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A Teaching Experience through the use of Tasks: Limits and possibilities for learning Mathematics in a university context

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ABSTRACT

Context: Rethinking mathematics teaching practices in a university context is an emerging research theme. **Objectives:** In this article, we aim to discuss the limits and possibilities of using mathematical tasks in the teaching and learning processes of the concepts of Derivative, Integral and the Fundamental Theorem of Calculus. **Design:** The study is based on a qualitative-interpretative perspective of research, with methodological procedures inspired by a Design-Based Research. **Environment and participants:** The research was developed with students attending a Functions of a Variable class in a public university in the state of São Paulo. **Data collection and analysis:** Data were collected through mathematical tasks on Differential and Integral Calculus solved by students. The protocols produced were analysed, pointing out the main aspects identified, which led us to organize categories of analysis and dimensions (i) knowledges mobilized and developed by students in relation to mathematical concepts; (ii) main errors and difficulties presented by students in the development of tasks; (iii) limits and possibilities of the practice of exploratory teaching in the university context. **Results:** The results reveal aspects that characterize a process of resignifying the mathematical concepts discussed with the students and a deepening of their knowledge about the concepts of the DIC. **Conclusions:** As future notes, we suggest rethinking university teaching practice, since the study indicated possibilities and potentialities of the use of exploratory tasks in the teaching of Differential and Integral Calculus.

Keywords: Mathematical tasks; differential and integral calculus; design-based research; university teaching practice.

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Uma Experiência de Ensino por meio do Uso de Tarefas: limites e possibilidades para a aprendizagem de Matemática em um contexto universitário

RESUMO

Contexto: Repensar as práticas de ensino da Matemática em um contexto universitário é um tema emergente de pesquisa. **Objetivos:** Neste artigo temos por objetivo discutir os limites e as possibilidades do uso de tarefas matemáticas nos processos de ensino e de aprendizagem dos conceitos de Derivada, Integral e do Teorema Fundamental do Cálculo. **Design:** O estudo enquadra-se em perspectiva qualitativa-interpretativa de pesquisa, com procedimentos metodológicos inspirados em uma Investigação Baseada em Design (IBD). **Ambiente e participantes:** A pesquisa foi desenvolvida com estudantes da disciplina de Funções de uma Variável (FUV) de uma universidade pública no estado de São Paulo. **Coleta e análise de dados**: Os dados foram coletados por meio de tarefas matemáticas sobre Cálculo Diferencial e Integral (CDI) resolvidas pelos estudantes. Os protocolos produzidos foram analisados apontando-se os principais aspectos identificados, o que nos levou à organizar categorias de análises as dimensões (i) conhecimentos mobilizados e desenvolvidos pelos estudantes em relação aos conceitos matemáticos; (ii) principais erros e dificuldades apresentados pelos estudantes no desenvolvimento das tarefas; (iii) limites e possibilidades da prática de ensino exploratório no contexto universitário. **Resultados:** Os resultados apontam para aspectos caracterizadores de um processo de ressignificação dos conceitos matemáticos abordados com os estudantes e um aprofundamento do conhecimento deles sobre os conceitos do CDI. **Conclusões:** Como apontamentos futuros, sugere-se um repensar a prática docente universitária, uma vez que o estudo apontou possibilidades e potencialidades do uso de tarefas exploratórios no ensino de Cálculo Diferencial e Integral.

Palavras-chave: Tarefas matemáticas; cálculo diferencial e integral; investigação baseada em design; prática docente universitária.

INTRODUCTION

Research has pointed out and argued how students' difficulties with mathematics in elementary school reflect on their performance in different higher education subjects (Castro, 2008; Farias, 2015), especially those addressing concepts worked on Differential and Integral Calculus (CDI in Portuguese acronym) (Marin, 2009; Hitt & González-Martín, 2016; Trevisan & Mendes, 2018).

It is recurring to come across research reports that point to "traditional" teaching practices in university education (Speer, Smith, & Horvath, 2010), mainly regarding subjects that involve the contents of CDI (Morelatti, 2001; Jaworski, Mali, & Petropoulou, 2016). Classes in university education usually follow a lecture model in which teachers present the content to be learned orally using some technology (black and/or white board, overhead projector), while students should listen, take notes and do exercises afterwards (Speer et al., 2010).

Immersed in such concerns, both regarding mathematical learning in the discipline of CDI, and rethinking teaching practices at the university, we discuss in this article the results of a study conducted with students from a public university in the State of São Paulo, aiming to investigate the possibilities and limitations of the use of exploratory mathematical tasks in the discipline of CDI.

With this, the present article is constituted by a section presenting the theoretical framework, in which we debate characteristics of the teaching and learning processes of CDI; next, we discuss the concept of exploratory mathematical tasks and how they are used in research; and, finally, we discuss an overview of university teaching practices. Then, we expose the context of research development, bringing the methodological assumptions that guided the study, with special attention to the particularities of the processes of elaboration, development and analysis of the mathematical tasks used. Next, we develop the analysis and discussion of the data. We ended the article with conclusions and future directions about the role of tasks of this nature; we highlight the possibilities and limits of such use, based on the results of the research; and finally, we predict ways and give suggestions to (re) think university practices in disciplines of/about mathematical content.

