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Early Years Math: A Study with Pre-service teachers in Mathematics based on the Principles of Cultural - Historical Theory

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ABSTRACT

Background: It is common the idea that the responsibility for learning mathematics worked in the first years of schooling rests solely with the teacher at this stage. This is a mistake since these contents run throughout basic education, and it is important that teachers at other levels understand the mathematics of the early years of elementary school. **Objectives:** The article aims to discuss the knowledge of mathematics undergraduate students about mathematics in the early years of elementary school. **Design:** The theoretical and methodological support is based on the assumptions of cultural-historical theory (CHT), which shows that the subject develops and impacts the reality where he/she lives by interactions with others through the acquired knowledge. **Setting and participants:** The study space consisted of workshops on the operations of addition, subtraction, multiplication, and division, carried out in the project “Knowing the mathematics taught in the early years” and the participants were seven undergraduates from the mathematics degree course. **Data collection and analysis:** The empirical data were systematized by the organization of the episode “Understanding the teaching of basic mathematics”, composed of four scenes. **Results:** They reveal that when it comes to teaching mathematics in the early years, students show a weakness in relation to basic mathematical knowledge and didactic knowledge to teach them. **Conclusions:** Although the results come from a specific context, we concluded that it is important for mathematics undergraduates to be able to discuss mathematics teaching in all stages of basic education.

Keywords: Basic math; Early years math; Math operations; Pre-service teachers in mathematics; Cultural-historical theory.

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Matemática dos anos iniciais: um estudo com licenciandos em matemática partindo dos princípios da Teoria Histórico-Cultural

RESUMO

Contexto: É comum a ideia de que a responsabilidade pela aprendizagem da matemática trabalhada nos primeiros anos de escolarização é unicamente do professor desta etapa. Trata-se de um equívoco, uma vez que estes conteúdos perpassam toda a Educação Básica, sendo importante que os professores de outros níveis compreendam a matemática dos anos iniciais do Ensino Fundamental. **Objetivos:** É intenção do artigo discutir sobre os conhecimentos de licenciandos em matemática sobre a matemática dos anos iniciais do Ensino Fundamental. **Design:** O respaldo teórico e metodológico é fundamentado pelos pressupostos da Teoria Histórico-Cultural (THC) o qual apresenta que o sujeito se desenvolve e impacta a realidade que vive pelas interações com os demais por meio dos conhecimentos adquiridos. **Ambiente e participantes:** O espaço do estudo foram oficinas sobre as operações de adição, subtração, multiplicação e divisão, realizadas no projeto "Conhecendo a matemática ensinada nos anos inicias" e os participantes foram sete acadêmicos do Curso de Licenciatura em Matemática. **Coleta e análise de dados:** Os dados empíricos foram sistematizados pela organização do episódio "Compreensões sobre o ensino da matemática básica" composto por quatro cenas. **Resultados:** Nos revelam que, em se tratando do ensino da matemática nos anos iniciais, existe fragilidade nos estudos por parte dos licenciandos tanto em relação ao conhecimento matemático básico, quanto ao conhecimento didático para ensiná-los. **Conclusões:** Apesar dos resultados serem oriundos de um contexto específico, conclui-se sobre a importância de os licenciandos em Matemática terem possibilidades de discutir sobre o ensino de matemática em todas as etapas da Educação Básica.

Palavras-chave: Matemática básica; Matemática dos anos iniciais; Operações matemáticas; Licenciandos em Matemática; Teoria Histórico-Cultural.

INTRODUCTION

We understand that teaching mathematics means being committed to keeping constant movement towards learning and relearning strategies that enable new generations to appropriate historically and culturally established knowledge. This implies that the subjects responsible for teaching have access to the means that make them understand their object, as they need to appropriate the meanings of what they teach so that their students can see sense and need in what teachers say it is essential for them to learn (Moura, 2004).

Let us take the context of basic mathematics teaching for the early years of elementary school. It is a common thought that the teachers who work with this age range (the pedagogues) are responsible for such knowledge. However, they are not alone, as this knowledge accompanies the individuals throughout their school lives. This leads us to realize how vital the role of the teachers who teach mathematics in the final years of elementary school and high school is when they need to go further into more elementary mathematics. It is with this concern that we built this article.

When observing mathematics degree course students, we seek to break the paradigm that the mathematics teachers should only be concerned with the teaching stage in their area of expertise, which involves the final years of elementary and high school education. We believe and defend that math teachers should understand the teaching of elementary mathematics as a unit among all stages of basic education.

When these professionals understand this knowledge, materialized in teaching in its entirety, and know how it occurs - whether in early, elementary, or high school education - they will have enough subsidies to organize their teaching for all grades, beyond fragmentation. In this way, they will be better able to overcome the prevailing view in the school context that teachers of the early years are the only ones responsible for the learning - or the failure to learn - concepts that are considered basic in school mathematics. Mathematics teachers of the final years of elementary and high school typically consider these concepts *a priori* learning.

