Linking inquiry and transmission in teaching and learning mathematics and experimental sciences[1]

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ABSTRACT

Different theories assume that the learning of mathematics and sciences should be based on constructivist methods, where the students inquire about problem – situations and assign a facilitator role to the teacher. From a contrasting view, other theories advocate for giving a more central role to the teacher, which involves the explicit transmission of knowledge and the students' active reception. In this paper, we reason that the optimization of learning requires adopting an intermediate position between these two extreme models, because of the complex dialectic between the students' inquiry and the teacher's transmission of knowledge. We base our position on a model with anthropological and semiotic assumptions about the nature of mathematical and scientific objects, as well as on the structure of human cognition.

Keywords: Theories of instruction. Inquiry learning. Knowledge transmission. Onto-semiotic approach. Scientific and mathematical knowledge.

Articulação entre a exploração e a transmissão de conhecimentos no ensino e aprendizagem da matemática e das ciências experimentais

RESUMO

Diversas teorias postulam que a aprendizagem de matemática e das ciências experimentais deve estar baseada numa pedagogia construtivista, orientada pela exploração de situações problema por parte dos estudantes, atribuindo ao professor um papel de facilitador. No extremo oposto

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situam-se outras teorias que defendem um papel de maior protagonismo da parte do professor, o que implicaria a transmissão explícita de conhecimentos e a recepção ativa por parte dos estudantes. Neste trabalho, baseando-nos numa síntese destas posições de ensino, induzimos que a otimização da aprendizagem requer a adoção de uma posição intermédia entre estas duas posições extremas, reconhecendo a dialética complexa entre a exploração por parte do estudante e a transmissão de conhecimentos por parte do professor. Fundamentamo-nos na asserção de pressupostos antropológicos e semióticos sobre a natureza dos objetos matemáticos e científicos, assim como em suposições relativas à estrutura da cognição humana.

Palavras-chave: Teorias de ensino. Aprendizagem construtivista. Transmissão de conhecimentos. Enfoque ontossemiótico. Conhecimento matemático e científico.

INTRODUCTION

The debate between the models of a school that "conveys knowledge" and others in which "knowledge is constructed" currently seems to tend towards the latter. This preference can be seen in the curricular guidelines from different countries, which are based on constructivist and socio-constructive theoretical frameworks:

> Students learn more and learn better when they can take control of their learning by defining their goals and monitoring their progress. When challenged with appropriately chosen tasks, students become confident in their ability to tackle difficult problems, eager to figure things out on their own, flexible in exploring mathematical ideas and trying alternative solution paths, and willing to persevere (NCTM, 2000, p.20).

In the case of mathematics education and experimental sciences, problem-solving and "mathematical investigations" are considered essential for both students' mathematical learning and teachers' professional development. Constructivist viewpoints of learning shifts the focus towards the *processes* of the discipline, practical work, project implementation and problem solving, rather than prioritizing the study of facts, laws, principles and theories that constitute the body of disciplinary *knowledge*.

Nevertheless, this debate is hiding the fact that students differ in skills and knowledge, and most of them need a strong guidance to learn; even when some students with high skills and knowledge can learn advanced ideas with little or no help. The issue of the type of aid needed, depending on the nature of what is to be built or transmitted is also missed in this debate. As a result of this situation, the question of the kind of help that a teacher should give to a usually heterogeneous class, when we want students to acquire mathematical knowledge, understandings, and skills, also arises.

The family of "Inquiry-Based Education" (IBE), "Inquiry-Based Learning" (IBL), and "Problem-Based Learning" (PBL) instructional theories, which postulate the inquiry-based learning with little guidance by the teacher, do not seem to take into account the described reality, namely the students' heterogeneity and the variety of knowledge to be studied. These models may be suitable for gifted students, but possibly not for the majority, because the type of help that the teacher can provide could significantly influence the learning, even in talented students.

In this paper, we analyse the need to implement instructional models that articulate a mixture of construction/inquiry and transmission of knowledge to achieve a mathematical instruction that locally optimizes learning. The basic assumption is that the moments in which transmission and construction of knowledge can take place are *everywhere dense* in the instructional process. Optimization of learning involves a complex dialectic between the roles of the teacher as an instructor (transmitter) and facilitator (manager), and student's roles as an active constructor of knowledge and receivers of meaningful information. Hiebert and Grouws (2007) state that "because a range of goals might be included in a single lesson, and almost certainly in a multi-lesson unit, the best or most effective teaching method might be a mix of methods, with timely and nimble shifting among them" (p.374).