THEORETICAL FRAMEWORK

a) Teaching and learning Differential and Integral Calculus

Researchers in the area of Mathematics Education have paid special attention to the processes of teaching and learning CDI concepts, addressing aspects related to teaching practice (Jaworski et al., 2016; Richit, 2010; Morelatti, 2001), to students' difficulties (Anacleto, 2007; Verzosa, Guzon, & De Las Peñas, 2014), to the different approaches to teaching CDI (Grande, 2016; Trevisan & Mendes, 2018), among other aspects.

Anacleto (2007) points out that the difficulties faced by students in disciplines that involve the concepts of the CDI are mostly related to the understanding of the meaning of the Fundamental Theorem of Calculus (FTC) and to the understanding of its key concepts: derivative, integral and continuity. Furthermore, even though students understand the relationship between derivation and integration operations, this is not a condition that guarantees them the skills to identify the connection between the concepts of area and rate of change, for example (Verzosa et al., 2014).

Ponce-Campuzano (2013) also highlights factors related to students' difficulties, which are often related to concepts considered fundamental for the understanding of the concepts of limit, integral and derivative. Among such factors, there is the limited knowledge that students have about the concept of function; the little understanding about the idea of the rate of change and accumulation; the few skills to work with the concept of covariation; and the understanding of the order in which the two "parts" of the FTC are presented in introductory Calculus courses.

We also highlight the need to pay attention to the training that is provided to students in the discipline of CDI, that is, it is necessary to identify what experience and what knowledge remains for them after taking such course. According to Bressoud (2011), many students are unable to understand and remember the subjects covered in the CDI, and they only have a procedural understanding of the discipline. For this reason, "as we think about how we should teach FTC, we must keep in mind what we want students to remember about this course and, then, we must work hard to ensure that this happens" (Bressoud, 2011, p. 113).

b) Mathematics teaching practices at the university

Although researchers in Mathematics Education seek alternative teaching approaches for students to overcome their difficulties in the discipline of CDI (Trevisan & Mendes, 2018), what prevails in higher education classrooms are approaches based on a rigor imposed by the presentation of the concepts and by the resolution of lists of exercises, "[...] of a purely algebraic and mechanical character, without considering the meaning of such concepts" (Richit, 2010, p. 27).

Morelatti (2001) highlights that the CDI discipline remains known for its high failure and dropout rate, as well as for a teaching methodology that prioritizes lectures centered on the teacher, in which "students, after class, solve a series exercises that often do not require creativity, reflection and new concepts" (p. 21). Thus, in general, CDI classes at the university follow the "traditional teaching" model, known for an approach based, most of the time, solely on the exposure of content, with little stimulation, without favoring creativity or a learning impact on students.

The study by Jaworski et al. (2016) points out that, in universities, most teaching is developed through an approach based on a unilateral lecture format, often described as transmission teaching, that is, "the teacher is the one who exposes mathematics to students, who listen, copy from the board and make their own meanings from the experience" (p. 169). Therefore, there are many students with difficulties with Mathematics as it is presented, as they are not prepared to deal with certain university concepts and contents from their school experience, due to the degrees of abstraction and formalism experienced during classes at the university.

In this sense, Marin (2009) summarizes some causes related to the problem of teaching and learning the concepts of CDI: the first of them refers to the fact that this subject addresses transition content between high school and higher education and, in many universities, is offered in the first semester of the undergraduate course, with a high number of students in the classroom; the second refers to the previous deficient training of students, which has impaired their performance in the discipline, that is, they start with precarious high school formation and, with this, to try to remedy some difficulties, the teacher ends up favoring the revision of some concepts and condensing the contents that need to be taught in the discipline; finally, as the workload is inadequate and the syllabus is extensive, classes end up following a fast pace, with few moments for questioning and developing critical thinking about what is being discussed, a fact that also reflects in other later subject matters that depend on the knowledge contemplated in CDI.

c) The use of exploratory tasks in the teaching of mathematics

Hitt and González-Martín (2016) point out that research in Mathematics Education has focused efforts on discussing the teaching practices of teachers of basic education, and research on practices in university education is scarce. Therefore, we bring aspects of teaching practice, from the use of exploratory-investigative mathematical tasks (Ponte, 2005, 2014; Ponte et al., 2015), as a possibility to contribute to the teaching and learning processes of CDI.

Seeking to characterize the teaching practice that privileges the use of tasks, Ponte (2005) discusses aspects of "curriculum management", that is, "the way the teachers interpret and (re) construct the curriculum, taking into account the characteristics of their students and their working conditions" (p. 11). For the author, when developing the planning of the didactic unit, the teacher makes use of a teaching strategy that can be (i) direct, privileging the lecture and the resolution of exercises, or (ii) exploratory teaching-learning, which has as a main characteristic the fact that the teacher does not explain everything, "but leave an important part of the work of discovery and knowledge construction for students to carry out" (p. 13), that is, it is based on the emphasis on exploration and research and teacher-student and student-student discussion.

According to the author, these are tasks that aim to support the learning of the students involved, and "are usually (but not necessarily) proposed by the teacher, but, once proposed, they have to be interpreted by the student and can give rise to very different activities (or no activities at all)" (Ponte, 2014, pp. 14-15). In this context, the task leads students to the development of different activities and towards their resolution. With this, "learning results from activity, not from tasks, and the most determinant are always the attitudes and concepts of the actors involved" (p. 15).

