ISSN: 2178-7727
DOI: 10.17648/acta.scientiae.5711

# Task (Re)Design to Enhance the Didactic-Mathematical Knowledge of Teachers

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*Received for publication on 27 Fev. 2020. Accepted after review on 21 Aug. 2020*
*Designated editor: Claudia Lisete Oliveira Groenwald*

**ABSTRACT**

**Background:** Currently, studies in the field of design of tasks, especially when linked to teacher training actions, have made significant contributions to the area of Mathematics Education. **Objectives:** This research aimed to point out the contributions that task (re)design can bring to enhance the didactic-mathematic knowledge of the teachers, as well as to discuss the difficulties and advances of educators when dealing with tasks characterised by challenging situations. **Design:** Using a qualitative approach, based on the Intervention Research format, we held training meetings with teachers in two main stages: the first consisted of meetings to work on the concept of the design and redesign of tasks; the second focused on redesigning a school project, putting into practice the concepts acquired in the first stage. **Setting and participants:** The intervention took place in an elementary school of the city of Jaguaquara, Bahia, Brazil, with three Mathematics teachers and the teacher-researcher. **Data collection and analysis:** The data were produced using a field diary, the materials produced by the teachers, and the audio recording of the training meetings. **Results:** The results show that designing tasks increases the didactic-mathematical knowledge of teachers. Also, it showed that the participants presented difficulties, some of which they could overcome during the process, while others remained after the training. **Conclusions:** The remaining difficulties leave room for new training actions. On the other hand, the training contributed to diminish the difficulties of the participating teachers and instigated innovative actions in Mathematics teaching.

**Keywords:** Task (re)design; Didactic-mathematical knowledge; Teacher training.

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**(Re)Desenho de Tarefas para Potencializar o Conhecimento Didático-Matemático de Professores**


**RESUMO**

**Contexto:** Atualmente, os estudos no campo do desenho de tarefas, principalmente quando ligados a ações formativas de professores, têm trazido importantes contribuições para a área de Educação Matemática. **Objetivos:** A pesquisa aqui apresentada buscou apontar as contribuições do (re)desenho de tarefas para potencializar o conhecimento didático-matemático de professores, assim como discutir as dificuldades e avanços de docentes quando (re)desenham tarefas caracterizadas por situações desafiadoras. **Desenho:** Por meio de uma abordagem qualitativa, no formato de Pesquisa Intervenção, realizamos encontros formativos com professors em duas etapas principais: a primeira, correspondendo aos encontros para trabalhar o conceito de desenho e redesenho de tarefas; a segunda destinada ao redesenho de um projeto escolar, colocando em prática os conceitos adquiridos na primeira etapa. **Ambiente e participantes:** A pesquisa foi realizada em uma escola da rede municipal da cidade de Jaguaquara, Bahia, envolvendo três professores-participantes e o professor-pesquisador. **Coleta a análise de dados:** Os dados foram produzidos utilizando um diário de campo, a gravação em áudio dos encontros formativos e os materiais produzidos pelos professores. **Resultados:** Os resultados revelam que desenhar tarefas incrementa o conhecimento didático-matemático de professores, que os participantes demonstram dificuldades, sendo que algumas delas foram superadas ao longo do processo, mas outras ainda permaneceram. **Conclusões:** As dificuldades que subsistem deixam espaço para novas formações. Por outro lado, a formação contribuiu para minorar dificuldades e instigar ações inovadoras no ensino da Matemática.



**Palavras-chave:** (Re)desenho de tarefas; Conhecimento didático-matemático; Formação de professores.


## INTRODUCTION

The question of what a teacher needs to know to teach Mathematics traverses a knowledge that goes beyond the "domain of content," taking into account contemporary social demands, the issue of innovation in the teaching-learning process, and students' needs, among other conditions. In this sense, the need for mathematics teachers' pedagogical/didactic training[1] has been hotly debated in recent years (Fiorentini & Lorenzatto, 2009; Godino, 2009; Ponte, 2014). Thus, other dimensions, such as the emotional (Gusmão, 2009), social (Skovsmose, 2008), and cognitive knowledge (Santos, 2015), among others, should be continually permeating the discussions on mathematics teaching.

In particular, we will assume that one of the ways to contribute to the teachers' mathematical and didactic training is through Task Design (Gusmão, 2014, 2019; Pochulu, Font & Rodriguez, 2013; Serrazina, 2010), inserted in enriching situations that refer to intra or extra mathematical contexts (Ponte & Quaresma, 2012; Sousa, Font, Gusmão & Roseira, 2019), and that allow discussions, interactions and (re)designs.

[1] In this article, we are treating didactic and pedagogical knowledge as similar theoretical constructs. Thus, pedagogical or didactic knowledge is the teacher's ability to, besides knowing Mathematics, produce learning situations that take into account the most appropriate ways to work on the knowledge of their field with the students.

From this perspective, we intend to *point out the contributions that task (re)design can bring to enhance the didactic-mathematical knowledge of the teacher,* aiming to understand to what extent the act of designing, redesigning, and reflecting on tasks causes improvement processes in the didactic and mathematical knowledge of teachers.

The work presented here corresponds to discussions that took place in a public school in Bahia, in which we held formative meetings in two stages. In the first set of actions, we conducted studies in the field of task design and, in a second moment, we redesigned the tasks built by the teachers in the year before the training, showing the relationship between task and context, enabling discussions, interactions and new learnings for those involved.

Consequently, in this article, we present a theoretical discussion on teacher training and its relationship to task (re)design, detail the carried-out intervention research, the data analysis, and the results achieved with the research.

