Respecting mathematical diversity: An ethnomathematical perspective

Arthur B. Powell

ABSTRACT

This paper presents a detailed example from the practice that encourages respect for a diversity of ways of knowing and for a diversity of knowledge. This approach represents an ethnomathematical pedagogy, and develops a theoretical discussion of an ethnomathematical perspective. Then this paper concludes with theoretical return to our original point about respecting mathematical diversity. It happens because not only must teachers respect their own and their students' knowledge, but students often need to learn to respect their own and others mathematical activities and products. There are many different ways, using the mathematical activities of peoples all over the world to enable students to develop this respect. The basic idea behind all of them is that peoples' cultural material context influences their knowledge and activities in the world. In particular, students learn the importance of respecting their own and other peoples' knowledge, and how all of our cultural backgrounds interact with the development of mathematical knowledge. Further, the examples show a variety of forms in which mathematical knowledge can be encoded differently from that in 'academic' textbooks.

Keywords: Mathematics Education. Ethnomathematics. Mathematical Diversity.

Respeitando a diversidade matemática: uma perspectiva etnomatemática

RESUMO

Este artigo apresenta um exemplo detalhado da prática que encoraja o respeito por uma diversidade de formas de conhecimento e de uma diversidade de conhecimentos. Esta abordagem representa uma pedagogia etnomatemática e desenvolve uma discussão teórica em uma perspectiva etnomatemática. Então este trabalho conclui com retorno ao ponto teórico original sobre a diversidade matemática. Isso acontece porque não só os professores devem respeitar o seu próprio conhecimento como o conhecimento de seus alunos. Entretanto, os alunos muitas vezes precisam aprender a respeitar as suas próprias atividades matemáticas e produtos, assim como de outros. Há muitas maneiras diferentes, utilizando as atividades matemáticas de povos de todo o mundo para capacitar os alunos a desenvolver esse respeito. A ideia básica por trás de todas elas é que o contexto material e cultural dos povos influencia o seu conhecimento e suas atividades em todo o mundo. Em particular, os alunos aprendem a importância de respeitar os seus próprios conhecimentos e de outros povos, e como todas as nossas origens culturais interagem com o desenvolvimento do conhecimento matemático. Além disso, os exemplos mostram uma variedade de formas em que o conhecimento matemático pode ser codificado de maneira diferente daquela apresentada em livros didáticos "acadêmicos".

Palavras-chave: Educação matemática. Etnomatemática. Diversidade matemática.

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INTRODUCTION: TEACHING RESPECT FOR PEOPLE'S MATHEMATICAL ACTIVITY

Stephen I. Brown has contributed much to a humanistic perspective on mathematics education. His concerns for the intellectual and emotional as well as ethical dimensions of the whole person engaged in mathematics intersects with our concerns for respecting the intellectual diversity of mathematical ideas among individuals and social groups and for the need to connect these concerns to social justice. Brown (1997; 1984) poses important questions such as how does mathematical thinking relate to areas of life beyond mathematics and what are the emotional experiences involved in doing mathematics (BROWN, 1997, p.37) as well as what is the purpose of solving particular mathematical problems and how are relationships of mathematics to society and culture illuminated by studying how individuals view historically the phenomenon in question (BROWN, 1984, p.13). Brown makes clear that an interdisciplinary, real-world mathematics curriculum must attend to humanistic as well as scientific problems: "I know of essentially no 'real world' problems that one decides to engage in for which there is not embedded some value implications" (BROWN, 1984, p.13). Moreover, Brown pushes mathematics educators toward a fundamental respect for intellectual diversity by calling attention to the need to engage learners in considering "[…] not merely complementary, but incompatible perspective on a problem or series of problems" (BROWN, 1984, p.11). He ventures further and criticizes mathematics curricula for not enabling students to "[…] *appreciate* irreconcilable differences rather than to resolve or dissolve them" (BROWN, 1984, p.11).

To realize Brown's educational perspective it is important for mathematics educators to begin by respecting their own mathematical activity and those of their students. An important prerequisite for learning is respect.[1] Not only must teachers respect their own and their students' knowledge, but students often need to learn to respect their own and others mathematical activities and products. There are many different ways, using the mathematical activities of peoples all over the world to enable students to develop this respect. The basic idea behind all of them is that peoples' cultural material context influences their knowledge and activities in the world. In particular, students learn the importance of respecting their own and other peoples' knowledge, and how all of our cultural backgrounds interact with the development of mathematical knowledge. Further, the examples below show a variety of forms in which mathematical knowledge can be encoded differently from that in 'academic' textbooks.