According to this approach, the proposed tasks aim to "[...] provide a consistent learning process to facilitate the construction of concepts and the understanding of procedures and to broaden the knowledge of relevant representations and connections between Mathematics and others areas" (Ponte et al., 2015, p. 112).

Based on the discussions developed here, we argue that a teaching practice that favors working with exploratory mathematical tasks can contribute to breaking with the "direct teaching" approach, or at least, proposing alternatives for a new approach. When working in the university classrooms from this perspective, it is possible to favor the discussion of mathematical concepts among students, always encouraged by the teacher, and contribute to their learning process in a different environment.

CONTEXT OF RESEARCH AND METHODOLOGY

We developed our study at a public university in the state of São Paulo, with students from an Interdisciplinary Bachelor (IB) course, in which "freshmen" are admitted to two types of IB, the Bachelor of Science and Technology (BC&T) and the Bachelor of

Science and Humanities (BC&H). After completing one of the interdisciplinary bachelor's degrees, the student chooses another course to pursue[1].

When starting university education, students find a curriculum with several subjects that are common and mandatory to the two IBs, as is the case of the One Variable Functions (OVF) discipline, in which the concepts of derivative, integral and FTC. This was the context in which we developed our investigation.

In 2017, one of the OVF disciplines offered by the university was developed by the authors of this article. 69 students participated in this course, 21 from BC&T and 48 from BC&H. With these students, we developed three exploratory mathematical tasks and, in this article, we chose to analyze only the tasks developed by BC&T students, as they are those who, in general, continue in professional careers related to Mathematics.

From a methodological point of view, the present investigation is part of a qualitative-interpretative study (Crotty, 1998; Esteban, 2010), whose procedures were inspired by a design-based investigation (DBI) (Ponte et al. 2016). Such methodology is suitable for the purposes of this study, as it is a research approach in which "educational interventions are studied in order to promote certain learning or systematic changes and understand the processes that underlie them" (Ponte et al., 2016, p 77). In addition, in this type of research, after identifying the research problem, one proceeds to the development of "[...] intervention, which must be materialized through some type of educational product. This goes through the process of analysis and refinement, so that, at the end of the investigation, it can be used by other people in other contexts" (Barbosa & Oliveira, 2015, p. 530).

With regard to data collection, we use participant observation and data collection procedures through student protocols, when performing the mathematical tasks developed. To this end, three exploratory mathematical tasks that addressed the concepts of derivative and integral of a function and the FTC were developed. Such tasks sought (i) to resume students' previous knowledge related to the concept of function, (ii) to provide the construction of new knowledge of the concepts contemplated, from the design of the tasks and the moments of discussion between students and between them and the teacher. The students, during the tasks, worked in pairs and the teachers (researchers) observed the development of the tasks.

MATHEMATICAL TASKS AND THE DATA ANALYSIS PROPOSAL

The tasks were developed with the students at different moments of the discipline: (i) the tasks on Derivative (DT) and FTC (FT), worked after the discussion of these concepts with the students, thus being characterized as application/validation tasks of concepts,

[1] Upon completing the IB, BC&T students can choose between courses in engineering; mathematics, biology, chemistry and physics (bachelor's and teaching degree); computer science; and cognition.

while (ii) the task on Integral (IT) was developed before the discussion of this concept with students and is characterized as a task of introduction to the concept.

The first task (DT) contained three parts: (i) interpretation and application of the derivative concept as slope of the line tangent to the graph of a function at a given point; (ii) interpretation and use of the derivative concept to study the behavior of the graph of a function; (iii) problem situation involving the idea of optimization, with the purpose of using an application of the derivative concept.

The second task (IT), in which we sought to address the concept of area calculation of a region limited by the graph of a function and the *x-axis* in a given range of the function domain, was also divided into three parts: (i) different situations for calculating the area of a region limited by the graph of a constant function and the *x* axis in a given interval; (ii) different possibilities to calculate the area of the region limited by the graph of a linear function and the *x* axis in a given interval; (iii) a situation to calculate the area of the region limited by the graph of a quadratic function and the *x* axis in a given interval, using the division of the region into rectangles, in order to implicitly address the concept of Riemann sum to calculate areas.

Finally, the third task (FT) explored the concept of definite integral calculus and the establishment of the relationship between differentiation and integration as inverse operations. This task was divided into four parts: (i) and (ii) addressed the relationship between the concept of primitive and derivative of a function, seeking to address the first part of the FTC; (iii) contemplated the graphic representation of the defined integral and its application, focusing on the second part of the FTC; and (iv) questioned the knowledge mobilized by the students during the performance of the three tasks performed during the course of OVF.

The data collected were analyzed from a movement to identify the students' learning when developing the tasks and the way such learning related to the exploratory approach and the use of tasks. From this analysis movement, three dimensions emerged:

i. aspects related to the knowledge mobilized and developed by the students, with regard to the mathematical concepts involved in the tasks;

ii. errors and difficulties presented by students when developing the tasks;

iii. limits and possibilities of an exploratory teaching practice, through mathematical tasks, identifying whether and how these favored the learning of the concepts of CDI in a class of students of higher education.

These dimensions were not established a priori, but rather, they became eminent during the process of reading and analyzing the data collected through the students' protocols, as well as from the notes arising from the participant observation of the researchers throughout the process.