The discussion raised on the inquiry and transmission instructional models can be related to the ongoing debate between constructivism and objectivism (JONASSEN, 1991), as well as to the distinction between teaching models that are focused on the learner or the teacher, respectively (STEPHAN, 2014). The different varieties of constructivism share, among others, the assumptions that learning is an active process, that knowledge is constructed rather than innate or passively absorbed and that it is necessary to pose significant open and challenging problems to the students to achieve effective learning (FOX, 2001). Within Objectivism, particularly in its behaviorist version, knowledge is publicly observable, and learning is the acquisition of that knowledge through interaction between stimuli and responses. Often direct instruction or lecture – based teaching is the type of conditioning used to achieve the desirable learning behavior (BOGHOSSIAN, 2006).

Below we first summarize the main features of instructional models based on inquiry and problem solving and secondly of models that attribute a key role to transmission of knowledge. We then present the case for a mixed model that combines dialectically inquiry and transmission, basing on the epistemological and didactical assumptions of the onto-semiotic approach to mathematical knowledge and instruction (GODINO; BATANERO; FONT, 2007). This mixed model is exemplified with research related to a process of training future teachers in statistics. Finally, we include some additional reflections and implications.

INQUIRY AND PROBLEM-BASED LEARNING

As indicated above, the acronyms IBE, IBL, PBL designate instructional theoretical models developed from several disciplines, which have parallel versions for the teaching of experimental sciences (IBSE) and mathematics (IBME). They attributed a key role to solve "real" problems, under a constructivist approach. In some applications in mathematics education, it is proposed that students construct knowledge following the lines of work of professional mathematicians themselves. The mathematician faces non-routine problems, explores, searches for information, makes conjectures, justifies and communicates the results to the scientific community; mathematics learning should follow a similar pattern. “Learning science and technology is to learn to participate in the communities of practices of scientist and technologist respectively” (MURPHY; MCCORMICK, 1997, p.462).

Using problem-situations (mathematics applications to everyday life or other fields of knowledge, or problems within the discipline itself) to enable students to make sense of the mathematical conceptual structures is considered essential. These problems are the starting point of mathematical and scientific practice, so that problem-solving activities, including formulation, communication, and justification of solutions are keys to developing the ability to cope with non-routine problems. This is the main objective of the “problem solving” research tradition (SCHOENFELD, 1992), whose focus is on the identification of heuristics and metacognitive strategies. It is also essential for other theoretical models such as the Theory of Didactical Situations (TDS) (BROUSSEAU, 1997), and Realistic Mathematics Education (RME) (FREUDENTHAL, 1973; 1991), whose main features are described below.

Problem solving

The importance given to problem-solving in the curricula and educational research is the result of a view that considers this activity as the essence of mathematics and experimental science. Polya’s seminal work on how to solve problems provided the initial impetus for a wide research program that took place in the following decades, and which analyzed issues such as solving simulated problems with the aid of computers, expert problem solving, problem-solving strategies and heuristics, metacognitive processes and posing problems. Artigue and Blomhoj (2013) relate the tradition of “problem-solving” with IBL:

Students facing non-routine problems have to develop their own strategies and techniques; they have to explore, conjecture, experiment and evaluate; they are given substantial mathematical responsibilities and generally encouraged to generate questions themselves and to envisage possible generalizations of the results they obtain. (p.802).

English and Sriraman (2010) analyse several reflections and evaluations of the effectiveness of problem-solving research and conclude on their little relevance for school practice. These authors believe that “Unfortunately, there is a lack of studies that address the conceptual development based on problem-solving in interaction with the development of problem-solving skills” (ENGLISH; SRIRAMAN, 2010, p.267).

Theory of Didactical Situation (TDS)

In TDS (BROUSSEAU, 2002), problem – situations should be selected in order to optimize the adaptive dimension of learning and students’ autonomy. The intended mathematical knowledge should appear as the optimal solution to the problems; it is expected that, by interacting with an appropriate *milieu*, students progressively and collectively build knowledge rejecting or adapting their initial strategies if necessary. According to Brousseau (2002),

> The intellectual work of the student must at times be similar to this scientific activity. Knowing mathematics is not simply learning definitions and theorems in order to recognize when to use and apply them. We know very well that doing mathematics properly implies that one is dealing with problems. We do mathematics only when we are dealing with problems—but we forget at times that solving a problem is only a part of the work; finding good questions is just as important as finding their solutions. A faithful reproduction of a scientific activity by the student would require that she produce, formulate, prove, and construct models, languages, concepts, and theories; that she exchange them with other people; that she recognize those which conform to the culture; that she borrow those which are useful to her; and so on. (p.22).