I present a detailed example from my own practice that encourages respect for a diversity of ways of knowing and for a diversity of knowledge. This approach represents an ethnomathematical pedagogy, and in the next section I develop a more theoretical discussion of an ethnomathematical perspective. Then I conclude with a more theoretical return to our original point about respecting mathematical diversity.

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[1] Shulman (2003, p.3), a lawyer who focuses on work-related issues, emphasizes the importance of respect in her book, *The Betrayal of Work: How Low-Wage Jobs Fail 30 Million Americans*. She presents a case study illustrating how lack of intellectual respect was the catalyst for a group of African American women, certified nursing assistants in rural Alabama, to organize a union: "none had ever gotten a raise of more than 13 cents. Some who had been there ten years were still making $6.00 an hour. But it was the lack of respect from their employer that motivated these women. They would tell their supervisors something important about patients but, they said, no one listens".

TEACHING MATHEMATICS WITH RESPECT FOR PEOPLE'S MATHEMATICAL ACTIVITIES

Respecting people's mathematical activities and products influences teaching and learning activities. This means that it is important to consider the politics of knowledge and the interaction of culture in the development of ideas. Questions such as which knowledge and whose knowledge is considered legitimate are an integral part of learning mathematics in the example that I discuss. Furthermore, questions concerning the interactions of culture with the development of mathematical ideas are also integral to the activities. These instructional considerations represent teaching from an ethnomathematical perspective.

The following example gives a much more detailed learning experience which moves from a broad review of various algebraic concepts in Ancient Egyptian culture to an application of those concepts in the contemporary cultural context of a USA algebra classroom. Finally, the curriculum unit returns to an exploration of the specific presentation in a papyrus of the original Ancient Egyptian mathematical idea underlying the teaching module. Students here deepen their understandings of the contemporary academic algebra, as well as develop a respect for the different uses of the concepts in the algebraic studies of the Ancient Egyptians. This outline of a lesson treats the mathematical ideas of another culture with respect, showing the complexities of its knowledge, and deepening learners' own knowledge through an analysis of varying representations and solution methods.

USING EGYPTIAN ALGEBRAIC AND NUMERICAL-PUZZLE IDEAS TO TEACH AND DEEPEN UNDERSTANDING OF EQUATIONS

In this example, I use algebraic and numerical-puzzle ideas from ancient Africa for an algebra course module. It incorporates scholarship based on mathematical insights documented in an ancient Egyptian text. Unfortunately, the importance of Africa's contributions to mathematics and the central role of these contributions to the academic mathematics studied in schools have not received the attention and understanding that befit them.[2] Important projects to redress this state of affairs have been initiated by some historians (DIOP, 1974; JACKSON, 1970), historians of mathematics (GILLINGS, 1972, 1982; KATZ, 1998) as well as by scholars of ethnomathematics (EGLASH, 1999; GERDES, 1989, 1992, 1999; JOSEPH, 1991; LUMPKIN, 1985, 2002; LUMPKIN; POWELL, 1995; ZASLAVSKY, 1999). Documentary evidence of insightful and critical algebraic ideas developed in ancient Egypt exists, but little of this information is taught to students studying mathematics, at any level.

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[2] Diop (1974, p.xiv) argues a more general point about historiography: "The history of Black Africa will remain suspended in air and cannot be written correctly until African historians dare to connect it with the history of Egypt.… The ancient Egyptians were Negroes. The moral fruit of their civilization is to be counted among the assets of the Black world.…that Black world is the very initiator of the 'western' civilization flaunted before our eyes today. Pythagorean mathematics…and modern science are rooted in Egyptian cosmogony and science" .