Since the analysis of tasks takes as principle to characterize the limits and possibilities of a practice founded on exploratory teaching, based on the use of mathematical tasks involving concepts of CDI, we searched in the relevant literature some theoretical and methodological subsidies to understand better the evidence identified and the analysis developed from the students' resolutions. We also tried to consider aspects that have not been favored during the development of the tasks and that can subsidize a new design of the tasks that we carry out.

ANALYZING THE DATA

We developed the analysis pointing out the main aspects identified in the protocols produced by the students, when developing the mathematical tasks, during the OVF discipline. This analysis contemplates, as explained above, three dimensions organized through an attentive look at the data, and they are related to the *mathematical knowledge mobilized and elaborated by the students during the development of the tasks*; *to the students' mistakes and difficulties in the development of tasks*; *to limitations and difficulties arising from the practice of exploratory teaching in university classrooms*.

Regarding the *knowledge mobilized and elaborated by the students*, it was possible to identify, in the task about FTC, that they established the relationship between the function $f(x)$ and its primitive, that is, they came to the conclusion that the result for the generic interval $[0, x]$ corresponded to the primitive of the function ($F'(x) = f(x)$ e $\int f(x)dx = F(x)$). The same happened for the generic interval of $[a, b]$, because the students realized that they were calculating the definite integral of function $f(x)$ for that interval and then used the second part of the FTC for their rationale ($\int_a^b f(x)dx = F(b) - F(a)$).

As for the task on FTC, students were able to understand the relationship between the concepts of derivative and integral, and the fact that FTC establishes such a relationship. This understanding is based on the work developed during the application of the tasks, their *design*, as well as the way they were developed, thinking about the collective discussion and the construction of knowledge in collaboration with peers.

We also observed that the relationship between the concept of derivative and integral, present in the task that mainly addressed the concept of FTC, led students - when asked about the relationship between the displacement of a particle that moves according to the function $s = f(t)$ in the interval [0.4] and the value of the defined integral $\int_0^4 v(t)dt$, being $v(t)$ the derivative of the function $s = f(t)$ - to the conclusion that the value of the displacement of the particle was the same, when calculated either from the position function or from the defined integral of the velocity function. In this sense, the students' resolutions demonstrate their understanding of the relationship between differentiation and integration as inverse operations, that is, they understood that $s(t) = \int v(t)dt$ e $s'(t) = v(t)$.

The OVF discipline, the context of our investigation, had the participation of students who had already taken this course. Thus, in the development of the task for the introduction of the concept of integral - which addressed the calculus of the area of a flat region limited by the graph of a function and the *x-axis* in a given interval - when asking students about the different possibilities of calculating this area, the analysis of the resolutions revealed that they used the concept of integral to propose a way to calculate the area of that region. Furthermore, some students calculated first the area using the concept of integral and, when asked about a new way of calculating this area, they identified it as a geometric figure and that they had another possibility to calculate the area. We believe that this happened because many students were not taking the course of OVF for the first time, that is, they had already hadsome contact with the application of the integral concept to calculate the area of plane figures.

Therefore, among *the knowledge mobilized and developed by the students regarding the mathematical concepts involved in the tasks*, we can highlight the knowledge prior to the discipline in relation to the concept of function, since the use of mathematical tasks led students to develop sketches of graphs of different elementary functions and to identify points in the Cartesian plane. In addition, tasks, whenever possible, asked students to determine the domain of different functions, and to study and interprete the behavior of a function from its graph.

Regarding the *knowledge developed by students in relation to the derivative concept*, it was possible to identify that they understood the application of the derivative concept as the angular coefficient of the line tangent to the graph of a function at a given point. The students also understood the interpretation of the derivative concept of a function to identify the growth and decrease intervals of the function and used the derivative concept to solve a proposed problem situation involving optimization.

Regarding the *knowledge developed by students in relation to the concept of integral*, we could observe that the pairs came to understand the intuitive idea of the Riemann Sum concept, realizing that the greater the amount of rectangles inserted below the graph of a function, the closer it is to the exact value of the area of the figure consisting of the region limited by the graph of the function and the *x* axis in a given interval.

Dealing with the *students' mistakes and difficulties in the development of tasks* regarding the learning process of the concept of a derivative, we highlight that not all pairs were able to construct generalizations of the concepts present in the task about derivatives from the calculations and results they obtained. This indicates a certain difficulty in generalizing the application of the derivative concept, that is, some pairs calculated the derivative of the function in each of the points and for the generic point, but were unable to express through words, the calculations they had developed:

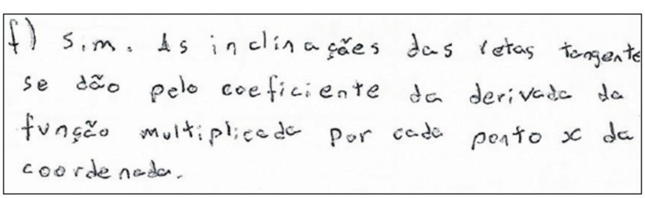
f) Sim. As inclinações das retas tangente se dão pelo coeficiente da derivada da função multiplicada por cada ponto x da coordenada.

Figure 1. Gustavo and Kelvin's answer to DT (survey data)[2]

[2] Yes, the slopes of the tangent lines are given by the coefficient of the derivative of the function multiplied by each *x* point of the coordinate.