To allow such activity, the teacher should conceive problem – situations in which they might be interested and ask the students to solve them. The notion of *devolution* is also related to the need for students to consider the problems as if they were their own and be responsible for solving them. The TDS assumes a strong commitment with mathematical epistemology, as reflected in the meaning attributed to the notion of the fundamental situation, which Artigue & Blomhøj (2013, p.803) describe as "a situation which makes clear the *raison d'être* of the mathematical knowledge aimed at".

Another important feature of the TDS is the distinction made between different dialectics: action, formulation, and validation, which reflect important specificities of mathematical knowledge.

Realistic Mathematics Education (RME)

In RME, principles that clearly correspond to IBME assumptions are assumed. Thus, according to the "activity principle," instead of being receivers of ready-made mathematics, the students, are treated as active participants in the educational process, in which they develop all kinds of mathematical tools and insights, themselves. According to Freudenthal (1973), using scientifically structured curricula, in which students are confronted with ready-made mathematics, is an 'anti-didactic inversion.' It is based on the false assumption that the results of mathematical thinking, placed on a subject-matter framework, can be transferred directly to the students. (VAN DEN HEUVEL-PANHUIZEN, 2000).

The principle of reality is oriented in the same direction. As in most approaches to mathematics education, RME aims at enabling students to apply mathematics. The overall goal of mathematics education is making students able to use their mathematical understanding and tools to solve problems. Rather than beginning with specific abstractions or definitions to be applied later, one must start with rich contexts demanding mathematical organization or, in other words, contexts that can be mathematized. Thus, while working on context problems, the students can develop mathematical tools and understanding. The guidance principle also stresses the same ideas. One of Freudenthal’s (1991) key principles for mathematics education is that it should give students a “guided” opportunity to “re-invent” mathematics. This implies that, in RME, both the teachers and the educational programs have a crucial role in how students acquire knowledge. According to Artigue and Blomhøj (2013, p.804), “RME is thus a problem-solving approach to teaching and learning which offers important constructs and experience for conceptualizing IBME”.

TRANSMISSION BASED LEARNING IN EDUCATION

We consider as models based on knowledge transmission, various forms of educational intervention in which the direct and explicit instruction is highlighted. A characteristic feature of strongly guided instruction is the use of worked examples, while the discovery of the solution to a problem in an information-rich environment is similarly a compendium of discovery learning minimally guided.

For several decades these models were considered as inferior and undesirable regarding different combinations of constructivist learning (learning with varying degrees of guidance, support or scaffolding), as shown in the initiatives taken in different international projects to promote the various IBSE and IBME modalities (DORIER; GARCIA, 2013). Transmission of knowledge by presenting examples of solved problems and the conceptual structures of the discipline is ruled by didactical theories in mathematics education with the strong predicament, as mentioned in the previous section.

The uncritical adoption of constructivist pedagogical models can be motivated by the observation of the large amount of knowledge and skills, in particular, everyday life concepts, that individuals learn by discovery or immersion in a context. However, Sweller, Kirschner and Clark (2007) state that:

There is no theoretical reason to suppose or empirical evidence to support the notion that constructivist teaching procedures based on the manner in which humans acquire biologically primary information will be effective in acquiring the biologically secondary information required by the citizens of an intellectually advanced society. That information requires direct, explicit instruction. (p.121)

This position is consistent with the argument put forward by Vygotsky; scientific concepts do not develop in the same way as everyday concepts (VYGOTSKY, 1934). These authors believe that the design of appropriate learning tasks should include providing students an example of a completely solved problem or task, and information on the process used to reach the solution. As Sweller, Kirschner, and Clark (2007) observe, “we must learn domain-specific solutions to specific problems and the best way to acquire domain-specific problem-solving strategies is to be given the problem with its solution, leaving no role for IL [inquiry learning]” (p.118). According to Sweller et al., empirical research of the last half-century on this issue provides clear and overwhelming evidence that minimal guidance during instruction is significantly less effective and efficient than a guide specifically designed to support the cognitive process necessary for learning.Similar results are reflected in the meta-analysis by Alfieri, Brooks, Aldrich &Tenenbaun (2011).

