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In-service Teachers' Didactic-Mathematical Knowledge on Elementary Algebraic Reasoning. The Case of the Shanxi Province of China

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Abstract

In this study, the Chinese in-service teachers' didactic-mathematical knowledge on elementary algebraic reasoning is assessed using a questionnaire developed in Spain. The first aim is to diagnose the current situation in China of in-service primary school teachers' knowledge regarding elementary algebra and its teaching. The second aim is to provide enlightenment to teacher training so as to make the concept of early algebra popularized among Chinese primary school teachers. The sample included 264 primary school teachers from the Shanxi province in China. The questionnaire consists of two parts: part 1 comprises 5 items about teachers' gender, teaching experience, professional certificate, educational background, and region; part 2 comprises 25 items from the original version in English. The difficulty, discrimination indexes as well as reliability of the instrument have been determined, reflecting the psychometric properties of the instrument. The dependent variables include 7 aspects divided in 2 dimensionalities apart from the total score. The results reveal Chinese primary school teachers have solid knowledge of algebra but lack pedagogical knowledge in early algebra. This suggests that it would be necessary to set clear training objectives on the core idea of the algebra in national teacher training programs; in addition, our findings emphasize more attention should be given to teachers who have less than 10 years of teaching experience. Finally, we conclude teacher training would be more effective if highly experienced teachers—more than 20 years, became trainers to teach the practical modules.

Keywords: Algebra Knowledge; Didactic-mathematical Knowledge; Elementary Algebraic Reasoning; Primary School Teacher; Teacher Training.

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INTRODUCTION

In recent years, the scientific community has been very interested in understanding the complexities of mathematics teaching and the didactic-mathematical knowledge that teachers need, either in general or specific topics. Results of these investigations have had a direct impact on the teacher training programs as a key point for educational improvement (Ponte & Chapman, 2008; Norton & Zhang, 2018; Stylianides, Stylianides, & Shilling-Traina, 2013; Wu, Hwang, & Cai, 2017).

China's education policies have not been kept aside, so large-scale teacher training began in 2010 (Xinhua News Agency, 2018). A leading example of this effort is the *National Training Program* for primary and secondary school teachers (NTP)—implemented by the Ministry of Education of the People's Republic of China (MOE, 2011). In addition, it proposed to take improving quality as the core task in the *Outline National Medium–and Long–Term Education Reform and Development Plan* (2010-2020) (MOE, 2010).

The present study is consistent with the aforementioned line of research, although restricted to a specific aspect: in-service primary school teachers' comprehension of algebra, algebraic thinking, and its instruction. Although it is clear there has been progress in the understanding of elementary algebraic reasoning, the interconnection between primary and secondary education algebra, as well as the knowledge of its teaching. are not solved yet (Artigue, Assude, Grugeon, & Lenfant, 2001; Godino, Neto et al., 2015; Li, Ma, & Pang, 2008). Several studies claim "the teachers' knowledge and practices and their development for the teaching of algebra have been largely unexamined in the research literature" (Doerr, 2004, p. 268). We consider it is necessary to apply tools that allow making large-scale diagnoses about in-service teachers' didactic-mathematical knowledge and thus influence teacher education and take measures in training programs.

To conclude, in this work we address the following research question in the context of the Shanxi Province of China:

> What is the in-service primary school teachers' knowledge regarding elementary algebra and its teaching? And in consequence, how could these results influence the professional education of Chinese elementary teachers?

To follow, section two synthesizes the background and goes further into the research problem; section three presents the theoretical framework on which the evaluation instrument is based–designed in order to explore key features of teachers' didactic-mathematical knowledge of elementary algebraic reasoning. Sections four and five present respectively the research method used and, the results in terms of the analysis of in service teachers' knowledge revealed from the applied questionnaire. Finally, section six presents a number of conclusions and revels some limitations derived from the obtained results.

BACKGROUND

Early algebra, also known as early algebra thinking, has been becoming an international teaching idea (Kaput, 2008). By the 1970s, there were discussions about the inclusion of algebraic ideas in mathematics curricula at earlier grades in some countries of Europe and North America. The earliest one is the Principles and Standards of School Mathematics Curriculum issued by the National Council of Teachers of Mathematics (NCTM, 2000), in which algebra strand for all grades has been proposed. Since then, more and more people have emphasized and widely accepted that students can develop algebraic thinking in earlier grades, which is reflected in some influential policy documents (Cai & Knuth, 2011).

From the Early Algebra Work Group (EAWG) implementation as part of 12th International Conference on Mathematical Education (ICME12) in 2001, early algebra has become a hotspot in mathematical education research (Chick et al., 2001). Since then, various research lines have led to a large increase in the amount of knowledge for early algebra teaching and learning (e.g. Blanton et al., 2015; Cai et al., 2005; Carraher, Schliemann, Brizuela, & Earnest, 2006). Doerr (2004) reports the investigations on teachers' knowledge and practice and its development with respect to the teaching of algebra focusing on three broad areas: "teachers' subject matter knowledge and pedagogical content knowledge, teachers' conceptualizations of algebra, and teachers learning to become teachers of algebra" (p. 270). In addition, the report from a specific workshop given in the 13th International Congress on Mathematical Education (ICME13) in 2016, showed a theoretical model of the classification of algebraic reasoning levels in primary and secondary schools developed by Godino and colleagues (Kaiser, 2017), a very important theoretical contribution on which various works of research have been based.