In addition, we have the exclusively algebraic answer developed by one of the student pairs. We see that the students understood the application of the derivative concept as slope of the line tangent to a point, however, they chose to bring an answer with mathematical expressions, instead of using their own words:

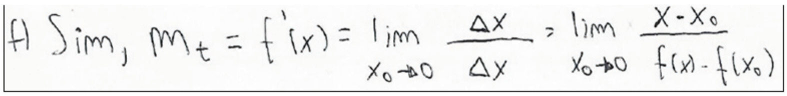
f) Sim, $M_t = f'(x) = \lim_{x_0 \to 0} \frac{\Delta x}{\Delta x} = \lim_{x_0 \to 0} \frac{x - x_0}{f(x) - f(x_0)}$

Figure 2. Gabriela and André's answer to DT (survey data)[3]

The concept of optimization was also addressed in the DT task, where the difficulty of the pairs to both determine a mathematical expression (function) that represented the problem situation and identify the domain of this function was observed. Some pairs solved the optimization problem using the derivative concept, but others opted for the solution by *trial and error*, checking which value of *x* would make the box have as much volume as possible (problem to be solved in that situation).

We highlight the students' confusion to establish a relationship between function *f*(*x*) and its primitive *F*(*x*) during the development of the task that involved the concept of the FTC, since we asked them to find the results inherent to the generic intervals for each of the functions provided and, soon after, establish a relationship between the results found and the function f(x). We noticed that they developed the algebraic calculations correctly, but when they were about to conclude the reasoning on what was being questioned, they ended up making an inversion and saying that "*f*(*x*) is the primitive of *F*(*x*)":

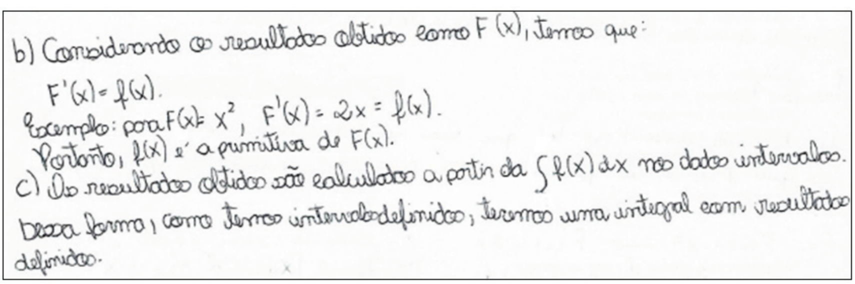
b) Considerando os resultados obtidos como F(x), temos que:
$F'(x) = f(x)$.
Exemplo: para $F(x) = x^2$, $F'(x) = 2x = f(x)$.
Portanto, f(x) é a primitiva de F(x).
c) Os resultados obtidos são calculados a partir da $\int f(x)\,dx$ nos dados intervalos. Dessa forma, como temos intervalosdefinidos, teremos uma integral com resultados definidos.

Figure 3. Fernanda and Thainara's aswwer to FT (survey data)[4]

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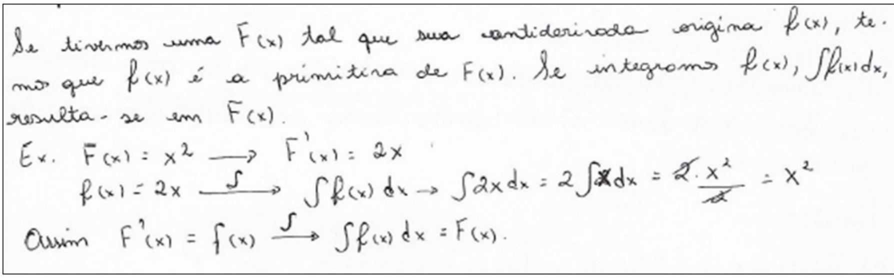
[3] f) Yes, $M_t = f'(x) = \lim_{x_0 \to 0} \frac{\Delta x}{\Delta y} = \lim_{x_0 \to 0} \frac{x - x_0}{f(x) - f(x_0)}$.

[4] b) Considering the results obtained as *F*(*x*), we have: $F'(x) = f(x)$
Example: for $F(x) = x^2$, $F'(x) = 2x = f(x)$.
Therefore, *f*(*x*) is the primitive of *F*(*x*).
c) The results obtained are calculated from $\int f(x)dx$ in the intervals given. Thus, as we have defined intervals, we will have an integral with defined results.

In the resolution that we present below, the students manage to develop a coherent answer based on algebraic calculations, however, they do not go so far as to explain in writing what they had developed:


Se tivermos uma $F(x)$ tal que sua antiderivada origina $f(x)$, temos que $f(x)$ é a primitiva de $F(x)$. Se integramos $f(x)$, $\int f(x)dx$, resulta-se em $F(x)$.

Ex. $F(x) = x^2 \longrightarrow F'(x) = 2x$

$f(x) = 2x \xrightarrow{\int} \int f(x)\,dx \rightarrow \int 2x\,dx = 2\int x\,dx = 2\cdot\frac{x^2}{2} = x^2$

Assim $F'(x) = f(x) \xrightarrow{\int} \int f(x)\,dx = F(x)$.