The course module that I have implemented contributes to redressing this lack of scholarship in school algebra courses by incorporating and appreciating mathematical ideas from the *Ahmose Mathematical Papyrus*[3] (DIOP, 1985; JOSEPH, 1991; LUMPKIN, 1985), now housed and displayed in the British Museum (except for a few fragments in the possession of the Brooklyn Museum).[4] It engages students in reflecting on their daily, informal experiences of performing (doing and undoing) physical and mental operations and relates these operations to algebraic techniques by building on the following three mathematical ideas present in algebraic equations extant in ancient Egypt and suggested in Problems 24 through 34 of the *Ahmose Mathematical Papyrus*:[5] (a) the concept of unknown or variable quantities (BOYER; MERZBACH, 1989; GILLINGS, 1972, 1982); (b) undoing or, equivalently, inverse operations (KATZ, 1998); and (c) "think of a number" problems (GILLINGS, 1972, 1982).[6] In what follows, these ancient algebraic ideas are used to develop a course module, the last part of which returns to analyze specific problems presented in the *Ahmose Mathematical Papyrus.*

To engage students in developing an awareness[7] of the role undoing or inverse operations play in solving certain types of equations, one can present a type of "think of a number problem" as appears in Figure 1:

"I'm thinking of a number. I subtract 11 from it, multiply by 3, and add 2. The result is 80. What's my number?"

FIGURE 1 – A "think of a number" problem.

[3] This text, found at Thebes in the ruins of a small building near the Ramesseum, is often referred to as the Rhind Mathematical Papyrus since, in 1858, Alexander Henry Rhind (1833-1863), variously described as a Scottish antiquary and Egyptologist, purchased it in Luxor . (ARNOLD BUFFUM CHACE; MANNING; ARCHIBALD, 1927; ROBINS; SHUTE, 1987) The naming of such text is another example of how cultural imperialism obliterates the authorship of knowledge of the peoples it dominates. In this instance, the text is correctly named the *Ahmose Mathematical Papyrus* after the Egyptian scribe-scholar who authored it.
[4] The location of many important ancient African cultural products in Western museums, as well as those of other Third World cultures, evidences the endurance of cultural imperialism.
[5] In the preface to their book, Chace, Manning, and Archibald (1929) indicate that in the original papyrus the problems are not numbered and credit Eisenlohr with numbering them.
[6] In the preface to their book, Chace, Manning, and Archibald (1929) indicate that in the original papyrus the problems are not numbered and credit Eisenlohr with numbering them. Space does not allow us to elaborate on where and how each of these ideas represent themselves in the *Ahmose Mathematical Papyrus*; however, for such details, see Powell and Temple (2004).
[7] We use the term 'awareness' or 'mathematical awareness' in the technical sense suggested by Gattegno (1987) and elaborated on by Powell (1993, p.358): "Gattegno makes clear that, for learners, learning or generating knowledge occurs not as a teacher narrates information but rather as learners employ their will to focus their attention to educate their awareness. Learners educate their awareness as they observe what transpires in a situation, as they attend to the content of their experiences. As a learner remains in contact with a transpiring experience, awareness proceeds from "a dialogue of one's mind with one's self" about the content of that which one experiences (GATTEGNO, 1987, p.6). One's will, a part of the active self, commits one to focus one's attention so that the mind observes the content of one's experience and, through dialogue with the self, becomes aware of particularities of one's experience. Specifically, in mathematics, the content of experiences, whether internal or external to the self, *can be feelings, objects, relations among objects, and dynamics linking different relations* [italics added] (GATTEGNO, 1987, p.14)".

This is an example of a "What's my number?"[8] problem that is likely to trigger in students an awareness of undoing that includes the ideas of using inverses: multiplicative and additive inverses or inverse operations as well as the reversal of the order of operations. Undoing is a mathematical process related to many aspects of daily life: dressing and undressing, wrapping and unwrapping parcels, or either physically or mentally retracing one's steps to find a misplaced or lost item, and so forth. In schools and textbooks, this quotidian process is often not related to mathematics but, of course, is fundamental to it. Resurrecting and rediscovering such links between quotidian and formal knowledge schemas is one project of ethnomathematics that involves respect for intellectual activities and connect to Brown's concern for relating mathematics to aspects of life beyond mathematics.

Students are asked to reflect on their undoing technique so as to specify completely its constituent processes and ideas. After students discuss their ideas, I introduce conventional terminology for labeling processes and ideas.[9] For the "think of a number" problem in Figure 1, students eventually might describe their process for solving it as follows:

> The first operation or action performed on the original number thought of was to "subtract 11." The second action was to "multiply by 3." The third and last action was to "add 2," and the result was "80." To undo the problem, we need to start at the end. Immediately before the result 80 was obtained, the last action performed was to add 2. To undo *add* 2 we can *subtract* 2 from 80, which is 78. Now, the action performed before 78 was obtained was to multiply by 3. To undo *multiply by* 3 we can *divide by* 3. So, we take 78 and divide it by 3, which gives 26. The action performed before 26 was obtained was to subtract 11. To undo *subtract* 11 we can *add* 11. So, the original number thought of was 37.

