6)



Some children formulated problems such as: Student E1. *Juan goes to the barbershop to get a haircut; how much does he pay if he goes three times a week?* Usually no one goes to get a haircut three times a week. E2: *With my mom, we make biscuits, we use eggs, 1 pound of flour and salt. How many biscuits do we make per day?* E3: *a car 21 goes straight away to 80 km/h, how*

²Taken and adapted from: <https://graffica.info/la-barberia-del-mono-art-show/>

³Taken from: <https://sp.depositphotos.com/186650702/stock-illustration-cartoon-of-mother-and-son.html>

⁴Taken from: <http://melhores-desenhos-para-colorir.blogspot.com/2017/04/5-desenhos-de-carros-para-colorir.html>

many kilometres does it go? From this information, we can infer that little work has been done with these students on the structural and developmental complexities that a problem-situation must have for it to be well formulated and have a solution. After the intervention with the DS implemented based on the problems they invented, the relationship presented in table 3 was found.

Table 3

Changes in the PAEV Formulation

	Diagnosis	Post-test
Mathematical structure	<p>The levels of success in creating a problem were between 19 and 40%. Student productions tend to be of the additive type, followed by the multiplicative type.</p> <p>40% of the productions manage to have more than one stage.</p> <p>One-fourth of the problems posed require different arithmetic operations for their solution.</p>	<p>An inverse variation appears between the additive and multiplicative structure problems.</p> <p>The creation of problems that require different operations increases by 47%.</p> <p>Problem creation involving more than one operation increases by 55%</p>
Semantic structure	<p>In additive structures, combination predominates, followed by change. None is comparable.</p> <p>The predominant multiplicative structure was isomorphism of measure, followed by multiplicative comparison and none of product of measure.</p>	<p>The additive structure of combination continues to predominate, while that of change decreases by a low percentage.</p> <p>The structure of the product of measure is still absent, and that of multiplicative comparison disappears. Only the isomorphism of measure predominates.</p>
Syntactic structure	<p>64% of productions use at least three propositions. However, less than half of the students verify that the formulated problem has a</p>	<p>The use of propositions equal to or greater than three increases, maintaining the use of at least two quantities in the statement.</p>

coherent solution with the numerical and semantic relationships they have established.

The percentage of written problems that are soluble increases by 55%.
The conditional interrogative proposition appears for the first time.

From Table 3 we can infer that from the cognitive complexity related to the processing associated with the linguistic structures used “semantics and syntax,” it is necessary to consider other issues that have more to do with the mathematical meaning of the text. We are speaking about the semantic operators and the represented situations. The former is a series of words whose importance for understanding the problem is crucial, since they are responsible for establishing the connections between the unknown and the data. In our case, the most used operators were “*how much does it cost*” followed by the word “*more*”. The meaning of these operators is defined by the function they play in the problem statement, and not by their own informative attributes. For example, “more” clearly suggests a sum. However, “how much does it cost” becomes an ambiguous term, because it implies both a loss (subtraction) for the person who performs the action and an increase (sum), in the case of the person who sells. Therefore, neither of the two terms is fully contradictory with the operation that we must carry out. In these cases, it is said that the statement is congruent or consistent when the student considers that when formulating the question, it is oriented to the buyer or the seller.

Concerning the development’s complexity, same as with the operators, the situations that the statements recreate are also one of the causes that, for example, not all the problems created by them can be solved by an arithmetic operation. However, for the student to achieve the ability and skill to create PAEV, he/she will need support from the teacher who will direct him/her to learn to pose these types of school problems and correctly understand the meaning and use of each of the four basic operations. The first variable that the teacher must consider is to teach the child to consider whether the situation stated by him is really a mathematical problem. Then teach him to recognise the difference in classification, i.e., if the problem is solved by more than one arithmetic operation. In this way, the child will distinguish between simple (or one-stage) PAEV, in whose resolution only one of the four elementary operations is needed, and compound (or multi-stage) PAEV, in the case that it requires the use of several operations.

Of the three structures that make up the PAEV formulation process, the main variations after the pedagogical intervention were concentrated in the syntactic structure related to the writing of the statement and the interrogative proposition of the problem. Despite this, the mathematical and semantic structures did not achieve variations greater than 50%.

CONCLUSIONS

In the PAEV formulation, of the three structures that are studied, according to Espinoza et al. (2015), Table 4 shows how students reach an understanding of the importance of considering the four phases that the mathematical structure must have so that the problem posed has a solution; “structural complexity”. We must note that the mathematical structure was strengthened, but less variation was obtained in the semantic and syntactic structures “cognitive and developmental complexities”.