Figure 4. Fernanda and Thainara's aswwer to FT (survey data)[5]

Many students have shown that the understanding of the application of the derivative concept is only related to the concept of slope of the line tangent to a given point. This could be observed because the students were not able to identify that the behavior of the graph of the derivative function can provide information on the intervals of growth and decrease of the function, an application that had been discussed in the classroom.

Also in the task on FTC, we identified the students' difficulty to develop the mathematical expression that related the highlighted area in the graphical representation of the position function and the speed function provided to students, the calculations they developed and the FTC. Many students demonstrated that they understood the relationship between the definite integral of the velocity function, the position function and the displacement value of the particle – $\int_0^4 v(t) = s(4) - s(0)$. However, when elaborating a mathematical expression that represented the relationship, most of them chose to use the mathematical expression from the second part of the FTC discussed in the classroom[6], using generic functions, instead of the functions that were being addressed in the task:

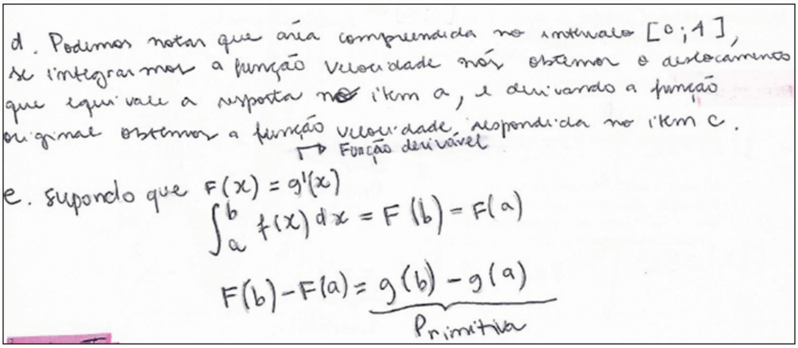
[5] If we have a $F(x)$ such as its antiderivative originates $f(x)$, we have that $f(x)$ is a primitive of $F(x)$. If we integrate $F(x)$, $f(x)$, $\int f(x)\,dx$, it results in $F(x)$.

Ex:: $F(x) = x^2 \rightarrow F'(x) = 2x$

$$f(x) = 2x \rightarrow \int f(x)\,dx \rightarrow \int 2x\,dx = 2\int x\,dx = 2\frac{x^2}{2} = x^2$$

Thus $F'(x) = f(x) \rightarrow \int f(x)\,dx = F(x)$.

[6] $\int_a^b f(x)dx = F(b) - F(a)$.


d. Podemos notar que área compreendida no intervalo [0;1], se integrarmos a função velocidade nós obtemos o deslocamento que equivale a resposta no item a, e derivando a função original obtemos a função velocidade, respondida no item c.
↳ Função derivável

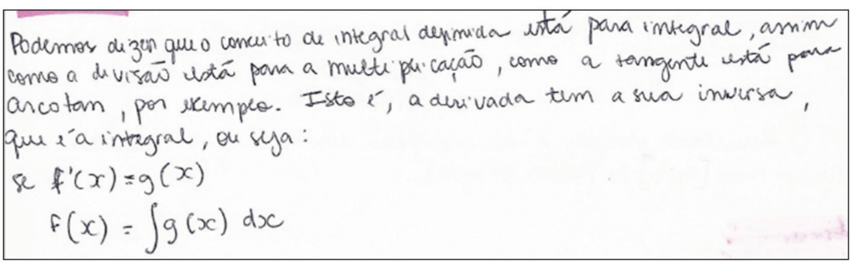
e. supondo que $F(x) = g'(x)$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$F(b) - F(a) = \underbrace{g(b) - g(a)}_{\text{Primitiva}}$$

Figure 5. Juliana and Kelvin's answer to FT (survey data)[7]

An erroneous understanding is characterized by the expression "the integral is the inverse of the derivative", presented by the students when we questioned whether from the developed tasks it was possible to establish a relationship between the concept of integral and the concept of derivative. In fact, they were expected to conclude that, as established by the FTC, derivation and integration are inverse operations, and to explain the algebraic representation $\int f(x)dx = F(x)$, in which $f(x) = F'(x)$:


Podemos dizer que o conceito de integral definida está para integral, assim como a divisão está para a multiplicação, como a tangente está para arcotan, por exemplo. Isto é, a derivada tem a sua inversa, que é a integral, ou seja:

se $f'(x) = g(x)$

$$F(x) = \int g(x)\,dx$$

Figure 6. Juliana and Kelvin's answer to FT (survey data)[8]

In this case, the students tried to establish a parallel between other operations, to explain the relationship between the derivation and integration operations. We could also observe they did not know exactly how to express the answer with an algebraic representation.