The above description reveals an awareness that the mathematical ideas of inverse operations as well as reverse order of operations are useful for solving the "think of a number" problem given in Figure 1. This awareness is generalizable, and students use its generalization to handle more complex "think of a number" problems such as the one in Figure 2:

"I'm thinking of a number. I raise it to the third power, add 8, raise it to the one-half power, add 1, and multiply it by 6. My result is 30. What's my number?"

FIGURE 2 – A "think of a number" problem involving integral and fractional exponents.

[8] We use interchangeably the phrases "think of a number" and "What's my number?" problems.
[9] This pedagogical approach is akin to the "[…] five crucial steps in the Algebra Project curriculum process" (MOSES; COBB, 2001, p.120). In both approaches, one begins with the experiential, mathematical world of learners and assists them to connect their ideas to the symbolic world of formal, academic mathematics.

As the number and the complexity of the operations increase, students tend to develop *ad hoc* notational devices or inscriptions to represent “think of a number” problems. One notational device that I offer is circle equations (HOFFMAN; POWELL, 1988, 1991). The circle equation for the problem in Figure 2 is given in Figure 4.

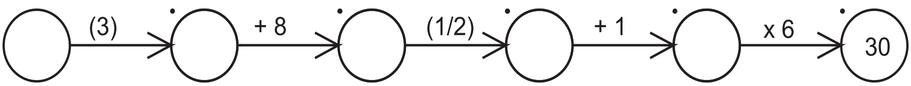

FIGURE 3 – The circle equation of the “think of a number” problem given in Figure 2.

In circle equations there are several conventions. The first circle represents the number referred to by the phrase “I’m thinking of a number.” The expressions above the right-facing arrows correspond to the operators in the “think of a number” problem in the order in which they are mentioned. Finally, numbers placed inside parentheses are considered to be exponents.

Besides being a tool to represent graphically “think of a number” problems, circle equations can also be used, with slight modifications, to depict a process of solving such problems. Explicit in the structure of circle equations, read from left to right, is the order in which operations are performed in “think of a number” problems. Similarly, but reading from right to left, the process of undoing or solving can also be represented with left-facing arrows with inverse operations written below them.

Moreover, in the circle equation, the circle that stands for the phrase, “I’m thinking of number,” can contain a variable, which means that the circle equation can be translated into the standard notation of academic mathematics (see Figure 4).

(a) $3(x-11)+2=80$ (b) $6\left\{\left(y^3+8\right)^{\frac{1}{2}}+1\right\}=30$

FIGURE 4 – The standard notation for the “think of a number” problems given in Figures 2 and 4, respectively, (a) and (b).

These examples suggest that a variety of equations can be dealt with through an adaptation and combination of Ahmose’s mathematical insights. Eventually, students are able to translate among standard and circle equations, and “think of a number” problems.[10]

$– 2 = 1;$ $5\left(\sqrt{\frac{\left(\left(\log_5 y\right)-1\right)^7}{32}}\right)=10$

FIGURE 5 – Two complex looking equations whose solution can be found using Ahmose’s mathematical insights.

[10] On the same point, Hoffman and Powell (1991) indicate that “with practice, pupils can imagine that a given equation is expressed as a circle equation and solve the equation, as it were, by sight” (p.95). In this sense, then, circle equations assist learners to develop dynamic imagery of equations that allows them to solve equations without pencil and paper.!!!

By working among these representations, students learn that equations such as the ones in Figure 5 simply appear complicated and are conceptually no more difficult to solve than an equation such as . The difficulty of the equations in Figure 6 arises from the amount of time required to solve them rather than any intrinsic complexity that they or their solution possess. In short, students learn that equations such as the ones in Figure 6 are, in fact, not difficult but only perhaps laborious. Moreover, once students develop facility solving equations like the ones in Figure 5, they deepen their algebraic understanding even further by, among a number of considerations, analyzing the use of circle equations to solve the following "think of a number puzzle" (Problem 28) from the *Ahmose Mathematical Papyrus*, as given in Gillings (1982, p.182):

