Design of self-instruction textbooks on number and operations by preservice elementary school teachers: A preliminary Study

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ABSTRACT

Context: Study framed in the area of teacher training. **Objective**: To analyze the self-instruction booklets that elementary education future teachers designed and implemented to teach a subject related to the Numbers and Operations axis. **Design**: The research follows a qualitative, exploratory-descriptive methodology, using the content analysis method. **Setting and participants**: Sample composed of 85 preservice teachers who studied *Numbers and Operations in Elementary School*, attending the first semester of *Pedagogy in Basic General Education with Mention* in a Chilean university. **Data collection and analysis**: The activities and conclusions of the booklets are analyzed, considering the units of analysis: implemented learning objective, detected obstacles, type of activities and contexts, assessment of the pedagogical knowledge of the content and general experience. **Results**: The work emphasizes that future teachers mostly: design booklets associated with the first two years of elementary school; consider the objective associated to counting to 1,000; declare the epistemological obstacle; the activities are exercise-like; activities are posed without context; indicators associated to the subdomains of knowledge of content and of the students, and content and teaching of the pedagogical knowledge model of mathematics are not mentioned; value this resource to encourage mathematics learning. **Conclusions**: The work with booklets has allowed preservice teachers to approach teaching work for the first time, involving mastery and application of both didactic and knowledge of the subject matter for teaching numbers, which will improve during their training.

Keywords: Self-instruction textbooks; Teacher training; Elementary education.

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Diseño de cuadernillos de auto instrucción sobre número y operaciones por futuros profesores de Educación Primaria: un estudio preliminar

RESUMEN

Contexto: Estudio enmarcado en la línea de formación de profesores. **Objetivo**: Analizar los cuadernillos de auto instrucción que han diseñado e implementado futuros profesores de Educación Primaria para la enseñanza de un tema relacionado con el eje Números y Operaciones. **Diseño**: Se sigue una metodología cualitativa, de nivel exploratorio-descriptivo, utilizando el método de análisis de contenido. **Entorno y participantes**: Muestra formada por 85 futuros profesores que cursaron *Números y Operaciones en Primaria*, del primer semestre de *Pedagogía en Educación General Básica con mención*, de una universidad chilena. **Recopilación y análisis de datos**: Se analizan las actividades y conclusiones de los cuadernillos, considerando las unidades de análisis: objetivo de aprendizaje implementado, obstáculos detectados, tipo de actividades y contextos, valoración del conocimiento pedagógico del contenido y la experiencia en general. **Resultados**: Se evidencia que los futuros profesores, mayoritariamente: diseñan cuadernillos asociados a los dos primeros cursos de primaria; consideran el objetivo asociado a contar hasta 1000; declaran el obstáculo epistemológico; las actividades son del tipo ejercicios; las actividades son planteadas sin contexto; no se mencionan indicadores asociados a los subdominios de conocimiento del contenido y los estudiantes, y del contenido y la enseñanza, del modelo de conocimiento pedagógico de la matemática; valoran este recurso para incentivar el aprendizaje de la matemática. **Conclusiones**: El trabajo con cuadernillos ha permitido que los futuros profesores tengan una primera aproximación a su labor docente, implicando el dominio y la aplicación de conocimientos didácticos y disciplinarios sobre la enseñanza de los números, lo que se irá afianzando durante su formación.

Palabras clave: cuadernillo de auto instrucción; formación de profesores; Educación Primaria.

Elaboração de cartilhas de autoinstrução sobre número e operações de futuros professores do ensino fundamental: um estudo preliminar

Contexto: Estudo enquadrado na linha de formação de professores. **Objetivo**: Analisar as cartilhas de autoinstrução que os futuros professores do ensino fundamental projetaram e implementaram para ensinar um tópico relacionado ao eixo Números e Operações. **Modelo**: Segue uma metodologia qualitativa, exploratória-descritiva, utilizando o método de análise de conteúdo. **Ambiente e participantes**: Amostra composta por 85 futuros professores que cursaram *Números e Operações no Fundamental*, no primeiro semestre de *Pedagogia no Ensino Geral Básico com menção*, de uma universidade chilena. **Coleta e análise de dados**: As atividades e conclusões das cartilhas são analisadas, considerando as unidades de análise: objetivo de aprendizagem implementado, obstáculos detectados, tipo de atividades e contextos, avaliação do conhecimento pedagógico do conteúdo e da experiência em geral. **Resultados**: É evidente que os futuros professores, em sua maior parte: elaboram cartilhas associadas aos dois primeiros anos do ensino fundamental; consideram o objetivo associado à contagem até 1.000; declaram o obstáculo epistemológico; as atividades são do tipo exercício; as atividades são propostas sem contexto; não são mencionados indicadores associados aos subdomínios do conhecimento de conteúdo e dos alunos e do conteúdo e do ensino do modelo de conhecimento pedagógico da matemática; valorizam este recurso para incentivar o aprendizado de matemática. **Conclusões**: O trabalho com

as cartilhas permitiu que futuros professores se aproximassem pela primeira vez de seu trabalho docente, envolvendo o domínio e a aplicação de conhecimentos didáticos e disciplinares no ensino de números, que serão fortalecidos durante sua formação.

Palavras chave: cartilha de autoinstrução; formação de professores; educação fundamental.

INTRODUCTION

In Chile, in recent times, the training of future teachers (FT) has been the object of research interest, with emphasis on both public policy and the general and specific knowledge that they must have (Ávalos, 2014).

The literature shows the existence of problems regarding the FT's didactic training and training in the subject matters (e.g., Sotomayor-Echenique, Coloma-Tirapegui, Parodi-Sweis, Ibáñez-Orellana, Cavada-Hrepich & Gysling-Caselli, 2013), as well as the poor relationship between practice and teacher knowledge (Vaillant, 2007). In the Chilean case, those who are trained to be elementary school teachers obtain poor results in the area of mathematics, motivating the change in curricula and teaching methodologies (Estrella, Olfos & Mena-Lorca, 2015; Olfos, Zakaryan, Estrella & Morales, 2019).

