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Tachymetry Production and Circulation Background

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ABSTRACT

Context: The 19th century was marked by the circulation of French pedagogical proposals in the area of Brazilian education. **Objectives:** To continue the historical investigation published by Pais (2019) on the geometry teaching method called Tachymetry, created by engineer Lagout and adapted by Dalsème, both French authors. **Design:** This text analyzes the criticism received by Tachymetry in France, revealing struggles over representation (Chartier) in the production and acknowledgement of a new teaching method and its appropriation in the state of São Paulo, Brazil. **Data collection and analysis:** This study collected new research sources which allowed an in-depth discussion about similarities and differences between Tachymetry and the intuitive method, with Brazilian sources collected by the Brazilian author, and French sources obtained by the French author. **Setting and participants:** Early analyses and a first draft of the article were developed during the Brazilian researcher's visit to Université de Limoges, in 2019. **Results:** An analysis of the criticism received by the tachymetric method and by its creator, Lagout, allowed us to reflect on how the clash between concrete and abstract, in relation to geometry teaching methods, is longstanding and has offered resistance. Lagout and Dalsème can be considered relevant agents in the hard and challenging task of producing geometry teaching for the early primary school years. **Conclusions:** This research work concludes that Tachymetry is based on the defense of visual observation in geometry teaching and on the possibility of manipulating concrete objects while preserving the argumentative exercise of demonstrative reasoning.

Keywords: Lagout, Dalsème, geometry teaching, intuitive method.

INTRODUCTION

Acta Scientiae's special issue on the History of Mathematics Education published the article *"Tachymetry as a Resource for Teaching Mathematics in Brazil at the End of the Nineteenth Century"* by Luis Carlos Pais, in which readers could find the historical elements to the production of the *tachymetric method*, created by Édouard Lagout[1] and how this method circulated in Brazil.

[1] A French engineer (1820-1885), head of the bridge and sidewalk service department in Paris, in the early second half of the 19th century (Pais, 2019, p. 150).

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Pais (2019) considers tachymetry a resource to highlight the properties of geometric figures through the visualization of manipulable materials or through illustrations, so that students can apprehend, in a fast and direct way, geometric properties and statements contained in theorems. Based on Schelbauer (2016), Pais believes that the tachymetric method expresses a superficial appropriation of the intuitive method and object lessons. He also emphasized that its creator was in close contact with other engineers, astronomers and mathematicians, but he did not discuss how much the material proposed by Lagout was in line with the principles of the intuitive method.

In an attempt to continue a historical investigation, this article aims at analyzing new research sources which were not approached by Pais (2019)[2] and which enable us to better explain the similarities and differences between the tachymetric method and the intuitive method[3], a symbol of modern education in the late nineteenth century. This text presents the criticism that Lagout and his tachymetric method received, revealing the struggles over representation[4] in the production and acknowledgement of a new teaching method. Moreover, our analysis offers readers other appropriation processes of tachymetry in Brazil, specifically a broader set of regulations produced in the state of São Paulo, which were not studied by Pais (2019) in his article.[5]

Tachymetry – place of origin

The production of the tachymetric method created by Lagout happened in a context of professional training for adult technicians, an experience lived by Lagout in Italy. Pais (2019) clearly described the moment:

> Édouard Lagout taught a course in Italy aimed at training professionals for the construction of bridges, sidewalks, squares and fountains. … Italian professionals should finish the course in a reduced number of hours. Therefore, considering the little time available, he had to devote considerable effort to transmit practical geometric and algebraic knowledge. This was the challenge that led him to devise a teaching method with a powerful visual appeal. (pp. 152-153, our translation)

[2] The books written by Lagout (the creator of the tachymetric method) were not analyzed by Pais. They have been analyzed by the two authors of this article since the first author's visit to Université de Limoges, in 2017, and included in Moyon's *Des savoirs en circulation: transmissions, appropriations, traductions en histoire des mathématiques*, *Mémoire d'habilitation à diriger des recherches*, 2019, pp. 68-70.

[3] One of the main principles of the intuitive method is the centrality of objects in the education of the senses, which makes teaching materials essential to teaching. (Souza, 2013).

[4] Investigations into representations assume a field of competition whose challenges are expressed in terms of dominating power. Struggles over representation have as much importance as economical struggles to understand the mechanisms through which a group imposes, or tries to impose, its conception of social world, its values and its domination (Chartier, 1990, p.17).

[5] The author discusses that specific circulation through newspaper news in publications from Paris, France, (*La Croix*,1884), from the Brazilian states of Rio de Janeiro (*Diário do Janeiro*, 1877; *Gazeta de Notícias*, 1883; O *Programma Guiador*, 1887) and Maranhão (*O Paiz*,1883). Pais makes a reference to a specific decree from 1912 in the state of São Paulo, Brazil, in which Practical Geometry (tachymetry) is found.

The need for reducing course duration seems to be a decisive element for the new method, which carried the mark of briefness in its designation:

> Rather than persisting with the classical approach of the logical-deductive method originated in Euclidean tradition, the new type of material aimed at making learning faster, justifying its Greek-origin prefix *tachy*, which means fast, and which works fast and regularly such as the *ticking* of old mechanical clocks or as in tachycardia, when the hearts accelerates beyond its normal rhythm. (Pais, 2019, p. 162, our translation)

Such teaching method was born in response to urgent and practical demands for technical training. The book that Lagout considered fundamental to transmit his proposal is called *Tachymétrie – Géometrie concrète en trois leçons – Cahier d'un soldat du génie* (1874), which emphasized its concrete character as innovative right from its title.