> Two thirds is to be added. One-third is to be subtracted.
>
> There remains 10.
>
> Make $\overline{10}$ [one-tenth] of this, there becomes 1. The remainder is 9.
>
> of it namely 6 is to be added. The total is 15.
>
> of this is 5. Lo! 5 is that which goes out, and the remainder is 10.
>
> The doing as it occurs!
>
> Let us state this in modern terms, adding a few clarifying details:
>
> Think of a number, and add to it its $\overline{\overline{3}}$. From this sum take away its $\overline{3}$ [one-thirds], and say what your answer is. Suppose the answer were 10. Then take away $\overline{10}$ of this 10, giving 9. Then this was the number first $\overline{\overline{\varepsilon}}$ thought of.
>
> *Proof*. If the number were 9, its is 6, which added makes 15. Then $\overline{3}$ of 15 is 5, which on subtraction leaves 10. That is how you do it!

FIGURE 6 – Problem 28 of the *Ahmose Mathematical Papyrus*.

Students successfully handling problems from the *Ahmose Mathematical Papyrus* such as the one in Figure 6 and complex-looking ones similar to those in Figure 6 increase their mathematical self-confidence. For they are able to solve equations that more advanced mathematics students initially find baffling. Students build and deepen their mathematical understanding and, thereby, own the knowledge they employ. They empower themselves while appreciating historical connections between the *Ahmose Mathematical Papyrus* and their own intellectual work. For instance, when students solve Problem 28, they encounter certain differences between it and the equations that they had been solving. In a circle equation, the unknown in Problem 28 needs to appear above the arrows and students may find it convenient to combine algebraic expressions to simplify what is contained in each circle. Students realize that at some point they must combine algebraic expression to find the number originally thought of. Also students become aware that they had been simplifying arithmetic expressions at each stage of solving a circle equation. Students then notice that problems such as this one illustrate that a numerical solution can provide a proof for whole class of problems. The *Ahmose*

Mathematical Papyrus has other kinds of algebraic problems of varying complexity. Studying these problems provides opportunities for students to deepen their understanding by analyzing the complexity of problems and different solution methods, such as false position, Problems 24-27, and division, Problems 31 to 34 (GILLINGS, 1972, 1982). Based on African algebraic techniques, students develop sophisticated mathematical insights and abilities and greater self-confidence as learners of mathematics. Since all students have a common biological heritage in Africa, they gain an increased appreciation and respect for the mathematical accomplishments of their ancestors and for the diverse cultural manifestations of mathematical ideas.

Moreover, students inquire into the politics of social structures that devalue the mathematical contributions of Africans and engage more deeply in academic mathematics. Egyptian algebraic knowledge has been devalued as being only practical and not employing proofs. As Gillings points out, examining the answers to problems they pose (for example, the answer to Problem 31 is $14\frac{28}{97}$) and even how many are posed abstracted from any measure make it clear that the problems were not posed to answer practical questions. Rather, as Gillings observes, "[…] they were meant to illustrate one method for the solution of simple equations of this type, and although they did this, the simplicity of the method has been masked by the complexity of the unit fraction that arise in the process and by the unexpected operations to which the scribe was to forced to resort" (GILLINGS ,1972, p.159).

This relates to the other criticism of ancient Egyptian mathematics, concerning the nature of proof or lack thereof. Gillings (1972) and Joseph (1991) both treat this issue in some detail. Gillings's observation concerning the rigor of Egyptian proof is worth noting:

> Twentieth-century students of the history and philosophy of science, in considering the contributions of the ancient Egyptians, incline to the modern attitude that an argument or logical proof must be *symbolic* if it is to be regarded as rigorous, and that one or two specific examples using selected numbers cannot claim to be scientifically sound. But this is not true! A nonsymbolic argument or proof *can* be quite rigorous when given for a particular value of the variable; the conditions for rigor are that the particular value of the variable should be *typical*, and that further generalization to *any* value should be *immediate*. In any of the topics mentioned in this book where the scribes' treatment follows such lines, both these requirements are satisfied, so that the arguments adduced by the scribes are already *rigorous*; the concluding proofs are really not necessary, only confirmatory. The rigor is implicit in the method. (GILLINGS 1972, p.233-234)

For our students, some of whom have experienced repeated failure in their trajectory studying mathematics and, in the process, position themselves as well as are positioned as mathematically underprepared, this augmentation in their confidence and self-respect is nontrivial and, in some cases, leads to their willingness to pursue mathematics beyond mere institutional requirements.