## THEORETICAL BACKGROUND

### Didactic, mathematical, and didactic-mathematical knowledge in teacher training

The need for a more comprehensive education for mathematics teachers, beyond the content-based vision of this discipline, has marked the reflections of authors such as Fiorentini and Lorenzatto (2009), Godino (2009, 2013), Ponte (2014), and Serrazina (2010, 2012). In these studies, the importance of the articulation between two essential aspects of the mathematics teacher's formation is highlighted: the disciplinary domain and their pedagogical training. These, of course, are not the only aspects of studies involving mathematics teachers' training: some issues pertaining to a more general approach to mathematics education, such as those arising from teacher professionalisation (Imbernóm, 2011) and the knowledge involved in teacher training (Saviani, 1996) should also be noted.

In the field of mathematics, authors like Godino (2009), Ponte (2014) and Serrazina (2010, 2012) suggest two complementary categories of knowledge for the teacher: *didactic knowledge* and *mathematical knowledge.* The first refers to the preparation of proper, creative, and innovative learning situations, in addition to the capacity of managing them reflectively and adequately to evaluate their results (Godino, 2009; Guerreiro, 2011; Serrazina, 2010). Besides, it is necessary to observe other types of knowledge linked to the socio-historical conditions that surround teaching work (Saviani, 1996). *Mathematical knowledge*, on the other hand, is "a specialised knowledge that involves understanding the explanations and non-conventional methods of problem-solving by students and the construction and evaluation of multiple representations of mathematical concepts"[2] (Serrazina, 2010, p. 10, our translation). We must bear in mind that "knowing mathematics"

[2] Um conhecimento especializado que envolve a compreensão das explanações e métodos não convencionais de resolução de problemas pelos alunos e a construção e avaliação de múltiplas representações de conceitos matemáticos.

is different from "knowing mathematics to teach," as mathematical knowledge requires skills that go beyond knowing how to perform procedures effectively (Serrazina, 2010, p. 11).

Godino (2009) clarifies that *mathematical knowledge* and *didactic knowledge* are complementary categories that should be interconnected, as a *didactic-mathematical knowledge*. This author then emphasises that the teaching of mathematics is not only constituted of epistemic reference knowledge but also of teaching and learning processes, which involve the collaboration of other areas, such as pedagogy, psychology, epistemology, sociology, among others; hence the expression

> "Didactic-mathematical knowledge of the teacher" to refer to this complex of professional knowledge and skills. We include, therefore, in didactic knowledge, the knowledge of mathematical content when said content is contemplated from the perspective of its teaching. The control of the transformations that must be applied to the mathematical content for its diffusion and communication in the different school levels must also be a competence of the mathematics teacher. (Godino, 2009, p. 15, our translation)[3]

Unlike what one might think, these - specific and pedagogical - aspects are not separated and divided into "drawers"; they are interrelated in a continually developing process. Serrazina (2010) points out that the didactic and mathematical knowledge develops in teaching, through the teacher's constant *planning-action-reflection* activity about their practice[4], turning it possible to make new decisions considering internal and external conditions of the classroom, which triggers processes of teacher knowledge growth.

## Tasks: Design and Redesign

The tasks that a teacher takes to the classroom will reflect the nature of their didactic-mathematical formation, as they will demonstrate, besides the concern with the development of concepts, other needs such as care with the used language, contextualisation of the subjects, and the emotional involvement of their students. The relationship between task design and teacher training has been discussed by many researchers (Gusmão, 2014, 2019; Pochulu et al., 2013; Serrazina, 2010). Research also shows that the use of appropriate tasks, contextualised and focused on the development

[3] "conocimiento didáctico-matemático del profesor" para referirnos a dicho complejo de conocimientos y competencias profesionales. Incluimos, por tanto, en el conocimiento didáctico, el conocimiento del contenido matemático en cuanto dicho contenido se contempla desde la perspectiva de su enseñanza. El control de las transformaciones que se deben aplicar al contenido matemático para su difusión y comunicación en los distintos niveles escolares debe ser también una competencia del profesor de matemáticas.

[4] The term practice in this paper, is similar to that given by Roseira (2010), where practice goes beyond the simple act of doing something, since it involves the dialectical relationship between this doing and the teacher's conceptions of mathematics and their teaching.

of cognition, strongly influences students' learning (Moreira, Gusmão & Font, 2016, 2018a, 2018b; Stein & Smith, 2009).

But what are tasks? Far from the common-sense view (understood as "homework," exercise, "mechanical" repetition of procedures), we base this paper on the concept that tasks are learning situations the teacher proposes to trigger the mathematical activity of the student (Pochulu et al., 2013). It is seen as "a sequence of didactic moments" to be used in a "classroom context" (Godino, 2013, p. 12), including from the planning of the activities to communicative processes and resolution of conflicts of meaning (Gusmão, 2014, 2019; Pochulu et al., 2013; Stein & Smith, 2009).

For Ponte (2014), a task is different from an activity, although the two concepts are interrelated. Activity is related to the student's active role facing a learning situation, since it is in the task that the mathematical activity is realised. The tasks' role is formative and reflective for the teacher, but for the student, it is primarily to instigate and regulate mathematical activity (Gusmão, 2019; Pochulu *et al.*, 2013; Ponte, 2014).

And what do task design and redesign consist of? The answer to this question is in Pochulu *et al.* (2013, p. 2), who state that designing is the process of "elaboration, creation and preparation of mathematical situations to be applied in the classroom and the redesign of tasks refers to the process of their adaptation, suitability, and adjustment" (our translation)[5]. Thus, it is in the process of designing and redesigning, here treated as *(re)design*, that the possibility to increase the knowledge arises for the teachers.

Thus, (re)design assumes a training role, since it can trigger reflection processes in the teachers, through a dynamics of task planning, the implementation of what was planned, and the consequent evaluation of the results, providing the *planning-action-reflection* process, as described by Serrazina (2010).