According to Kirschner, Sweller, & Clark (2006), we are skillful in an area because our long-term memory contains huge amounts of information concerning the area. That information permits us to quickly recognize the characteristics of a situation and indicates to us, often unconsciously, what to do and when to do it. (p.76).

In Table 1 we present the main features of the objectivist and constructivist perspectives considered in this paper as extreme ideal alternatives when applied in instructional design. They are grouped into three dimensions: epistemic (knowledge which is the object of instruction), cognitive (learning of knowledge, skills, and dispositions), and instructional (means and modes of interaction). These features have been summarized from the works of various authors (JONASSEN, 1991; ERNEST, 1994; MURPHY; MCCORMICK, 1997; BOGHOSSIAN, 2006; ANDREW, 2007).

TABLE 1 – Characteristics of objectivism and constructivism as the basis of instruction.

Dimensions	Objectivism	Constructivism
Epistemic (Nature of knowledge object of instruction)	- Knowledge has an existence external to the subject. - Knowledge structure is determined by concepts, properties, and relationships. - Meaning corresponds to world entities and categories, independent of the understanding of any subject - Symbols represent reality.	- Knowledge depends on the subject's mental activity. - The structure of knowledge depends on experience and personal interpretations. - Meaning is not based on a correspondence with the world; it depends on personal understanding. - Symbols are tools to construct reality.
Cognitive (Learning of knowledge, skills, and dispositions)	- The mind is a symbol processor and reflects reality. - Thinking is based on the structured, recognizable and transmissible accumulation of factual knowledge. - Prior students' knowledge and the answers they provide during the learning process are accepted if they agree with the objective knowledge. - Student's reflection is irrelevant and unnecessary.	- The mind is a builder of symbols and a conceptual system that builds a reality. - Thinking is based on the perception and grows from physics, physical and social experience. - Prior students' knowledge and the answers they provide condition instruction, which must be adapted to the students' conceptual frameworks. - Students' reflection and exploration are the engines of problem solving and inquiry into the situations posed.

Dimensions	Objectivism	Constructivism
Instructional (Means and interaction pattern)	- The teacher is the source of the knowledge that is the object of teaching. - Student exploration is unnecessary and, therefore, is not stimulated. - Interactions between students are unnecessary because learning is an individual act. - Reproduction of objective knowledge culturally accepted is emphasized. - Errors are used as an opportunity to reinforce the right behavior.	- The teacher acts as a coach, leading the emerging knowledge in cooperative contexts. - Student exploration is essential and, therefore, is stimulated. - Interactions between students inside and outside the classroom are promoted because learning is a social act. - The construction of knowledge is emphasized by solving tasks in contextualized environments. - Students take responsibility for their own learning.

Source: The authors.

A DIDACTICAL MODEL BASED ON INQUIRY AND TRANSMISSION

In the two previous sections, we described some basic features of two extreme models for organizing mathematics instruction: discovery learning versus learning based on the reception of knowledge (usually regarded as traditional whole-class expository instruction). In this section, we describe the characteristics of an instructional model in which these two models are combined: the students' investigation of the problem situations with the explicit transmission of knowledge by the "teacher system"[2] at critical moments in the mathematical instruction process. We consider that it is necessary to recognize and address the complex dialectic between inquiry and knowledge transmission in learning mathematics. In this dialectic, *dialogue* and *cooperation* between the teacher and the students (and among the students themselves), regarding the situation – problem to solve and the mathematical content involved, can play a key role. In these phases of dialogue and cooperation, moments of transmitting knowledge necessarily happen.