Table 4

Changes in the PAEV Resolution

Phases	Diagnosis	Post-test
Analysis phase	The student exposes irrelevant information in the statement without coordinating what the interrogative statement indicates.	The student identifies relevant information to formulate mathematical problems. Advance is achieved in that at least 30% of them identify changes related to the interrogative proposition.
Exploration phase	Absence of numerical strategies to approach the solution of the problem. 14% of students manage to write a sequence of logical steps to solve the problem. The use of diagrams or plans to organise information is absent.	Use of numerical strategies, such as relating “similar” quantities to solve a problem. The use of logical sequences that allow solving the problem posed is evidenced. The absence of the use of diagrams or plans that allow him to address the problem persists.
Execution	Students prefer to approach the results through additions or	The tendency to prefer additions or subtractions continues.

Phase	<p>subtractions, avoiding multiplications and divisions. Only a third of the students execute the plan as it is written in the exploration phase. There is identification of similarities when comparing two resolution processes.</p>	<p>Multiplication appears as an operation to use. Awareness is raised about the importance of following a sequence of logical steps. The identification of similarities and differences between two resolution processes is prioritised.</p>
Verification Phase	<p>There is no indication of writing correct and logical answers to the interrogative sentence. An approach to numerical results with no writing at all is perceived. 19% of the children consider that their solution process is correct.</p>	<p>Awareness is achieved when writing the PAEV, in such a way that it is consistent with the use of numbers. The answer obtained is validated as correct.</p>

From Tables 3 and 4 we can infer that the resolution presents higher variations than the formulation, because the students expressed that in mathematics classes it is not usual for teachers to ask them to invent problems and least of all from images; regularly the teacher provides them. The arithmetic problems with which they are familiar are reduced to the use of additive operations. As a consequence of the constant individual work alternated with teamwork that the DS encouraged, the interaction between students somehow forced them to improve their communication and exchange of ideas, generating greater confidence regarding the proposed mathematical tasks and their own identity as academic peers to their classmates. This effect could somehow be associated with what is called, in Schoenfeld model, a belief system (related to the confidence gained towards oneself) and heuristic strategies (in the dialogue and exchange of ideas with academic peers).

For the PAEV resolution, significant progress was evidenced in the analysis, exploration and verification phases when there was the possibility of comparing the process carried out with that of another academic peer. We emphasise that the analysis phase was strengthened because the students learned how to identify relevant information in a statement and the relationship it had with the formulation of the question. We believe that this change in the syntactic structure is attributed to the possibility that existed, during the development activities of the DS, of showing the group of students some of their own productions anonymously and having the students judge them. In this

activity, students expressed whether a statement made sense, whether it was understandable, whether the interrogative proposition was related to the statement or other opinions that the exercise elicited. Apparently, the fact that they identified themselves as the authors of a production that was not classified as a PAEV motivated the students to invest more concentration in the formulation tasks in the exit test. The few problems in the post-tests that correspond to the multiplicative semantic structure makes it possible to note that the idea of direct proportionality operates in the minds of the students, but that they have not yet conceived the possibility of comparing two quantities through a scalar or a situation where there is the product of measures, thus being a task to strengthen and subject for another investigation.

In contrast, the mathematical and semantic structures did not achieve a significant variation, since they require dedicating more time to their work. According to Espinoza et al. (2015), for the formulation of a PAEV, the change presented in relation to the resolution process was evidenced in the variation in the analysis and verification phases when alternating individual and group work, where sometimes it was necessary to provide more time for the group discussion in which several students were compromised by the proposed tasks. The fact that the teacher only gave a definite response to the students' results shortly before the end of the class sessions generated both emotions and assumptions regarding the problems posed.

Regarding the sequence of steps proposed to address a PAEV, we remark that the identification of primary and secondary information caused quite a lot of conflicts in group discussions, thus evidencing the students' effort to understand the relationship between the quantities that appear in the statements and the relationships established with them, leading them to identify whether the situation presented a simple PAEV or a compound PAEV. Another step that represented difficulty for the students was the clarification of the logical sequence to solve a problem "phases." On the one hand, when comparing the work with another colleague, it generated discussions because the older ones tried to be more specific in their indications "analysis and exploration" and, on the other hand, assuming specific "execution" interpretations as obvious caused discontent in group work, particularly in younger children. Those are elements to be considered by the teacher in charge to avoid causing discomfort in the students.