The context in which the tachymetric method was developed highlights the differentiation between concrete geometry and the knowledge consolidated and acknowledged by the scientific community at that time (Euclidean Geometry). There was a direct confrontation between concrete and abstract, a contrast between a logical deductive sequence with all argumentative steps forming a demonstrative path and a coherent and reasoned process in which speed was the priority, so it had to be short due to time restrictions. In Lagout's words (1872), "*la tachy-métrie, c'est la géométrie concrète, c'est le premier essai de concrétisme dans les sciences exactes*"[6] (p. 5). The author defended the reform of methods, so that abstract aspects should be abandoned in order to prioritize concrete ones. By doing so, he believed that both primary school students and professionals would be able to understand it:

> *J'ai entendu un de nos premiers mathématiciens-philosophes dire en comité que l'enseignement de la géométrie d'Euclide ne convient qu'aux esprits mûrs pour les abstractions philosophiques et nullement aux esprits des enfants des écoles primaires ou professionnelles.*[7] (Lagout, 1872, p. 6)

The target audience – primary school children and technical professionals, in this case – determines the knowledge that should be taught; what both have in common is the necessity for making ideas, concepts, properties and especially geometry formulas concrete in an understandable way, based on arguments and evidence, so that learners could find meaning in concepts without having to make use of memorization processes.

[6] Tachymetry (the word is represented through different spellings, such as *takimétrique*, *tachymétrie*, *tach-métrie* and other forms) is concrete geometry, the first attempt to make exact sciences concrete (our translation).

[7] I heard one of the first mathematical philosophers say, during a commission meeting, that Euclidean geometry teaching is only suitable for mature minds due to its philosophical abstraction, but not suitable for primary school children or professionals (our translation).

In a very particular way, formulas used for measuring land, angles and volume needed to be intelligible to learners.

Even though Lagout did not make it clear, he provided clues indicating that the rigor and details of Euclidean geometry should be studied in secondary school or in college, not by primary school children or by technical professionals.

Therefore, the geometry to be taught, as proposed by Tachymetry, distanced itself from Euclidean Geometry and, in this sense, represented a new understanding of Euclidean Geometry, in line with what Barbin et al. (2013) consider "*réinterprétations et de réappropriations des savoirs géométriques, qui correspondent à des conceptions épistémologiques sur la géométrie et sur son enseignement, marquées par leur contexte historique*[8]" (p. 58).

In the book *Panorama de la Géométrie Tachy-métrie – Géométrie en trois leçons*, Lagout (1872) made a list of ten items that characterize his method:

> 1. ***Reform of words*** – school geometry contains a big number of Greek words, which are very long and hard to be learned, thus a dead language should be removed;
>
> 2. ***Reform of sterile truths*** – they should be removed so that fertile truths, which point the main utility, can be highlighted;
>
> 3. ***Aesthetic Reform of Drawing*** – geometry is the art of thinking fairly based on false figures, but not on ugly figures; that is to say, truth and beauty are intimate: beauty attracts us and reality retains us;
>
> 4. ***Indicative Reform of Drawings*** – students have difficulty with figures whose parts are named A, B, C, D; they must be replaced by colors that clearly divide the figures and make them ***intelligible at first sight;***
>
> 5. ***Reform of teaching by hand or geometric manipulations*** – movable figures from the ***Tachymetric box,*** set in motion by hand, spiritually transform the enchanting figure of truth, which would be just an imprecise draft without it;
>
> 6. ***Mnemonic Reform*** – we must make numeric approximations to compose formulas that are simple and easy to remember instead of complicated expressions;
>
> 7***. Reform of continuous syllogism*** – when the proof is short, it is firmly attached to the rule, and both the rule and the proof must be presented in the same cerebral box that becomes a type of mental library, with books that are always open;
>
> 8. ***Reform of all equivalence proofs –*** geometric figures must be reduced to a single proposition of movable partitions based on the back-and-forth movement of a drawer;

[8] Reinterpretations and reappropriations of geometric knowledge, which correspond to epistemological concepts of geometry and its teaching, marked by their historical context (our translation).

9. ***Reform of the fundamental basis*** – Tachymetry, in opposition to Euclid's Postulates, must rest on a visible support, the square, whose existence is proved in a clear and sensible way;

10. ***Reform of artificial division and of the complex classification of truths*** – a new division established on the nature of things: what is accessible, what is inaccessible and round shapes. (pp. 6-7, our translation)

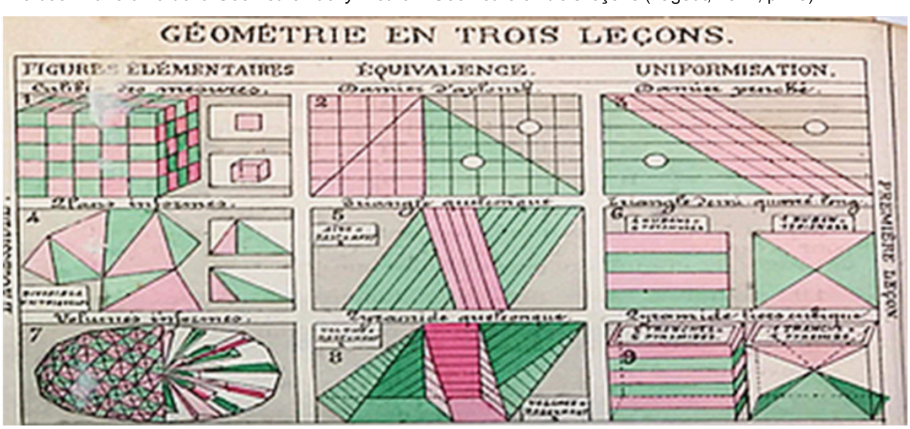
Seemingly, the conception of the method developed by Lagout did not start from an appropriation of the intuitive method, which had been introduced by Pestalozzi (1746-1827) and circulated in the 19th century, especially because the birth of Lagout's method was linked to the training of professionals. Nevertheless, the author included children as learners of his method, so it was possible to identify occasional connections between the two proposals.

According to Trouvé (2008), Pestalozzi's principles emphasize the prevalence of a sensitive experience as a starting point for knowledge and for the representation of reality, the principle of sensitive intuition. The logic of fundamental teaching was based on a triad (head, heart and hand), which corresponded to three basic activity forms: to know, to want and to be able to do something.