In this article, which is part of a master's degree research, we aim to discuss mathematics undergraduates' knowledge of mathematics in the early years of elementary school. Also, we intend to carry out a movement of reflection and provocation about the way those students may be understanding and appropriating that knowledge. We initially present some theoretical assumptions of the research that was based on the premises of Vigotski's Cultural-Historical Theory (CHT) (1998, 2009). Then, we expose the methodological organization conducted within the scope of the project *Conhecendo a matemática ensinada nos anos iniciais* (Knowing the mathematics taught in the early years). In the following, we analyze the empirical data from the systematization of an episode composed of four scenes from Moura's (2004) perspective. Finally, we reflect on the results, pointing out how important it is that mathematics degree students approach mathematics teaching in the early years of elementary school.

THEORETICAL FRAMEWORK

The Cultural-Historical Theory (CHT), whose greatest idealizer is Vigotski (1896-1934), guided the theoretical assumptions of this study, helping us to understand that human beings learn and develop through the relationships they establish with others in the different spaces they occupy. This premise leads us to realize that at birth, the subjects do not have the qualities that characterize them as belonging to the human species. Such attributes will appear when the individuals start to be in contact with other human beings, living in a society, in a process we understand, from Leontiev's perspective (1978), as humanization.

During this process, they appropriate an entire cultural heritage and the qualities that distinguish them from other living beings. Moura (2007) mentions that being part of a specific culture means to have the right to appropriate some knowledge that was built as the humankind needed it. This allowed humans to live with other humans and exchange meanings, which helped the group to discover together new ways of living and develop.

Among so much knowledge produced by humanity, we have mathematical knowledge, which we believe that, historically, allowed for human beings to develop intellectually, and to meet the integrative needs that the world had imposed. It is essential to reflect on when mathematics started to be part of the subjects' lives so that it is not seen only as something that society determines they should learn, but rather, something that they could apprehend in its essence. In this perspective, Araújo (2014) explains that the child

> comes into contact with mathematics from its birth and even before it. From the moment it reaches the world, it is inserted in a society in which numbers, space, shapes, i.e., mathematical quantities, are part of it. But from the moment it enters the school, either in early childhood education or in the early years of elementary school, it is faced with another way of learning, different from the one it knew in the family life to which it was used. (p.4)

Thus, mathematics will not necessarily be seen only when the child enters a proper place for teaching, i.e., the school; it is part of the child's daily life. However, every learning outside of school is decisive for the subject to appropriate a specific type of knowledge: scientific knowledge. The role that the school plays in fostering the generalization of the concepts that are part of the mathematical content becomes significant.

Vigotski (1998) helps us to differentiate spontaneous concepts from scientific concepts. The subject appropriates concepts of a spontaneous nature through lived experiences, empirical generalization, based on the relationships established in daily life. Scientific nature concepts are those apprehended in the school environment through a guided and organized process, respecting the main elements that are characteristic of their definition.

In this way, Vigotski (2009) makes a list of these concepts explaining that

> the development of scientific and spontaneous concepts follows different paths in the opposite sense, both processes are internally, and most profoundly, interrelated. The development of the child's spontaneous concept must reach a certain level so that the child can grasp the scientific concept and become aware of it. In its spontaneous concepts, the child must reach that threshold beyond which awareness is possible. (p. 349)

The appropriation of such knowledge is imperative because it allows for the development of mental functions that help in the construction of instruments that improve life in society. In this respect, about mathematical content, Moura (2007) lists that

> it consists of signs articulated by rules that, operated logically, produce a result that has support in objective reality. In other words, when applied to the solution of concrete problems, the concepts should allow an objective intervention in reality. By this, we mean that the knowledge that prevails is that which has concrete proof when tested to solve objective problems. (pp. 50-51)

Starting from this premise and anchored in this author, we see that there is not a single mathematical knowledge studied until today that has not been a response to meet some human need. Thus, based on the mathematics taught in the early years of elementary school, we understand that they are configured as fundamentals everybody should learn.

They are essential not only to meet the pragmatic needs of different natures but also to assist with the cognitive development of the individuals who appropriate them. In this process, the teachers, as the ones who are responsible for teaching, must have strategies that enable their students to lay hands on them.

Based on what has been exposed so far, we assume that the mathematical content discussed in this study, i.e., the mathematical operations - addition, subtraction, multiplication, and division - are fundamental elements in mathematics teaching, as students' understanding of those operations goes beyond the initial years. Therefore, if we expect that the organization of teaching mathematical content - such as operations – leads students to learn, the teacher must understand that content in its entirety to plan such organization. Since the four basic operations are vital for all schooling years, the teacher cannot justify not addressing them, claiming that they are only in the scope of the initial years. Hence, the teacher must apprehend that content in its essence, which demands knowing the necessity that drove its creation, the logical-historical movement behind each operation and the methodological strategies to be used to teach the content.

