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Differential Equations: Between the Theoretical Sublimation and the Practical Universalization

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"Great mathematicians have acted on the principle of ***Divinez avant de démontrer,*** *and there is no doubt that almost all important discoveries are made this way."*
(Edward Kasner)

ABSTRACT

In this paper, we present, briefly, the bifront character of the ordinary differential equations (ODE): on the one hand the theoretical specialization in different areas and on the other, the multiplicity of applications of the same, as well as some reflections on the development of a course of ode in this context.

Keywords: Ordinary Differential Equations. Applications of ODE. Mathematics education.

Equações Diferenciais: entre a Sublimação Teórica e a Universalização Prática

RESUMO

Neste artigo, apresentamos sucintamente o caráter de duas faces de equações diferenciais ordinárias (EDO): uma experiência teórica em diferentes áreas e, por outro, a multiplicidade de aplicações, bem como algumas reflexões sobre o desenvolvimento de um curso de EDO neste contexto.

Palavras-chave: Equações diferenciais ordinárias. Aplicações de EDO. Educação Matemática.

INTRODUCTION

One of the foundations of the current reform of teaching mathematics is the concept that part concerning the nature of mathematical knowledge. The historical perspective allows us to show, among other things, that mathematics is a body of knowledge constantly evolving and this evolution often plays a major role their interrelationship with other knowledge and the need to resolve certain practical

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problems. Another important consideration is derived from the use, in the historical process of constructing mathematical knowledge, empirical-inductive reasoning in not less than deductive reasoning degree.

All of the above can reaffirm the fact that the development of mathematics has followed a heuristic, historically demonstrated process (Farfán, & Hitt, s.d.). Contrary to the claims of proponents of deductivist style, who claim that the deduction is both the pattern of mathematics as the logic of discovery, like most of the concepts developed by an isolated mathematician. Thus, when the historical roots of the concepts are evident, they clearly observed the circumstances that originally led and promoted its development towards becoming an essential part of coherent and meaningful theories. Although it is necessary to emphasize, is not enough to analyze the facts and the evolution of ideas and concepts; an evaluative and critical view is also necessary to select and reconstruct the problems that can really help develop a creative activity.

The main problem in mathematics education, is that these models or methodologies based on the use of historical resources have not been except very few cases- the field of teaching mathematics, being undeniable need them, because the success of the teaching and learning of mathematics depends on a successful combination of logical, historical and pedagogical.

In this direction, it should be noted that the didactic-methodological framework in which our conference is framed is as follows:

1. Conceiving dynamically mathematics, which is expressed in the famous phrase by the French mathematician of Philip E. Jourdain (1879-1919), in the introduction to his text "The nature of mathematics" when declaring the central objective noted: "I hope I get to show that the process of mathematical discovery is alive and developing " (Jourdain, 1976). This view is reflected in a teaching based on problem solving, both for the development of various logical abilities of students, and to clarify what those events were that led to the emergence of a concept and why, what was the frame of rigor at the time, what methodology, concepts and how these factors influenced the development of mathematics was given in one direction and not another.

2. Accept the triple meaning of mathematical objects: institutional, personal and temporal.[1]

3. Distinguish between an argument and a demonstration test, and the necessary dosage of these in the school curriculum, as well as discussions about classical conceptions

[1] Knowledge is produced with temporal continuity and not only in the institutional area recognized for that purpose, occurs in all areas of human life. Different knowledge produced can be parceled for analysis and rate each facet separated with a different name, but in the subject who knows such separation is impossible so that the mathematical activity must take into account such diversity knowledge sources and the conditions that have the mathematical knowledge immersed in the powerful cultural knowledge. Both levels of analysis within and outside of schools, we believe that may be valid to shed light on the cognitive processes. See Díaz & Batanero (1994) and Nápoles (1997a).

of mathematical proof and rigor within the same.[2] The concept of mathematical proof not only as formal verification of a result but as a convincing argument as a means of communication has become more important lately mainly linked to certain problems of mathematics education. Thus, sometimes they prefer to explain evidence rather than evidence just "try". Both tests as evidence proving that explain valid. They have become important in recent times even calls wordless tests where geometric representations would come to play the role of the necessary explanations.

4. That there are qualitative differences between academic performance (at the research level, as "knowing wise") of certain knowledge and teaching operation thereof because, for various reasons, uses and connotations of mathematical notions treated in institutions teaching are necessarily restricted.