The head, as a central element for knowing something, not for merely reproducing it, highlights that Lagout favored intelligibility over memorization. Words that did not contribute to understanding should be avoided, and useful truths that could be assimilated by children should be preferred. An example is the Indicative Reform of Drawings, which favors the use of colors to clearly divide figures, thus making them intelligible at first sight.

Figure 1

The book Panorama de la Géométrie Tachy-métrie – Géométrie en trois leçons (Lagout, 1872, p. 23)



The heart, as an element of will, is revealed due to the emphasis on the importance of beautiful, pleasant and desirable aspects in the aesthetic reform of drawing, which focuses on beautiful figures and on the choice of friendly colors (green and pink, for example, as in Figure 1).

Finally, the hand represents power in the motion of figures harnessed in geometric manipulations. It is a fundamental element to change from abstraction to the concretization of concepts as a form of justifying geometric properties. Lagout (1872) emphasized that it was necessary to connect this proposal with the *boîte à manipulation* (manipulation box):

> *Pour apprendre vite et à jamais la géométrie en trois leçons, il est indispensable d'avoir manié les figures mobiles de la boîte à manipulation – Une seule boîte peut suffire à toute une classe, et la dépense ainsi répartie sera minime et le résultat en sera considérable*[9] [emphasis added]. (p. 17)

The mnemonic reform and the reform of continuous syllogism make use of the head in order to provide ready responses and stress visualization speed without considering, however, the different steps from the intuitive teaching process, going from brute intuition to simple concepts, as instructed by Pestalozzi. Such aspects differ from the intuitive method because the briefness highlighted by Lagout is not present in Pestalozzi's method. It is important to emphasize that Lagout did not use the term "intuitive geometry". He described his proposal as "concrete" and "based on arguments".

Lagout's principles (1872) stress what he expressed from the beginning, when he affirmed his respect for children:

> *Ou renoncer à enseigner la géométrie abstraite et raisonnée d'Euclide et alors supprimer le raisonnement de la Reine des Sciences du raisonnement, ou bien agréer une géométrie raisonnée, mais concrète, que tout enfant de 10 ans peut apprendre sans effort.*[10] (p. 6)

It is clear that his proposal was in opposition to classic and traditional Euclidean Geometry, so it was obvious that his method would be heavily criticized.

[9] To learn geometry quickly and for good in three lessons, it is essential to manipulate the movable figures from the manipulation box – a single box might be enough for the whole class, so the expenses will be very low, and results will be significant (our translation).

[10] We should either abandon Euclid's abstract and argument-based teaching and then remove reasoning from the Queen of the Reasoning Sciences, or we should approve a type of geometry that is based on arguments, but concrete, which any ten-year-old child is able to understand with no effort (our translation).

Criticism against Tachymetry

The articles published in the journal *Nouvelles Annales de Mathématiques* recorded the tension caused by Lagout's new proposition, Tachymetry. In 1875, Casimir Rey wrote an article under the title "*De la tachymétrie*", in which he sharply criticized Lagout's method. Two years later, in 1877, Lagout replied to Rey's criticism in the same journal with an article called "*Correspondance".*

The first tachymetric resource questioned by Casimir Rey involved the objective of studying Geometry, which should be to think precisely and not to propose "proof by eye" – it was unacceptable in Euclidean Geometry. Tachymetry did not exclude thinking or reasoning as important skills, but according to the new proposal, thinking should be primarily based on visual, and in this sense, it removed rigor from the process of proof. In fact, it was a new version of Euclidean Geometry aimed at its target-audience and at its objectives, which were clearly marked by practice and visual observation, not by argumentative thinking, according to Rey (1875):

> *L'un des buts les plus importants de l'étude de la Géométrie est d'habituer l'esprit à raisonner avec la plus complète précision en ne s'appuyant que sur des vérités indémontrables, non pas à cause de leur obscurité, mais à cause de leur extrême évidence; ce but est laissé de côté par la Tachymétrie, dont les raisonnements, conduisant à un résultat juste dans le cas spécial choisi par l'auteur, conduiraient à des résultats faux dans mille autres cas. La Tachymétrie donne démonstrations par l'œil comme irréfutables et très supérieures aux démonstrations d 'Euclide.*[11] (p. 436)

Another aspect stressed by Rey (1875) while he discussed the differences between Euclidean Geometry and the new method was that Tachymetry ignored the study of drawing and the tracing of lines and figures in order to emphasize the study of measurements. The practice and use of measurements and briefness are demands that emerged from practice, so they became a priority for Lagout, which resulted in the exclusion of other objectives of geometry teaching, for example, its relation to drawing. It is necessary to mention D'Enfert's (2007) study, in which the author discusses the invention of linear drawing in France and highlights that it mostly had no connection with professional reality. According to the same author (D'Enfert, 2003), at the turn of the century, that is, from the 19th to the 20th century, classic geometry teaching was intensely challenged, because there were demands for a form of explanation that privileged a concrete experience based on drawing activities – linear or geometric drawing – in order to clarify and apply the concepts that were studied. Tachymetry did not consider linear drawing a part of its methodology, so it opposed the movement mentioned by D'Enfert.

[11] One of the most important objectives of studying geometry is to make the mind used to thinking with the highest precision based only on indemonstrable truths – not because of their obscurity, but due to extreme evidence; this objective is not considered by tachymetry, whose thinking leads to a fair result in the case chosen by its author, but would lead to false results in thousands of other cases. Tachymetry considers proof by eye as irrefutable and greatly superior to Euclid's proofs (our translation).