The place where the teachers intervene is school, and, following the premises of the CHT, we understand it is the space for social formation that provides subsidies that allow students to appropriate the knowledge produced historically by humanity. It is in the school that teachers perform the far-reaching function of bringing students closer to them. Therefore, we advocate that the teachers – who are responsible for teaching - must understand that their actions must be intentionally organized for that purpose (Moura, Araújo, Ribeiro, Panossian, & Moretti, 2010).

Thus, it is not only the methodological approaches the teachers choose that will be key as they guide their actions within the classroom, but also the way they perceive the world around them.

> The way we look at the world and conceive it can define the purposes of the knowledge that we seek at each moment. The growing complexity of human

> relationships is certainly lavish in the example of how we have been changing throughout human history by defining and redefining the role of knowledge conveyed in school. ... Teaching, or better yet, the pedagogical project is possible to be conceived when an individual perceives him/herself as an agent that creates and impacts on reality. (Moura, 2001, p. 146)

Therefore, we believe that teaching should take place through the articulation of intentional actions that enable it. And as a result, that the professional responsible for education has the necessary knowledge that not only allows the student to learn but also allows his/her development, impacting on the reality of which he/she is part.

However, as Vigotski (2009) stated, it is not just any teaching that leads to development. Therefore, if we have this purpose in mathematics teaching, we need to stop looking at it in its fragmented stages exposed through basic education, but as a unified process, which requires further theoretical deepening.

Having exposed our theoretical position, we will discuss how the methodology was organized in this study.

METHODOLOGY

The method adopted here aligns with the studies carried out in the CHT, by its research method, the historical and dialectical method. When investigated, this relationship allows the phenomenon to be exposed in such a way that it will enable us to understand it in its entirety (Cedro & Nascimento, 2017). Regarding the research in this proposition, Freitas (2007) mentions that

> this theoretical perspective has implications that are reflected in the procedural and ethical characteristics of doing research in the humanities, requiring researcher coherence in the design and use of methodological instruments to collect and analyze data, as well as to construct texts with the discussion of the findings. (p. 5)

By assuming CHT as guiding principles, we agree with Perlin (2018), who states that this "theoretical and methodological approach allows us to investigate *with*, not *about*[1], theory" (p. 39). Thus, we affirm that the results presented later did not follow some specific linearity, but the elements that compose them helped us understand the object in question.

This research was developed throughout the second term of 2018, within the scope of the project *Conhecendo a matemática ensinada nos anos iniciais* (Knowing the

[1] (Translator's emphasis)

mathematics taught in the early years), organized through different actions so to foster a study concerned with relevant concepts of fundamental mathematics. The participants in the research were seven students attending the fourth semester of the Mathematics degree course of the Federal University of Santa Maria (UFSM). All signed a Free and Informed Consent Form (ICF)[2], and they chose fictitious names to preserve their identities.

In this article, we bring the results from workshops about the four operations, addition, subtraction, multiplication, and division, where we explored historical aspects, logical and historical movement, use of didactic materials, and theoretical studies involving the research theme. We used audio and video recordings to capture the entire development of the research, which enabled us a more careful analysis, seeking to contemplate the whole.

To systematize the analysis of the material, it was necessary an organization that allowed us to reveal the phenomenon studied, aiming at the possibility to elaborate theoretical abstractions based on the particular extensions of the empirical phenomenon. This will be revealed, through the idea of episode proposed by Moura (2004) as a methodological contribution, composed of scenes. For him, an episode can be "written or spoken phrases, gestures and actions that constitute scenes that can reveal interdependence between the elements of an educational action" (Moura, 2004, p. 276). Yet, for the author, scenes are moments that reveal the subjects' formation movement.

After having introduced our methodological organization and its theoretical framework, we now present an episode composed of four scenes, where we discuss the undergraduates' knowledge of mathematics teaching in the early years of elementary school.

REFLECTING ON MATHEMATICS TEACHING IN THE EARLY YEARS OF ELEMENTARY EDUCATION

It is in the learning movement of teaching that teachers can receive the necessary conditions that help them understand their responsibilities when teaching, and, thus, realize that the way they learned a concept in their formative process cannot be the same they will use to teach in the area of expertise, i.e., basic education.