In several previous works (Nápoles, 1997b, 1988, Nápoles & Negrón, 2002), we have presented other results on the historical focus problem at hand; that although they are independent of this work, they serve as a prologue to it. In particular, we require a little more didactic indications of ordinary differential equations.[3]

Moreover, we can ensure that sensitive situation currently changes the methodological principles of the specific teaching of this subject, motivated by studies that have penetrated even the ontology and epistemology of mathematics teaching and learning is experienced. It is tangible from the effort to introduce advanced undergraduate ideas openly defend the metaphor of research professor, sustaining a dynamic stance of mathematics, which is the punishable methodology of teaching, problem-solving.

We would like to echo the words of the National Program Problem Solving when he postulates:

> The math is a mindset, a way of reasoning. It decides whether an idea is reasonable or at least to establish whether an idea is probably adequate for what you are looking for.The math is open to exploration and research field and every day new and fruitful ideas are produced. It is a mindset that serves to solve the problems of science, administration, industry, etc. (See details in http://www.me.gov.ar/curriform/publicaciones2002/reuegb3poli/estrategias.pdf)[4]

Undoubtedly the mathematical sciences, as well as the exercise of his teaching, throughout its history, has always had as its main means and end solving mathematical

[2] We can briefly say that an argument is the action to let you know something to someone, maybe yourself that a test is a special type of argument that incorporates a true epistemic value and demonstration is a logically conclusive evidence.

[3] Such indications have been outlined in (Nápoles, 2000a).

[4] Other issues related to mathematical problems (classification, strategies for their solution and construction) can be consulted in (Nápoles, 1999, 2000b, Nápoles & Cruz, 2000). Many of these issues are reflected in (Nápoles, 2005a).

problems. Halmos cannot be more eloquent about when he says that problems are "the heart of the math" (Halmos, 1980, 524). Problem-solving involves dissimilar gear cognitive resources from the resolver. For this, solve a problem should serve not only to a simple intellectual training but also for healthy and enjoyable entertainment. To illustrate this clearly enough, we have taken a not too heterogeneous subject, ordinary differential equations.

In this paper, we present some reflections on non-routine problems in a ODE course, and how they can illustrate the dual character of ODE: on the one hand, the theoretical specialization in various areas and on the other, the multiplicity of applications thereof.

FIVE NONROUTINE PROBLEMS

Although ODE is an important issue in the engineering curriculum, students experience difficulties in conceptual understanding of them.[5] Hence the need to develop problems arising out of the books of ODE standard texts available to students.

We must make clear that, in our work, the not routine problem we consider those issues in which the information provided is not adequate to solve the same, either because data are lacking because they have redundant data and even that may appear contradictory data. Sometimes they may be ill-defined problems.

Problem 1. Determine the limit as $x\to\infty$, of the general solution of the differential equation $y' = y^2-1$.

If we analyze the algebraic framework, its solution is very basic; it is an equation in separable variables and solvable in quadratures, whose solution can be expressed by $y=\frac{1+ce^{2x}}{1-ce^{2x}}$. It is clear that this term students do not tell them much about the behavior of solutions: thus declare that the limit is -1. Even with graphical behavior (Figure 1), the analysis is necessary when the limit is -1, an occasion to reveal the need to take into account the initial conditions and the overall analysis of the problem. Again, we stress the need for integration of different approaches to complete the analysis.

[5] See for example, (Arslan, 2010) and (Rowland, & Jovanoski, 2004)

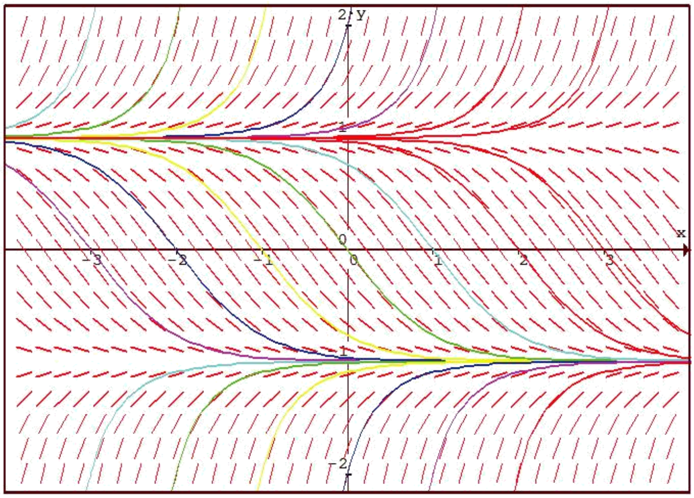

Figure 1. Graphical behavior of the solution $y = \frac{1 + ce^{2x}}{1 - ce^{2x}}$.

