

## Why Newton's Second Law is not $F = ma$

Camila Maria Sitko<sup>1</sup> 

<sup>1</sup> Mestrado em Ensino de Ciências e Matemática, Centro de Ciências Exatas, Universidade Estadual de Londrina/UEL, Londrina, PR, Brazil

*Received for publication on 2 Oct. 2018. Accepted, after revision, on 23 Nov. 2018.*

### ABSTRACT

The second law enunciated by Isaac Newton in the *Principia* is not equivalent to  $F = ma$ , as it is popularly known. The latter was described by Leonhard Euler, in 1752. However, for some historians, this formulation would be implicit in the statement proposed by Newton, and in this way,  $F = ma$  is considered by some of them only as a mathematical reformulation of Newton's law. In this paper, initially, the considerations of these historians are used and is it is performed an interpretation in order to establish a connection between the law proposed by Newton and  $F = ma$ , using current mathematical language. Based on this interpretation and on a bibliographic research, made from primary and secondary historical sources, reasons for the non-equivalence of these laws are discussed, as well as aspects that lead to this confusion of interpretations; limitations of Newtonian mechanics are discussed too, which are about mathematical method, natural coordinates, and Newton's conceptions of force. Next, the elements not covered by Newton's mechanics are indicated, thus showing that, in fact,  $F = ma$  is a much more general law than that proposed by Newton. Finally, it is briefly discussed the importance of using the historical approach in Science Teaching, especially with respect to the episode discussed here.

**Keywords:** Newton's Second Law.  $F = ma$ . Leonhard Euler. History of Mechanics.

## Por Que a Segunda Lei de Newton não é $F = ma$

### RESUMO

A segunda lei enunciada por Isaac Newton nos *Principia* não é equivalente a  $F = ma$ , como popularmente é conhecida. Esta última foi descrita por Leonhard Euler, em 1752. Entretanto, para alguns historiadores, essa formulação estaria implícita no enunciado proposto por Newton, e dessa forma  $F = ma$  é considerada por alguns apenas uma reformulação matemática da lei de Newton. Neste trabalho, inicialmente são utilizadas as considerações desses historiadores e tento fazer uma interpretação de modo a estabelecer uma conexão entre a lei proposta por Newton e  $F = ma$ , utilizando a linguagem matemática atual. Com base nessa interpretação e numa pesquisa bibliográfica, a partir de fontes históricas primárias e secundárias, são discutidos motivos para a não equivalência das leis, assim como aspectos que levam a essa confusão de interpretações; também são discutidas as limitações da mecânica newtoniana, que são a respeito do método matemático, coordenadas naturais e das concepções de força de Newton. Em seguida, são indicados os elementos não abrangidos pela mecânica de Newton, evidenciando assim, que de fato,  $F = ma$  é uma lei muito mais geral do que a proposta por Newton. Por fim, é discutida rapidamente a importância do uso da abordagem histórica no Ensino de Ciências, especialmente com relação ao episódio aqui tratado.

**Palavras-chave:** Segunda Lei de Newton.  $F = ma$ . Leonhard Euler. História da Mecânica.

Corresponding author: Camila Maria Sitko, Email: camilasitko@yahoo.com.br

## INTRODUCTION

Conceived by Isaac Newton, the three laws of motion were presented in his masterpiece, *Mathematical Principles of Natural Philosophy*, or also known as *Principia*, in 1687. Of these laws, to this work interests the second law. Newton wrote several forms for the second law between one edition and another of the *Principia* (three were made in 1687, 1713, and in 1726); in the end, he chose not to make any change in his statement. Newton adopts the impulse-momentum form as fundamental.

A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed. (Newton, 1999, p.415)

This formulation is entirely different from the format that we know,  $F = ma$ , which was written only in 1752, in the article *Découverte d'un nouveau principe de Mécanique* (Discovery of a new principle of Mechanics), by Leonhard Euler,<sup>1</sup> in 1752. In fact, the text of Newton's statement seems to correspond to  $F = \Delta(mv)$ , not to  $F = ma$ , as Dias (2006) already pointed out before.

However, in some textbooks, both of High School and Higher Education, the statement of the second law is written in words others than those of Newton, followed by the modern formulation, inducing the reader to believe that they are the words and the mathematization of Newton. As Sears, Zemansky, Young and Freedman (2008, p.116) point out,

Newton synthesized all these relationships and experimental results into a single formulation called Newton's second law:

Newton's Second Law: When a resulting external force acts on a body, it accelerates. The acceleration has the same direction of the resulting force. The resultant force vector is equal to the product of the body mass and the body acceleration vector.