Some countries have launched special research projects, such as the New Zealand Numeracy Development Project, Victorian Early Numeracy Research Project and the Count Me In Too program in New South Wales, Australia, recognizing the importance of the early childhood years in the development of numeracy (Bobis et al., 2005). In addition, there are several comparative studies that reflect the importance of advancing in teacher training to address the problems of early algebra teaching. For instance, Norton and Zhang (2018) examine the basic content knowledge of trainee teachers in Australia and China about number and early algebra.

Interest in exploring the current situation in China and comparing of early algebra in primary school education has grown steadily among mathematics education researchers recently (Pu, 2014; Chen, 2018). The number and algebra have been integrated as a whole content in the mathematics curriculum standards for compulsory education (CMOE, 2001), pointing out an important direction for Chinese primary school mathematics teachers. However, they also pointed that there was little research on early algebra in China, the ideas and practices of early algebra did not seem to be engrained. Many changes have been made in the training programs, but few have been analyzing on the specialized knowledge of the Chinese teachers (CMOE, 2012). Despite the rather large body of research on this topic, it is necessary to analyze the knowledge that teachers have about this content so justified by literature to generate effective changes in teaching (Li, Zhao, Huang, & Ma, 2008).

Understanding the current situation in China on the in-service mathematics teacher' knowledge, gives us valuable information to make a justified contribution in the field of professional development. Facing this problematic area, we present below our theoretical position that supports the aforementioned research objective.

THEORETICAL FRAMEWORK

This research is supported on two key theoretical perspectives. On one hand, the conception about the *nature of algebraic reasoning in primary education* (Carraher & Schliemann, 2007; Cai & Knuth, 2011) that is based on a conception of algebra that recognizes signs of algebraic thought in mathematical activities of initial educational levels, as NCTM (2000) indicates. On the other hand, the model of mathematics teachers' knowledge known as *Didactic-Mathematical Knowledge* (DMK) that has been developed in several studies by Godino and colleagues (Pino-Fan, Godino, & Font, 2018). The DMK model is based upon theoretical assumptions and theoretical–methodological tools of the theoretical framework Onto-Semiotic Approach (OSA) to mathematical cognition and instruction (Godino, Batanero, & Font, 2007). Next, we synthesize the fundamental aspects of both aforementioned theoretical components—on which the evaluation instrument is based.

Elementary Algebraic Reasoning

The questionnaire used in this research is based on the conception of elementary algebraic reasoning (EAR) developed by Godino and collaborator (Aké, Godino, Gonzato, & Wilhelmi, 2013; Godino, Neto, Wilhelmi, Aké, Etchegaray, & Lasa, 2015). The nature of the EAR is a special perspective that considers a mathematical practice as algebraic, based on the intervention of generalization and symbolization processes, along with other objects usually considered as algebraic, such as binary relations, operations, functions and structures. Specifically, it is a method to define degrees of algebrization of mathematical activity. In their views, children's algebraic thinking should be cultivated by mathematical practice and not be told the algebraic concepts or use formal symbolic language during the elementary education.

This conception of school algebra allows to Godino, Ake, Gonzato & Wilhelmi (2014) to develop a model of EAR structured into four algebrization levels, taking into account the objects and processes involved in mathematical activity. The criteria to delimit the different levels are based on (Burgos, Godino, & Rivas, 2019, p. 66):

– Type of objects: concepts (mathematical entities that can be introduced by descriptions or definitions), propositions (properties or attributes, statements about concepts), procedures (calculation techniques, operations and algorithms), arguments (statements required to justify the propositions or explain the procedures).

– Type of representations used (languages in their different registers).
– Generalization processes involved.
– Analytical calculation put into play in the corresponding mathematical activity.

At level 0 there are no algebraic traits in mathematical activity, whereas level 3 is clearly algebraic. The intermediate levels 1 and 2, considered as progressive algebrization levels, display some objects and processes, which are algebraic in nature.

Didactic Mathematics Knowledge (DMK model)

Regarding the modelling tool adopted for the teacher's didactic-mathematical knowledge (Pino-Fan, Godino, & Font, 2018), two types of variable were taken into account: algebraic content and didactical content. In terms of the algebraic content three values or categories with different subcategories were taken into consideration (Godino, Wilhelmi et al., 2015, p. 204):

– *Structures* (equivalence relation; properties of operations, equations, ...);
– *Functions* (arithmetic patterns, geometric patterns, linear, affine, quadratic functions, ...);
– *Modelling tool* (context problems solved via equations or function relationships).
– Regarding the variable 'didactical content' (based on an algebraic content associated to primary school level or higher) the following categories were considered:
– *Epistemic*. Recognition of objects and algebraic processes (representations, concepts, procedures, properties, generalization, modelling); recognition of algebrization levels.
– *Cognitive*. Personal meanings inferred by the students (knowledge, understanding and competence in elementary algebraic contents); learning conflicts at the level of algebraic objects and processes.
– *Instructional*. Resources for teaching algebra in primary school (situation-problem, technical resources), as well as its adaptation to the school curriculum.

METHOD

Participants

In this study, considering the representativeness and convenience of the samples, some primary school mathematics teachers were selected in the south, the middle, and the north of the Shanxi province, respectively, including the cities of Linfen, Jincheng, Jinzhong, Datong. All these participants are specialized mathematics teachers, as in the rest of China.