Problem 2. The differential equation describing the growth of a fish population on a farm, in time is $\frac{dP}{dt} = P_0 - 5000P$. Give the biological significance of each of the three dP/ dt, P_0 and -5000P terms in the differential equation.[6]

Problem 3. Find the explicit general solution of the differential equation y '= ylnx, x>0. Contrast the solution obtained with Existence and Uniqueness Theorem (TEU).

Problem 4. Verify that $y = \frac{2}{x} - \frac{C}{x^2}$ is the general solution of the differential equation $\frac{dy}{dx} + \frac{2y}{x} = \frac{2}{x^2}$. Prove that the initial conditions y(a) = a and y(-a) = - 3a, with a$\in$**R**, result in the same particular solution. Does this violate the TEU? Can we use any real value of a?

Problem 5. Get the approximate solution by the method of Euler,[7] for the van der Pol equation x''-b (1-x2) x'+ x = 0.

This equation has a limit cycle (Figure 2). This model is used to describe the behavior of electric circuits, certain types of pulsating stars and many other phenomena. With b = 1, it is solvable by the Euler method, and one obtains

[6] Additional details can be found in (Guerrero Ortiz, Mejía Velasco, & Camacho-Machín, 2015).
[7] See for example, (Hall, Keene, & Fortune, 2016) and (Tournès, 2018).

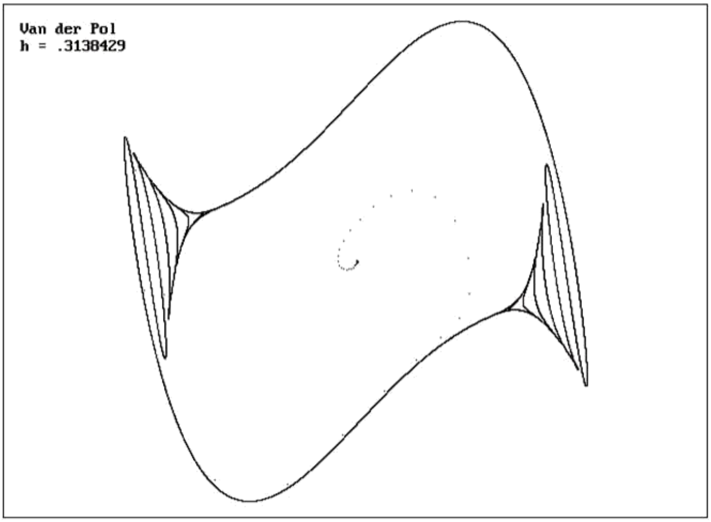

Figure 2. Solution of Van der Pol oscillator with b = 1 and h = 0.3138429.
Source: http://sprott.physics.wisc.edu/chaos/eulermap.htm

For 0.1 h less than the solution is reasonably accurate, but when h increases, the typical limit cycle (Coddington & Levinson, 1955) becomes a chaotic behavior (see the Figure 3 obtained with h = 0.168).