In symbols,

$$\sum \vec{F} = m\vec{a} \quad (1)$$

<sup>1</sup> However, Euler's elaboration is due not only to the scientist himself, but also to his interactions with other physicists, such as d'Alembert, Johann and Daniel Bernoulli, Maupertuis, etc. For the sake of stylistic economics, when we refer to  $F = ma$ , we will use only the name of Euler, without disregard other contributions.

<sup>2</sup> The concept of vector is already known since Aristotle, which was also used by Newton in the *Principia*. Descartes proposed to work with these quantities just as we do with arithmetic operations. However, this idea was achieved in algebraic terms only in the nineteenth century (CHAPPELL et al., 2016), and that is why we are not using vectorial notation ( $\vec{F}$ ) in this paper, because this notation does not appear in Euler's work with this format (nor in other works before nineteenth century). This notation appears in equation (1) because it is a quote from a current book on Physics.

The *Fundamentos de Física* states that Newton's second law is "*the relation between the total resultant force acting on a body of mass  $m$  and the acceleration produced by the force*" (Halliday, 2016, p.237), and then the formulation is presented. There is no incompatibility between the statement (already reformulated) and the equation; however, this does not reflect the Newtonian formulation, which was presented at the beginning of the paper. The problem with this suggestion is that  $F = ma$  cannot be considered a mathematical translation of the Newtonian statement, because some definitions, concepts and mathematical tools to  $F = ma$  were not yet used at the moment Newton presented his second law, which entails an erroneous conception about the construction of  $F = ma$ .

On the other hand, in other books, as in *Curso de Física Básica*, Nussenzveig defines the linear moment, and then deduces the second law of motion, however, associating to this deduction the statement "*the variation of momentum is proportional to the impressed force, and has the direction of the force*" (2002, p.72). Thus, Newton's statement is presented together with the Eulerian formulation, thus suggesting that Euler's elaboration is nothing more than a notational variant of the Newtonian statement. However, as we shall see below, the term momentum variation is not directly related to acceleration, but to speed. In this way, we can establish the existence of what could be freely called

of "incompatibility problem" between Newton's statement and  $F = ma$ . Based on this problem, it is relevant the question: what were the main scientific occurrences that led to Newton's statement being modified by Euler? That is, what were the limitations of Newton's work so that it needed improvements?

Maltese (1992) and Dias (2006) have already done studies in this sense (much more profound than the one accomplished in this paper) concerning the law of the movement proposed by Euler, using the point of view that the second law of Newton and  $F = ma$  are not the same principle. This paper is concerned with explaining why these are not the same.

This is a fragment from bibliographical research of a doctoral thesis. The research is based on the delimitations of research in History of Science, using primary and secondary historical sources as a basis (Martins, 2005). From this, it was analyzed the Newtonian work (1687), as well as that of Euler (1752), and it was perceived an incompatibility of meaning between the two, that has been already commented here. On the other side, some historians as Cohen (1970) defend that if some considerations are taken into account, it could lead us to a relationship between force and acceleration, which in modern terms and be understood as  $F = ma$ .

So, in this work, at first an attempt is made to establish this connection between the law proposed by Newton and  $F = ma$ , by using the current mathematical language and taking into account the interpretation of Cohen (1970), that Newton maintained the variation in time implicit in his problems of discrete forces acting on impulses of very short duration. In doing so, it is performed an interpretation of Newton's law, and with

the intention of showing that, indeed, it is possible to obtain the formula  $F = ma$  using those considerations, although, as we will see, even so, this result does not contain the conceptual foundations nor the generalization that the law proposed by Euler offers. From this observation, elements that are missing so that the law proposed by Newton can be equivalent to  $F = ma$  are listed: it is briefly discussed the geometric method used by Newton and its limitations and the ambiguous concept of force presented by him, which gives scope for different interpretations regarding the second Law; after, it is presented what are the main elements that differentiate Newton's statement from  $F = ma$ , that is, what lacked to Newton does to generalize his statement as a fundamental principle of mechanics, and which was finally presented in Euler's work. Finally, it is discussed the importance of using the historical approach in teaching science, specially in this episode.