From the curriculum point of view, after the last changes, math classes for elementary education has been declared a priority, granting a minimum of 6 hours per week for its academic work, aiming to:

> (…) Enrich the understanding of reality, facilitate the selection of strategies to solve problems and contribute to the development of critical and autonomous thinking in all students, whatever their life and study options at the end of the school experience (MINEDUC, 2018, p. 214).

Math teaching is organized in five thematic axes: 1) Numbers and Operations; 2) Patterns and Algebra; 3) Geometry; 4) Measurement; and 5) Data and Probabilities (MINEDUC, 2012a). The importance of the first axis is that:

> (...) it covers both the development of the concept of number and the skill in mental calculation and the use of algorithms. Once students assimilate and construct the basic concepts, with the help of metaphors and representations, they learn the algorithms of addition, subtraction, multiplication and division, including the positional system of writing numbers. It is expected that they develop mental calculation strategies, starting with small numerical areas and expanding them in the upper courses, and that they come closer to rational numbers (such as fractions, decimals and percentages) and their operations. In all axes, and especially in numbers, learning must begin by having students manipulate concrete or didactic

material and then moving on to a pictorial representation that, finally, is replaced by symbols (MINEDUC, 2018, p. 128).

Concomitantly, the guiding standards for graduates in basic education pedagogy have been established (MINEDUC, 2012b). Those standards consider that the pedagogical components and components related to the subject matters are elements that define the basic knowledge that FTs must have when graduating. Specifically, they aim to "guide teacher training institutions, by establishing standards that instruct what every teacher is expected to know at the end of their initial training years" (MINEDUC, 2012b, p. 3). The pedagogical standards are the knowledge, skills and attitudes common to all teachers, no matter the specialty of study, that is, the teacher must know their students, the elementary education curriculum, as well as the fundamental elements of the instruction process (planning, teaching, evaluation and reflection). The FTs are expected to demonstrate, among others, that (MINEDUC, 2012b, p. 17):

- They know the elementary education students and know how they learn;
- They know the elementary education curriculum and use its various curriculum instruments to analyse and formulate teaching and assessment proposals;
- They know how to design and implement teaching-learning strategies, appropriate for the learning objectives and according to the context;
- They are prepared to manage the class and create an appropriate environment for learning according to contexts;
- They know the evaluation methods and know how to apply them to observe students' progress and know how to use the results to provide feedback on learning and teaching practice;
- They are prepared to address diversity and promote integration in the classroom.
- They communicate orally and in writing effectively in various situations associated with their teaching work;
- They learn continuously and reflect on their practice and their insertion in the educational system

On the other hand, the disciplines standards are divided into four axes: Numbers (6 standards); Geometry (5 standards); Algebra (3 standards); and Data and Probabilities (3 standards); highlighting, in this way, the importance of the Number axis and its associated contents.

From an international perspective, the knowledge that teachers who teach mathematics must have at different educational levels has been the subject of research and reflection by various authors (e.g., Ball & Bass, 2000; Ball, Hill & Bass, 2005; Ball, Lubienski & Mewborn, 2001; Ball, Thames & Phelps, 2008; Carrillo, Climent,

Contreras & Muñoz-Catalán, 2013; Godino, 2009; Gómez, 2007; Hill, Rowan & Ball, 2005; Pino-Fan & Godino, 2015; Ponte & Chapman, 2008), motivated by Shulman's works (1986, 1987), proposed, from various epistemological perspectives of knowledge and education, models that seek to describe, assess and guide the teaching and learning processes, highlighting the knowledge and its use in the different teaching situations. In this sense, the Mathematical Knowledge for Teaching (MKT) (Ball et al., 2008) introduces the notion of mathematical knowledge for teaching, assuming it as "the mathematical knowledge that the teacher uses in the classroom to produce instruction and development in the student" (Hill, Ball & Schilling, 2008, p. 374). This type of knowledge is what characterizes the teacher, of any educational level, who teaches mathematics (Carrillo et al., 2013; Varas, Lacourly, López & Giaconi, 2013), allowing teachers to anticipate possible conflicts of meaning that emerge during the student's resolution of mathematical tasks and thus predict the complexity of the teaching process.

Also, two types of knowledge have been emphasized: the mathematics knowledge and the teaching knowledge of mathematics (Ball, 1990; Ball et al., 2008; Ball et al., 2001; Hill & Ball, 2004). The latter considers the following subdomains:

Knowledge of the content and of the students. It implies the "knowledge of the content that is intertwined with the knowledge of how students think, know or learn a specific content" (Hill et al., 2008, p. 375), composing the following indicators: 1) Recognizes frequent errors of the students; 2) Anticipates student answers; 3) Recognizes students' difficulties or misconceptions to understand a content or concept; 4) Selects the most appropriate activities to build a mathematical knowledge, by virtue of knowing what will seem easy, difficult, interesting, boring, overwhelming or motivating; 5) Identifies the strategies they use to solve the problems.

Knowledge of the content and of teaching. It combines knowledge about teaching with knowledge about mathematics (Ball et al., 2008), allowing us to understand the teachers' decisions when using strategies, powerful examples, analogies and teaching resources, among others (Carrillo et al., 2013). What is specified in the following indicators: 1) Demonstrates mastery in the election of representations to teach a specific content and the use of different methods and procedures to teach that mathematical content; 2) Sequences the tasks, choosing the examples to start with the activity and the examples they use to help students deepen the content; 3) Considers in experience the appropriate and relevant resources and teaching materials to teach an idea or a mathematical situation; 4) Plans the activities, considering the different learning contexts.

With the above, we observe the relationship between the *guiding standards* for the FTs (MINEDUC, 2012b) and the domains *of pedagogical content knowledge* (Ball et al., 2008), where it is established that the FTs, upon graduating, look for the best ways to teach mathematical content, learn how students learn and how to help them overcome the difficulties they face in their academic work.