WHAT IS AN ETHNOMATHEMATICAL PERSPECTIVE?

Ethnomathematics is a discipline that emerged from a politically engaged multicultural perspective on mathematics and mathematics education. Theorization about ethnomathematics was initiated and elaborated on by, mathematician and educational theorist, Ubiratan D'Ambrosio (1985; 1987; 1988; 1990; 2001, 2008). In Powell and Frankenstein (1997), we review the development of various definitions and associated perspectives on ethnomathematics. For us, ethnomathematics attempts to correct the history of mathematics for African and other formerly colonized peoples disempowered by a varied, a violent, and an avaricious European colonial process and currently threatened and plummeted by a pernicious imperial project euphemistically called globalization, one ancillary tentacle of which is the current misdirected "war against terrorism." In this perspective, they also include concerns about the politics of knowledge and cultural underpinnings and interactions of mathematical ideas. In his book, *Etnomatemática: Elo entre as Tradições e a Modernidade*, D'Ambrosio (2001, p.9) articulates his viewpoint on the political nature of the discipline:

> Etnomatemática é a matemática praticada por grupos culturais, tais como comunidades urbanas e rurais, grupos de trabalhadores, classes profissionais, crianças de uma certa faixa etária, sociedades indígenas, e tantos outros grupos que se identificam por objetivos e tradições comuns aos grupos. Além desse caráter antropológico, a etnomatemática tem um indiscutível foco político. A etnomatemática é embebida de ética, focalizada na recuperação da dignidade cultural do ser humano. A dignidade do indivíduo é violentada pela exclusão social, que se dá muitas vezes por não passar pelas barreiras discriminatórias estabelecidas pela sociedade dominante, inclusive e, principalmente, no sistema escolar.

In this viewpoint, a conceptually fruitful consequence of defining ethnomathematics as specific mathematical practices constituted by cultural groups is the scope of such activities. One can theorize that in ethnomathematics, the prefix "ethno" not only refers to a specific ethnic, national, or racial group, gender, or even professional group but also to a cultural group defined by a philosophical and ideological perspective. The social and intellectual relations of individuals to nature or the world and to such mind-dependent, cultural objects as productive forces influence products of the mind that are labeled mathematical ideas. For example, Dirk J. Struik (1997), an eminent mathematician and historian of mathematics, indicates how a particular perspective—dialectical materialism—decisively influenced Marx's theoretical ideas on the foundation of the calculus. The calculus of Marx (1983) represents the ethnomathematical production of a specific cultural group. [11]

Another important consequence of D'Ambrosio's viewpoint is that by highlighting the culturally damaging consequences of social exclusion, ethnomathematics breaks

[11] For a popular account of Marx's calculus, see Gerdes (1985, 2008).

with attributes of Enlightenment thinking. It departs from a binary mode of thought and a universal conception of mathematical knowledge that privileges European, male, heterosexual, racist, and capitalistic interests and values. D'Ambrosio (2001, p.42) puts it this way:

> A etnomatemática se encaixa nessa reflexão sobre a descolonização e na procura de reais possibilidades de acesso para o subordinado, para o marginalizado e para o excluído. A estratégia mais promissora para a educação, nas sociedades que estão em transição da subordinação para a autonomia, é restaurar a dignidade de seus indivíduos, reconhecendo e respeitando suas raízes. Reconhecer e respeitar as raízes de um indivíduo não significa ignorar e rejeitar as raízes do outro, mas, num processo de síntese, reforçar suas próprias raízes.

As D'Ambrosio (2001) notes, this research program contains other dimensions including conceptual, historical, cognitive, epistemological, and educational. As an ethnomathematics educator, I am not neutral academics, but activist academics, committed to finding ways to contribute to struggles for justice through our educational work. I am not just interested, for example, in the mathematics of Angolan sand drawings, but also in the politics of imperialism and globalization that arrested the development of this cultural tradition, and in the politics of cultural imperialism that discounts the mathematical activity involved in creating Angolan sand drawings. Further, I am alert for ways that this contextualized mathematical knowledge can be used in educational settings to encourage greater justice in society culturally, socially, economically, and politically.