Casimir Rey believed that constructing volumes with students for an initial study of shapes and decomposition, and for the manipulation of geometric solids would cause "no considerable inconvenience". However, he stressed that the proposal should not replace the study of Euclidean Geometry:

> *Que les autorités imposent dans les écoles primaires cette fabrication, qui repose quelques instants l'attention de l'élève et lui fait mieux saisir la forme ou la décomposition d'un volume nouveau pour lui, nous n'y voyons pas grand inconvénient; mais que la Tachymétrie ne remplace pas ces petites Géométries primaires, souvent signées par des géomètres éminents, qui enseignent la Géométrie d'Euclide. ... la Géométrie est la science qui apprend à raisonner juste, même sur des figures qui sont fausses, tandis que la Tachymétrie est un art qui apprend à raisonner faux, même sur des figures qui sont justes.*[12] (Rey, 1875, p. 437)

Finally, not only did Rey criticize the method, but also the people Lagout employed to assess his work. According to Rey, they had legal authority to impose the methodology because they were ministers, bishops, generals, deans, mayors and councilors, but they were not authorities on Geometry.

The argumentation presented by Lagout (1877) did not reply to all of Rey's accusations, only to two of them: the one that legitimized his method and the one involving the comparison between Euclidean Geometry (which reasons fairly based on false figures) and Tachymetry (which reasons falsely based on fair figures).

Thus, Lagout's first reply to Rey consisted of arguing that his work had been submitted to and approved by engineers who had authority to assess a study of Geometry. He offered a list containing the different institutions that assessed and approved his method:

> 1. *Au Conseil académique de Clermont, en 1872, le Recteur, des Inspecteurs d'Académie, les deux professeurs de Mathématiques du lycée, appelés par le Recteur à la Commission du Conseil académique, laquelle a assisté à plusieurs conférences d'initiation et a constaté les résultats par un examen;*
>
> 2. *Les Ingénieurs professeurs de l'École des Ponts et Chaussées;*
>
> 3. *Le Directeur de l'École des maîtres mineurs d'Alais, Ingénieur en chef des Mines (sorti le premier de l'École Polytechnique);*

[12] We do not see considerable inconvenience in the construction imposed by authorities in primary school, for it makes students focus their attention for some time and makes them understand better a shape or the decomposition of a new volume, but tachymetry does not replace the little primary geometry, frequently supported by distinguished geometers who teach Euclidean geometry. […] Geometry is the science that teaches correct reasoning, whereas Tachymetry is an art that teaches false reasoning, even though it uses fair figures (our translation).

> 4. *Le Conseil de l'École des Mines, qui m'a admis à l'honneur d'exposer ma méthode aux élèves, en vue de l'instruction technique des maîtres mineurs.*[13] (Lagout, 1877, pp. 274-275)

Lagout's second refutation considered "the false reasoning of Tachymetry" and was supported by the circulation of his work in France and other countries:

> 1. *L'épuisement de la 4e édition de l'opuscule critiqué, le* Panorama de la Géométrie*, dont le texte scientifique sera conservé intact, mais dont les accessoires indépendants seront, grâce à M. Rey, refondus ou supprimés*;
>
> 2. Le Cahier du soldat du Génie*, qui était à son quatrième tirage, est arrivé au neuvième;*
>
> 3. *II est publié en anglais, par la maison Collins de Londres, sous le nom de Livre fondamental de Tachymétrie, et se vend trois fois plus cher qu'en France (nation pratique!).*[14] (Lagout, 1877, pp. 275-276)

It is safe to say that Casimir Rey's (1875) criticism focused on the method and on its principles, almost always in comparison with the geometry teaching based on Euclid, whereas Lagout's (1877) replies were based on the authority of distinguished institutions and professionals who approved and used his work in France, as well as on the circulation of his method outside France in acknowledgement of its quality. In his article, Lagout did not discuss the reasons for the creation of his method, however, he stressed that his opponent had criticized many of his publications, but he had never mentioned the book that was fundamental to his method, *Cahier d'un soldat du Génie*, which had had its 4th edition published by that time.

The *Revue Pédagogique* also participated in the debate on Tachymetry with articles written in 1879 by M. Bovier-Lapierre[15], also considered by Lagout as an opponent. Lapierre's criticism was countered by Lagout in an article published in the same journal in 1880.

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[13] 1. No Conselho Acadêmico de Clermont, em 1872, o Reitor, Inspetores da Academia, os dois professores de matemática da escola, da Comissão do Conselho Acadêmico, que participaram de várias conferências de iniciação e tomaram nota dos resultados por exame; 2. Os professores de engenharia da Escola de Pontes e Estradas; 3. O diretor da Escola de Alais, engenheiro-chefe de Minas (formado pela primeira Escola Politécnica); 4. O Conselho da Escola de Minas, que admitiu a honra de expor meu método aos alunos, com vistas à instrução técnica dos mestres menores (tradução nossa).

[14] 1. O esgotamento da 4ª edição do panfleto criticado, o *Panorama de la Géométrie*, cujo texto científico será mantido intacto, mas cujos acessórios independentes serão, graças ao Sr. Rey, reformulados ou excluídos; 2. O *Cahier du soldai du Génie*, que estava em sua quarta edição, chegou à nona; 3. É publicado em inglês, pela casa de Collins, em Londres, sob o nome de *Livre fondamental de Tachymétrie*, e é vendido três vezes mais caro que na França (nação prática!) (tradução nossa).

[15] We were not able to find the articles written by Bovier-Lapierre, only Lagout's reply.

Again, there was a comparison between Tachymetry (concrete and argument-based geometry) and Euclidean Geometry. In relation to Bovier-Lapierre, Lagout (1880) was more incisive in his reply, claiming that his rival did not have geometric knowledge:

> *Pour fortifier sa raison, il n'existe, je crois, que deux traités: la Logique d'Aristote et la Géométrie d'Euclide interprétée par de savants professeurs de l'Université. Mais combien d'écoliers sont aptes à s'assimiler Aristote et Euclide? Un sur mille, tout au plus. La presque totalité des enfants a donc été privée de la culture régulière de sa raison. Et il est bien facile de voir que mon adversaire n'a jamais connu ni Aristote, ni Euclide, ni leurs interprètes: car ses ataques sont dénuées de tout esprit géométrique.*[16] (p. 303)