Under this premise, Elementary Education FTs were asked to design and implement a self-instruction resource (Babakhani, 2011; Montague, 2008; Steedly, Dragoo, Arafeh & Luke, 2008), specifically, a booklet to guide the work of some content considered in

the axis *numbers and operations*. Through the self-instruction booklets, FTs should be able to incorporate mathematical content in situations that students of a given course can develop autonomously (Olivares-Escanilla, 2012).

In accordance with the above considerations, we aim *to analyse the self-instruction booklets the Elementary Education FTs have designed and implemented for teaching a topic related to the Numbers and Operations axis*.

METHODOLOGY

This research follows a qualitative methodology (Pérez-Serrano 1994), of an exploratory-descriptive level (Hernández, Fernández & Baptista, 2010), using the content analysis method (Cohen, Manion y Morrison, 2011), seeking to analyse the booklets of self-instruction created and implemented by 85 FTs who attended the subject of *Numbers and Operations in the First Years of Education*, corresponding to the first semester of the *Pedagogy career in Basic General Education with mention*, from the Catholic University of Maule (Chile). This task considers, in the first place, designing a self-instruction booklet that details: data on the subject matter, name of the booklet, description of the resource in its pedagogical and didactic aspects (learning objectives (LO), achievement indicators, description of the mathematical object and progression of contents) and learning activities (introduction to the mathematical object, reproduction, and exercise). Secondly, the application of the resource is required in a student of the course associated with the LO selected. Finally, the PT must prepare a report, detailing the elements described above, as well as the difficulties and obstacles observed in the implementation, and a general conclusion about the task.

This resource aimed at being a facilitating instrument of students' autonomous learning, of some course of 1st to 4th of Elementary Education, specifically in the *Numbers and Operations* axis.

For this research, based on the type of design of the task and the instructions given by the teacher of the subject matter, the first units of analysis are defined, whose categories are established a priori:

Learning Objective (LO). The LOs established in the *Curriculum Bases* for Elementary Education (MINEDUC, 2018) "define the purposes and achievements of the process and establish what the student's performance would be that will allow verification of learning achievement" (p. 13). The 44 LOs proposed in the *Number and Operations* axis were considered for the first four grades of Elementary Education (10 for the 1st grade, 11 for the 2nd grade, 11 for the 3rd grade and 12 for the 4th grade). To cite them, throughout the work the nomenclature LO was considered, accompanied by the letters C (corresponding to the course) and N (corresponding to the number of the objective in that course). For example, the LO 4.7 corresponds to Basic 4 and objective N°7 of this axis (solving routine and non-routine problems in everyday contexts that include money, selecting and using the appropriate operation). The Annex bring the complete list of the Os.

Obstacles. They are related to the difficulties recognized by the FTs when designing and/or implementing the booklet, which prevents progress in the tasks proposed and in the construction of new knowledge. In this work, we consider those defined by Brousseau (1998): *ontogenetic*, *epistemological* and *didactic* obstacles.

Type of activity. Situations proposed by the FTs to be developed by the students in the different sections of the booklet. These were classified into *exercises* and *problems.*

Type of contexts. To analyse the situations in which the work on the issues of *Number and Operations* in the booklets is proposed, we consider the contexts proposed in PISA (OECD, 2013): 1) *personal;* 2) *labour;* 3) *social;* and 4) *scientific*.

In addition, the conclusions drawn up by the FTs, which allowed us, from the above, to define the following units of analysis and their components (a posteriori) were considered a relevant aspect in this task.

Assessment of the pedagogical knowledge of the content. The research considered elements related to the pedagogical knowledge of the content of the Ball et al. (2008), in particular: 1) knowledge of the content and the students; 2) knowledge of the content and of teaching. To do this, the conclusions drawn up by the FTs about the design and implementation of the booklet are assessed.

Assessment of the activity. The conclusions de FTs drew on the design and implementation of the self-instruction booklet are analysed through their assessment of aspects of the mathematical content, the work as a future teacher, among other aspects.

All units of analysis and their respective categories have been defined based on the contributions of the theory and relevance within the Didactics of Mathematics.

ANALYSIS AND RESULTS

In this section, we present the results regarding the LOs considered in the booklets, the obstacles declared by the FTs, the types of activities declared, the types of contexts, the assessment of the pedagogical knowledge of the content and, finally, the assessment of the professional activity.

Firstly, the FTs selected the educational level (grade) to which this resource is intended, being able to choose one or more LOs related to the *Numbers and Operations* axis. Table 1 presents that the highest percentage corresponds to the 2nd grade, with 32.9%, followed by the 1st grade with 30.6%.

Table 1
Distribution of booklets according to course.

Grade	Frequency	Percentage
1st	26	30.6
2nd	28	32.9
3rd	18	21.2
4th	13	15.3
Total	85	100

Objectives considered in the self-instruction booklet

According to the working instructions, the FTs could choose one or more LOs proposed in the curriculum framework to design various activities related to the *Numbers and Operations* axis. Table 2 shows the distribution of the objectives selected, with LO 2.1 and 3.5, corresponding to the 2nd and 3rd grades the ones with the highest frequency, with 9.4% and 8.2%, respectively.

Table 2

Frequency and percentage of LO selected.

Objective	Frequency	Percentage (n=85)
2.1	8	9.4
3.5	7	8.2
1.1 and 1.9	6	7.1
1.3	5	5.9
2.3, 3.3, 4.9 and another axis	4	4.7
2.2, 2.9 and 4.1	3	3.5
1.4, 1.5, 1.6, 2.5, 2.7, 2.8, 2.11, 3.1, 3.8, 4.3 and the union of LO	2	2.4
1.10, 3.2, 3.10, 4.6, 4.8, 4.10 and does not declare	1	1.2

In addition, 35 FTs will alter something at the LO selected from the *Curriculum Bases, such as* eliminating or altering words, or merging with another LO. For example, FT6 changes the word *number* by *element* in LO 2.1. This change apparently highlights the conceptual mastery of the FT, by recognizing that the number is an abstract representation (Aharoni, 2012; Alsina, 2012; Baroody, 1997; Benacerraf, 1983), therefore impossible to read. Only the graphic or numeral record can be read. FT83 restricts the numerical scope of LO 2.1 to 50, and not to 1.000, although retaining the mistake of considering the number as an object that can be read and not as the representation of a quantity.