What students learn is strongly related to the quality of the tasks the teacher offers them; thus, if well planned, such tasks help in the development of the teacher's knowledge (Steiner as cited in Gusmão, 2014). In the same direction, Godino (2013) stresses the relationship between tasks, student learning, and the teacher's didactical-mathematical training. For this author, it is in the task that the teacher encourages students' activity, seeking to create appropriate situations, making changes to fill perceived gaps in the process, to refocus the learners' attention. At the same time, this teacher must have theoretical tools that guide changes, analyses and reflections, redirecting routes, wherefrom the potential of the tasks to improve the teacher's knowledge arises.

Ponte and Quaresma (2012), Font (2005, 2006), and Sousa et al. (2019) argue that tasks should: be contextualised, allowing us not only to understand the contents better, but the world around us; and bring intra mathematical (internal, relative to associations of mathematical content to each other) and extra mathematical (external, associations with the environment, or with other areas of knowledge) relationships. Font (2005) points

[5] "elaboração, criação e preparação de situações matemáticas a serem aplicadas em sala de aula e o redesenho de tarefas, faz referência ao processo de adaptação, adequação e ajustes das mesmas."

out that contextualisation is efficient when it allows a later content generalisation (or globalisation), relating it to other situations and issues.

To (re)design enriching tasks that may promote student learning, some authors argue that there should be criteria, or guiding principles, in these constructions, thus avoiding the commonplace of repetitive and dull exercises. Pochulu et al. (2013) and Gusmão (2014, 2019) argue that good tasks should be based on guidelines, such as: being open tasks, that is, admit more than one way to be solved; propose significant contexts, with situations experienced by the students; request justifications and explanations as to the choice of the steps taken; not consisting in routine; being adequate, neither too easy to be uninteresting nor too difficult, to avoid discouragement; they should be fun, thought-provoking and lead to the development of cognition through generalisation processes (Gusmão, 2014, 2019; Pochulu et al., 2013).

The Ontosemiotic Approach points to the Didactic Suitability Criteria[6] as theoretical-methodological tools for the guidance and regulation of mathematical activity in the classroom (Godino, 2009, 2013; Breda, Font & Pino-Fan, 2018; Godino, Batanero & Font, 2019). These criteria are represented by aptitudes: *Epistemic* – Institutional Mathematics transposed to the curriculum; *Cognitive* – the degree to which learning is in the students' proximal development zone; *Emotional* – relationship between learning and emotions; *Interactional* – teacher-student and student-student relationship; *Mediational* – availability and adequacy of resources and time; and *Ecological* – adequacy to the curriculum and the social environment.

Didactic Suitability Criteria give a greater dimension of the complexity that involves the planning and execution of tasks in mathematics. Godino (2009) suggests working with the proposed characteristics as beacons, regulators of planning and execution of situations, permeated by constant evaluations of mathematical activity, observing the relationship between what was planned and the achieved results. Teachers also use these criteria to justify the quality of the tasks they design and implement (Breda, 2020; Hummes, Font & Breda, 2019; Hummes, Breda, Seckel & Font, 2020; Morales-López & Font, 2019).

*Didactic analysis* is a part of this process (Godino, 2009, 2013), the dynamics of planning, analysis, and constant reflection on the practice that permeates task (re)design, the reflection on errors, and achievements that occurred in the process; the resolution of *semiotic conflicts* (Godino, Batanero & Font, 2008), considered as differences of comprehension between the personal (student) and institutional (epistemic) meanings of mathematical objects, where the teacher should identify inconsistencies and perceive knowledge gaps that are causing them, creating alternatives to address them.

[6] Suitability in Godino's texts has the meaning of quality, correctness and adequacy.

## METHODS

This research, with a *qualitative approach* and descriptive character (Ludke & André, 2006), was based on an *Intervention Research,* understood as "research on the action when it comes to studying it to understand and explain its effects" (Chizotti, 2006, p. 80, our translation)[7]. The intervention took place in a school of the municipal network of the city of Jaguaquara, Bahia, Brazil, with the participation of teachers and the teacher-researcher, who conducted the meetings. The teachers worked in the final years of elementary school in the curricular component Mathematics, in the day shift.

We used three instruments for data production: a *field diary,* the *materials produced by the teachers*, and *the audio recording of the training meetings.* The recorded *audio* dialogues were transcribed and then identified: PP for the research teacher (*Professor Pesquisador* in Portuguese) and the codenames Dalva, Nilzete, and Samuel[8] for the participating teachers. Dalva has been teaching for 32 years, has a degree in Pedagogy, and was acting outside her field of training for a short time; Nilzete, a 33-year-old teacher, educator, and undergraduate in mathematics; and Samuel, who was the only mathematics graduate, teaching for 19 years. In the field diary, we recorded observations such as their gestures and emotional expressions. The materials produced during the training concern the tasks (re)designed by these teachers.

The intervention activity was divided into two main stages, each consisting of "training meetings" (the name we called the meetings held with teachers for discussions, training, reflections). In the first (preparatory) stage, we studied the literature on task design in three training meetings: in the first, we worked on the concept of task; in the second, (re)design; and in the third, the personal criteria for task (re)design. In these meetings, we read texts, held theoretical discussions, and, logically, studied (re)design, and from them, we raised difficulties/resistances and conceptions of these teachers about the studied subject.

The second stage was made up of four meetings, corresponding to the redesign of the task sequences that made up a school project[9] called *O Homem do Campo*, (The Country Man), conducted the year before[10] in the school, along with in-depth discussions of the Didactic Suitability Criteria and the intra and extra mathematical contexts.