In the process of urbanization development in China, it is also generally divided into urban-dominant region, urban-rural transition region and rural dominant region (Wang, 2018), according to the tripartite method of the urban-rural division system of the World Economic Cooperation and Development Organization (OCED) (Veiga, 2004). Considering the geographical distribution, the samples we collected from five kinds of public schools based on the actual situation of the Shanxi Province in Table 1. Additionally, the gender, teaching experience, professional certificate, and educational background of the primary school mathematics teachers are considered.

Table 1
Statistical table of the basic information of samples.

Category	Option	Frequency	Percentage (%)
Gender	Man	33	12.5
	Woman	231	87.5
Teaching experience	1-3 years	46	17.4
	4-10 years	59	22.3
	11-20 years	97	36.7
	More than 20 years	62	23.5
Professional certificate	No	33	12.5
	Third-level	6	2.3
	Second-level	138	52.3
	First-level	76	28.8
	High-level	11	4.2
Educational background	Below high school	9	3.4
	High school	7	2.7
	Junior college	56	21.2
	Undergraduate	169	64.0
	Master graduate	23	8.7
	Doctor graduate	0	0
Region	City	50	18.9
	Urban (or county)	84	31.8
	Rural-urban continuum	77	29.2
	Township central	42	15.9
	Village	11	4.2

Note. n = 264

There is a clear imbalance between the proportion of male teachers and female teachers (about 1:9), but this phenomenon is consistent with the reality—male teachers are seldom seen in primary schools.

Teachers with teaching experience between 11 and 20 years represent approximately 40% of the entire sample. In addition, about 85% of teachers are junior college graduates or undergraduates, and there is no doctor graduate in primary school at present.

In terms of professional certificate, 12.5% of the participants have no professional certificate because 17.4% of them have less than 4 years of teaching experience. In this sense, it is important to note, that in China, professional certificates are related to teaching experience. An ordinary new teacher who has just been employed will have a professional certificate one year later generally. But there is a special kind of teacher named *Special Post Teacher* (SPT). What's a SPT?

In China, with the development of urbanization, the proportion of rural teachers once declined. To improve the overall quality of the rural teachers, and promoting the balanced development of the urban and rural education, the government has issued a special policy named 'Special Post Plan for school teachers in rural compulsory education' (SPP), with the support of the central financial department, publicly recruiting college graduates to teach in western rural schools. The SPP begins from 2006 including the Hebei and the Shanxi provinces (Zhang, 2012). For those SPTs, they must serve rural education for three years before they have a professional certificate.

On the other hand, the teachers from urban-dominant region (city and country), rural-urban region, and rural-dominant region (township and village) are, respectively, about 50, 30, and 20 percent of the total. Especially, teachers coming from village schools are only 4.2%, consistent with the reality trend of urbanization development in China. Therefore, the sample in this study well reflects the reality.

Instrument

The instrument used for data collection was the Didactic-Mathematical Knowledge on Elementary Algebraic Reasoning (DMK-EAR) a questionnaire originally developed by Godino and colleagues as part of a research project on this topic (Godino, Aké et al. 2015; Godino, Wilhelmi et al., 2015) in addition to some basic information about teachers' gender, teaching experience, professional certificate, educational background, and region (see Appendix).

The original questionnaire has been used to assess 597 prospective primary school teachers with an acceptable reliability as well as good construct validity (Godino, Wilhelmi et al., 2015). It is composed of a set of 10 tasks, each of which consists of items that evaluate different aspects of algebraic and didactic-algebraic content (25 items total).

Dependent variables

The first objective is to define the quantitative variable—degree of accuracy of the answers given to the 25 items of the questionnaire. Partially correct answers were assessed positively, so the score assigned to each item was:

– 0 points, if the answer is incorrect;

– 1 point, if it is partially correct;

– 2 points, if it is correct.

Besides the variable 'Total Score', i.e. sum of scores obtained in all 25 items (0-50 points), the other quantitative variables are defined regarding 'common and advanced mathematical knowledge' and 'didactical content'.

a) Variables concerning common and advanced mathematical knowledge

ALG: Assess knowledge of *algebra* characteristic in Primary school (common knowledge) or Secondary school (advanced knowledge). This scale includes items 3a, 4a, 5a, 6a, 7a, 8a, 9a, and 10b.

EST: Evaluates knowledge related to properties of the algebraic structures used in equation solving. It includes items 1a, 1b, 2a, 2b, 3a, 3b, 4a, 4b, 4c, 7a, 7b, 7c, and 8b.

FUN: Includes knowledge related to geometric patterns and functions. It includes items 5a, 5b, 5c, 6a, 6c, 8a, 8b, 9a, 10a, and 10b.

MOD: Evaluates knowledge related to algebraic modelling (using equations or functions). It includes item 8b, 9a, 9b, 9c, and 10a.

b) Variables concerning didactical content

EPI: Incorporates knowledge on the epistemic facet of the DMK–EAR and includes items 2b, 4b, 5c, 6b, 7b, 7c, 9b, and 10b.

COG: Contains knowledge on the cognitive facet and includes items 1a, 1b, 2a, 3b, 4c, and 6c.

INS: Assesses knowledge on the instructional facet and includes items 5b, 8b, 9c, and10a.