We intend to discuss the degree students' knowledge when exposed to situations connected to basic mathematics teaching, concerning mathematics teaching in the early years of elementary school. This will be consolidated with the episode *Compreensões sobre o ensino da matemática básica* (Understanding the teaching of basic mathematics), through its four scenes, namely: The trivial is not necessarily easy: the unexpected; When

[2] We emphasise that this research, in particular, did not pass through the Research Ethics Committee, because it is part of a broader research entitled: A licenciatura em matemática em questão: de que formação falamos? (The Mathematics Degree at Stake: What Training are we Talking About?) We exempt Acta Scientiae from any consequences arising therefrom, including full assistance and eventual compensation for any damage resulting to any of the research participants, in accordance with Resolution No. 510, of April 7, 2016, of the Conselho Nacional de Saúde (National Health Council).

explaining is natural, but it is not: discussing the algorithm; Zero does not fall from the sky! When we stop accepting the saying: "it is the rule"; When the simple becomes difficult: by explaining what we think we know.

In our first scene, we talked about a discussion on teaching based on one of the undergraduates' report.

Figure 1

Scene 1: The trivial is not necessarily easy: the unexpected. (Researcher's diary, 2018, p. 2*)*

Description (Scene 1) - This scene was based on a discussion the subjects of the first two chapters of Ifrah's text (2013) held about a memory game. In this text, they discussed elementary mathematics concepts. The discussion was raised when they were asked about the importance of the most elementary concepts.

1. Luke: Even the abstract concepts that we learn here in college, those that, if we don't... it's not palpable, the things that we learn are very easily lost.

2. Researcher: There are things that we take for granted and don't question ourselves.

3. Luke: Yeah, you don't question yourself, whether it really is or not.

4. Researcher: And many times, you will question yourself later when a student asks you, and you keep thinking [...] what am I going to answer? It's definition! Of course, it is not easy to explain to a student.

5. Luke: Yes, I know because, when I tried to explain it to my godson, I only gave, let's say, private lessons to my godson and my cousin. So, I was trying to explain what was even and odd, right? [pause] Yeah, what was even and odd to him. I said so, I got kind of 2^n.

6. Everyone: Laughter.

7. Luke: And I started laughing to myself, because there was a "delay", a "delay" in my head, so I kind of [...].

8. Researcher: What do you mean2^n?

9. Elídio: $2^n, 2^{n+1}$?

10. Researcher: Hold on, 2^nor $2n$?

11. Luke: No, for even, so to speak.

12. Elídio: But then it is $2n$.

13. Luke: But, it was to explain all the numbers, see? The one, what is it? Odd. Two, what is it? Even. So, I thought, and stopped, and kept thinking, well, but how am I going to explain it to him, that is so easy. Then, from the one, what is one? Odd, then two is even.

14. Researcher: Why?

15. Luke: He got it, like, at once, I don't know.

16. Researcher: Why?

17. Luke: I don't know [laughs]. Then, what about three? He stopped and was [...] It's odd. And the four? Even.

18. Researcher: So, you tried to explain it like that, starting from one, that would be even, and then you skip one?

19. Luke: It's an order, then the next [number] is always even, and if the next is even, the other is odd, this kind of logic.

20. Someone: And did he understand?

22. Luke: I don't know [laughs].

23. Monique: It's just that things that are so simple for us, I think they are the hardest for you to pass on.

24. Monique: It's just such a simple thing you're going to transmit, so, how can I teach it, it's so obvious, it's so simple.

25. Pitágoras: It is so trivial.

26. Monique: Yeah, I think that's what ends up being the most difficult.

The central point discussed in this scene is the account presented by Luke (f.5). We can see some elements in his statement, in an attempt to explain what an even number and an odd number was. The first element was to try to understand why the first thing he thought was to explain it by using the term 2^n, since, even trying to rely on the generalized form, this is not the correct way, mathematically. As he mentioned, this may have happened because the only occasions when he came across a teaching situation was when he gave private lessons to his godson and cousin. Support in generalization – even if it is not the correct one - may have been a result of the way he learned, either in basic or in higher education.

We have another element that appears in the sequence. When Luke realized how he would approach this concept, he started to "laugh" and, later, to think how he would convey it. As he said, he had a "*delay*,"[3] which means that, for a few moments, he could not find the explanation he wanted to give. We noticed that Luke knew what an even number and an odd number was, he also knew how to perform a generalization demonstration to find any of these numbers, however, being in a teaching situation - which demanded that his actions be focused on the other's possibility of appropriating the knowledge - he felt insecure, and kept wondering how he could do it.

This scene makes us reflect on the incompleteness of what is necessary to teach when the future teacher faces this kind of situation. It is not enough to have specific knowledge; the future teacher is required to master the methodological teaching strategies to assist him/her in this process. This feeling of incompleteness was also evidenced in Perlin's research (2018), carried out with future mathematics teachers who were going through the supervised preservice practice. They also faced situations that made them wonder what elements would be needed in the act of teaching.