As in his reply to Rey, Lagout refused that "en dehors de la rigueur absolue, pas de mathématiques!"[17] (Lagout, 1880, p. 304), and he found it absurd that the deans, university professors and Polytechnic School engineers were considered false scientists because they had approved his method. He continued his direct personal accusations against M. Bovier-Lapierre by questioning his critic's geometric knowledge:

> *On peut s'étonner à bon droit qu'un homme déclarant avoir une longue expérience de l'enseignement et être auteur d'un livre de géométrie applicable au dessin, etc., n'ait pas trouvé l'occasion de voir que l'on pouvait décomposer un triangle quelconque en deux équerres.*[18] (Lagout, 1880, pp. 305-306)

Lagout (1880) also emphasized the arguments he presented in his defense in 1878, like the number of editions of his books and their broad circulation, especially their translations into other languages; he mentioned that *La Takimétrie* had been translated into English, Russian, Spanish and Italian. Moreover, Lagout referred to M. Dalsème as a supporter of his method. Dalsème was a teacher at Normal School of Seine who taught Tachymetry courses. In terms of authority to legitimize his method and works, he highlighted the presence of Tachymetry in a brochure to promote the Exposition Universelle (1878) at the request of the General Commissioner, specifically in an extract that contained a summary of documents.

In summary, it is safe to say that the criticism against Tachymetry in mathematics and pedagogy journals is mainly centered on the absence of rigor in its logical-deductive

[16] To emphasize the reasons, I believe there are only two treaties: Aristotle's logic and Euclidean Geometry, interpreted by university professors. However, how many students are able to assimilate Aristotle and Euclid? One in a thousand, at most. Almost all children were, therefore, deprived of a regular reasoning development. It is very easy to see that my opponent does not know Aristotle, Euclid or their interpreters because his attacks are deprived of geometric spirit (our translation).
[17] Without absolute rigor, there is no mathematics! (our translation)
[18] It is surprising that a man who claims to have long teaching experience and to be the author of a book on geometry applied to drawing, etc., has not had the opportunity to understand that we can decompose any triangle into two rectangles (our translation).

processes. The articles did not emphasize or discuss children's learning processes or Pestalozzi's intuitive method. Seemingly, the criticism against the new method occurred because it disrespected the rules of mathematical science.

Jules Dalsème's Tachymetry

Jules Dalsème worked as a primary teacher trainer and published the book *Éléments Takymétrie (Géométrie Naturelle)* in 1880. It was an adaptation of Lagout's proposal aimed at primary teachers, with focus on pedagogical aspects for the teaching of children.

Not only did Dalsème adapt Lagout's work, but he also criticized his method in order to justify his own interpretation of the tachymetric method:

> *M. Lagout, malheureusement, n'était qu'à demi pédadogue; la phraséologie tourmentée et bizarre qui entourait ses démonstrations en rendait souvent l'accès pénible. Il ne faut pas, d'ailleurs, abuser des néologismes, et l'on doit reconnaître que takymétrie sonne dur. Du vocable, donc, il convient de faire assez bon marché. L'expression: géométrie intuitive vaudrait assurément mieux, car il s'agit précisément d'un ensemble de procédés destinés à rendre plus aisément assimilables, en les matérialisant, les règles essentielles de la géométrie des arts et métiers.*[19] (Dalsème, 1889, pp. 22-23)

In comparison with Lagout's book, the first difference contained in Dalsème's work is the number of lessons. The book *Panorama de la Géométrie Tachy-métrie – Géométrie en trois leçons*, written by Lagout (1872), proposes his method in 3 lessons, whereas the book *Éléments Takymétrie (Géométrie Naturelle) à L'USAGE DES INSTITUTEURS PRIMAIRES, DES ÉCOLES PROFESSIONNELLES DES AGENTS DE TRAVAUX PUBLICS, etc.*[20] by Dalsème (1880) contains 7 lessons, as commented by Claude Georgin (1991) in *Dictionnaire du Buisson*:

> *Nous avons dit que le créateur de la tachymétrie comme système d'enseignement est M. Lagout. Un autre mathématicien, M. Dalsème, s'est aussi occupé de ce sujet, et a apporté quelques modifications à la méthode de M. Lagout, pour la rendre plus accessible aux instituteurs. M. Lagout prétend enseigner la takymétrie en trois*

[19] Unfortunately, Mr. Lagout was only partially Pedagogue; the tormented and bizarre phraseology that surrounded his manifestations often made the access to his ideas painful. Furthermore, one should never use neologisms excessively, and it is necessary to admit that tachymetry seems to be difficult. Therefore, the vocabulary used should be easy. The expression *intuitive geometry* would certainly be better because it is a set of processes that aim at making the essential rules of the geometry of arts and work more easily assimilable by materializing them. (our translation)

[20] A similar book was also published in 1880 under the title *Premières Notions de Takymetrie (Géométrie Naturelle) à L'usage des Écoles Primaires* by the same publisher, Librairie Classique D'Eugène Belin.

leçons, «que tout enfant de dix ans peut apprendre sans effort », dit-il ; M. Dalsème, dans son manuel publié en 1880, divise la matière en sept leçons.[21] (p. 2)

To better understand and highlight the differences in structure between the two books, we present their indexes. Lagout's book is organized in three lessons, as follows (Figure 2):

Figure 2
Index of Panorama de la Géométrie Tachy-métrie – Géométrie en trois leçons by Lagout (1872)

Première Leçon L'accessible
1. Figures uniformes et informes
2. Figures uniformes – Règle des mesures
3. Figures mixtes dérivées des précédentes
4. Figures informes – Plans et Volumes ; EQUIVALENCE
5. Figures uniformes – Equivalence
6. Figures informes - Equivalence

Deuxième Leçon L'inaccessible
7. Règle unique des mesures
8. RESSEMBLANCE
9. Précieuse vertu d'équerre
10. LES TROIS QUARRES de L'EQUERRE
11. L'angle au ciel
12. Perspective

Troisième Leçon Formes rondes
13. Polygone à 6 pans (hexagone)
14. Polygone à 12 pans
15. Cercle – contour
16. Cercle – aire
17. Abat-jour, mesure immédiate
18. Abat-jour, mesure transformée
19. Sphère – aire
20. Sphère – volume

Dalsème's book is organized into seven lesson, as shown in Figure 3.