## **TRYING TO ESTABLISH A CONNECTION BETWEEN NEWTON'S LAW AND $F = ma$**

Newton defines *motion*, or *quantity of motion* (1990), as obtained from the velocity and the quantity of matter conjointly. Thus, as Dias (2006) showed previously, the law proposed seems to say, in a modern form, that  $F \propto \Delta(mv)$ . According to Cohen (1970), Newton usually omitted time when dealing with discrete forces acting on impulses of very short duration.

So, if we agree with Cohen and consider that the expression *change in motion* on the second law means the *rate* of change in motion, we could think in a compatibility relationship between Newton's formulation and  $F = ma$ .

Thus, if we would adopt this perspective, that quantity of motion is a product between velocity and quantity of matter (denoting it by  $p$ ), and that Newton wanted to mean rate of the quantity of motion (that in modern notation means a derivative in time), we would arrive at  $F = \frac{dp}{dt}$ , which we can easily relate to  $F = ma$ . Let us see.

Let us denote the quantity of motion by  $p$ :

$$p = mv \tag{2}$$

where  $m$  is the mass (or quantity of matter) of the body and  $v$  is its velocity. Thus, the rate of change of the quantity of motion can be written as:

$$\frac{dp}{dt} = \frac{d(mv)}{dt} \tag{3}$$

In the Newtonian mechanics, the quantity of motion can change because of both velocity and mass, and so, it is not correct simply assume that mass is constant in all situations. However, the purpose of this section is to do whatever algebra we need to see if it is possible, somehow, to obtain a formula similar to the principle  $F = ma$ , from Newton's statement, based on what Cohen (1970) says about it. To do that, we are

adopting a situation where the mass  $m$  of the body is constant,<sup>3</sup> and thus, it can be put out of the derivative,

$$\frac{dp}{dt} = \frac{mdv}{dt} \quad (4)$$

Moreover, knowing that  $a = \frac{dv}{dt}$ ,

$$\frac{dp}{dt} = ma \quad (5)$$

Furthermore, since the variation of the quantity of motion in time is equal to the resultant force, according to this interpretation of Newton's second law, we finally have

$$F = ma \quad (6)$$

However, it is important to realize that what was done here is a reading *interpretation* of the Newtonian text (by assuming some considerations that are not contained in the *Principia*), not a logical inference from it. Nevertheless, even if this interpretation were adopted as correct, this would not solve the problem of the incompatibility between Newton's statement and the second law of motion as we know it today, since Euler's elaborations that culminated in  $F = ma$  were constructed from tools not used by Newton and conceptual elements not yet available at that time. In other words, there was a conceptual evolution between the statement of Newton and  $F = ma$ , and therefore,  $F = ma$  cannot be understood as a notational variant of the statement.

The fact is that the fundamental principle of mechanics appeared in 1752, written by Euler. As Truesdell points out (1968), no one doubts about the law proposed by Newton, of its truthfulness, but what no one had realized before Euler have proposed the principle, is that the last one was a general principle over others, which could be applied to every part of each system.

## GEOMETRIC METHOD AND ITS LIMITATIONS

For Newton, mathematics was essentially geometric, and he believed that this was the natural method. Newtonian mechanics was based on Cartesian geometry because it solved problems that other methods were not capable of. For example, in this method, different quantities could pass through mathematical operations, resulting in other physical quantities, something that previously could not be done. However, difficulties in the

---

<sup>3</sup> By stipulating mass as a constant, we are based on the same reasoning that Euler would have about sixty years after Newton in writing his work *Decouverte d'un nouveau principe de Mécanique* (1752), when he considers "an infinitely small body, or one whose mass is contained in a single point, that mass being =  $M$ " (1752, p.195). This new principle proposed by Euler and its differences in relation to Newton's law will be discussed in a future paper.

method were found by Newton: Cartesian definitions were able to provide the conditions for an amount to be a product or a ratio, but they did not provide the product, that is, they were not able to compare, for example, velocities or accelerations.

In order to solve this difficulty, still using the constructs of Descartes, Newton begins to treat each mechanical quantity as a measure of a segment. Thus, in the case of measures of the segments of space travelled and segments of the time spent for a particular movement, it is possible to determine velocities and accelerations also in the form of segments, and this is what makes possible to obtain algebraic expressions to denote the object (Panza, 2002).