In her research on mathematics teaching, Lopes (2009), observed that the future participating teachers highlighted two critical points:

> the first is that being a teacher who teaches mathematics requires knowledge of the content in this discipline; the second is that this is not enough, since the teacher's content knowledge does not guarantee the student's learning (p. 184).

This perception confirms what we have just exposed in this scene, i.e., although Luke knew what an even number was, it did not guarantee that he had actually managed to teach his godson.

[3] TN: Brazilians sometimes use this English word as slang to explain they are taking some time to carry out an action or remember something. Usually pronounced "delei".

This makes us consider the relevance of specific content to go together with didactic and pedagogical knowledge. As taught by Libâneo (2014):

> In the traditional teaching conception, the teacher is seen as someone who transmits knowledge based on the logic of the discipline taught, and it is very common to affirm that to teach a subject matter, it is enough to master its content. Pedagogical knowledge is understood, in this case, merely as a repertoire of teaching techniques. Thus, most teachers ignore that the professional knowledge of those who dedicate themselves to teaching is made up of at least two requirements, mastering the content of a discipline, and mastering the knowledge and skills to teach that content. (pp. 4-5)

Teachers cannot disregard any of the requirements to obtain a good performance in teaching, because, as Perlin (2018) states, "to teach mathematics, specific knowledge of the area and experience with teaching are not enough. Pedagogic and didactic knowledge are also necessary" (p. 145).

In the second scene, we present a conversation about the importance of explaining an algorithm organization, regardless of the operation to be discussed.

Figure 2

Scene 2: When explaining is natural, but it is not: discussing the algorithm. (Researcher's diary, 2018, p. 4)

Description (Scene 2): The scene is the result of a moment of reflection on a workshop held in the space of the extension course in which the subjects worked with the addition and subtraction operations.
1. Elídio: I liked it a lot; sometimes, we treat things as very natural, the problems, those mental actions that were previously unknown to be able to solve a problem, you think it's all the same, but, it's not.
2. Eduardo: We think it's very easy to build the algorithm, but when it comes to explaining to the child how that happens, when one goes up etc., then it complicates, and we could get a sense of it using the materials to solve it.
3. Researcher: Do you think the material helped you understand the algorithm?
4. Eduardo: It helped to understand that part of when it [the number] goes up, it goes to the other [side].
5. Leão: And borrowing, too, when I take a larger unit and I want to transform it into portions of a smaller unit.

One of the actions developed in the research was to carry out workshops involving basic mathematics operations exploring historical aspects, the structure of algorithms, the use of teaching materials, and the different mental actions that accompany each operation.

Through the fragments presented in this scene, we can observe an indication of a possible change in quality (Moura, 2004) concerning the way of dealing with the object - in this case, the addition and subtraction algorithms - especially in Eduardo's speech (f.2). Usually, these are understood as ready-made and finished mechanisms that work just by following the rules. Through realizing that the organization we use today to solve mathematical operations represents syntheses of a historical construction process, we now may give it another meaning. That is, from pre-established rules, they start to be recognized as human production, the result of the effort of past generations, being constituted as human culture.

In this sense, we apprehended that the study through the logical and historical movement of knowledge can help us appropriate it, considering all its particularities and rules. In this respect, Kopnin (1978, p. 187) points out that the

> The logical reflects not only the history of the object itself but also the history of its knowledge. Hence, the unity between the logical and the historical is a necessary premise to allow us to understand the movement process of the thought, the creation of the scientific theory. (p. 187)

In other words, it is the possibility of learning beyond what has already been crystallized.

Interconnected with this, we also have the didactic material that, as the subjects of the research pointed out, helped them understand better how some rules that appeared in the algorithm worked. Let us return to Vigotski's (1998) remarks when he explains that "properly organized learning results in mental development and sets in motion several development processes that would otherwise be impossible to happen" (p. 118). We can infer that the didactic material – thought as an instrument - can contribute to the teacher's teaching organization. In the author's perspective, the instrument has the function of directing the individual's influence in the face of the object of the activity, thus "it is *externally* guided; it must necessarily lead to changes in objects. It constitutes a means by which external human activity is directed towards the control and mastery of nature" [emphasis in the original] (Vigotski, 1998, pp.72-73).

Through the educational material as an instrument in the teaching and learning process, teachers can plan actions to contribute to their students' learning. However, we cannot forget that all materials are limited, so students' learning cannot be restricted to them, since the content to be dealt with is not in the material but in the relationships that the individuals establish through it. In this way, the material may not fulfill its mediating function. In the following scene, we discuss a stage that can happen in the division operation and which, unfortunately, is often taken as a rule without justification.

Figure 3

Scene 3: Zero doesn't fall from the sky! When we stop accepting the saying: "it is the rule." (Researcher's diary, 2018, p. 4)

Description (Scene 3): At one point during a workshop held on the multiplication and division operations, two subjects exposed the operation on the board412 ÷ 4, using the algorithm and didactic material as support, which, in this case, was the golden material. After the explanation, the following situation occurs.