The different dependent variables comprise a number of different items scoring 0, 1 or 2, so its variation range is distinct. To ease the comparison and interpretation of the scores they were converted to the interval [0, 10].

RESULTS

Item Analysis

Table 2 presents the difficulty indices of the items included in the questionnaire calculated on the entire sample. This index does not correspond to the percentage of correct responses as scores of 0, 1, and 2 were ascribed based on the degree of correction. To make the analysis simpler, the mean values of each item and the total scores were converted to interval [0-100].

Table 2
Difficulty index of the items.

Item description	Difficulty index		Discrimination: Mean Difference ($P_{33} - P_{66}$)	
	Mean	Standard error	t	p
1a. Explain equality arithmetic result	63.64	1.57	2.813	.005
1b. Illuminate equality arithmetic result	58.90	1.66	4.896	.000
2a. Explain equality equivalence	44.70	2.68	8.342	.000
2b. Arithmetic properties	35.61	2.75	8.872	.000
3a. Generalize sum three numbers	70.27	1.86	8.306	.000
3b. Reasoning type	71.97	2.27	10.016	.000
4a. Solve and explain partial sum	59.28	2.26	13.630	.000
4b. Algebraic solution	38.83	2.21	6.604	.000
4c. Pupil’s solution	49.06	2.73	16.510	.000
5a. Continue two terms of hexagonal pattern	82.39	1.97	8.229	.000
5b. Algebraic generalization	68.37	2.78	13.485	.000
5c. Type of algebraic objects	49.24	3.02	11.789	.000
6a. General solution of square pattern	79.17	2.42	16.017	.000
6b. Possible solution	81.82	2.31	12.138	.000
6c. Pupil’s solution	67.81	2.51	10.809	.000
7a. Solve food cost	81.63	2.38	10.904	.000
7b. Arithmetic solution	77.65	2.38	13.316	.000
7c. Algebraic solution	70.08	2.54	8.053	.000
8a. Interpreting expresses	60.04	2.49	15.431	.000
8b. Problem statement	59.09	2.89	16.323	.000
9a. Explain graph function	70.46	2.53	19.574	.000
9b. Mathematical basis	59.85	2.93	13.971	.000
9c. Curriculum provision	44.89	2.79	17.940	.000
10a. Design question about linear function	52.65	2.69	13.594	.000
10b. Algebra recognition	40.34	2.30	13.368	.000
Average Difficulty Level	61.52	1.45	37.976	.000

Note. n = 264

As indicated in Table 2, the mean score was 61.52 (standard error is 1.45), indicating that the general level of knowledge in the whole sample is relatively high. In fact, 13 items had a difficulty index above 60, three of which had a difficulty index above 80 (5a, 6b, 7a). In addition, 3 items had a difficulty index below or about equal to 40 (2b, 4b, 10b) (at least six of the 10 answers were incorrect), indicating that the geometric model is the easiest and the algebraic structure is the most difficult. The last column is the item discrimination, that is, the mean difference between the low group (percentile 33) and the high group (percentile 66). Except the significance of 1a ($p = 0.005$), the p value of all the other items are less than 0.001, indicating that each item and the total score have a good degree of discrimination.

Reliability

In this study, using 264 in-service primary school mathematics teachers as the sample, the Cronbach's alpha (α) value of the total questionnaire was 0.9207. The Cronbach's α value of the dimension of algebraic knowledge was 0.846. The Cronbach's α value of the dimension of didactic knowledge was 0.881. All of the Cronbach's α values were greater than 0.800, demonstrating a strong internal consistency of the instrument.

Analysis of Dependent Variables

To simplify the analysis, the eight dependent variables–including the total score variable and 7 partial variables, were firstly transformed to the interval [0, 10]. Table 3 shows the mean score (M) and standard deviation (SD) of each dependent variable, and in Figure 1, there is a comparison of the frequency distributions of the total score obtained in the seven partial variables using box plots.

Table 3

Descriptive statistics of dependent variables.

Dependent variable	M	SD
ALG-Common and advanced knowledge in algebra	6.79	2.58
EST-Knowledge of equations and relations	6.01	2.23
FUN-Knowledge of functions	6.47	2.80
MOD-Knowledge of modeling	5.74	3.35
EPI-Knowledge of epistemic aspects	5.78	2.55
COG-Knowledge of cognitive aspects	6.17	2.28
INS-Knowledge of instructional aspects	5.63	3.34
TOTAL-Total score	6.15	2.35

From Figure 1 shown below, we can conclude that there are no outliers for all the variables, except for ALG. If they were suppressed from the calculations the mean value of ALG would be even higher than that presented in Table 3. This finding indicates that teachers of our sample have solid algebraic knowledge.