Another difficulty arises due to the fact that the use of segments was not able to represent the situation of non-rectilinear movements. To solve the problem, Newton uses his conceptions of inertia and force, decomposing the movement: if no force acts on the punctual body, it is in rectilinear motion; if forces act on the body, the movement is a combination of these forces. In the case of curvilinear motion, continuous forces act by changing the direction of motion. Thus, at each instant of curvilinear motion, velocity and force acting on the point can be expressed, the velocity is represented by a segment tangent to the curve, and the force also represented by a segment, but in another direction, both originating from the punctual body. These segments turn out to be called vectors. Thus, Newton describes his mechanics by a “*geometry of trajectories*” (Panza, 2002, p.18).

For Newton, movements are effects of forces (or inertia), and so, he calls the study of dynamics. Forces and velocities are no longer understood as qualities of movements, but as mathematical objects, and thus, movement is no longer seen as an external phenomenon. His method allowed him to extend mathematics to the treatment of the movement of natural bodies from axioms and definitions that dictated how movements occur due to forces. In this way, Newtonian mechanics becomes a science of mathematical objects, that is, the result of a process of mathematization (Panza, 2002).

However, the geometric approach was not enough to solve more complex mechanical problems. For example, problems in which the extension of bodies should be taken into account, that is, rigid bodies, could not be solved, much less the fluid and deformable bodies; more complex problems brought very large resolutions, always using segments. Thus, Newtonian mechanics was limited to punctual bodies. According to Panza, one of the aims of analytical mechanics of the eighteenth century was to make this science of geometrization of motion a science of an analytical object, so that these resolutions could cover a more general class of problems, and more practically and systematically.

## USE OF NATURAL COORDINATES

The problems at that time were treated with the so-called natural coordinates. To use these, at each point in the trajectory, it was necessary to establish two reference frames

“naturally” chosen by the tangent and normal curves of the situation, according to the context of the problem (Stan, 2017). According to this use, it was necessary to have a certain ability to perceive which was the most natural referential, and this method was not applicable to extensive bodies nor three-dimensional movements. In this way, a very large class of problems was not covered.

## CONCEPTS OF FORCE

One of the fundamental concepts present in the Newtonian work is that of force. In the *Principia*, Newton worked with two general conceptions of force: the force of inertia and the impressed force,<sup>4</sup> which would have several subtypes, such as the centripetal, of pressure, percussion, and so forth.

Chang (2014) discusses the ontology of Newton’s concept of the force of inertia, which is given as obvious in the textbooks, and even brings to light the erroneous conception that inertia is what makes the body maintain its state of uniform motion or rest. Newton believed that inertia is what made the body persevere in movement (Martins, 2012). This idea of perseverance in the movement arises with Descartes, who strongly influenced Newton’s ideas, especially in relation to the first law, as Cohen (1964) has already shown previously, and is also evidenced in Martins (2012).

However, inertia has to do with mass, the resistance of the body, and not with perseverance (the way Newton used to think about perseverance), because, once there are no forces, the movement simply does not change, and perseveres. Halliday and Walker, for example, discarded the terms “inertia” of their books in order to avoid confusion about this subject (Chang, 2014).

The first time Newton worked with the concept of impressed forces<sup>5</sup> was from the problem of collision between bodies. Collisions are treated in Newton as a discrete situation of a dynamic case, that is, as an instantaneous action (equivalent to force proportional to velocity,  $F \propto v$ ). Using the same basis of this discrete analysis, Newton later dealt with the case of continuous forces, also in a dynamic way, which are situations in which the forces act for a certain period of time, as in the free fall, for example (equivalent to proportional force to acceleration,  $F \propto a$ ). Thus, the problem is that, logically, the treatment of the continuous and discrete situation, as done by Newton, results in dimensionally different quantities, which made Newton’s definition confuse to the relation.

Thus, the two notions of force (discrete and continuous) were used in the *Principia*, revealing the ambiguous conception of the concept of force by Newton. In his proposition 1 on *Principia*, which deals with the problem of collision, Newton considered the forces as small discrete impulses; and in his scholium, he deals with accelerated movement

---

<sup>4</sup> In the *Principia*, Newton deals with central forces, but without worrying about their nature. This does not mean that Newton has never expressed his conceptions about the nature of force, but that these were not necessary for scientific explanations, for example, for gravitational theory.

<sup>5</sup> Which means change in the state of motion of a body.

(continuous force) and also with the proportionality between body weight and mass, which is also a case of continuous forces (Maltese, 1992).