[21] We have said that the creator of tachymetry as an educational system is Mr. Lagout. Another mathematician, Mr. Dalsème, also approached the same subject and made some modifications in Mr. Lagout's method in order to make it more accessible to teachers. Lagout affirms that he is able to teach tachymetry in three lessons and that "any ten-year-old child can learn it without effort"; M. Dalsème divided the subject in seven lessons in his manual published in 1880. (our translation)

Figure 3
Index of Éléments Takymétrie (Géométrie Naturelle) by Dalsème (1880)

Première Leçon.
Définitions – Volume, surface, ligne et pont – Ligne droite. Ligne brisée – Plan – Angles. Angle droit ou d'équerre. Perpendiculaires – Parallèles – Lignes courbes – Les deux règles fondamentales de la tachymétrie

Deuxième Leçon.
Naissance du rectangle. Sa division en deux équerres égales. Mesure du rectangle et de l'équarri parfait. Parallélogramme et équarri droit. Equarri oblique.

Troisième Leçon.
Le triangle – Somme des angles du triangle – Précieuse propriété de l'équerre – Mesure du triangle – Polygones et prismes.

Quatrième Leçon.
La circonférence et le cercle – Mesure des arcs et mesure des angles – Polygones réguliers – Tour du cercle – Volume et surface latérale du cylindre.

Cinquième Leçon.
La pyramide – Équivalence des pyramides – Décomposition d'un prisme triangulaire – Volume et surface de la pyramide.

Sixième Leçon.
Figures tronquées – Trapèze – Tas de cailloux. Décomposition d'un tas de cailloux en 9 parties se reconstituant en un équarri et une pyramide d'angle – Equivalence des tronquées – Troncs de pyramide – Volume et surface du tronc de cône.

Septième Leçon.
La ressemblance. Caractères précis de la ressemblance – Principales applications de la tachymétrie – Cubage d'un massif de maçonnerie – Cubage des bois – Jaugeage des tonneaux.

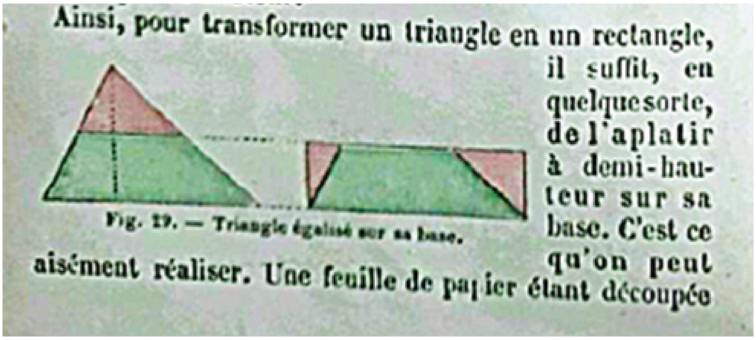
An analysis of Figures 2 and 3 highlights the level of detail inserted by Dalsème in the study of different concepts throughout his seven lessons. In addition to the number of lessons, Dalsème added pedagogical elements to the development of his proposal. For instance, in the *Première Leçon* (First Lesson), he introduced and defined the main concepts that were used throughout the book, and explained what he understood by *Takymétrie:* "*La Takymétrie nous apprend à mesurer les lignes, les surfaces et les volumes, par des règles simples et justes*"[22] (Dalsème, 1880, p.5).

Another adaptation in Dalsème's work is the distribution of illustrative figures throughout the text, unlike Lagout's (1872) book, in which the collection of figures is in a single page at the end of the publication, as shown in Figure 4:

[22] Tachymetry teaches us to measure lines, surfaces and volumes through the use of simple and fair rules (our translation).

Figure 4

Measuring a triangle in Éléments Takymétrie (Géométrie Naturelle)(Dalsème, 1880, p. 18*)*



Ainsi, pour transformer un triangle en un rectangle, il suffit, en quelque sorte, de l'aplatir à demi-hauteur sur sa base. C'est ce qu'on peut aisément réaliser. Une feuille de papier étant découpée

Fig. 17. — Triangle égalisé sur sa base.

At the end of each lesson, Dalsème summarized the main ideas, concepts and properties discussed in that section. Dalsème's book clearly highlighted the special care with which he developed the lessons. His themes are carefully explained, commented and summarized at the end of each lesson. It can be considered a more didactic version of Lagout's proposal. The use of colorful figures throughout the text contributed substantially to the understanding of his work. Lagout's (1872) 23-page book was adapted and expanded by Dalséme (1880) into 60 pages.

It is important to highlight the title chosen by Dalséme, *Géométrie naturelle*, which is different from the one used by Lagout, *Géométrie concrète et raisonnée*. The term *tachymetry* is the same in both books, but the way the two authors described geometry stated the difference between them. Another relevant aspect is that, although Dalsème's work was characterized by the pedagogical elements that it contained, he chose not to use the term *Intuitive Geometry,* but *Natural Geometry.*