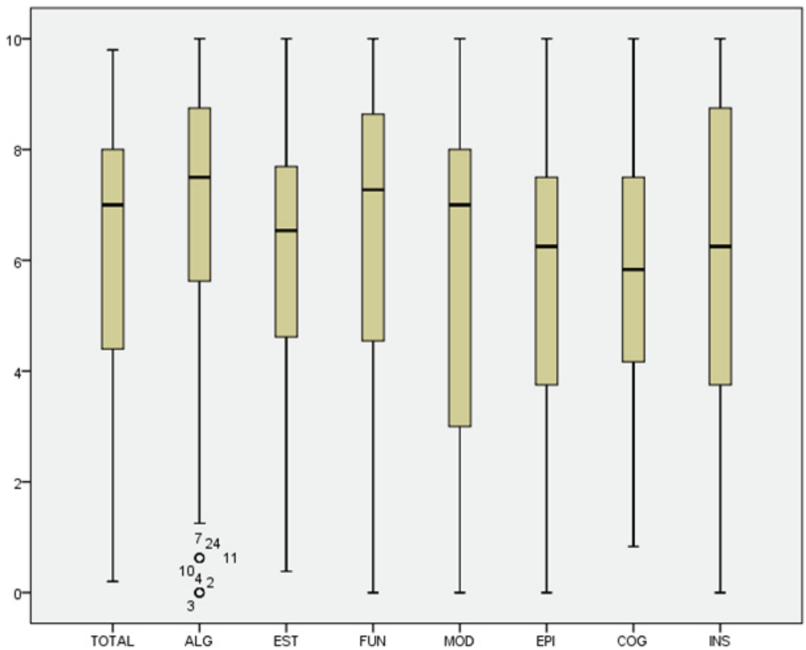


Figure 1. Comparison of the distribution of the different variables using box plots

The total average score is 6.15 points. It can be concluded that the in-service primary school mathematics teachers' DMK on EAR is relatively high especially on ALG. Scores below 7 highlight the difficulties faced by teachers answering questions of instructional facet (INS), i.e. those that require specialized training of early algebra.

Analysis of Algebraic Knowledge

The mean score obtained in the dependent variables was insufficient. The best result was obtained in ALG (6.79 points), and it is also the best result if we consider all 7 variables although it has some low outliers. Take item 6a for example, generalizing the solution and its possible solution of square pattern. For the generalization from particular to general, the teacher must carry out a series of calculations and then come up with a result. Six kinds of solving strategies were found as following in this study:

a. 3, 3 + 5,   3 + 5 + 7,   3 + 5 + 7 + 9 … add the next odd number every time;

b. 1 + 2,   2 + 3 + 3,   3 + 4 + 4 + 4,   4 + 5 + 5 + 5 + 5,   5 + 6 + 6 + 6 + 6 + 6 … (look at the line from right to left and write down the number of the line successively);

c. 1 + 2,   1 + 2 + 3 + 2,   1 + 2 + 3 +4 + 3 + 2,   1 + 2 + 3 + 4 + 5 + 4 + 3 + 2,

1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3 + 2 ...;

d. 1 x 2 + 1,   2 x 3 + 2,   3 x 4 + 3... $n(n+3) + n$ look at the line from left to right);

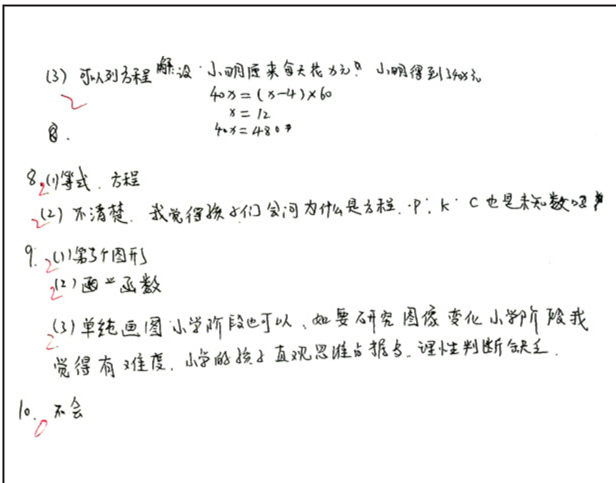
e. $2^2 – 1, 3^2 – 1, 4^2 – 1, ..., (n + 1)^2 – 1$ with $n \geq 1$;

f. 2 x 2 – 1 = 3,   3 + 2 x 3 – = 8,   8 + 2 x 4 –1 = 15, 15 + 2 x 5 – 1 = 24 ...

$f(n – 1) + 2 \text{ x } (n + 1) – 1$ with $n \geq 1, f(0) = 0$.

The first three solving strategies are similar to enumerate numbers from different partition methods. These solving strategies can indicate the rules of square pattern intuitively with low abstract degree. The abstract degree of the last three solving strategies increases gradually. Both addition and multiplication are used in the strategy d. However, square is used in the strategy e. The last one has the highest abstract degree using iteration method.

Following FUN (6.47 points). 50% of teachers have scored between 4.5 and 8.5, indicating that teachers are good at identifying geometric patterns and functions. Nevertheless, the ability to use functions is very weak, as reflected in items 8, 9, 10 (Figure 2):



8. (1) Equation or equation with unknown numbers.

(2) I don't know. But I think children maybe ask why they are equations and are PK and C also unknown numbers?

9. (1) The third figure.

(2) Function.

(3) It is suitable for pupils to learn some simple drawings. However, I think it is a little difficult and not suitable for them if you want them to study the changes in the graphics. After all, primary school children's thinking is mostly in intuitive one, lacking rational judgment.