It can be noted the ambiguity in other cases too, as in Newton's *Waste Book*, which is where the relation between force and motion first appears; this corresponds to  $F = \Delta(mv)$ .<sup>6</sup> On the other hand, due to the use of Galileo's law, when treating centripetal force and gravity in the *Waste Book*, Newton also used the reference that force is proportional to acceleration, that is, the modern equivalent of  $F = ma$ .

Then, it is possible to see that we are dealing with two formulations equivalent to what Newton describes in his drafts:  $F = ma$  and  $F = \Delta(mv)$ . However, the statement of second law of motion, which appears in the *Principia*, will be equivalent to  $F = ma$ .<sup>7</sup> Moreover, nowhere of his work is  $F = ma$  or the conceptual principle that gives it a meaning as the law of motion. Westfall (Maltese, 1992) notes this ambiguity in Newton's work, which is the cause of much controversy today regarding his conceptualization of force, and what leads to confusion about his enunciation of the second law.

From the statement of the second law proposed by Newton, if we take into account some assumptions, as we did previously, even so, the path to  $F = ma$  is not trivial, and now we have just shown why there is no consensus about whether Newton meant  $F = ma$  or  $F = \Delta(mv)$ . His conceptions of force were confuse because of the ambiguity maintained between the use of continuous and discrete forces, and much still needed to be thought out, elaborated and unified, until one could arrive at the principle of mechanics in the modern form,  $F = ma$ .

## DEPENDENCE BETWEEN THE LAWS

In Newton's second law, the motion of a body would be a sequence of small "strokes", and the law would refer to what happens when the stroke is applied, and the first law is valid when there is no stroke. For Newton, the first law would be required when working with the second one. Nowadays, we use the first law to define inertial reference frames in which the second law is valid.

The principle proposed by Euler expresses Newton's second law related to an independent reference frame system, in orthogonal Cartesian coordinates. Not only that, but if the resultant force on the body is null, it is noted that its velocity becomes constant, that is, if no force acts, the motion is uniform rectilinear, agreeing with Newton's first law. From the second law written by Newton, one cannot deduce it (Maronne & Panza, 2014): with it, we know that if the force is zero, the motion is not altered. However, it cannot be said that there is a uniform rectilinear motion. For this, it would be necessary to rely on the first law concomitantly, which states that the body remains in its state of motion, if no force acts on it, or if the resultant of the forces acting on it is zero.

<sup>6</sup> For several reasons (of the nature of matter and mathematics), for Hankins (in Maltese, 1992, p.27), Newton carried forward  $F = \Delta(mv)$  as the statement of the second law.



Thus, it is perceived that there is a dependence between the laws proposed by Newton. These and Euler's elaborations are then substantially different,<sup>7</sup> that is, there has been a conceptual evolution between the two forms of writing (not only of writing but of meaning) the law of movement (Maltese, 1992), as well as also with the first law: inertia is no longer the cause of the first law, but it offers a definition of inertial reference frame.<sup>8</sup>

## ANGULAR CASE

In Newton's enunciation of the second law, he states that the variation of motion occurs in the direction of the force's action. However, this is not valid for other types of problems, such as the rotation cases, where is needed the introduction of a concept of motion appropriate to the rotation movement.<sup>9</sup> Later, Euler realizes that new principles are then needed to solve, along with this case, a larger class of problems.

## WHAT NEWTON DIDN'T DO?

Newton left a great legacy in mechanics, although it cannot be said that it was an exclusive work since his contemporaries and successors also left great contributions in the area. Although a great work, the *Principia* were still a material to be perfected and completed. Despite the fecundity of the Newtonian program, according to our bibliographic research, it left many open questions due to the conceptual and mathematical limitations of that time.

For example, in addition to the fact that the *Principia* does not contain differential equations, which could be the argument that some thinkers use by claiming that  $F = ma$  was merely a change in mathematical formalism, such a work does not contain: 1) the treatment of any other type of problem other than of material points: the dynamics of rigid, elastic bodies, the dynamics of bodies in interactions, fluids<sup>10</sup> are excluded, since neither Newton nor any other contemporary scholar dominated the principle of quantity for solving problems with respect to deformable bodies; 2) the concept of inertia, which is not defined in Newton; it is only said that for a body to move, an impressed force is necessary; as soon as it stops acting, the body maintains its state due to its inertial force;<sup>11</sup> 3) laws of angular motion: Newton even comments on the motion of a top, which would

---

<sup>7</sup> The second law can be seen as the result of a series of experiments and observations, which is the so-called synthetic view, defended by Newton, or can be seen as a definition, a function, which is the functional analytic proposition defended by Euler (Buchdahl, 1951).