## DATA DISCUSSION AND ANALYSIS

The tasks chosen for the training work should satisfy two important aspects: they had to be "enriching situations" and have training potential. The information analysed was

[7] "uma pesquisa sobre a ação quando se trata de estudá-la para compreendê-la e explicar seus efeitos".
[8] The codenames of the participating teachers were chosen as a tribute to teachers who marked the school trajectory of the teacher-researcher.
[9] In this work, we are using the term "school project" without theoretical depth, since what interests us most here are the sequences of tasks that constitute such projects. Thus, we are relying on Ponte (2013), which considers projects as long-term task sequences.
[10] For more details on intra and extra-mathematical contexts see Sousa et al. (2019).

grouped into four categories (Table 1), according to the difficulties/resistances found: a) traditional conceptions of tasks[11]; b) difficulties in solving and redesigning tasks; c) gaps in mathematical knowledge; d) difficulties in communicative mathematical processes.

Here, we take difficulties/resistances jointly, because we understand that most of the time resistance comes from the lack of knowledge of a specific issue, which ends up causing problems in solving some mathematical situations. By way of example, we have the traditional conception of tasks rooted in the training of the teacher-participant that generated difficulties in the resolution of open and innovative situations.

**Table 1**
*Difficulties/Resistances Found in Didactic-Mathematical Training*

| Difficulty/Resistance | Characteristics |
|---|---|
| (a) Traditional conceptions of tasks | - View of tasks as old-fashioned, compulsory or traditional; |
| | - View of tasks as a synonym for exercise and activity. |
| (b) Difficulties in solving and (re)designing tasks | - Difficulties in interpreting tasks that have no restrictions in their statements; |
| | - Search for a single answer when solving tasks. |
| | - Search for numerical answers; |
| | - Difficulties in turning "closed" tasks into "open" ones. |
| (c) Gaps in mathematical knowledge | - Semiotic conflicts (doubts like "is every square a rectangle?" Or "is the chayote tendril a type of propeller?"); |
| | - Lack of knowledge of concepts and definitions, mainly related to geometry; |
| | - Difficulties in generalizing resolution procedures, using a given task or context to solve others. |
| (d) Difficulties in communicative math processes | - Difficulties in expressing themselves mathematically; |
| | - Difficulties in discussing tasks using clear mathematical language, with correct epistemic foundation. |

Some of our participants' speech fragments illustrate the first category under analysis, *(a) traditional conceptions of tasks,*

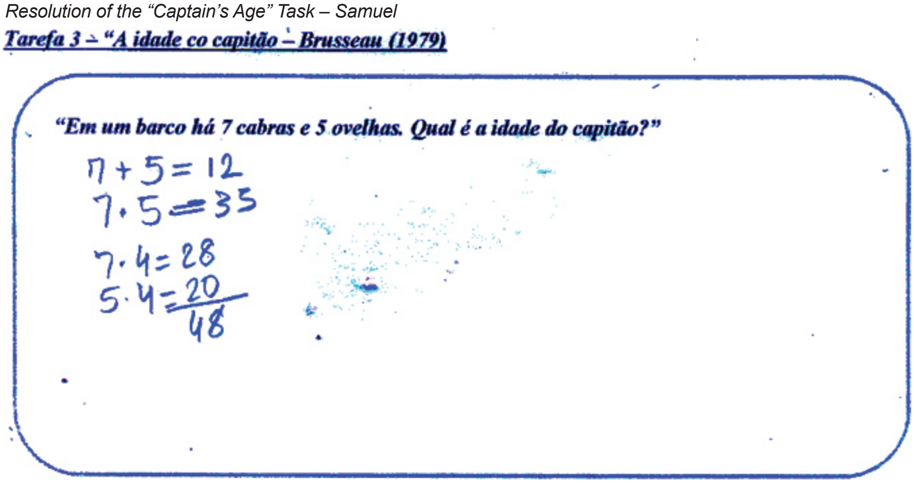
> *Samuel: I don't usually use this word 'task,' I use the word 'exercise' more, the word 'activity,' right. [...] And, in my view, I don't know if it is true, but 'task' is something like…, more used, more... in the old days, right?*
>
> *Nilzete: I always used it: homework task, class assignment task... because, erm, the student had to do it, commit to doing something, the task then was a commitment... Because, to me, the concept of a task is something you have to do, right?... a goal to meet.*

[11] Here we are using the term 'conceptions' in the same sense as Roseira (2004), as the "particular philosophy" of the mathematics teacher, involved in his way of explaining and working the subject matter.

Also, the abovementioned excerpts reveal an initial conception of tasks similar to that described by Gusmão (2014, p.1), as “old practices of compulsory activities imposed by the teacher in the classroom, the old lists of exercises, review activities, content fixing” (our translation),[12] and, although they do not express a clear concept, they indicate tasks like something obligatory and outdated.

Teachers with a closed conception of tasks are more likely also to use open resolution procedures when faced with types of tasks they are not used to, such as the one presented by Brousseau (1979 as cited in Santos, 2016): “there are 7 goats and 5 sheep in a boat. How old is the captain?” The written and spoken records of our participants in solving this question are examples of a restrictive conception of tasks (number manipulation).

**Figure 1**
*Resolution of the “Captain’s Age” Task – Samuel*




> *Samuel: I thought like this: as there are 7 goats and five sheep, what is the total? So, it would be 12. Then I thought: 7 times 5, thirty-five. Then I thought like this: 7 goats, each one has 4 legs (laughs...) so I said 7 times 4, 28 (...) and I just thought this here (shows the numbers 12, 35, 28, 20 and 48, as possible ages of the captain).*
>
> *Nilzete: What do goats and sheep have to do with this guy’s age?... absurd (laughs)... Then, when I saw Samuel adding (laughs) here 7 and 5: 12. Then I said: well, a 12-year-old in command of a boat? I thought about the age of the person to perform the profession, in the case of the task, the activity. A child, a 12-year-old driving a boat? (...) .*

[12] “velhas práticas de atividades obrigatórias impostas pelo professor na sala de aula, as velhas listas de exercícios, às atividades de revisão, de fixação de conteúdos”