10. I cannot do it.

Figure 2. Prototypical response given by a participant to items 8, 9, and 10

In item 8, many teachers look at the expressions '$4x + 5 = 25$, $y = 2x + 13$...' (see appendix) as equations, not functions. So, they don't know what questions pupils would ask about those expressions. In item 9, most such teachers respond appropriately to items a and b; in item c, participants respond, in general, that this task is not suitable for elementary school children. Their reasons are, for example: linear function is not, an elementary school content; children cannot understand two quantities which are changing at the same time, etc. We believe that there are strong arguments to justify that this task is appropriate for elementary school students, for example: direct proportion is put into play, which is adequate content to be learned; relationships are raised between distance and speed, which can be understood by analogy; the tabulation method is a strategy that the primary school students must master. The teacher in Figure 2 is able to understand this problem dialectically. He points out that simple drawing could be accepted by primary school students, but it is difficult to study the changes of images, because pupils have more intuitive thinking and less rational judgment.

In item 10, most teachers write down the word 'no', as shown in Figure 2, directly because this problem involves the rounding or the piecewise function.

To follow is the score in EST (6.01 points). 50% of teachers have scored between 4.5 and 7.5. Finally, the lowest score was obtained in MOD (5.74 points), which shows the highest degree of dispersion (highest standard deviation). This even dispersion (without outliers) justifies the conclusion that teachers' ability to apply algebra knowledge is weak because the items in this variable only refer to simple models studied in secondary school. Take the items 10a and 10b for example, establishing a linear function model for a shopping phenomenon and naming the algebraic skills used. The percent of the complete accuracy of 10a and 10b are 40.9% and 20.1%, respectively. This means that only half of the teachers who correctly built a model were able to solve the problem he/she stated and to name the algebraic skills used correctly. Among those models built correctly, a complete piecewise function is seldom found in this study. And, it seems that it is difficult for teachers to distinguish the linear function from the direct proportion function. A plausible explanation for this result is that the primary school teachers lack specialized training in algebra.

The variables EST, FUN, and MOD are about the application of algebraic knowledge. Item 8b asks to enunciate three problems that may be proposed to elementary students and whose solution leads to specific algebraic expressions. According to its frequency statistics, the percentages of incorrect, partially correct, and correct answers are 36.4%, 9.1% and 54.5%, respectively. It means that 45.5% of the teachers could not or could not correctly use a functional expression to construct a real problem situation for primary school students.

Analysis of Didactic Knowledge

As is showed in Figure 1, among the variables of didactic content, the best score was obtained in COG (6.17 points), with the third best score if we consider all 7 variables, equaling the total average score (6.15 points). It is evenly distributed, showing no outliers:

50% of teachers are within a score (4 - 7.5) and the remaining 50% are evenly distributed in the two whiskers (box plot).

The other two variables rank the first and the third from the bottom of the 7 partial variables, respectively.

The lowest score was obtained in INS (5.63 points), indicating teachers' lack of instructional practice of early algebra. And it turns out again that the lower the mean, the higher the standard deviation just like in MOD.

Lastly, EPI (5.78 points) is the second in the lower scores, with the third lowest score if we consider all 7 variables. Keeping in mind the box plot, 25% of teachers have scored less than 4. This variable includes items 2b, 4b and 10b, whose difficulty indexes are the lowest of all of the 25 items showed in Table 2. It is necessary to analyze their descriptive statistic of frequencies. The results indicate that the percentage of teachers getting 0 points in items 2b, 4b and 10b are 58.3, 39.4 and 39.4, respectively; the percentage of teachers getting 1 point are 12.1, 43.6 and 40.5, respectively; the percentage of teachers getting 2 points are 29.5, 17.0 and 20.1, respectively. That is to say, about 60 percent of teachers cannot understand the arithmetic properties of the equality equivalence (13 + 11 = 12 + 12). About 40 percent of teachers have no algebraic method to solve the partial sum; and also, about 40 percent of teachers are unable to name the algebraic skills used correctly, indicating an absence of deep understanding of the basic principles of arithmetic operations. In addition, 43.6% of teachers know, but cannot correctly write a ternary equation. No more than 30% of the answers to these three items are completely correct, especially item 4b. A plausible explanation for this result is that the core idea that early algebra has not been put into practice.

To sum up, didactic knowledge is worse than the algebraic. This result was to be expected, since Chinese elementary teachers have a profound understanding of fundamental mathematics (Li et al., 2008; Norton & Zhang, 2018). The lower scores in modeling and epistemic are the affiliated phenomenon to the absence of teaching early algebra. The epistemic facet allows us to be more specific about what we understand by knowledge for teaching of algebra (Breda, Pino-Fan, & Font, 2017).

CHANGE TREND WITH TEACHING EXPERIENCE

As a reconfiguration of pedagogical content knowledge, teacher pedagogical constructions result mainly from planning, but also from the interactive and post-active phases of teaching (Hashweh, 2005). Teachers' DMK on EAR should be changed with their teaching practice including planning, implementation, reflecting and so on. We analyzed the change trend of teachers' DMK on EAR with teaching experience using multiply line graphs as is shown in Figure 3.

It is possible to notice that, when increasing the teaching experience, the change curve of all the means of dependent variables shows a pattern of descending firstly, then, ascending and descending lastly. Compared to the initial one, most of the mean values finally increase.