Finally, FT21 generates a new objective by putting LO 1.1 and 1.3 together, although with inaccuracies:

> "Quantify numerals from 1 to 100 and represent them in concrete, pictorial or symbolic form" FT21.

Obstacles

To study the possible obstacles that students may face, we used Brousseau classification (1998). In relation to the *epistemological obstacles*, they are understood as those attributed to the student that account for a conceptual vacuum that prevents the acquisition of new knowledge. In this context, FT 30 indicates that a difficulty can be generated in the understanding of the multiplicative algorithm of the natural numbers.

> "A possible mathematical error is that students do not understand that the product of a multiplication will always be larger than the multiplication and the multiplier" FT30

The second is the *ontogenetic obstacle*, which refers to those specific genetic conditions of the students that make learning difficult or impossible. FT 14 mentions the difficulty in writing the numbers, which, as a motor perceptual ability, must be developed gradually to interpret the information.

> "Alteration in the writing of the numbers, for example: in the case of the number 3, one should write it simulating the E (handwritten capital letter)" FT14

The *didactic obstacle* originates from the decisions that the teachers make in the teaching practice and the way in which they develop the teaching and learning processes, in this case, the authors of the booklets. FT16 mentions as a possible limitation the way in which the mathematical content is presented in the booklet.

> "That the delivery of the content has not been clearly or easily understood by the student" FT16.

Finally, the *other* category was generated, which includes those aspects not considered in the previous categories (e.g., attitude, motivation or affective) and that hinder the construction of new knowledge. For example, FT4 mentions as a possible obstacle the student's motivation to face this type of learning situations.

> "The student show no interest in elaborating the booklet" FT4.

In Table 3, we see the distribution of 289 obstacles recognized and/or exemplified by 70 of the 8FTsPF. 62.3% of them have been classified as epistemological, followed by those classified as other, with 23.2%. *Ontogenetic* and *educational* obstacles are poorly recognized (7.3% each).

Table 3

Distribution of obstacles identified by the FT

Type of obstacles	Frequency	Percentage
Epistemological	180	62.3
Ontogenetic	21	7.3
Didactic	21	7.3
Other	67	23.2
Total	289	100

Type of activities

The different activities included in the 85 works were analysed considering the type of task proposed. For this, two types were considered:

Exercises. A task is considered repetitive when the student knows in advance what to do to solve a proposition (Isoda & Olfos, 2009). An exercise is that activity that has an immediate resolution strategy (Díaz & Poblete, 2001). Figure 1 shows a task that was classified in this category. Here, the student had to replicate previously worked activities, involving giving the sum, and then colouring the boxes to represent each figure of the addition.
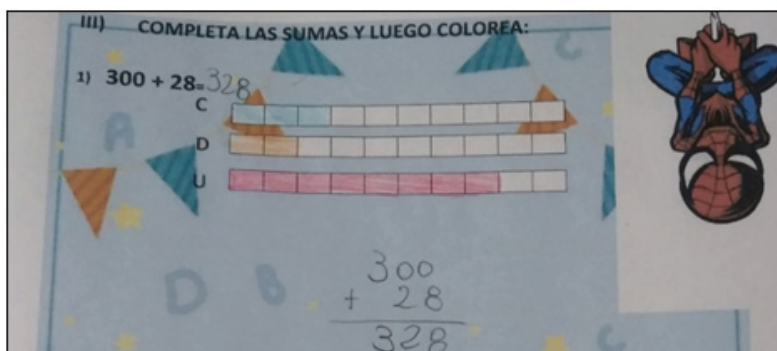

Figure 1. Activity considered as an exercise (PF40)

Problems. Problems are those tasks that do not have a known resolution strategy (e. g., Alsina, 2019; Díaz & Poblete, 2001; Polya, 1945; Santos-Trigo, 2014; Schoenfeld,

1985). Figure 2 shows a situation considered a problem, raised by the FT62, where the student must calculate the total to pay, the change the person should receive back, and whether the change is enough to buy something else.

4. Pamela tiene una pastelería, en la cual vende pie de limón a $250, repollitos a $150 y roscas a $120 c/u. Si Bastián quiere comprarle a su polola un pie de limón, dos repollitos y dos roscas. ¿Cuánto es el total que tiene que pagar? Si pagó con $1000, ¿Cuánto vuelto tiene que entregarle Pamela? Con el vuelto ¿Le alcanza algo más?

Figure 2. Activity considered a problem (FT6)

Table 4 shows the distribution of teaching activities, according to their classification in exercises or problems. In this table, we observe that the FTs focus, in their self-instruction booklets, activities classified mostly as *exercises* (78.8%); and that those considered *problems* reach 21.2%.

Table 4
Type of teaching activities

Activity	Frequency	Percentage
Exercises	681	78.8
Problems	183	21.2
Total	864	100

Contexts

A fourth unit of analysis corresponds to the type of context used to present the activity, where the data and the result make sense. For this case, we considered the contexts described in PISA (OECD, 2013):

Personal. Considers those activities that are related to situations close to the students. For example, the activity we see in Figure 3 is based on the organization of a girl's birthday (María).