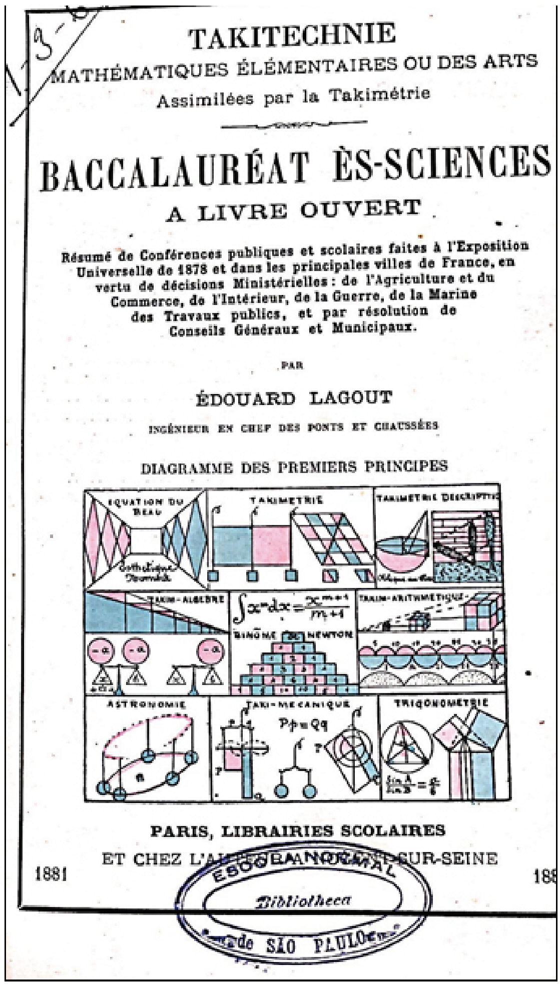
The circulation of Tachymetry

As stated several times by Lagout in response to the criticism he received in journals, his books on the tachymetric method had numerous editions and translations into different languages, which supported the international circulation of his proposal. Again, it is important to emphasize the context in which his method was produced, that is, the second half of the 19th century, which corresponded to the "first globalization", a historical period marked by trade and interchange of experiences among different countries (Matasci, 2015).

One of the examples that highlighted the connections between different countries at that time was *Exposition Universelle*. The book *Takitechnie – Baccalauréat ès-sciences – A Livre Ouvert* by Édouard Lagout (1881) circulated in the federal capital of Brazil, in the state of Rio de Janeiro and in São Paulo[23], an economically strong state that was at the forefront of educational reform in Brazil in the end of the 19th century. His book cover made reference to the *Exposition Universelle* of Paris, 1878, which proves that the publication had gained visibility (Figure 5).

Figure 5

Cover of Takitechnie – Baccalauréat ès-sciences – A Livre Ouvert (Livres – Banco de Dados de Livros Escolares, da Universidade de São Paulo (Livres – Textbook Database, São Paulo University, in free translation)



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[23] Copies from 1881, published by Paris, Librairies Scolaires, were found in Brazil. One of them at Biblioteca Nacional do Rio de Janeiro (Rio de Janeiro National Library, in free translation) and one at Livres – Banco de Dados de Livros Escolares, da Universidade de São Paulo (Textbook Database, São Paulo University, in free translation), related to the São Paulo Normal School collection. The book *Premières Notions de Takymétrie (Géométrie Naturelle) – à l'usage des Écoles Primaires* by Dalsème, published in 1880, is also part of the São Paulo Normal School collection.

Moreover, in Brazil, the 19th century was quintessentially considered a century of Francophonie, when Brazilian culture absorbed much of what was produced in France. In the field of education, French influence is significant in terms of ideas and pedagogical innovations: Brazilian intellectuals, in search of scientific support for development, appropriated French intellectuals' ideas to voice the proposals that they considered meaningful and relevant to their country (Bastos, 2000).

As mentioned in the beginning of this text, Pais (2019) analyzed newspapers from Rio de Janeiro and Maranhão, in which he identified the presence and promotion of Lagout's and Dalsème's tachymetric method in the end of the 19th century. However, Tachymetry could be found in other places in Brazil, as in the standardizations that supported the republican primary school model.

Still in the imperial times, decree No. 7.247 of April 19th, 1879, signed by minister Carlos Leôncio de Carvalho, marked the introduction of the "object lessons"[24] method in primary school regulations in Brazil. The decree generated serious discussions and was sent to the Public Instruction Commission of the Brazilian House of Representatives. The commission was composed by Rui Barbosa (rapporteur), Thomaz do Bonfim Spínola and Ulisses Vianna. Rui Barbosa's[25] report stated that:

> it constitutes a significant text due to the deep approach adopted by the legislator while discussing the theme and to the influence exerted by the document, which became a reference to republicans in their discussions and propositions about popular education in the Empire. … having considerable bibliographical material from abroad as a reference. (Souza, 2009, pp. 74-75)

Chapter 7 of volume II had the title *Matemáticas Elementares* (Elementary Mathematics, in free translation) and approached the teaching of mathematics. Barbosa (1946)[26] argued that:

> Tachymetry is the concretization of geometry, it is the teaching of geometry through material evidence, the adaptation of geometry to rudimentary intelligences: object lessons applied to measuring extensions and volumes. Due to the intervention of Edouard Lagout, a bridge and sidewalk engineer, the method provides less developed minds with ready access to the truths and fundamental rules of geometric calculation, while it offers the advantage of a more complete security in processes and perfect precision in results. … In France, tachymetry attracts the attention

[24] "The centrality of objects in the education of senses constitutes one of the main principles of the intuitive method, considered a symbol of modern school in the end of the 19th century. Also known as object lessons and sense teaching, the method became an important discursive matrix in the internationalization process of education at that period (Souza, 2013, p. 105)."

[25] Rui Barbosa (1849-1923) was a lawyer, a statesman, a lecturer, a legal expert, a flawless literate man in the Portuguese language. He had a library with 35,000 volumes, which he started in 1871 (Bastos, 2000).

[26] Rui Barbosa's quotes about the reform of primary education, from 1883, are referred in this text according to the edition *Obras Completas, volume X, Tomo II,* published in 1946.

> of central administrative councils of agriculture, commerce, war, Navy, public instruction [emphasis added]. (pp. 290-291)

In fact, Barbosa linked Tachymetry with the object lessons method, also called intuitive method in Brazil. It was intuitive and concrete geometry, which represented the *appropriation*[27] of modern pedagogical principles in the study of Tachymetry. As summarized by Barbosa (1946), "the tachymetric method is, therefore, the most rigorous, the most feasible adaptation of intuitive pedagogy laws to the popular teaching of geometry, to the instruction of children on geometry" (p. 292).