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[7] d. We can observe that the area comprehended in the interval $[0; 1]$, if we integrate the function velocity, we obtain the dislocation that is equivalent to the answer in item a, and by getting the derivative of the original function, we obtain the function velocity answered in item c.

e. Let's assume $F(x) = g'(x)$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$F(b) - F(a) = g(b) - g(a)$$

[8] We can say that the concept of defined integral is for integral, as division is for multiplication, as tangent is for arc tangent, for example. That is, the derivative has its inverse, that is the integral, that is:

If $f'(x) = g(x)$

$F(x) = \int g(x)\,dx$

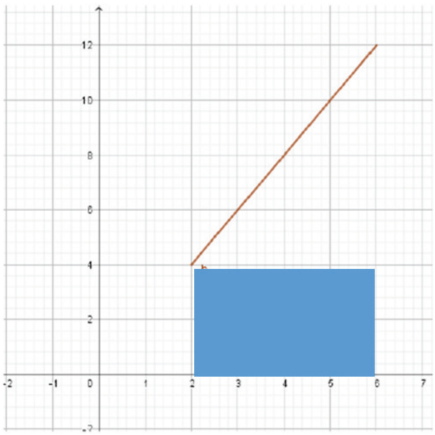
Based on the previous discussions, we can say that the data analysis revealed different *errors and difficulties presented by the students when developing the tasks*, especially when dealing with the sketching of graphs and the identification of points in the Cartesian plane, a fact that contributed to a full development of the tasks. We also emphasize that in the learning process, not all the pairs were able to construct generalizations of the concepts present in the tasks from the calculations and the results they obtained. In addition, the students found it a little difficult to formalize and systematize the relationships established with the development of the tasks. Often students chose to develop exclusively algebraic responses, using mathematical expressions, instead of using their own words.

Finally, regarding the *limitations and difficulties arising from the practice of exploratory teaching in university classrooms, we highlight aspects of the* task developed for the introduction of the concept of integral that focused on the rescue of students' previous knowledge in relation to the concept of function and the calculation of areas of plane figures. The difficulty level of this task evolved as the students progressed in their development, in order to arrive at the calculation of areas below the function graph, in which the plane figure did not have a known format for the area calculation, and would need other resources.

This structure of the task reflects the good performance of the students in their development, both in relation to the calculation of the area of the figure that was formed under the graphical representation of the function constant at a certain interval - a rectangle, in relation to the figure below the linear function in the interval established.

Also regarding the task of introducing the concept of integral, we emphasize that the students, even though they identified that the quadrilateral present in the figure limited by the graph of the linear function and the x axis in the given interval have equal sides, only two pairs named this quadrilateral as square, while most of the pairs named it a rectangle. We believe that this is because the presentation of the graph of the linear function was developed out of scale, and in this case, visually, the figure seemed to be a rectangle.

Figure 7 illustrates the task proposed, highlighting the figure limited by the graph of the linear function and the x axis in the interval [2,6], noting that the highlight was offered to students when the task was proposed.


Tarefa 2: Considerando o gráfico da função $h(x) = 2x$, no intervalo de [2,6], a seguir:

a) Calcule a área da figura limitada pelo gráfico da função e pelo eixo x, no intervalo em que a função está definida.

b) Calcule novamente a área da figura descrita no item a, utilizando-se de procedimentos distintos daqueles utilizados no item anterior.

Figure 7. Task 2 that compose the Task on Integral[9]

Also regarding the task on integral, the students found a little difficult to determine the value of the measure of the base of the rectangles that would be inserted below the graph of the quadratic function. We believe that this is due to the function provided to students ($g(x) = x^2$) and because the interval starts at 0, that is, the first rectangle would have a base measurement equal to 1 and a height measurement equal to 0. This also helps to understand why some students chose to insert rectangles that go beyond the function graph. We understand this fact as a limitation of the task, which, in a new version, may indicate a different interval and/or function than the ones presented. In addition, we consider it necessary, in a next version, to explore both the rectangles inserted below the function graph and the rectangles with height that exceeds the limit of the function graph, so that students can compare the area values in each situation.

Finally, dealing with the *limits and possibilities of practice based on exploratory teaching*, we realize that the use of mathematical tasks, such as those we develop with students, can collaborate with the construction of mathematical knowledge in relation to the concepts of derivative and integral. We credit this construction to the way these tasks were developed and worked with students, by privileging a task *design* that sought to

[9] Task 2: Considering the graphic of the function $h(x) = 2x$, in the interval of $[2, 6]$, as follows:
Calculate the are of the figure limited by the graphic of the function and by axis *x*, in the interval in which the function is defined.
Calculate again the area of the figure described in item a, using different procedures from the ones used before.

rescue students' previous knowledge, as well as leading them to build new knowledge, through problem situations and discussions in pairs.

DISCUSSION OF THE RESULTS

Considering the focus of the article, we have chosen to reflect on the main aspects of the practice based on exploratory teaching, through the development of tasks on CDI, and on how those tasks can be modified and/or expanded, in a way that guide students in understanding the concepts covered.

We identified, based on the analysis of the different errors and difficulties in the development of tasks, errors related to the concepts seen previously to the OVF discipline, such as, for example, determining the domain and sketching the graph of different functions, as well as errors related to the concepts of derivative and integral. In addition, the students demonstrated a certain lack of rigor with the mathematical language, not using the mathematical symbols correctly and providing summarized answers.

We found that the nature of the tasks proposed and the approach adopted by teachers (Ponte et al., 2016) led students to a new dynamic in the learning process, fostering the elaboration of written answers, conjectures and systematizations based on peer discussion (Jaworski et al., 2016). However, there have been some difficulties, such as the ones we reported, since most students had never had this type of experience with tasks. Therefore, we emphasize that, over time, students evolve to answer the questions of the tasks, since they recognize that the answers help them articulate what they think or what they want to say (Jaworski et al., 2016).