Work-related. Considers those situations that are based or related in the world of work, involving sales, inventory making, among others. For example, the activity in Figure 4 shows the amount of fruits (oranges) at a fair stand (inventory).
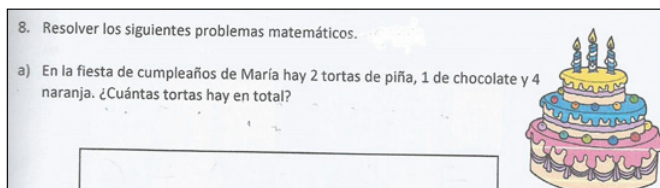

8. Resolver los siguientes problemas matemáticos.

a) En la fiesta de cumpleaños de María hay 2 tortas de piña, 1 de chocolate y 4 naranja. ¿Cuántas tortas hay en total?

Figure 3. Activity proposed in a personal context (FT15)

Figure 4. Activity proposed in a working-related context (FT11)

Social. Considers those activities that refer to the local or a wider community, with which students observe a certain aspect of their environment. An example of this context is seen in the activity of Figure 5, referring to a two-team soccer match, which can be considered local communities.

Figure 5. Activity proposed in a social context (FT38)

Scientific. Those activities that refer to the application of mathematics in other disciplines or subjects, such as science, technology or mathematics. An example of this context is seen in Figure 6, where the student is asked to create a problem from the multiplicands.

No context. Those activities that are not framed within any of the previous contexts, generally associated with algorithmic tasks. For example, the activity in Figure 1, which asks the student to solve an addition and paint the squares according to the numbers of the addition, without posing a greater cognitive challenge.
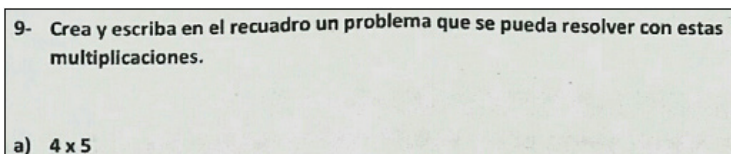

9- Crea y escriba en el recuadro un problema que se pueda resolver con estas multiplicaciones.

a) 4 x 5

Figure 6. Activity proposed in a scientific context (FT30)

Table 5 shows the distribution of teaching activities according to the contexts in which they are framed. We see that most of them are not proposed within a given context (72.8%), since they are limited to replicating certain algorithms. Then, there are the activities that refer to some aspect close to the student (*personal context*) (21.8%) and the other contexts are poorly represented in the activities proposed.

Table 5
Frequency and percentage of contexts identified in the activities of the self-instruction booklets

Context	Exercises		Problems		Total	
	Frequency	Percentage	Frequency	Percentage	Frequency	Percentage
Personal	82	12	106	57.9	188	21.8
Working-related	30	4.4	7	3.8	37	4.3
Social	1	0.1	2	1.1	3	0.3
Scientific	1	0.1	6	3.3	7	0.8
No context	567	83.3	62	33.9	629	72.8
Total	681	100	183	100	864	100

Knowledge of content and students

This subdomain intertwines knowledge of content with knowledge of how students think, know or learn a specific subject (Hill et al., 2008). In the case of the self-instruction booklet, those elements that the FTs highlight in their proposal linked to this subdomain were identified. For example, the FT32 recognizes the importance of the design of activities, referring to the indicator *selects the most appropriate activities to build mathematical knowledge by knowing what will seem easy, difficult, interesting, boring, overwhelming or motivating*.

> "creating activities to make math learning fun, that it is applied in an easier and more playful way [...] to make this content more enjoyable for students" FT32.

Regarding *identifying the strategies they use to solve the problems,* FT3 mentions the importance of the development of the activities of the booklet and its analysis to interpret the way of thinking of the student when facing certain problems.

"the importance of this booklet is the performance of activities, the development and analysis used by the student based on the mathematical reasoning used" FT3.

FT20 conjectures about the student's difficulties in some of the booklet activities has designed. Situation related to the indicator *recognizes frequent student errors*.

"once the child performs the activities, we conclude that the student was confused with some numbers, which caused him to feel uncertain" FT20.

Regarding the indicator *anticipating the students' answer*, FT64 mentions the relevance of being prepared for the possible difficulties that they have, to have the tools to help students to overcome them.

"be precise with the statement of the activity, taking into account immediately the possible difficulties or obstacles they may have and thinking about the possible answers" FT64.

The indicator *recognizes the difficulties or misconceptions to understand a content or concept*, we observe it in what was mentioned by the FT9, when identifying difficulties in the tasks and finding ways to solve it.

"there were very few difficulties in the process, anyway, we knew how to solve them in a successful and useful way" FT9.

In the case of not being able to link the conclusion with any previously mentioned indicator, the *unobserved indicators category* was included. An example of this situation is recognized in FT1.

"the student who is not clear about the content prior to the subject harms his/her ability to move forward with the contents" FT1.

Table 6 shows the distribution (frequency and percentage) presented by each of the indicators related to the knowledge of the content and the students. It is observed that the FTs do not recognize indicators related to this sub-domain (*unobserved indicators*) (63.5%), while only 17.6% highlight the importance of selecting activities, according to their level of complexity or interest. The remaining indicators are poorly observed.

Table 6
Frequency and percentage of indicators of knowledge of the content and students

Indicators	Frequency	Percentage
Recognize frequent student errors	3	3.5
Anticipate students' answers	4	4.7
Recognize the difficulties or misconceptions to understand a content or concept	7	8.2
Select the most appropriate activities to build a mathematical knowledge by knowing what will seem easy, difficult, interesting, boring, overwhelming or motivating.	15	17.6
Identify the strategies they use to solve problems	2	2.4
Not observed	54	63.5
Total	85	100

Knowledge of content and teaching

This subdomain combines knowledge about teaching with knowledge about mathematics (Ball et al., 2008), allowing us to understand the teachers' decisions when using strategies, examples, teaching resources, among others (Carrillo et al., 2013).

The indicator *show mastery in the choice of representations, use of methods and procedures to teach specific content*, as indicated by FT48, who emphasizes the need to design activities with creativity and dedication.