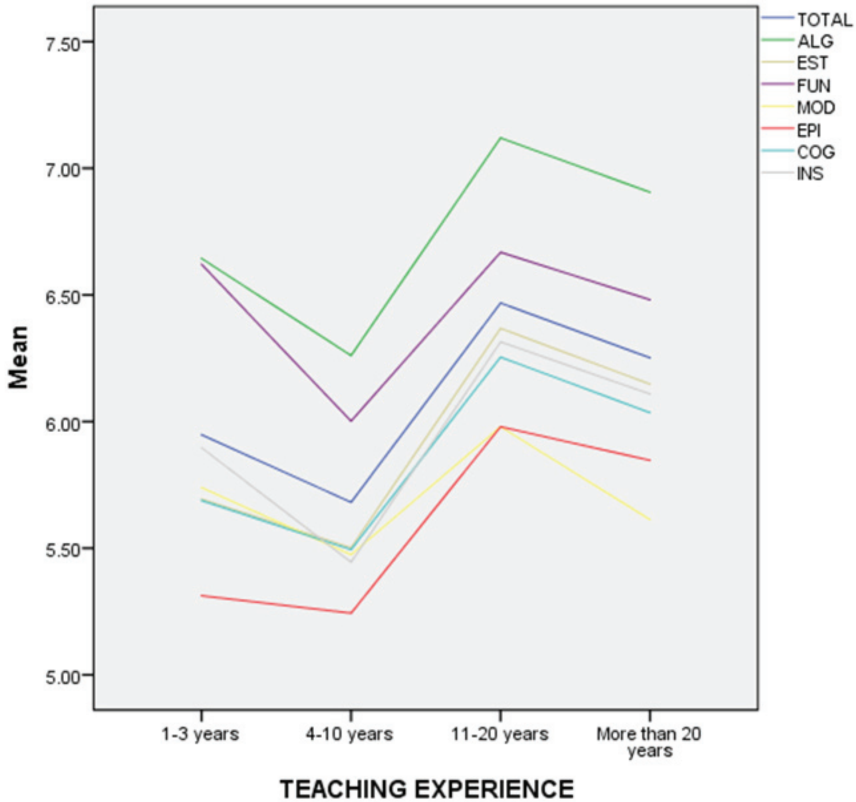


Figure 3. Change trend of teachers' DMK on EAR with teaching experience

Specifically, in the first 10 years of teaching experience, the mean values of the total score variable and the 7 partial variables all show a significant downward trend. In comparison, the variables of FUN and INS have the largest decline, while EPI has the smallest decline, with basically no change. During the period of 10 to 20 years of experience, all of the dependent variables recover significantly and exceed the initial level except for FUN. After 20 years, all variables show a downward trend, of which MOD drops the most. While comparing the initial to the final level, Figure 3 shows that, except for MOD and FUN, the original level of all variables is improved despite the twists and turns; this result suggests that teaching practice has a direct impact on teachers' DMK on EAR, thus supporting Hashweh (2005)'s opinion. This conclusion can be used to explain the retrogress phenomenon of the MOD and FUN level, that is, the lack of relevant teaching practice is the main reason. We believe teachers' algebraic knowledge on EAR will be improved increasing the practice on early algebra. In addition, the analysis presented in the Figure 3 reflects the changing characteristics of primary school teachers' professional development in stages, that is, the first 10 years was a retrogressed stage, the golden period of professional development was 10 to 20 years, and the stage of job burnout is coming after 20 years.

FINAL CONSIDERATION

Some significant information can be found in this study by assessing in-service primary teachers' didactic-mathematical knowledge (DMK) on elementary algebraic reasoning (EAR). This study justifies the reliability of the original instrument again with high Cronbach's alpha values whether it is the whole questionnaire or its dimensions.

Overall, Chinese in-service primary school mathematics teachers' DMK on EAR is relatively high especially on the variable ALG, indicating a solid foundation in algebra. The mean values of all the dependent variables are between 5.63 and 6.79 points. However, some weaknesses have also been shown. Teachers have a better mastery of the knowledge of algebra itself than its application level, reflecting evidently in two facets of EST and MOD. That is to say, teachers are not very clear about the properties of the algebraic structures used in equation solving, cannot build a suitable mathematical model in a real-life situation, and especially have difficulty in designing a real problem situation for primary school students. Perhaps, the absence of deep understanding of the basic principles of arithmetic operations inhibits teaching practice, and vice versa.

In conclusion and according to Chen (2018), this research indicates that the teaching of early algebra is still incipient in China. It is the lack of algebra teaching in the early stage that makes it difficult for teachers to be able to design a question related to elementary algebra for pupils, such as 5b, 8b, 9c, and10 items. It is also because of the lack of early algebra teaching that teachers cannot grasp how and on what grounds pupils answer such questions as 1a, 1b, 2a, 3b, 4c, and 6c. It is time to start doing specialized training on early algebra with the current education boom. Firstly, it is necessary to spread the early algebra idea in the group of primary school teachers in the period of NTP. Only if teachers comprehend an idea, they may use it to guide practice effectively. Only if a specific goal for teacher training at the national level, such as NTP, is clearly defined, the idea of early algebra will be popularized among the in-service teachers. Secondly, following the results obtained, teachers with less than 10 years of teaching experience should be given more training to help them overcome the retrogression of this period. Strengthening the training of these teachers will provide a high base for their professional development in the next stage, thus contributing to the improvement of the quality of the entire teaching staff. Teachers with high level professional certificates, especially those with more than 20 years of teaching experience, should be involved in the training. In this sense, it is time for in-service teachers with rich practical experience to play an important role as practical experts in the National Training Program NTP (Tian, 2016).