We managed to overcome the difficulties that existed in the analysis and exploration phases, teaching the children to identify the semantic structure that a situation that we want to be a problem in mathematics must have, related

to the length of the statement, the order of presentation of the data, the context of the question, and the size of the numbers to use. In this section, we identified difficulties in using the execution and verification phases related to the choice of the resolution strategies that the students followed. Something logical, if we consider that they learn problem-solving almost at the same time as they learn to read. In this sense, it should be noted that elementary verbal arithmetic problems are expressed through the language of arithmetic instruction and that their statements are usually not intended to clarify the understanding of the problem, but to be part of the task that one must tackle to solve it.

The elements described above allow us to infer that regarding thinking skills, from the perspective of Márquez (2014), students progressed in *describing, ordering, and predicting* from the implementation of tasks that required description and synthesis (related to the analysis and exploration phases) of a problem situation, with emphasis on the prediction skill as the best one developed. The execution and verification phases related to the skills of *synthesising* and *comparing* require more work with this type of population, because children are in transition from a stage of concrete operations to one of formal operations. However, we note that the self-imposed prescriptions that an adult makes, placing the focus on what really interests him/her is not present in the children's minds. We believe that this happens because the reading experience of these children is reduced to children's literature and some adapted informative texts in previous school years, and the semantic features that stand out during narrative reading are different from those we must emphasise when we tackle the text of school math problems.

We believe that the content analysis developed in this work is useful for elementary school mathematics teachers as a tool to classify the different additive and multiplicative PAEVs, considered as a teaching model to promote problems from the simplest to the most difficult one. We reiterate that emphasis should be placed on the semantic structure and the syntactic component. In this sense, the way of outlining is another element that contributes to the understanding and representation of the problem; elements that make it necessary to articulate areas of knowledge, minimum literacy, and mathematics.

During the development of this work, we noticed that students' low level of problem-solving competence may be influenced by the natural way in which teachers directly use the problems posed in textbooks, without considering the planning and development phases thereof. According to

Mateus-Nieves and Rojas (2020), these elements may impair the development of specific thinking skill.

RECOMMENDATIONS

The articulation of different areas of knowledge is imminent so that students at this level achieve an adequate management of semantic and syntactic structures in a coherent way, which will facilitate an adequate mathematical structure.

It is important that the teacher who teaches mathematics in primary school recognises that the PAEV can entail difficulties that we will be unable to solve correctly if we only do a superficial study of them, since elementary verbal arithmetic problems are the first contact children have with problem-solving. Hence, it is very important that their first approaches to this new field of activity do not start in the wrong way. Here, it is important that the teacher guides the student to recognise when a problem is not a problem, until the child understands its characteristics and rules. That is, until he/she assumes that what is in front of him/her is a school mathematics task and nothing else.

The PAEVs are necessary, thanks to them the boys and girls can begin to be competent in that particular task of “writing” in mathematics through problems. For this reason, it is necessary that the teacher considers in detail the semantic and syntactic characteristics that form a coherent and soluble mathematical problem. That is, when teaching, that the teacher emphasises on whether the student understands and identifies the semantic structure of the problem that he/she has created or that has been offered to him/her. By not understanding it, the student may choose an inappropriate procedure to solve it, or simply abandon it, as is reflected today in most students in high school.

An aspect that we consider is pending for in-depth studies is related to the formulation process, particularly in descriptors 3, 7 and 13. The interpretation of the present situation with these descriptors is presented in Table 5.

Table 5

Interpretation of negative or null variation in the formulation

Descriptor	Interpretation
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3	The students abandoned the additive structure of change. Some are presumed to have transitioned from change problems to combination and comparison problems.
7	Descriptor referring to the multiplicative structure. Very few students managed to transition to the isomorphism of measure structure.
13	Referring to the multiplicative mathematical structure of the PAEV. Some students who had presented elementary handling of this structure, either abandoned it, or returned to the formulation of PAEV with a clearly additive mathematical structure.

AUTHORS' CONTRIBUTION STATEMENT

EMN directed the research project, planned background, question, objectives, theoretical framework, methodology, triangulation of the information, collected results, conclusions, and the formal structure of this document. HRDV developed the research project, and carried out the field work, collecting information on site, helped to design, apply instruments, applied the phases of the methodology, planned the results, conclusions, as well as organise the syllabus, and subject guidelines of the elective seminar.

DATA AVAILABILITY DECLARATION

The data supporting this study will be made available by the corresponding author (EMN), upon reasonable request.

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