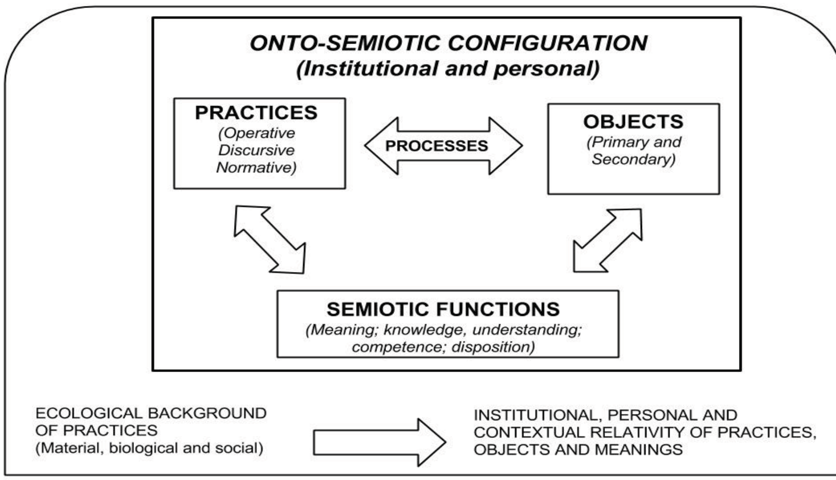
Onto-semiotic complexity of knowledge

Godino and Batanero (1994) started to lay the foundation for an ontological, epistemological and cognitive model of mathematical knowledge, based on anthropological and semiotic assumptions, which has been expanded and systematized in later works (GODINO; BATANERO; FONT, 2007; FONT; GODINO; GALLARDO, 2013), resulting in the Onto-semiotic Approach to mathematical knowledge and instruction (OSA).

[2] This system can be an individual teacher, a virtual expert system, or the intervention of a "leader" student in a team working on a collaborative learning format.

In Figure 1, we present the key elements of the epistemological and cognitive modeling of mathematical knowledge: the notions of practice, object, process (sequence of practices from which the object emerges) and semiotic function (where the various entities and operations are related). Operative, discursive and regulatory practices are conducted in solving certain problems, which may have a mathematical or extra-mathematical character; and therefore the onto-semiotic modeling of knowledge can be applied not only to mathematics but also to other disciplines.

FIGURE 1 – Primary entities of the OSA ontology and epistemology.



Source: The authors.

Within the OSA it is postulated that the systems of practices and emerging objects are relative to the contexts of use, the institutions in which the practices take place and the subjects involved in these practices (language games and forms of life, according to Wittgenstein 1953). In this way, in this framework, assumptions from the objectivist (practices, objects, and institutional meanings) and constructivist approaches to knowledge (systems of practices, objects, and personal meanings) are combined.

The OSA provides the basis for an instructional model that recognizes a key role in both the inquiry and the transmission of knowledge in teaching and learning mathematics and experimental sciences. Moreover, the nature of mathematical and scientific objects involved in practices whose competent performance by students is intended, is considered.

The way a person learns something depends on what has to be learned. According to the OSA, students should appropriate (learn) the onto-semiotic institutional configurations involved in solving the proposed problem – situations. The paradigm of “questioning the world” proposed by the Anthropological Theory of Didactics (TAD) (CHEVALLARD, 2013), and, in general, by IBE models is assumed, so that the starting point should be the selection and inquiry of “good problem – situation”.

The key notion of the OSA for modelling knowledge is the *onto-semiotic configuration* (of mathematical practices, objects, and processes) in its double version, institutional (epistemic) and (cognitive). In a training process, the student’s performance of mathematical practices related to solving certain problems brings into play a conglomerate of objects and processes whose nature, from the institutional point of view is essentially normative (regulative)[3] (FONT; GODINO; GALLARDO, 2013). When the student carries out no relevant practices, the teacher should guide him/her to those expected from the institutional point of view. Thus each object type (concepts, languages, propositions, procedures, argumentations) or process (definition, expression, generalization) requires a focus, a moment, in the study process. In particular, regulative moments (institutionalization) are *everywhere dense* in the mathematical activity and the process of study, as well as in the moments of formulation / communication and justification.

Performing mathematical practices involves the intervention of previously known objects to understand the demands of the problem – situation and implementing an initial strategy. Such objects, their rules and conditions of application, must be available in the subject’s working memory. Although it is possible that the student him/herself could find such knowledge in the “workspace,” there is not always enough time or the student may not succeed; so the teacher and peers can provide invaluable support to avoid frustration and abandonment. These are the moments of remembering and activation of prior knowledge, which are generally required throughout the study process. Remembering moments can be needed not only in the exploratory / investigative phase, but also in the formulation, communication, processing or calculation, and justification of results phases. These moments correspond to acts of knowledge transmission and may be crucial for optimizing learning.

Results of mathematical practices are new emerging objects whose definitions or statements have to be shared and approved by the community at the relevant time of institutionalization carried out by the teacher, which are also acts of knowledge transmission.