The quote also allows us to identify that Rui Barbosa's arguments stemmed from France, more specifically, they were based on Edouard Lagout, sidewalk and bridge engineer. It is very highly likely that Rui Barbosa was responsible for starting the process to legitimize the teaching of tachymetry in Brazil.

In 1889, the year of the Brazilian Independence and the beginning of the Republic, Brazilian primary education started to be organized. Decree 144-B of December 30th, 1892 authorized the preliminary school program in the state of São Paulo. Among other disciplines, article 56 included practical geometry (tachymetry) with the necessary notions for its application to volume and surface measures. That is to say, the term was used as a synonym for practical geometry and was linked to surface and volume measures. It is important to consider that, in the 1892 program, the contents were not organized according to school years.

With the creation of the educational model called "*grupo escolar*" in 1893, which organized primary school in grades and disciplines, new programs were approved and reformulated until the mid-twentieth century (1894, 1905, 1918, 1921, 1925, 1934 and 1949/50)[28]. Only the 1925 program resumed the use of the term tachymetry (in Portuguese, it was written with *qu* in 1925, *taquimetria*, not with *ch* like in 1892, *tachimetria*), which was proposed for the discipline of Geometry in the 4th grade:

> The general education common base that preliminary schools must comprise is not complete if students do not acquire some elementary preparation for calculating and measuring after discerning, drawing and modeling the geometric combinations of lines, surfaces and solids. To this end, we have introduced tachymetry in schools. Entirely ignored by us in educational practice so far, tachymetry constitutes the only system able to transform the geometric science into a universal component of popular education. Tachymetry is the concretization of geometry, it is the teaching of geometry through material evidence, the adaptation of geometry to rudimentary intelligences: object lessons applied to measuring extensions and volumes. Every time we materialize figures in order to make our explanation clear and evident,

[27] Appropriation, as we understand it, aims at a social history of interpretations focused on their fundamental determinations (social, institutional and cultural determinations) and inscribed within the specific practices that produce them (Chartier, 1990, p. 26).

[28] A complete analysis of tachymetry in all these programs can be found in Frizzarini (2014).

> we will practice this proof process. Thus, we will be able to demonstrate areas easily and objectively by making triangles, squares, etc., from sheets of paper; if we obliquely cut a rectangle, we can form an equivalent parallelogram with the two pieces; similarly, we will transform a triangle into a rectangle, a trapezium into a triangle, a rhombus into a triangle, etc. [emphasis added]. (Primary School Teaching Program[29], 1941)

The example in the end of the quote (the transformation of a triangle into a rectangle refers to the illustration in Figure 2, from Dalsème's book). The program emphasizes that Tachymetry still had not been incorporated, but it justified its importance because it simplified proof processes. There was no mention of its briefness.

Moreover, there is a reference to authors Lagout and Dalsème in the preface of the book *Noções Intuitivas de Geometria Elementar*, published in 1895 by the headmaster of *Escola Normal de São Paulo*, Gabriel Prestes (1895)[30]:

> I heard about publications by Edouard Lagout and by mathematician Dalsème, the creators of the tachymetric system. … Both books are excessively reduced and exclusively practical: the former contains only thirty-six pages, and the second has seven lessons. … As it was impossible to obtain the publications I have just mentioned, I do not have any further information on them. (p. 10)

In summary, the analysis of São Paulo state primary school programs indicates the circulation of Tachymetry. Even though they were not translated into Portuguese, the books circulated, and Lagout's and Dalsème's ideas and proposals gained new interpretations in Brazil. Unlike what occurred in France, in Brazil tachymetry was seemingly adapted as an application of the intuitive method to geometry teaching and to calculations of areas and volumes. Nevertheless, the speed of the method or its establishment as a substitute for Euclidean Geometry were not discussed as it had been done in France. Brazilian tachymetry was proposed in association with Euclidean Geometry and, in the 1925 program, it was recommended in the end of primary school, in the 4th year of school, after children had studied Euclidean Geometry. In other words, it was a series of reinterpretations and resignifications of a method that had been created in a distant place for other purposes.

[29] The 1925 program for *grupos escolares* was published as *Programa Mínimo de 1934* (1934 Minimum Program, in free translation) in the book *Programa de Ensino para as Escolas Primárias* (Primary School Teaching Program, in free translation) in 1941.

[30] A detailed analysis of Prestes's book can be found in Leme da Silva (2019).

CONCLUSION

This study aimed at discussing the background to the production, acknowledgement and circulation of a new geometry teaching method – Tachymetry. Created for the purposes of professional training and inserted into a context of pedagogical changes connected and articulated with the "first globalization", the method circulated, received criticism and acknowledgement outside France and was consequently reinterpreted, in a continuous process of creative consumption.

An analysis of the criticism received by the tachymetric method and by its creator, Lagout, allowed us to reflect on how the clash between concrete and abstract, in relation to geometry teaching methods, is longstanding and has offered resistance. Altering representations – like the one that considers Euclidean Geometry perfect for all cultures – involves establishing contact with historical processes and knowing characters that made contributions, recorded their proposals and left a legacy, in spite of the criticism they received. Lagout and Dalsème can be considered relevant agents in the hard and challenging task of producing geometry teaching for the early primary school years.

It is important to highlight the Brazilian alignment with international proposals since the 19th century. However, the appropriations produced in Brazil were numerous due to the dimensions of our country. The example contained in the São Paulo programs expresses the movement of permanence and ruptures in the production of school knowledge in search of a type of geometry for children.

Finally, the term Tachymetry, which is understood today as "fast measuring", can be considered completely separated from the geometry teaching process. Nevertheless, at its core, we can find the argument in favor of visual observation in geometry teaching, and the possibility of manipulating concrete objects built for educational purposes (manipulation box); in addition, there was the argumentative reasoning exercise for children, adapted for students in the early years of school.

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AUTHOR CONTRIBUTION STATEMENT

M.C.L.S. inventoried the sources related to the states of São Paulo and Rio de Janeiro. M. M. collected the sources related to French authors. Both authors were responsible for the conception, organization and data analysis performed in this work.

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