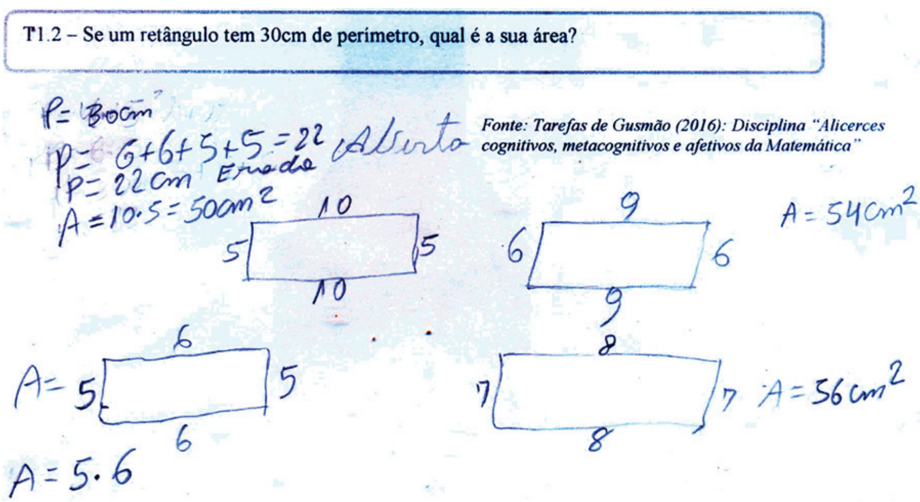
The teacher questions, but also ends up presenting a numerical answer: “That’s when I thought, here, when I saw... The date, right!” (Referring to the date indicated in the statement of the task of Brusseau, 1979). And then, she subtracts the current year (2017) from the task creation year (1979), finding 38 as a result.

The relationship between the initial conception of tasks (closed propositions) and how they solve these learning situations (seeking a unique, preferably numerical result) is noticeable. Although in some moments they worked on activities of creating problems, there was always an expectation of a standard reply, within a particular right or wrong logic. Hence, there was a process of “initial strangeness” when faced with problems whose data were not connected, problems without a numerical solution, or with an open resolution, which gave rise to discussion and reflection. Later, as one of the enhancements of this intervention, the teachers reformulated the concept of tasks.

It is not difficult to infer that the next category *(b) difficulties in solving and (re) designing tasks* is directly related to the difficulties pointed out in the previous category, since traditional conceptions of tasks lead to less creative resolution strategies. In solving the task of Sullivan and Clarke (1992) during the first meeting – “a rectangle has 30 perimeter units, what could be its area?” – the teachers initially proposed unique or limited answers. At first, when we always asked to solve the question individually, we found answers such as:

**Figure 2**

Resolution of the “Area Calculation” Task – Samuel




Initially, Samuel presented only the response ‘22 cm’ for the perimeter and ‘30 cm²’ for the area. Then, the teacher-researcher asked new questions, “could this area be different from 30 cm²? Could this perimeter have another configuration?" And Samuel goes on presenting other responses (Figure 2), elaborated now with his colleagues, and he realises that "with a fixed perimeter the area may vary," generating several possible outcomes. In the resolution of the situation, understanding the meaning of the task (the communication between the participant teacher and the task) was the first obstacle: what is "being asked" in the question? How to proceed? In this regard, Nilzete's statement is elucidative:

> *If I were to think about the area, an answer for the perimeter could be five by five and ten, ten, that is thirty. But five times ten is fifty. That's past 30 a long time ago... (laughs).*

At first, resolving individually, the teachers found it difficult to understand that the area measurement was "open" to various possibilities. With the evolution of the resolution and the "task management" (Gusmão, 2019) by the researcher, they realised that, although the perimeter was fixed, there were no restrictions on the area configuration.

In this sense, Gusmão's (2014) reflection on the difficulties in dealing with open tasks, which is the result of our traditional education, is valid.

> We are the result of a training in which our practice is reduced to simply giving answers. Everything comes ready, we just have to answer. We are not used to creating, designing tasks, especially when it comes to open-ended questions or multiple answers. (Gusmão, 2014, p. 7, our translation)[13]

Breaking this kind of thinking was one of the challenges of this research. At the second training meeting, we began to notice changes:

> *Samuel: If you don't question, just like you did, right? Questioning, to go get (...) Look, this here, is there no other way to resolve it? Let's think? Could it be that taking it here and there... because students today like things already done.*

The third category, (c) *gaps in mathematical knowledge,* is verified at some moments of the intervention, when discussing concepts, in solving tasks, doubts, etc. A participant

[13] "Somos frutos de uma formação em que a nossa prática se reduz/reduziu a simplesmente dar respostas. Tudo vem pronto, é só responder. Não estamos acostumados a criar, a desenhar tarefas, ainda mais quando se tratam de perguntas abertas ou que admitem múltiplas respostas."

shows that doubts about concepts that should be part of the teachers' knowledge repertoire are more common than imagined:

> *Nilzete: Here, every ten I get from here, I will add vertically (shows a task proposal: a rectangle in the shape of a flowerbed, where the resolver would have to distribute 860 tomato seedlings every 10 meters, forming a rectangle). Then I can only go up to x, because if it's up to y, I'll have a square, and the figure will no longer be a rectangle.*
>
> *Samuel: It is no longer a rectangle, because a square would have all four sides equal, right? And so, no. Got it.*

Despite the task presented by the teacher at the fifth formative meeting[14] being quite interesting, even bearing an open resolution, a *semiotic conflict* appears (Godino et al., 2008): the understanding that a square is not a rectangle. And this is not just a difficulty for Nilzete, as her fellow students agreed with the controversial statement. But when questioned by the researcher-teacher, they realised the main flaw in the question of not including the square as a "special kind of rectangle," which could lead to an important communication conflict in task management in the classroom.

This situation corroborates what Gusmão, Ferreira, and Fagundes (2013) discuss: in many situations, teachers have similar difficulties to those that students experience in the tasks, and such difficulties, if not worked on properly, can directly interfere in the learning process of the students.