Types of didactical trajectories

Under the OSA framework, other theoretical tools to describe and understand the dynamics of mathematics instruction processes have been developed. In particular, the

[3] This view of mathematical knowledge is consistent with that taken by Radford s objectification theory. Radford (2013) writes: “Knowledge, I just argued, is crystallized labor – culturally codified forms of doing, thinking and reflecting. Knowing is, I would like to suggest, the instantiation or actualization of knowledge” (p.16). He adds: “Objectification is the process of recognition of that which objects us – systems of ideas, the cultural meanings, forms of thinking, etc.” (p.23). In our case, such crystallized forms of work are conceived as cultural “rules” fixing ways of doing, thinking and saying faced to problem – situations that demand an adaptive response.

notions of *didactical configuration* and *didactical suitability* (GODINO; CONTRERAS; FONT, 2006; GODINO, 2013). A didactical configuration is any segment of didactical activity (teaching and learning) between the beginning and the end of solving a task or problem – situation. Figure 1 summarizes the components and the internal dynamics of a didactical configuration, including the students' and the teacher's actions, and the resources to face the joint study of the task.

A priori, neither the transmission nor the inquiring interaction format is only proposed for managing the didactic trajectory; instead, the notion of didactical suitability (GODINO, 2013) serves as a guiding criterion to optimize this management. Achieving a high didactical suitability requires a good balance among the epistemic, ecological, cognitive, affective, interactional, and mediational facets. Depending on the content, the students, the available resources and other factors that influence the instructional processes, a high didactical suitability may involve the implementation of a mixed instructional model, where inquiring, transmission, and dialogic / cooperative moments are articulated. Cooperative moments involve the application of inquiry and reflective actions by the students and knowledge transmission actions by the teacher, and have therefore a mixed character.

It is interesting to distinguish four basic types of problem situations depending on the different role they play in the instructional process, and the main pattern of interaction required in each case to optimize learning:

- *Introductory situations*, through which students "meet the content for the first time" To get students involved in the study, the teacher should remember or communicate the basic information they need to understand the context and the task; in these situations, therefore, the transmissive and dialogic formats predominate.
- Drill and practice situations, focused on the mastering of procedural skills and on the retention of concepts, properties and justifications involved in solving the introductory situations. The personal interaction format is prevalent with possible reinforcements on the part of the teacher.
- Application situations, through which students more autonomously apply the knowledge learned and explore possible extensions. Inquiry and cooperative formats are common, where the problem is addressed by teamwork, without ruling out the participation of the teacher to guide and provide the necessary information.
- Assessment situations, through which systematic information on the student's degree of learning is obtained. The pattern of interaction is essentially individual; the student personally responds to a representative sample of tasks.

These types of situations give rise to the corresponding types of didactical configuration, which are sequenced along the didactical trajectory. As was previously indicated, in some configurations dialogue and knowledge transfer predominate, and in others inquiry. The change from one format to another is not a priori regulated, but instructional circumstances (basically the previous students' knowledge, their capabilities, temporary and material resources available) can determine the change, the optimization of the didactic suitability of the process being the basic criterion.