> "creativity and dedication is required to design an activity […] we must put ourselves in the place of the students and try to make the methodology suitable for any student who solves it" FT48.

On the indicator, *sequence the tasks, choosing the examples to start the activity and deepen the content*, the FT63 points out the need to use activities of gradual complexity.

> "the exercises I had to solve were ordered from the simplest to the most complex" FT63.

The FT55 mentions the need to apply mathematics in real situations, as well as the use of different resources, where the student's experience is important for learning. This is directly related to the indicator *consider, in the experience, appropriate resources and teaching materials to teach a mathematical idea.*

"by applying mathematics in real situations […] using the manipulation of concrete materials, exploring, questioning and discussing" FT55.

The FT72 mentions the need to use different elements, to facilitate learning gradually and respecting the different rhythms and ways of learning. This is related to the indicator *plan the activities considering the different learning contexts*

"on the other hand, the decomposition is a slightly more complex factor for children to understand, because separating an amount or object is more difficult [...] must be done in a more pictorial and concrete way to better understand" FT72.

If the indicators cannot be linked to any of the descriptors above, one called *unobserved indicators* has been created. An example of this situation is drawn, in part, from FT74's conclusion, who cites the students' prior knowledge and not the teacher's role in the teaching of mathematics.

"by having a good mathematical base at the beginning of formal regular teaching, better results will be obtained in future learning" FT74.

Table 7 shows the distribution of indicators related to content knowledge and teaching of the mastery of teaching knowledge of the mathematics of Ball et al. (Ball, 1990; Ball et al., 2008). We observe that, in general, the FT do not refer to this subdomain (55.3%). From the indicators observed, the most frequent are those related to the correct choice of activities and methods for the effective teaching of a subject (16.5%) and considering the experience an important resource for teaching and learning processes (14.1%).

Table 7

Frequency and percentage of knowledge of content and teaching indicators

Indicators	Frequency	Percentage
Master the election of representations, use of methods and procedures to teach specific content	14	16.5
Sequence the tasks by choosing the examples to start the activity and deepen the content	3	3.5
Consider the appropriate experience, resources and teaching materials to teach a mathematical idea	12	14.1
Plan the activities considering the different learning contexts	9	10.6
Not observed	47	55.3
Total	85	100

Assessment of the activity

We also analysed the PFs' assessment of the design and implementation of the self-instruction booklet, finding the following categories:

Management of the mathematical object. When the FTs considers relevant, when planning mathematical experiences, to master the topics to be taught, to be able to explain clearly to their students and, thus, help them build learning. This last aspect is clearly mentioned by FT60.

> "mastering numbers in the first years of schooling is fundamental for the next development of mathematical knowledge" FT60.

Professional work. This category considers the reflection on the teacher's work, the importance of developing these types of tasks, as a way of approaching their future professional work. As mentioned by FT12.

> "The development of this work serves to assess the work and effort that all professionals make to bring and present class-to-class material to their students" FT12.

Affective aspect. It considers the FTs' comments about the sensations produced by the elaboration or application of the self-instruction booklet. As mentioned by FT5.

> "it was a very nice experience to be able to work with a student and guide him […] I was fascinated to be able to work hand in hand with a child" FT5.

Students' attitude towards homework. This indicator includes those aspects observed by the FTs in relation to the student's way of working when developing the activities proposed in the booklet, as mentioned by FT82.

> "creative and entertaining activities are a tool of great benefit as a teaching resource because it allows students to arouse interest in the study of mathematics" FT82.

Not observed. This category considers aspects that are not linked to the other categories defined above. For example, what FT79 mentions, in the design and implementation of the task.

> "it was an extensive work" FT79.

Table 8 shows the distribution (frequency and percentage) of the assessment indicators based on the design and implementation of this resource, mentioned in the conclusions, and that can relate to more than one aspect. The table shows that the highest frequency indicator is associated with the *attitude of the students towards the task* (45.9%), followed by *management of the mathematical object* to be worked on (42.4%) and the assessment of the *professional work* (40%).

Table 8
Frequency and percentage of assessment indicators based on discourse analysis

Assessment	Frequency	Percentage (n=85)
Dealing with the mathematical object	36	42.4
Professional work	34	40
Affective aspect	28	32.9
Students' attitude towards the task	39	45.9
Not observed	11	12.9

CONCLUSIONS

The teachers' initial training should facilitate the development of different capacities, to allow them to demonstrate pedagogical, didactic and disciplinary competences. Cossío and Hernández (2016) sustain, from the review of the literature, that the way teachers signify teaching and learning determines the strategies and practices they carry out within their classrooms for their students to build learning from the contents established in the curriculum.

Thus, recognizing the importance of the initial teacher training discussed in the previous sections and the processes involved, we aimed to analyse self-instruction booklets that have been designed and implemented by Elementary Education FTs for teaching a topic in the *Numbers and Operations* axis. This activity enables us to observe the FTs' understanding of a mathematical content, the formulation of teaching activities, the contexts in which they are placed, the pedagogical knowledge of the content, the LOs they consider to design their proposal and the obstacles they identify, recognizing in this their ability to carry out a didactic transposition, that is, the adaptation of a wise knowledge for their teaching in a certain course (Chevallard, 1991).

In this context, in more than half of the booklets designed the FTs privileged those LOs of the *Numbers and Operations* axis suggested for the first two grades of Elementary Education. This can be explained by the professional training course they are attending (during the first academic semester). Besides, we could observe in their proposal that they delved into the content of natural numbers and their operation, especially addition and subtraction, a topic considered in the LOs proposed by the curriculum bases for the axis in question (MINEDUC, 2012a).

In relation to the obstacles, although the FTs reflect, identify and describe some of them, they consider the *epistemological* one of the most prevalent obstacles. The above can be explained from the FTs' little experience or lack of knowledge on teaching strategies and the ability to interpret the students' answers. Ontogenetic and didactic obstacles are very poorly visible, so we have a feeling that the FTs delegate responsibility for their work to the student. Teachers must realize the difficulties and obstacles students face, so that they can improve their teaching practices and ensure that each student masters the mathematical concepts and basic skills before they teach the new subjects (Abdullah, Abidin y Ali, 2015; Ashlock, 2005).