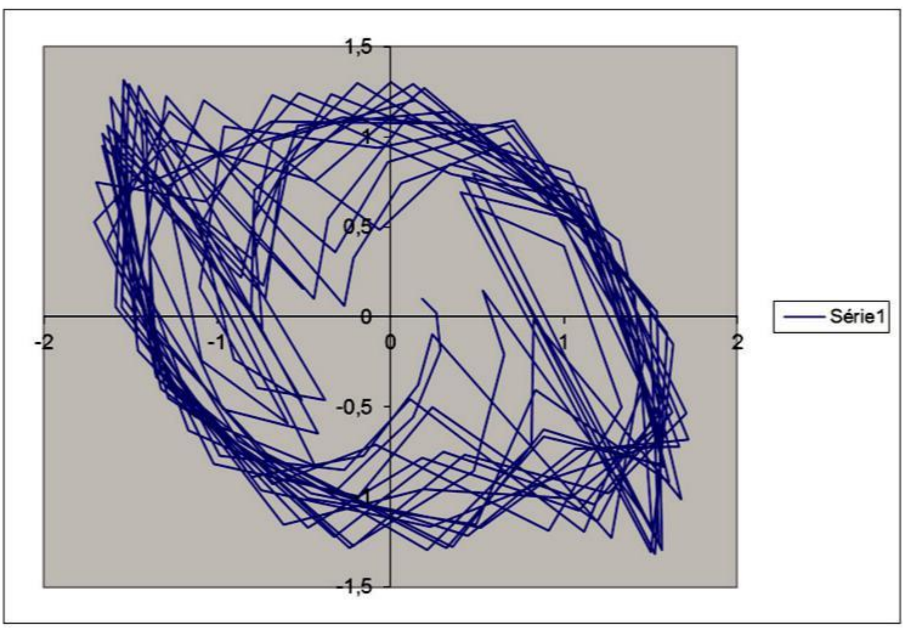

Figure 3. Chaotic behavior of the solution with h = 0,168

Concluding remarks

We must add a crucial issue in the current conception of the course of differential equations is its algorithmic-algebraic character, which is determined primarily by the close relationship between the development of algebra (as a search for the roots of a polynomial in terms of radicals) and linear differential equations (in terms of integration by quadratures) which we have already spoken and that even in the "modern" concept of linear operators, is present. A historical sketch of ordinary differential equations allows us to make the following observations on the current program:[8]

1. Differential Equation concept born (late seventeenth century) as a differential equation relating this concept remains stable until Cauchy (circa 1821) adds the derivative. This last definition is the one that remains today, "disappearing" the differentials, even when the methods of solving ordinary differential equations of the first order first conception without making it explicit (i.e., the derivative is no longer the derivative but a ratio between differentials), reviving the algebraic manipulation above mentioned, because this makes this method of solution an "appetizing" tool from the educational point of view, without forgetting the help that this work provides the Principle of Superposition.

2. The way of introducing Ordinary Differential Equations first order in the work of Euler and Cauchy is taking the differential expression pdx + qdy, as the differential of a certain function u = u (x, y), and hence du = 0 and finally to the general solution u (x, y) = c. In the case where it cannot find the function u(x, y), construct an integrating factor that makes exact the differential equation. After that, start studying the other types of differential equations of first order (e.g., linear, Bernoulli, homogeneous, etc.), always keeping in mind that need to build an integration factor. This situation generally is not retained in the current curriculum. The main facts that led to this are the development and demonstration of Fundamental Theorem of Algebra (the first true demonstration, Gauss offered at age 22 in his doctoral thesis) which, in its present form, states that every polynomial of degree n in C, has exactly n complex zeros (same or different); so, C is a numerical domain that provides solution to any algebraic equation and development the theory of Complex Variable Functions, which allowed to present a complete theory of "solubility" for linear equations of order n, thus providing a formidable educational tool for modeling many practical phenomena.

3. Regarding the current study program, there is a clear permanence of Algebraic scenario over the other two scenarios, which, in addition to the strength of the historical component as I mentioned earlier, due to other factors which point out the following:

a) The algorithmic-algebraic methods (directly linked to the instrumental understanding of ODE and its multiple applications) are easier to develop in students. Closely related to cognitive and behavioral trends in Mathematics Education that, gradually, and even more so thanks to the rejection of the "Modern Mathematics", has

[8] More information on this dominance, their scope and impact can be found in (Nápoles & Negrón, 2002, 2003, Nápoles, 2005b).

been disappearing and giving way to “Problem Solving”, with a totally different didactic and epistemic conception.

b) Implement the geometric and numerical scenarios in the classroom, necessarily it requires the counting means since otherwise, it is difficult to visualize, e.g. fields slopes and isoclines curves on the one hand, and the approximate solutions on the other.

c) With the addition of the Laplace transform, the second half of this century, algebraic procedures again gain new momentum in teaching. This received additional support with the new wave of modern mathematics, and the already known “Down with Euclid!”

In conclusion, we can recommend that in the course of ordinary differential equations, we must implement not only instrumental work but also conceptual understanding, for this we must create a new teaching paradigm that emphasizes modeling of learning, analysis of the differential equation the analysis of the qualitative behavior of the solution using software tools with graphical facilities, and finally to pass to “speak” a “talk about”. That will undoubtedly result in significant student learning (Bibi, Zamri, Abedalaziz, & Ahmad, 2017).

Even if it is a general affirmation, the inclusion of such issues in the curriculum is beneficial, as this will force students to move away from a purely instrumental approach to handling a focus on conceptual understanding (Davis, 1994).

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