1. Researcher: Isabela, you said it now made sense, what would it be?

2. Isabela: The zero, because I forgot how to make the algorithm, I had divided and when I tried to make the algorithm I didn't remember the zero, and I wrote what I did in the golden material in full to explain what I was doing, and when I got to a ten divided by four, I just did it like that, a ten is ten units, I didn't think like I wasn't going to deal with the ten. I automatically did that, but I didn't think about saying that I won't have any tens in the algorithm.

3. Teachers' Supervisor: This automatic you spoke about is very common.

4. Isabela: It's just that we learn: when this is not possible, put the zero there, but nobody says what happened.

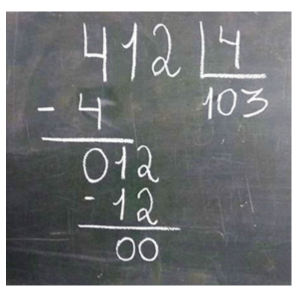
5. Teachers' Supervisor**: And then, in the middle of the way, you forget the zero.**

6. Isabela: Or you know you have to put it there, and you don't know why.

This scene shows signs that new meaning was attributed to something that is widely used in mathematics teaching, and because it is so common, we sometimes do not ask much about it. During the explanation of one of the operations, the students were extremely focused. Their attitude and the parallel comments revealed that from that moment on, they understood the reason for that zero (Researcher's diary, 10/20/2018, p. 4). To situate better the impact of this example, we present the operation in the image below.

Figure 4

Operation resolution 412 ÷ 4



The zero, addressed in Isabela's speech (f.2), is what appears corresponding to the ten in the quotient; this is because, when we perform the division previously shown after the division of the hundreds, we have a ten that cannot be divided into four whole parts, which gives us zero as quotient. Then, the ten is broken down into units to complete the operation.

Unfortunately, it is common to hear: “If you cannot divide it, put the zero and go on,” without a justification for it, which prevents the student from understanding the organization of the algorithm. This action does not contribute to the learning process, as the student cannot follow the movement of algorithm organization, whose referrals are syntheses that were historically constituted.

Noteworthy is that most students demonstrated that despite knowing that they should put a zero in that situation, they did not know why. What leads us to infer that as basic education - and now as higher education - students, they did not realize the essence of this knowledge.

For the future teacher to be able to appropriate a given scientific knowledge, it is essential that he knows all its particularities, knows how to apply it in different situations and not only in general cases. In this sense, Vigotski (2009) explains that

> the development of scientific concepts begins precisely with what has not yet been fully developed in spontaneous concepts throughout schooling. It usually begins by working the concept itself as such, with the verbal definition of the concept, through operations that imply the non-spontaneous application of that concept. (p. 345)

The author refers to the development of a scientific concept in the school period. Still, we understand that this also applies to future teachers who are in the process of learning to teach, since they have shown weaknesses in apprehending it. Although future teachers know the algorithm of the division operation, their speech revealed that this probably came from an empirical generalization. However, what allows the appropriation of an object of study, in the case of the algorithm, in all its essence, is a theoretical generalization (Davydov, 1988).

Thus, this leads us to reflect on the importance of the teachers appropriating concepts about the specific content that they will teach, as we agree with Moura (2004) when he states that a person who performs some action will impact another person through his/her symbol instruments. That is, the teachers, when teaching some content, directly affects their students; therefore, if knowledge derives from empirical generalization, it is from it that they will teach their students.

The last scene portrays the students’ positioning when thinking about knowledge that they believe they have.

Picture 5

Scene 4: When the simple becomes difficult: explaining what we think we know. (Researcher's diary, 2018, p. 10)

Description (Scene 4): This scene is the result of a moment of reflection when the subjects commented on what they thought of the actions carried out in the workshops on the four operations.
1. Researcher: Regarding the workshops developed, what did you think?
2. Isabela: Pretty cool! We struggled to make the algorithms, but it was very good.
3. Researcher: When you went to the board to build the algorithm, did you have any concerns?
4. Isabela: Yes! We even got lost [laughs].
5. Pitágoras: That was my biggest fear, of going to school and not being able to explain it to them.
6. Researcher: Nobody was doubting that you knew the operations, but when placed in the teaching situation, what knowledge do you think is needed when teaching? What do you think a teacher has to know, does he/she have to study to be able to teach a specific content?
7. Isabela: You have to know at least the most basic, because we didn't know the most basic and it was what we most needed to know.
8. Luke: And in a simpler way.
9. Eduardo: We didn't know how to explain what we already knew.
10. Pitágoras: From the most basic concepts to the end, we have to know.