Likewise, in their speeches the FTs point out that, in order to build mathematical learning, attractive and entertaining activities must be designed. Castro, Menacho-Vargas and Velarde-Vela (2019) and Palarea (2016), explain that practical and recreational activities, as well as motivating strategies, arise interest in mathematics learning. However, according to the FTs, 78.8% of the activities proposed in the booklets correspond to exercises. Moreover, 83.3% of the exercises lack context, just replicating algorithmic procedures, which prevents students from seeing how useful mathematics is (Alsina, 2019).

Regarding the assessment of the pedagogical knowledge of the content, the FTs mention the importance of the design of this type of teaching resources, which, by being implemented, help them recognize the obstacles that students present in the understanding of certain numerical ideas, and, in this way, propose different ways to overcome them. In addition, it is particularly interesting that the FTs, although very incipiently, recognize some mistakes that children can make when performing or answering to a task. Del Puerto, Minnaard and Seminara (2006) suggest the teachers must diagnose and treat seriously students' errors, discussing their misconceptions with them, and providing them with

mathematical situations to enable them to readjust their thoughts. This could encourage the teaching and learning processes very much.

Likewise, the FTs indicate that, to carry out the activities, it is necessary to consider the didactic material, as this is a good mediator for the construction of mathematical ideas and concepts. Numerous works have highlighted the importance of the teaching resources in learning activities (e.g., Aristizábal, Colorato and Gutiérrez, 2016; Barrantes & Blanco, 2004; Lezama & Tamayo, 2012; Tenelema & Tenelema, 2016; Vecino, 2005), meanwhile, they become good mediators for the construction and understanding of mathematical ideas and concepts by students, making sense of mathematics. On the other hand, they highlight the idea of a learning trajectory, which would allow progress in the students' constructions of more complex ideas and concepts, assuming that, for this, appropriate methodologies must be used. For Clements and Sarama (2014), the construction of student learning trajectories is one of the most urgent challenges that Mathematics Education is currently facing.

Regarding the overall assessment of the activity, the FTs mention that this type of activities contributes to the construction of the knowledge they are acquiring in their training process and that it resembles what their professional work will be, when they will have to be prepared to create strategies in a diverse context. They highlight the teaching role its implications, declaring the need to generate a methodology that is adequate to achieve quality learning. Concomitantly, they recognize that pedagogical knowledge of solid mathematical content is needed to design learning experiences (Hill et al., 2008; Hoover, Mosvold, Ball & Lai, 2016; Rojas, 2014; Rojas, Flores & Carrillo, 2015).

Based on this experience, we consider it necessary to continue working on a design of tasks that promote the construction and understanding of mathematical learning, mediated by games and the manipulation of materials, active and innovative strategies that allow teachers to obtain the students' attention (Ramirezparis, 2009), highlighting a pedagogical knowledge of the content (Ball et al., 2008). Only then can the teaching and learning processes be improved. (Engler, Gregorini, Müller, Vrancken & Hecklein, 2004; Franchi & Hernández de Rincón, 2004).

DECLARATION OF CONTRIBUTIONS OF THE AUTHORS

The three authors (J.P.F., M.S.S. and D.D.L.) participated in all stages of the research process, as well as in the creation, writing and correction of the article in an equivalent manner.

DECLARATION OF DATA AVAILABILITY

The data supporting the results of this study will be made available by the corresponding author J.P.F., upon reasonably previous request.

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Annex. Learning objectives related to the Number and Operations axis

First Grade (MINEDUC, 2012, p. 99)

1.1 Skip counting from 0 to 100 by ones, twos, fives and tens, counting backwards and forwards, starting with any number less than 100.

1.2 Identifying the order of the elements of a series, using ordinal numbers from the first (1st) to the tenth (10th).

1.3 Reading numbers from 0 to 20 and representing them in concrete, pictorial and symbolic form.

1.4 Comparing and order numbers from 0 to 20 from the lowest to the highest and/or vice versa, using concrete material and/or using educational software.

1.5 Estimating quantities up to 20 in concrete situations, using a reference.

1.6 Composing and decomposing numbers from 0 to 20 in an additive way, in concrete, pictorial and symbolic form.

1.7 Describing and applying mental calculation strategies for additions and subtractions up to 20: forward and backward counting; complete 10; double

1.8 Determine the units and tens in numbers from 0 to 20, grouping in tens, in a concrete, pictorial and symbolic way.

1.9 Demonstrate that they understand the addition and subtraction of numbers from 0 to 20 progressively, from 0 to 5, from 6 to 10, from 11 to 20 with two addends: using everyday language to describe actions from their own experience; representing additions and subtractions with concrete and pictorial material, manually and or using educational software; representing the process in symbolic form; solving problems in family contexts; creating mathematical problems and solving them.

1.10 Demonstrating that addition and subtraction are inverse operations, in a concrete, pictorial and symbolic manner.

Second Grade (MINEDUC, 2012, p. 103-104)

2.1 Skip counting numbers from 0 to 1,000 by twos, fives, tens and hundreds, counting backwards and forwards, starting with any number less than 1,000.

2.2 Reading numbers from 0 to 100 and representing them in concrete, pictorial and symbolic form.

2.3 Comparing and ordering numbers from 0 to 100 from the lowest to the highest and vice versa, using concrete material and national currencies manually and/ or through educational software.

2.4 Estimating quantities up to 100 in concrete situations, using a reference.

2.5 Composing and decomposing numbers from 0 to 100 in an additive way, in concrete, pictorial and symbolic form.

2.6 Describing and applying mental calculation strategies for additions and subtractions up to 20: complete 10; use doubles and halves; "one plus one less"; "two plus two minus"; use reversibility of operations.