The last category, *(d) difficulties in communicative math processes,* quite present in the training meetings, involved conflicts in the communication and interaction among the participants of the learning process. In these *communicative processes* (Guerreiro, 2011), there may be conflicts that cause the message to get distorted to the student, that is, understood adversely, generating comprehension difficulties among those who dialogue or with the proposed tasks. For example, taking up the perimeter task of the figure, mentioned above, Samuel tried to assist Dalva, who revealed to be having difficulties to differentiate perimeter from area.

> *Samuel: You have 30, you have to divide those 30 among these four parts, as long as one remains larger and the other smaller (Dalva, puts on paper 30:4 and finds 7.5).*
>
> *PP: Samuel, when you say she has to divide, are you talking about a division or are you talking about splitting in four...*
>
> *Samuel: In four parts, four sizes, two larger and two smaller.*

[14] The task to which we refer (distributing 860 tomato seedlings in a field, every 10 meters), and the comments related to it, are part of the second training stage. All other tasks or statements present regarding the teachers' difficulties (Table 1) were taken from the first training stage.

*Nilzete: If they were equal, it would be a square.*

*Samuel: It's like I told her before, it's to be divided into 4 pieces, two larger and two smaller.*

*Nilzete: ten, ten, and here, five, five (points to the rectangle drawn by Dalva).*

*Nilzete: What do you think? (Dalva then records the perimeter equal to 30, with sides measuring 10 and 5 centimeters, but continues having difficulties relating this to the area, which makes the teacher-researcher intervene).*

*PP: But based on this perimeter you made, will the area have the same value, or will it be different?*

*Dalva: It's different. Five times 10 would be fifty (she seems to realize that the area would have a value and the perimeter with a different configuration, but fixed).*

This dialogue reveals two interesting situations. First, a conflict in the process of interaction between colleagues: Dalva did not understand what Samuel said, and he used the term divide, which caused a communication conflict. Secondly, there was a difficulty in Dalva's mathematical knowledge, who did not seem to differentiate the area and perimeter easily in the task. She may have found it challenging to master the concepts involved or did not have experience in solving this type of question.

Another interesting situation arose when solving a task that required planting seedlings on an 860-meter plot. The participants revealed that communicative conflicts did not appear only in speech but also in writing the tasks.

*Nilzete: Did you understand what I'm talking about? (Asking the teacher-researcher). Because I can only [get] till here, which will give x, but if it passes x it will be a square. But I couldn't write it. It's the Portuguese language!*

*Samuel: Because I always, I say to my students: we have to know a lot of Portuguese because it helps a lot in the tasks.*

Although the teachers credit many of their difficulties to problems of mastering writing related to the Portuguese language, it seems more accurate to say that the difficulties in mastering the didactic-mathematical field itself are what bring most discomfort to the teachers, arising from the more profound knowledge they lack regarding concepts, themes, definitions, etc., in the field of the discipline they teach. Besides, there is a specificity of mathematical language that goes beyond learning codes and symbols, seeking relationships with language, and having a set of meanings of its own (Luvison, 2013).

Starting from the obstacles observed in the first stage, some situations were planned as a counterpoint for the second stage, regarding the meetings for the design and redesign of tasks to alleviate such difficulties. Among them were: encouraging the teachers to think about communicative processes (including in the writing of tasks), pointing out and studying the semiotic conflicts that emerged in the intervention, and providing tasks that would exploit the generalisation and contextualisation capacity.

Thus, we can highlight some improvements and advances that enhanced the teachers' didactic-mathematical knowledge at the end of the training meetings of the second stage, as shown in Table 2. It is important to note that we are using the concept of improvement used in the Ontosemiotic Approach, which is related to the need to achieve higher quality/suitability in educational processes (Breda, Font & Lima, 2015). This search is based on ideas already consensual in the academic community about mathematics teaching, such as the importance of adapting the curriculum to students' reality and the concern with affective issues, among others.

**Table 2**

*Advances and Improvements Observed in the Teachers' Didactic-Mathematical Knowledge*

| Difficulty/Resistance | Indicators of outlined improvements and advances |
|---|---|
| (a) traditional conceptions of tasks | - Changes in the conception of tasks and reflections on the need for the renewal in mathematics teaching; |
| | - Vision of tasks as learning situations, construction of personal concepts of tasks and redesign. |
| (b) difficulties in solving and redesigning tasks | - Slight improvement in open tasks solving ability; |
| | - Emergence of (re)designs proposing open and "creative" situations. |
| (c) gaps in mathematical knowledge | - Slight improvement in the teachers' mathematical knowledge and coping with semiotic conflicts; |
| | - Use of some generalizations and establishment of relationships between mathematical contents. |
| (d) difficulties in communicative processes | - Reflections on task management and communicative processes during the resolution of these tasks. |

Regarding category (a), *traditional conceptions of tasks,* the improvement process was related to a very significant change in the participants' conception of tasks, according to testimonials obtained during the meetings:

> *Nilzete: Today, after these meetings, one may think that it (a task) is something planned, executed, re-evaluated, and redesigned to be applied again.*
>
> *Dalva: Because a task has to elaborate one thing and give the results, you only show the paths (...)* ***In the case of the task it will create, and several paths may come, several answers, or an answer with many different interpretations*** *(our emphasis).*

In her speech, Dalva indicated an evolution of the concept of tasks, which went from a single and "closed" response to the possibility of multiple answers and/or interpretations, and Nilzete pointed out that she was marked by the theme of (re) designing. It is noteworthy that the participants moved from a traditional conception to a more open definition, adopting a comprehension of tasks close to what Gusmão (2014, p. 5) defines, as those that "admit multiple answers and multiple representations, allow greater interaction and communication in the classroom, require greater cognitive performance" (our translation)[15].