These fragments make us reflect on what we found. From the students' speeches, it becomes clear how important the students considered the opportunity to discuss the algorithms of the operations, this, perhaps, because they were allowed some space to do so. Although the four operations covered are considered elementary mathematics concepts, we cannot accept them as something innate.

Moura (2012) sees the concepts as

> syntheses produced by specific social groups when dealing with problems, the result of physical or psychological needs whose solutions could allow for a better life. These syntheses were chosen at a given time by a group of people who considered them relevant and, therefore, should be disclosed to enable the integration of new subjects in the dynamics of the society of which they are a part. (p. 148)

We think the mathematical concepts that are shared today in the school environment as syntheses of a logical and historical process that met a human need and that are learned by the individuals as a way to help them interact in the objective world. The future mathematics teacher must know how the history of concepts developed. This can be done through the study of its logical-historical movement, as "it allows to understand it, explaining relationships between its elements, highlighting the internal connections and not just its formalism" (Pozebon, 2017, p. 114).

Another element addressed in Isabela's speech (f.7), was that she considered that the teacher must have a minimum of knowledge, especially what is believed to be the most basic knowledge. Her speech was in line with Eduardo's position (f. 9) when explaining that there was a lack in this aspect, as they did not know how to explain what they believed they knew.

This aspect asserts the idea that to appropriate a little the concepts discussed, they had to comprehend them in their essence. As we have already reiterated previously, the study of the logical and historical movement can be configured as a crucial tool for future teachers when they think about the organization of teaching that enables the student to appropriate the concept through the need to study it. Therefore, we agree with Fiorentini (2012), supporting that

> to be a mathematics teacher, it is not enough to have a conceptual and procedural mastery of historically produced mathematics. Above all, the teacher needs to know mathematics epistemological foundations, its historical evolution, its relationship with reality, its social uses, and the different languages with which a mathematical concept can be represented or expressed. (p. 110)

Through its four scenes, the episode reveals how distant undergraduates are from valuable knowledge that is part of the teaching of the operations of addition, subtraction, multiplication, and division. This may happen because, by not being responsible for teaching them, they do not feel they must appropriate them - in essence. However, we are very concerned about how these professionals have understood this more elementary mathematics, and how it can influence their future role as math teachers.

SOME FINAL REFLECTIONS

Understanding and assuming that elementary knowledge of mathematics teaching is essential regardless of the teaching stage in basic education goes through the commitment that the professionals responsible for its teaching must be prepared to teach it. Although this knowledge is introduced in the early years of elementary school, it does not mean that the mathematics teachers working with the other school grades do not need to have it.

In this article, we presented data obtained from a master's research to discuss the knowledge of mathematics undergraduate students on mathematics in the early years of elementary school. The data were carried out in workshops developed with mathematics degree students, in the second term of 2018, focused on the four basic operations: addition, subtraction, multiplication, and division, which are part of the curriculum organization of the early years of elementary school.

According to the results we present, we found evidence that there are some difficulties concerning elementary mathematical knowledge and didactics on the part of the undergraduate students who participated in the research. At various times, during their discussions, we could identify signs of their anxieties that may reflect on their teaching/learning process.

As much as these subjects are still in the process of formation, this leads us to reflect on what perceptions they actually have about what is usually called elementary mathematics concepts, such as the case of the four operations, and our indifference, as teacher educators, when we regard them as “trivial.” Therefore, we assume that all the subjects who are willing to take a degree in Mathematics should have that knowledge. The data indicate that a considerable part of those students’ knowledge does not come from their understanding of the whole since they know (and very well) how to solve basic mathematical questions. However, resorting to ready-made models that lack the recognition of their organization indicates how frail that knowledge is.

This research reveals how important it is that the teachers and future teachers perceive that “the task of a person who teaches mathematics is to give meaning to what he/she teaches so that whoever learns can perceive, or better, take possession of the human production processes of concepts” (Moura, 2012, p. 8). This implies making a commitment to the educational environment and to an entire society that seeks, through knowledge appropriation, conditions that foster its development. If the mathematics teacher presents an incompleteness in this process, we run the risk of also providing the new generations with a teaching of incompleteness.

Knowing better your object of study, the mathematics - be it the most elementary or something more complex-, can allow for a new way of conceiving the teaching of mathematics, which may take place in different spaces that guide to learning for teaching. Thus, according to Pozebon (2017), we believe that this may happen through actions of intentional studies that lead degree students to “appropriate knowledge theoretically that subsidizes the different aspects permeating the teaching activity, it is essential for teachers’ work committed to education and their students’ humanization process” (Pozebon, 2017, p. 187).

Therefore, although the results obtained here refer to a specific context investigated, we reiterate our concern regarding how crucial it is that mathematics undergraduates are concerned not only with the teaching of mathematics in their area of activity, but rather to seek to understand it in the different stages of basic education.