CONCLUSION: RESPECTING MATHEMATICAL DIVERSITY

A cornerstone for a pedagogical approach that advances education for justice in the context of ethnomathematics involves actively respecting mathematical diversity. Accepting and respecting mathematical diversity leads us to reconsider all our knowledge of the world and to recognize that there is much about the world of which we have no knowledge. Pinxten, van Doren, and Harvey (1983, p.174) argue that "[a]s long as science cannot pretend to have valid answers to all basic questions…it is foolish to exterminate all other, so-called primitive, pre-scientific or otherwise foreign approaches to world questions"[12].

In 1982, in a course he gave at Boston College in Massachusetts, USA, Paulo Freire said the following about what counts as knowledge:

[12] An example of a scholar who would be labeled "primitive" or "pre-scientific" is discussed by Brown (1997, p.37): "The life of Ramanujan supports the view that innocence may be an asset in much of mathematical thinking. As a matter of fact, Ramanujan, who had received minimal formal mathematical training, came upon the most remarkable connections, and many of his arguments defied accepted cannons of proof. That many of his conclusion were wrong is beside the point, for given his untutored notion of proof and his lack of formal education, it is noteworthy that he was able to come upon so many discoveries and in fact to create so many new fields."

> Our task is not to teach students to think—they can already think, but to exchange our ways of thinking with each other and look together for better ways of approaching the decodification of an object.[13]

This implies a fundamentally different set of assumptions about people, pedagogy, and knowledge-creation. For example, one the one hand, some people in the United States need to learn to speak and write in “standard” American English. However, it does *not* follow that they cannot express in their own language complex analyses of social, political, economic, ethical, and other issues. On the other hand, many people with an excellent grasp of reading, writing, and mathematics need to learn about the world, about philosophy, about psychology, about justice, and about many other areas to deepen their understanding of the world. Marcuse (1964) argues that, for instance, in our society the rational, sophisticated calculations of nuclear kill is truly irrational, obscuring the only rational response to nuclear holocaust—namely, resistance:

> [I]n this society, the rational rather than the irrational becomes the most effective mystification.[…] For example, the scientific approach to the vexing problem of mutual annihilation—the mathematics and calculations of kill an over-kill, the measurement of spreading or not-quite-so-spreading fallout…is mystifying to the extent to which it promotes (and even demands) behavior which accepts the insanity. It thus counteracts a truly rational behavior—namely, the refusal to go along, and the effort to do away with the conditions which produce the insanity. (MARCUSE, 1964, p.189- 190)

In a similar vein, D’Ambrosio (1997, p.15-17) argues:

> The mission of bringing Western civilization to the planet has been the essence of conquest and of the colonial enterprise. Now we are at a crossroads. The human species and the planet itself are threatened […]The only possibility for survival depends on a better understanding of the entire set of possible explanations and views of the individual, of society, of nature, of the cosmos. Western mathematics, the most perfect embodiment of Western civilization, cannot be immune from the search for this deeper understanding. We can benefit much from understanding the workings of different systems of knowledge, the same way a stranger can tell us much about ourselves. […] There is no future in denying some successes in the science and technology developed following the Greek style. We will surely not be able to build faster jets and more powerful missiles using the male and female triangles of the Xingu ethnomathematics. But maybe the male and female triangles could help us *not* build the missiles and the jets carrying the bombs.

[13] This quote is from course notes taken by Marilyn Frankenstein from 5 to 15 July 1982.

In non-trivial ways, we all can learn a great deal from mathematical diversity. Most of the burning social, political, economic and ethical questions of our time remain unanswered. In the United States, I live in a society of enormous wealth. The country has significant hunger and homelessness; although its scientists have engaged in medical and scientific research for scores of years, the US society is not any closer to changing the prognosis for most cancers. Certainly we all can learn from the perspectives and philosophies of people whose knowledge has developed in a variety of mathematical and experiential conditions. Currently "the intellectual activity of those without power is always labeled non-intellectual" (FREIRE; MACEDO, 1987, p.122). When we see this as a political situation, we can realize that all people have knowledge, all people are continually creating knowledge, doing intellectual work, and all of us have a lot to learn.

Brown's attention to the emotional aspects of doing mathematics, the connection of mathematics to society and culture, the value implications of real-world problems, and studying incompatible perspectives on problems, all suggest a broader, political concept of humanistic mathematics education that I have developed here and in my other work. Respecting mathematical diversity is politically important precisely since the knowledge of non-Western and indigenous peoples has been violently devalued. In mathematics classrooms, the politics of knowledge needs to be part of the mathematical conversation and in this way we can develop respect for mathematical diversity.

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