Finally, this analysis represents an interesting approach to look beyond and the results (not conclusive) can thus be used as a diagnostic tool; as Blömeke, Suhl, and Döhrmann, (2013) point out, "the study can provide information for mathematics teacher educators and policy makers that allow thinking about foci to be set on teacher education" (p. 814). As implication in teacher education programs, we suggest that a series of theory and practice courses need to be introduced or developed related with: the nature of the EAR, the algebraic reasoning level model, the textbook analysis based on EAR, the school activities design as well as its demonstration teaching practice, etc. This is a crucial point for teacher education and future research on early algebra in China.

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AUTHORS' CONTRIBUTIONS STATEMENTS

All authors conceived the idea presented. G.T performed the activities and collected the data. All authors discussed the results and, through meetings, jointly elaborated the final version of the manuscript.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the first author, G.T., upon reasonable request.

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APPENDIX: DMK-EAR QUESTIONNAIRE

• Basic Information

1. What's your gender? ( )

A male **B** female

2. How many years have you been teaching? ( )

A 1-3 years **B** 4-10 years **C** 11-20 years **D** more than 20 years

3. What's your professional certificate? ( )

A No **B** Third- level **C** Second-level **D** First-level **E** High- level

4. What is your educational background? ( )

A Below high School **B** High school **C** Junior college **D** Bachelor's degree

E Master's degree **F** Doctor's degree

5. Where is your school? ( )

A City B Urban (or county) **C** Rural-urban continuum **D** Township central **E** Village

• Knowledge of DMK on Elementary Algebraic Reasoning

1. Consider the following question posed to a pupil of the first cycle of primary education:

What number should be placed in the box so that equality is true?

8 + 4 = ___ + 5

A pupil answers that the number is 12,

a) Explain the possible reasoning that led the pupil to give that answer.

b) Which interpretation of the sign = is being done by the pupil?

2. A pupil was asked to indicate whether the expression «13 + 11 = 12 + 12» is true or false.

The pupil answers the following:

It is true because we subtract one from twelve and add it to the other twelve, the result is what is there (on the left).

a) Explain the reasoning that could lead the pupil to come up with this response.

b) Which properties of addition led the pupil to justify their response?

3. A pupil made the following hypothesis: “I add three consecutive natural numbers. If I divide the result by three, I always get the second number”

a) Is the statement valid for all-natural numbers? Why?

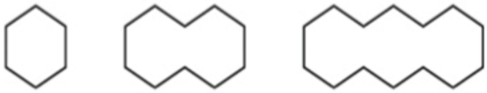
b) In your opinion, what kind of justification could a primary school pupil give to this hypothesis?

$$\begin{array}{cccc} & A & B & C \\ & A & B & C \\ + & A & B & C \\ \hline 2 & A & C & C \end{array}$$

4. Carefully analyze the following sum, and determine the number representing each letter. Consider that each letter has a different value.

a) What are the numerical values of A, B and C? How do you know they are correct? Explain your reasoning.

b) Can you solve the task using an algebraic procedure? What would that solution be and which algebraic concepts would be used?



c) What kind of response and justification do you think an elementary school student could give to this problem?

5. Consider the following sequence:

a) Represent the next two terms of the sequence and indicate the number of segments needed to build each one of them. Explain how you did it.

b) How would you change the statement of the task to hint a solution procedure, which involved algebraic knowledge?

c) What would the algebraic knowledge involved be?

6. Consider the following sequence of three shapes defined by dots:

a) Determine the number of dots that the shape placed on the twenty-fifth (25th) position of this sequence will have, assuming it continues with the same rule of formation. Support your answer.

b) Indicate techniques or different ways to solve the problem.

c) Do you consider that this task could be proposed to students in the 3rd cycle of primary school? How could they reach a solution?

7. A pupil received a certain amount of money to eat for 40 days from his parents. However, he found places where he could save 4 euros a day on food. Thus, the initial budget lasted 60 days.

a) How much Money did he receive?

b) Can you solve the problem using only arithmetic procedures? How?

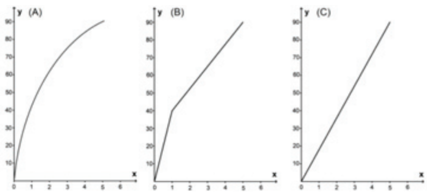
c) Can you solve the problem using algebraic knowledge? How?

1. $4x + 5 = 25$
2. $y = 2x + 1$
3. $P = 2c + 2l$

8. Analyze the following expressions and answers:

a) Describe your interpretation of each of the above expressions.

b) Come up with three problems that may be proposed to elementary students and whose solution leads to these expressions.



9. To fill a container with a maximum capacity of 90 liters with water from a faucet whose flow is constant and equal to 18 liters per minute.

a) Indicate which of the three graphical representations corresponds to the situation described above, the X axis representing time in minutes and the Y axis the volume of water in liters.

b) What mathematical knowledge or other type of knowledge is used to solve this task?

c) Do you consider that this task could be given to primary school children? If so, what cycle? Support your answers.

10. A teacher presents the following problem to his pupils: At a store they sell each kilo of pears for 4¥ and per bag for 0.1¥. Suppose each bag carries 2kg.

a) Come up with a variant of the problem, which could be used to introduce linear functions.

b) Solve the problem you stated and name the algebraic skills used.

Note. For the convenience of investigation in China, the original data and units of this task had been changed in this study.