<sup>8</sup> The new principle, somehow, modifies the meaning of the first law. Initially for Newton, one cause was needed to make the movement persevere. His studies about Descartes made him change his conception, but the idea of inertia as an internal force to the body remains in all his work (Chang, 2014).

<sup>9</sup> The inertia tensor will influence the movement, where once the mass exerted this role.

<sup>10</sup> Newton even addressed some of these cases, but from *ad hoc* hypotheses and sometimes did not lead to correct results.

<sup>11</sup> Which shows Newton's conception that inertial force would be necessary for the body to persevere in its motion.

only change its motion due to contact with the air, but does not specify which laws govern such a movement (which would be with respect to angular motion); 4) the treatment of celestial bodies as extensive bodies; 5) analytical form independent of geometry: no analytical formulation of the seventeenth century could be made independently of geometric representation. The mathematical object of Newtonian mechanics was the movement, and according to Panza, it was “*intrinsically geometric*” (2002, p.20), due to Newton’s conceptualization of his objects.

On the other hand, one cannot fail to mention that Newton organized and deduced laws and phenomena seemingly independent of each other, created new concepts, compared his predictions to measured values, brought genuine dynamic concepts, and no longer those derived from static, as was once done. However, he could not offer the classical form we use today of mechanics since his principles were not clear and precise (Truesdell, 1975). Thus, Newton can be considered as the one who initiated the preparations for the formulation of  $F = ma$ , but such formulation had countless successors for the “production of mechanics”.

As we saw earlier, there is an incompatibility between the statement proposed by Newton and what we know by Newton’s second law, namely,  $F = ma$ . Effectively, by doing a documentary analysis, we can see that neither  $F = ma$  nor any other differential equation are included in the *Principia*. In the following years, it was up to other scholars to seek principles that would show these equations for Newton’s systems and for problems that other scientists would still be studying. Names such as Euler have perfected the Newtonian developments, creating new fields, concepts, formalisms, etc., even because, in the eighteenth century, the *Principia* were not the only work of mechanics: there were still the Huygens and Leibniz sides (that had followers like Hermann and the Bernoullis) (Stan, 2017), Varignon and the tradition of static, Newtonian concepts and methods, the study of the relation between forces and its deformations of Johann Bernoulli, and finally, Euler and the joint between Newtonian concepts and the formalism of Leibniz. This narrative extends to 1788, with the work of Lagrange, *Mécanique Analytique*, which finalizes the developments of the today called Classical Mechanics.

## SCIENCE TEACHING AND THE SECOND LAW

To teach physical concepts from the history of its construction could be very helpful to present to students how Science is made. Besides the several arguments presented by Matthews (2015) that encourage this approach, according to him, it is necessary to introduce Science as knowledge in construction. Also, this kind of approach is necessary so that students do not develop a distorted view of science, without observing the ruptures in scientific knowledge, conflicting theories, and end up interpreting Science as a finished product, done by the genius of few scientists (Könhlein & Peduzzi, 2005).

This insertion aims to provide a more adequate view of science as a construction process, to serve as a basis for discussion of fundamental elements, to reveal epistemological obstacles through similarities between alternative conceptions and relative

to scientific ancestors, and to overcome the teaching models that aim at the transmission of knowledge.

In the episode of the second law, as we have seen previously, certain concepts and formalisms defined after Newton are presented as the creation of him, being omitted the contributions of other scientists for that question. This kind of historical omission can lead to a distorted image of how Science is constructed, bringing the image of isolated brilliant minds producers of knowledge, and ignoring the role of collective and cooperative work. Not only concerning the historical process but this omission, as we have seen, can difficult the understanding of that physical concept, while it says that the enunciate made by Newton corresponds to the equation proposed by Euler.

Therefore, it is argued that the use of historical approach is beneficial to teaching, and with this paper, it is expected that the idea of Science as a process of construction over time and by different scientists be better established.

## ACKNOWLEDGMENTS

The present work was carried out with the support of *Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES)* – Brazil.