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AUTHOR CONTRIBUTION STATEMENT

CPG was responsible for collecting and systematizing the article’s data, aligning the theoretical basis with the results, as well as the organization of the article. ARLVL was responsible for guiding the preparation of the article, the theoretical and methodological assumptions. Both authors were responsible for the final considerations.

DATA AVAILABILITY STATEMENTS

The authors agree to make their data available at the reasonable request of a reader. It is up to the authors to determine whether a request is reasonable or not.

REFERENCES

Araújo, E. S. (2014). O ensino de matemática na infância. *Didática e Prática de Ensino na relação com a Formação de Professores*, 2, 1-11. <http://www.uece.br/endipe2014/ebooks/livro2/O%20ENSINO%20DE%20MATEM%C3%81TICA%20NA%20INF%C3%82NCIA.pdf>

Cedro, W. L., & Nascimento, C. P. (2017). Dos métodos e das metodologias em pesquisas educacionais na Teoria Histórico-Cultural. In M. O. Moura (Org.), *Educação escolar e pesquisa na Teoria Histórico-Cultural* (pp. 13-45). Loyola.

Davydov, V. V. (1988, agosto). Problemas do ensino desenvolvimental: A experiência da pesquisa teórica e experimental na psicologia (J.C. Libâneo & R. A. M. da M. Freitas, Trads.). *Educação Soviética*, 8.

Freitas, M. T. A. (2007, janeiro/junho). A pesquisa em educação: questões e desafios. *Revista Vertentes*, *29*, 28-37.

Fiorentini, D. (2012) Formação de professores a partir da vivência e da análise de práticas exploratório-investigativas e problematizadoras de ensinar e aprender matemática. *Cuadernos de Investigación y Formación en Educación Matemática, 7*(10), 63-78.

Ifrah, G. (2013) *Os números: história de uma grande invenção.* 9.ed. Globo.

Kopnin, P. V. (1978). *A dialética como lógica e teoria do conhecimento* (Coleção Perspectivas do homem, v. 123). Civilização Brasileira.

Leontiev, A. N. (1978). *O desenvolvimento do psiquismo.* Horizonte Universitário.

Libâneo, J. C. (2014). A integração entre o conhecimento disciplinar e o conhecimento pedagógico na formação de professores e a contribuição da teoria do ensino de Vasili Davidov. In J. C. Libãneo, *Adeus professor, adeus professor* (pp. 1-20). Cortez.

Lopes, A. R. L. V. (2009). *Aprendizagem da docência em matemática: o Clube de Matemática como espaço de formação inicial de professores.* Ed. Universidade de Passo Fundo.

Lorenzato, S. (2006). Laboratório de ensino de matemática e materiais didáticos manipuláveis. In S. Lorenzato (Org.), *Laboratório de Ensino de Matemática na formação de professores* (pp. 3-38). Autores Associados.

Moura, M. O. (2001). A atividade de ensino como ação formadora. In A. Castro, & A. M. P. de Castro (Orgs.), *Ensinar a ensinar: didática para a escola* (pp. 143-162). Pioneira.

Moura, M. O. de. (2004). Pesquisa colaborativa: um foco na ação formadora. In: R. L. L. Barbosa (Org.), *Trajetórias e perspectivas da formação de educadores* (pp. 257-284). Editora UNESP.
Moura, M. O. (2007). A Matemática na Infância. In M. Migueis, & M. G. Azevedo (Orgs.), *Educação Matemática na Infância* (pp. 40-63). Gailivros.
Moura, M. O. (2012) Didática e prática de ensino para educar com a matemática. In *Anais do Encontro Nacional de Didática e Práticas de Ensino*, Araraquara, Unicamp.
Moura, M. O., Araújo, E. S., Ribeiro, F.D., Panossian, M. L., & Moretti, V. D. (2010) A atividade orientadora de ensino como unidade entre ensino e aprendizagem. In M. O. Moura (Org.), *A atividade pedagógica na teoria histórico-cultural* (pp. 81-109). Liber livro.
Perlin, P. (2018). *Constituindo-se professor de matemática: relações estabelecidas no estágio curricular supervisionado determinantes da aprendizagem da docência* (313 f.). [Doutorado em Educação]. Universidade Federal de Santa Maria de Santa Maria. Santa Maria, RS.
Pozebon, S. (2017). *A formação de futuros professores de matemática: o movimento de aprendizagem da docência em um espaço formativo para o ensino de medidas* (307 f.). [Doutorado em Educação]. Universidade Federal de Santa Maria, Santa Maria, RS.
Vigotski, L. S. (1998). *A formação social da mente*. Martins Fontes.
Vigotski, L. S. (2009). *A construção do pensamento e da linguagem*. Martins Fontes.