2.7 Identifying the units and tens in numbers from 0 to 100, representing the quantities according to their positional value, with concrete, pictorial and symbolic material.

2.8 Demonstrating and explaining in a concrete, pictorial and symbolic way the effect of adding to and subtracting 0 from a number.

2.9 Demonstrating that he/she understands addition and subtraction in the realm of 0 to 100: using a daily and mathematical language to describe actions from his/ her own experience; solving problems with a variety of concrete and pictorial representations, manually and/or using educational software; recording the process symbolically; applying the results of additions and subtractions of numbers from 0 to 20 without calculating; applying the algorithm of addition and subtraction without considering reserve; creating mathematical problems in family contexts and solving them.

2.10 Demonstrating that he/she understands the relationship between addition and subtraction when using the "family of operations" in arithmetic calculations and problem solving.

2.11 Showing that he/she understands multiplication: using concrete and pictorial representations; expressing multiplication as an addition of equal addends; using distributivity as a strategy to construct the 2, 5 and 10 times tables; solving problems that involve the 2, 5 and 10 times tables.

Third Grade (MINEDUC, 2012, p. 107-108)

3.1 Counting numbers from 0 to 1,000 by fives, tens, hundreds: starting with any natural number less than 1,000; by threes, fours..., starting with any multiple of the corresponding number

3.2 Reading numbers up to 1,000 and representing them in concrete, pictorial and symbolic form.

3.3 Comparing and ordering natural numbers up to 1,000, using the number line or positional table manually and or through educational software.

3.4 Describing and applying mental calculation strategies for additions and subtractions up to 100: by decomposition; completing to the nearest ten; use doubles; adding instead of subtracting; applying the associativity.

3.5 Identifying and describing the units and tens and hundreds in numbers from 0 to 1,000, representing the quantities according to their positional value, with concrete, pictorial and symbolic material.

3.6 Showing that they understand the addition and subtraction of numbers from 0 to 1,000: using personal strategies with and without concrete material; creating and solving problems of addition and subtraction involving combined operations in concrete, pictorial and symbolic way, manually and/or through educational software; applying the algorithms with and without reservation, progressively, in the addition of up to four addends and in the subtraction of up to one subtrahend.

3.7 Demonstrating that they understand the relationship between addition and subtraction when using the "family of operations" in arithmetic calculations and problem solving.

3.8 Demonstrating that they understand the multiplication tables up to 10 in a progressive manner: using concrete and pictorial representations; expressing multiplication as an addition of equal addends; using distributivity as a strategy to build the tables until 10; applying the results of the times tables up to 10 · 10, without calculating; solving problems involving the tables learned until 10

3.9 Demonstrating that they understand the division in the context of tables7 of up to 10 · 10: representing and explaining the division as equal distribution and grouping, with concrete and pictorial material; creating and solving problems in contexts that include distribution and grouping; expressing division as repeated subtraction; describing and applying the inverse relationship between division and multiplication; applying the results of the times tables up to 10 · 10, without calculating

3.10 Solving routine problems in everyday contexts that include money and involve the four (not combined) operations.

3.11 Demonstrating that they understand the fractions of common use: 1/4, 1/3, 1/2, 2/3, 3/4: explaining that a fraction represents the part of a whole, in a concrete, pictorial, symbolic way, manually and/or with educational software; describing situations in which fractions can be used; comparing fractions of a

Fourth Grade (MINEDUC, 2012, p. 113-114)

4.1 Representing and describing numbers from 0 to 10,000: skip-counting them by tens, by hundreds, by thousands; reading them and writing them; representing them in concrete, pictorial and symbolic form; comparing them and ordering

them on the number line or positional table; identifying the place value of the digits up to ten thousand; composing and decomposing natural numbers up to 10,000 in additive form, according to their place value.

4.2 Describing and applying mental calculation strategies to determine multiplications up to 10 · 10 and their corresponding divisions: counting forwards and backwards; folding and dividing by 2; by decomposition; using the double of the double.

4.3 Demonstrating that they understand the addition and subtraction of numbers up to 1,000: using personal strategies to perform these operations; breaking down the numbers involved; estimating sums and differences; solving routine and non-routine problems that include additions and subtractions; applying the algorithms in the addition of up to four addends and in the subtraction of up to one subtrahend.

4.4 Basing and applying the properties of 0 and 1 for multiplication and the property of 1 for division.

4.5 Demonstrating that they understand the multiplication of three-digit numbers by one-digit numbers: using strategies with or without concrete material; using times tables; estimating products; using the distributive property of multiplication with respect to the sum; applying the multiplication algorithm; solving routine problems.

4.6 Demonstrating that they understand division with two-digit dividends and one-digit dividers: using strategies to divide, with or without concrete material; using the relationship between division and multiplication; estimating the quotient; applying the dividend decomposition strategy; applying the division algorithm.

4.7 Solving routine and non-routine problems in everyday contexts that include money, selecting and using the appropriate operation.

4.8 Demonstrating that they understand fractions with denominators 100, 12, 10, 8, 6, 5, 4, 3, 2: explaining that a fraction represents the part of a whole or a group of elements and a place on the number line; describing situations in which fractions can be used; showing that a fraction can have different representations; comparing and ordering fractions (for example: 1/100, 1/8, 1/5, 1/4, 1/2) with concrete and pictorial material

4.9 Solving additions and subtractions of fractions with the same denominator (denominators 100, 12, 10, 8, 6, 5, 4, 3, 2) in a concrete and pictorial way in the context of problem solving.

4.10 Identifying, writing and representing proper fractions and mixed numbers up to 5 in a concrete, pictorial and symbolic way, in the context of problem solving.

4.11 Describing and representing decimals (tenths and hundredths): representing them concretely, pictorially and symbolically, manually and/or with educational software; comparing them and ordering them until the hundredth

4.12 Solving additions and subtractions of decimals, employing the place value to the hundredth in the context of problem solving.