## REFERENCES

- Buchdahl, G. (1951, November). Some Thoughts on Newton's Second Law of Motion in Classical Mechanics. *The British Journal for the Philosophy of Science*, 2(7), 217-235.
- Chang, W., Bell, B., & Jones, A. (2014, June). Historical development of Newton's laws of motion and suggestions for teaching content. *Asia-Pacific Forum on Science Learning and Teaching*, 15(1), Article 4, 1.
- Chappell, J. M et al. (2016, March). The Vector Algebra War: A Historical Perspective. *IEEE Access*, 4, 1997-2004.
- Cohen, I. B. (1964). 'Quantum in se est': Newton's concept of inertia in relation to Descartes and Lucretius. *Notes and Records of the Royal Society of London*, 19, 131-155.
- Cohen, I. B. (1970). "Newton's Second Law and the Concept of Force in the *Principia*". In R. Pelter, (ed.) *The Annus Mirabilis of Sir Isaac Newton (1666-1966)*, The M.I.T. Press, p.143-185.
- Dias, P.M.C. (2006).  $F = ma$ ?! O nascimento da lei dinâmica. *Revista Brasileira de Ensino de Física*. 28(2), 205-234.
- Euler, L. 1750 (1752). Decouverte d'un nouveau principe de mecanique, *Mem. Acad. Roy. Sci. Berlin*, 6, 185-217. Also in *Opera Omnia*, serie II, 25, 98-131.
- Halliday, D. (2016). *Fundamentos de física*, volume 1 : mecânica / David Halliday, Robert Resnick, Jearl Walker; translation Ronaldo Sérgio de Biasi. 10 ed. – Rio de Janeiro: LTC.

- Kohnlein, J.F.K. & Peduzzi, L.O.Q. (2005). Uma Discussão Sobre A Natureza Da Ciência No Ensino Médio: Um Exemplo Com A Teoria Da Relatividade Restrita, *Caderno Brasileiro de Ensino de Física*, 22(1), 36-70.
- Lagrange, J. L. 1788 (1811). *Mécanique Analytique*. Paris, New and revised edition.
- Maltese, G. (1992). *La storia di "F=ma"* – La seconda legge del moto nel XVIII secolo. Biblioteca di nunciis: Firenze.
- Maronne, S. & Panza, M. (2014). Euler, Reader of Newton: Mechanics and Algebraic Analysis. *Advances in Historical Studies*, 3(1), 12-21.
- Martins, L. P. (2005). História da Ciência: objetos, métodos e problemas. *Ciência & Educação*, 11(2), 305-317.
- Martins, R. A. (2012). *Estado de repouso e estado de movimento: uma revolução conceitual de Descartes*. In: Temas de História e Filosofia da Ciência no Ensino. Org: Peduzzi, Luiz O. Q., Martins, André Ferrer P., Ferreira, Juliana Mesquita Hidalgo. Natal: EDUFRN, 372p.
- Matthews, M. R. (2015). *Science teaching: The contribution of history and philosophy of science*. New York: Routledge.
- Newton, I. (1999). *The Principia*. Translated by I. Bernard Cohen and Anne Whitman. University of California Press, Berkeley.
- Newton, I. 1726 (1990). *Princípios Matemáticos de Filosofia Natural*. Book I. Translation: Trieste S.F. Ricci, Leonardo G. Brunet, Sônia T. Ghering and Maria Helena C. Celia. 1ª Edition. São Paulo: Nova Stella Editora.
- Nussenzveig, M. H. (2002). *Curso de Física Básica I: Mecânica*, 4th Ed., Editora Edgard Blücher.
- Panza, M. (2002). *Mathematisation of the Science of Motion and the Birth of Analytical Mechanics: A Historiographical Note*. In: Cerrai P., Freguglia P., Pellegrini C. (eds) *The Application of Mathematics to the Sciences of Nature*. Springer, Boston, MA.
- Sears, F. W., Zemansky, M. W., Young, H. D., & Freedman, R. A. (2008). *Física I: Mecânica*. 12th ed, Ed. Pearson Addison Wesley.
- Stan, M. (2017). Euler, Newton, and Foundations for Mechanics. In Chris Smeenk & Eric Schliesser (eds.), *The Oxford Handbook of Newton*. Oxford University Press. 1-22.
- Truesdell, C. (1968). *Essays in the History of Mechanics*. Springer-Verlag. Berlin Heidelberg, New York.
- Truesdell, C. (1975). *Ensayos de historia de la mecanica*. Translation: Juan Carlos Navascues Howard and Enrique Tierno Perez-Relaño. Madrid: Editorial Tecnos.