Regarding category (b)*, difficulties in solving and redesigning tasks,* we highlight a situation that illustrates the improvement in the teachers' didactic-mathematical knowledge, evidenced in the (re)design made by one of the participants of a task we created, which led (intentionally) to a closed resolution:

**Figure 3**
*"Fencing a Tomato Field"*

***Task 6: Fencing a tomato field***

*A small farmer from the city of Jaguaquara organizes his 10-hectare farm in the Baixão de Ipiúna town, between dairy cows and tomato fields. In the year 2017 he wants to fence part of the farm to plant tomatoes, with one dimension of 85 meters and the other with 40 meters. The remaining land will be used for breeding activities.*

*Source: http://revistagloborural.globo.com/Revista/GloboRural/foto/0,,69827725,00.jpg*

*6.1 Using the perimeter calculation of the part to be fenced, calculate the amount of wire required to surround the tomato field, bearing in mind that the wire is set in five rows on the fence.*
*6.2 Calculate also the farmer's spending with wires, as each roll of 500 meters costs around R$ 170.00.*

Through questions, we urged the teachers to propose changes in the structure of the task. Among the changes made is the use of Sullivan and Clarke (1992)'s problem, already commented, as an "inspiration." However, when it came to "write down" these suggestions, the teachers experienced difficulties.

Professor Nilzete confesses that she could not (re)design the task itself, but presented a second situation, based on Gusmão (2006)[16].

[15] "admitem múltiplas respostas e múltiplas representações, possibilitam uma maior interação e comunicação em classe, exigem maior desempenho cognitivo".
[16] A fisherman took 50 kilos of fish to the market. In the morning, he sold the fish for 3 euros per kilo, and in the afternoon, in order to sell everything, he reduced the price to 2 euros. If he sold all 50 kilos, how much money did he make?

**Figure 4**

*Redesign of the task "Fencing a Tomato Field," carried out by Nilzete*

| |
|---|
| *A tomato farm produced 1,800 crates. More than half of the production was first-class (large tomatoes and more or less equal maturation) and the remainder, second-class (smaller tomatoes). The first-class tomatoes were sold for R$ 50,00 and the second-class ones for R$ 30,00. How much was the landowner able to make by selling the tomatoes?* |

As a solution, the teacher exhibits a short outline showing the possibilities for varied answers that can be found. According to Nilzete, the initial values would be at least 901 (more than half of the "first") and 899 (the remainder of the "second") and that the answer margin could be up to a maximum of 1,799 crates of first-class tomatoes, and only one second-class crate. Note in the teacher's answer the understanding of the problem proposed by Gusmão (2006), since she followed a similar line of reasoning. The propositions proved to be quite coherent, especially in the way she explained and translated them in her writing, resulting in a good redesign of the task.

The situations highlighted here lead us to understand that the difficulties of (re)designing must be faced, that there are no simple recipes to overcome them, as everything is a process; after all, "task design is configured as a construction that does not occur quickly and lightly. It requires time and study" (Moreira et al., 2016, p. 789, our translation)[17].

In category *(c)*, *gaps in the teachers' mathematical knowledge*, there were some advances, albeit timid, in mathematical knowledge, which can be noticed in the redesign activities and the discussions among the teachers. For example, after solving the task of Sullivan and Clarke (1992), the participants began to use the involved property, a fixed perimeter can have a variable area, in other contexts, such as the task "Fencing a Tomato Field" (see Figure 3).

> *PP: What if instead of giving the size of the sides, you could do to let this task become more open?*
>
> *Nilzete: give the full perimeter, let them redesign, let them redesign the field according to their imagination, right? Square... rectangular... it's... triangular...*
>
> *PP: And that would open the task ... Would it make it more open?*
>
> *Nilzete: It would, because then everyone would think according to their own space or their imagination.*

As in the example above, in other moments of the intervention, the teachers demonstrated associating tasks from the literature with their redesigns, making relations between mathematical contents and presenting less semiotic conflicts.

[17] o desenho de tarefas se configura como uma construção que não ocorre de forma rápida e aligeirada, que demanda tempo e estudo

Finally, category (d), *difficulty in communicative processes,* very much present in the first meetings, showed a slight improvement in both discussions and writing. At the third meeting, in the discussion of task management, a teacher ponders:

> *Samuel: I, as a math teacher, I have an obligation... it's... to look for means, isn't it? Here comes the need... to explain to my student, if they did not understand it that way, in another way.*

In the same direction, Dalva points to advances in the process of analysis and resolution of tasks:

> *Dalva: Only in this meeting, so, I see that there is already something in the correction of the exercises. So, we are taking notes, to question within the activities that we correct. To base a new activity that will come, based upon the failures, the deficiencies that the student has.*

The teachers' statements highlight the importance of the task management process coupled with redesign (Gusmão, 2014, 2019; Ponte, 2005), involving listening, changing paths, seeking alternatives. At the same time, the attention that the teacher should pay to what the student says, to monitor their learning, coupled with the process of searching for new models, illustrations, contexts, is important both for the teacher's growth (didactic-mathematical knowledge), as well as for the increase of the student's learning (Gusmão, 2014, 2019; Ponte, 2014; Stein & Smith, 2009).

Although the advances discussed here show that the participants' knowledge evolved, some difficulties persisted: even though they developed, it was not possible to resolve some of the *remaining difficulties* (those that persist), such as the conflicts of meaning in communication, gaps in mathematical knowledge, and the difficulties to compose open tasks. Although these difficulties were alleviated with the discussions, the training failed to resolve them satisfactorily, leaving some fringes in the epistemic (development) and cognitive (learning gaps) fields.

In an attempt to understand where the teachers' difficulties in the process of redesigning tasks and proposing open tasks come from, we asked them some questions. According to their responses, three causes seem to be the most obvious: insufficient initial training, lack of continuing education policies, and the embarrassments that occur within the school environment.