Errors related to the concepts seen prior to the OVF discipline were raised and identified, such as, for example, determining the domain and sketching the graph of different functions, as well as errors related to the concepts of derivative and integral. Also, the students demonstrated a certain lack of rigor with the mathematical language: they did not use the mathematical symbols correctly and elaborated summarized answers.

Students' difficulties were also observed when developing written answers, especially in situations where we requested a generalization of what had been developed in the task, and we tried to identify if they had understood the concept discussed. Many pairs chose to bring answers using only mathematical expressions, while others often resolved only the items in which resolutions were requested using algebraic calculations, to the detriment of the use of natural language.

Thus, when looking at the practice developed, we consider the importance of formalizing and systematizing ideas for the construction of knowledge, since it is at that moment that the student elaborates a relationship between the algebraic calculations developed and the concept approached, culminating in the production of knowledge on that concept. Therefore, students recognize themselves as autonomous beings that are responsible for their learning (Trevisan & Mendes, 2018).

The analysis of tasks, as well as the experience and observation of researchers during their development revealed the need for some adjustments to avoid new mistakes, and expand the approach of some concepts, such as the concept of integral. In addition, we consider it important to develop two new tasks: (i) introduction of the concept of derivative and (ii) validation of the concept of integral[10].

The students' anxiety in relation to the development of the first task owes to the fact that they had had no experience at all with this methodological approach in OVF, since the classes follow in a traditional approach, with traditional lectures, the development of examples and exercises (Morelatti, 2001; Richit, 2010). Thus, a teaching practice based on exploratory teaching brings new possibilities to the teaching and learning processes, breaking away from classes that adopt a lecture model and end up limitng the development of different practices (Speer et al., 2010).

Furthermore, a teaching and learning proposal that favors the work with mathematical tasks needs to consider aspects indicated by research in Mathematics Education and differ from classes based on the unilateral lecture format and centered on the textbook. Therefore, as highlighted by Trevisan and Mendes (2018), such proposals need to meet the routines of the classroom and be "in line with the didactic-pedagogical organization proposed by the institution (committed to a mandatory curriculum, to the political-pedagogical project of the course, to the attribution of a grade at the end of a period)" (p. 210).

The use of mathematical tasks and the way they were worked with students contributed to a "coherent learning path" (Ponte, 2005, p. 18), since they allowed (i) the construction of mathematical knowledge about the concepts addressed by part of the students; (ii) the understanding of mathematical procedures; (iii) the partial mastery of notations and relevant forms of representation about the concepts of derivative and integral.

Finally, we emphasize that an exploratory teaching practice, with the use of mathematical tasks such as the ones we develop, is composed of teacher choices, which aims to "work in a natural way the different aspects of content" (Ponte, 2005, p. 18). Such choices come up against the different aspects that make the university context - in our case, for example, the students' especificities, the curriculum, the resources we can access to develop the tasks. In the research experience we are reporting, we always had in mind the need to establish an adequate strategy, favoring exploration, reflection and discussion on the mathematical concepts covered.

[10] A scientific initiation research is under development, advised by the second author of this article, aming at elaborating such tasks, besides developing interviews with university professors to identify (a) what they think of the tasks and (b) what they think about the way of working with them (to, once again, relate to "higher education teaching practices").

FINAL CONSIDERATIONS

When analyzing the use of mathematical tasks on CDI in a higher education classroom, we were able to identify some limits and possibilities of this approach regarding student learning. When unveiling and seeking to understand such aspects, we aim to suggest possible different approaches to be used in the teaching and learning processes of CDI. With that, we suggest alternatives for teaching practice based on exploratory teaching in the university environment. In this work, the tasks developed played a central role, as they allowed students to refine intuitive concepts and previous knowledge, as well as to establish relationships between the new concepts of the CDI discipline. In addition, we saw the involvement of students in the development of tasks, which resulted in the construction of mathematical knowledge, always prioritizing the discussion and systematization of concepts, besides enabling a different experience during the discipline, through collective discussions that took place over time.

On the other hand, when discussing the limits faced during the development of tasks, we refer, in particular, to the difficulties of students with content prior to the subject matter, such as, for example, the concept of function; students' resistance to a differentiated methodology in the classroom; the choice and organization of tasks, prioritizing the construction of knowledge in peer collaboration and the teacher's fulfillment of a specific curriculum of the discipline.

However, despite the aspects identified in the research, different questions remain open and are open to be investigated: How to prioritize a practice based on exploratory teaching that encompasses the development of tasks and moments of reflection and discussion, considering the different factors that compose the context of higher education, such as, for example, the number of students in the classroom? How to provide formative experiences that favor higher education teachers to work with the approach considered in this research?

We understand that this research opens space for different and new discussions on the practice of exploratory teaching in the university context, with regard to both the learning of students and the training profile and the practice of university professors who work in this perspective. Thus, we hope, based on the results discussed here, to contribute to research in university Mathematics Education regarding the teaching and learning processes of CDI concepts, and to provide reflections on/for other teachers' teaching practice, inspiring teaching work centered on task development and collective discussions between students and between them and their teachers.

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AUTHOR CONTRIBUTION STATEMENT

A. J. R. supervised the development of J. F. V. P.'s postdoctoral research project, in which the data analyzed in this article were constituted. A. J. R. was responsible for devising the format of this article. J. F. V. P. developed the organization and data analysis. Both authors discussed and contributed to the final version of this article.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the corresponding author, J. F. V. P., upon reasonable request.

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