> *Samuel: Unfortunately, our undergraduate courses... honestly, mine... (and shakes his head no). It didn't prepare me to be a primary school teacher or a*

*high school teacher. Very far from it! Very far! I think colleges, you see, colleges should think...*

*Dalva (speaks up): Because they prepare us to be scientists, it's not to be a teacher.*

The teachers' reports corroborate what Moreira and David (2005) and Pereira and Curi (2012) discuss, that teacher training courses end up prioritising formal mathematics, leaving pedagogical training in the background. Moreira and David (2005, p. 53) highlight the difficulties of graduated teachers in dealing with daily issues, such as the teaching of basic operations that, initially, they consider to be simple learning, but eventually realise how complex it is to teach them.

*Nilzete: We work... in our daily lives, and that does not allow us, thus, to reflect. I think this is a breakthrough (training promoted by research), we can get a sense of what we're doing, because we are trying to get it right [...] it's like: when we are keen to reflect, to seek other information, we can. Without this, it's impossible! It's impossible! How do you, alone, at home, with your books, with your material, give you the best of yourself? Isn't it? Me, myself, if I'm going to self-assess my work, I'll say, I'm doing my best. But is it really what I have as a potential? Or, would I have more potential to give if I found more stimulation?*

The teacher's statement leads us to think, as trainers, to what extent we can impute responsibility to teachers for a knowledge to which they were not given access? Moreover, based on Samuel's statement, we need to reflect on how the training courses are preparing their graduates to face the adversities of the educational process. And regarding untrained teachers, such as Dalva, who is a pedagogue and only conducts math classes since recently, how can one help these teachers with the challenges they face every day? Besides, there are complaints of daily problems within schools that the participants also pointed out, such as lack of material, difficulties in establishing an appropriate dynamic functioning of the institution, constant teacher shortages, among others.

Thus, we must recognise the difficulties exposed, but we have to think of alternatives to address them, which forces us to bring the discussion into the environment of continuing teacher education. Imbernón (2011) stresses that training should assist in professional development, while instigating educational innovation. Serrazina (2010, 2012) points to the need for a formation involved in the process of constant activities planning and reflection on the found results, enabling redesigns and proposals for improvement.

Hence, in this work, even partially, it is observed that "reflection plays a central role in the professional development of teachers, because, as they increase their mathematical and didactic knowledge, they deepen their reflection" (Serrazina, 2010, p.

11, our translation)[18]. In this sense, we realise that the discussions that take place show us the potential of task (re)design to leverage teachers' knowledge, especially when it is permeated by reflection.

## CONCLUSIONS

We realise from this study that the proposition of creative and innovative tasks by teachers requires time and much practice in this process. Even those who have adequate training or teaching experience have gaps in mathematical knowledge, which makes them insecure when the time to solve, analyse, and (re)design tasks comes.

We perceived not only the lack in their didactic training but difficulties that come from disciplinary training, reflected in questions like "is a bolt more like an ellipse or a cylinder?' Or "which is the geometric object that resembles the tendrils of the chayote?" uttered during the training. Some of these gaps have been narrowed in the training process, perhaps overcome – as was the traditional conception of tasks. Nonetheless, other difficulties, more rooted in the educational and academic training of the teachers, remain, if not in their original pre-training conception, but still included in the participants' didactic and mathematical training.

Thus, resuming the objective of this work, which is to *point out the contributions that task (re)design can bring to enhance the didactic-mathematical knowledge of the teacher,* the training based on activities of task construction/reconstruction brought some important advances, which can be perceived in the evolution of task understanding and contextualisation, in the emergence of self-produced tasks with a more varied display of contents, and especially in the reflections provoked, leading the participants to question established certainties about tasks, about communicative processes in the classroom, and about their own mathematical and didactic knowledge. Thus, this work shows us these teachers' ability to overcome part of their difficulties in a short time, which can be further explored if they keep their curiosity and extend their studies beyond the training meetings held.

Hence, the participants go out of the training with important strengths related to a "new" conception of tasks, their design, and redesign, the contextualisation of learning situations within the discipline they teach. And these are all essential increments in their didactic-mathematical knowledge.

On the other hand, the development of the training meetings led us to believe that for many years these teachers had not had access to discussions about the educational potential of tasks. Thus, as much as these professionals sought to be innovative in their practices, they lacked the simple realisation that mathematics, to be of quality, does not

[18] "a reflexão joga um papel central no desenvolvimento profissional dos professores, pois à medida que aumentam os seus conhecimentos matemático e didático conseguem aprofundar a sua reflexão".

have to be relentless, numerical, partial, and decontextualised. In this sense, the proposal of task (re)design contributed, if not to remedy the difficulties of the participating teachers, to diminish them and to instigate these teachers, provoking them in the direction of more open and innovative teaching, where Mathematics gives its valuable contribution.


## ACKNOWLEDGEMENTS

Our thanks to the coordination of the Programa de Pós Graduação em Educação Científica e Formação de Professores at the Universidade Estadual do Sudoeste da Bahia (UESB), for funding the translation of this article. The authors would also like to thank Dr. Bruno Ferreira dos Santos, from the same program, for yours suggestions and contributions that enriched this work. This work has been carried out within the framework of the Research Project on Teacher Training PGC2018-098603-B-I00 (MCIU/AEI/FEDER, UE).


## AUTHORS' CONTRIBUTIONS STATEMENTS

J.R.S., T.C.R.S.G. and V.F. conceived the idea presented and developed the theory. J.R.S. performed the activities, collected data and wrote the initial draft. J.R. S., T.C.R.S.G., V.F. and J.C.L. partipated in methodology construction, data analysis and jointly produced the final version of the article.

## DATA AVAILABILITY STATEMENT

The data used and analysed during the current study are available from the corresponding author, J.R.S., on reasonable request.

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