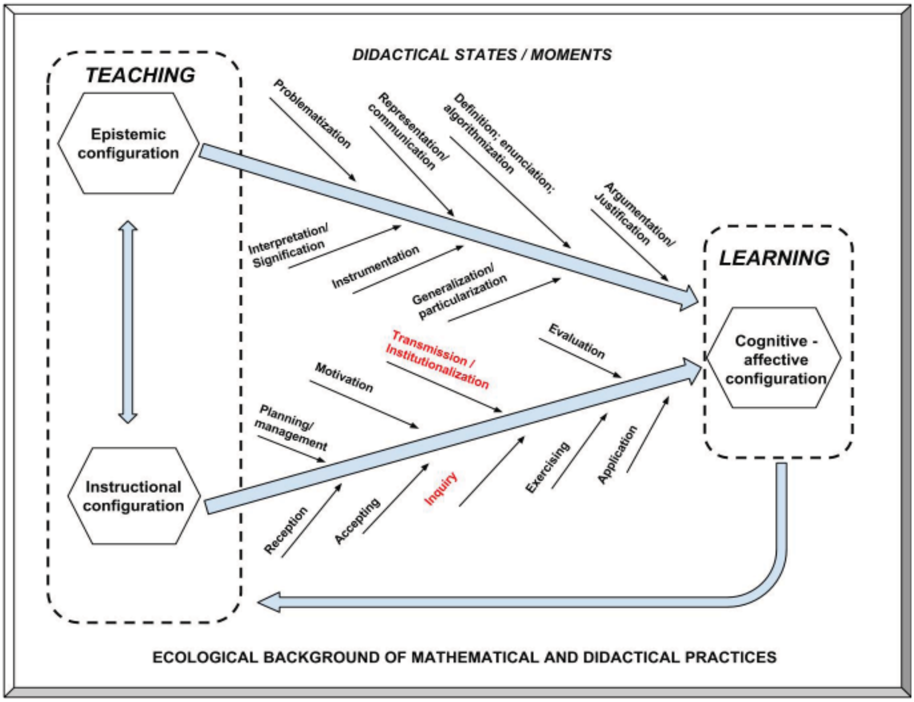
Dynamics of didactical configurations

Under the OSA framework, other theoretical tools to describe and understand the dynamics of mathematics instruction processes have been developed. In particular, the notions of *didactical configuration* and *didactical suitability* (GODINO; CONTRERAS; FONT, 2006; GODINO, 2013). A didactical configuration is any segment of didactical activity (teaching and learning) between the beginning and the end of solving a task or problem – situation. Figure 1 summarizes the components and the internal dynamics of a didactical configuration, including the students' and the teacher's actions, and the resources to face the joint study of the task.

The problem – situation that delimits a didactical configuration can be made of various subtasks, each of which can be considered as a sub-configuration. In every didactical configuration there is an epistemic configuration (system of institutional mathematical practices, objects, and processes), an instructional configuration (system of teacher and learner roles and instructional media), and a cognitive configuration (system of personal mathematical practices, objects, and processes) which describe learning. Figure 1 shows the relationships between teaching and learning, as well as with the key processes linked to the onto-semiotic modelling of mathematical knowledge (FONT; GODINO; GALLARDO, 2013; GODINO; FONT; WILHELMI; LURDUY, 2011). Such modelling, together with the teachers and learners roles, and their interaction with technological tools, suggest the complexity of the relationships established within any didactical configuration, which cannot not be reduced to merely inquiry and transmission moments.

The bottom arrow in Figure 1, directed from learning to teaching, points out that these relationships are not linear, but cyclical. At the time of inquiry, for example, the student interacts with the epistemic configuration without intervention (or with little influence) from the teacher. This interaction determines the teacher's interventions that, therefore, should be provided in instructional configurations. Cognitive trajectories produce examples, meanings, arguments, etc., that influence the study process and, consequently, the epistemic and instructional configuration, enabling or modifying–and then conditioning – the educational project planned.

FIGURE 2 – Components and internal dynamics of a didactical configuration.



Source: Godino et al. (2015, p.2646).

From our point of view, the "theory of didactical moments" proposed in the Anthropological Theory of Didactic (ATD) (CHEVALLARD, 1999) help to describe part of this internal dynamics of didactic configurations, which is described in this theory as *didactical praxeologies.* The onto-semiotic view of mathematical knowledge provides additional criteria to complement Chevallard's theory. Each type of mathematical object / process plays a role in the mathematical work and therefore requires a focus and time in the study processes. Such moments can be seen as states of the stochastic processes trajectories that we have used to model the mathematics teaching and learning in previous works (GODINO; CONTRERAS; FONT, 2006). There will be states (or moments) devoted to conceptual definitions, propositional statements, procedural routines, justification arguments, representation building and translation (DUVAL, 1995), generalization, and establishment of relationships and connections between different objects. There are also meta-mathematical moments (D'AMORE; FONT; GODINO, 2007) of planning, management, and evaluation, both for the students (who should manage their work to solve each problem or task) as well as for the teacher (who have to plan what and how to teach) (Figure 1).

The problem is not whether or not to guide the instruction, since nobody argues that some degree of guidance is necessary; the central question is when and how to guide, what aspects of the construction of knowledge can be left to the students' responsibility and for which direct instruction is preferable. No doubt, these are decisions that the teacher should take, depending on the cognitive – affective states of their students along the corresponding didactical trajectory. Hence, general recommendations on inquiry or transmissive models are simplifications of an extraordinarily complex reality.

Optimization of learning, that is, achieving high didactical suitability instructional processes in the various facets, especially regarding interactional – mediational and cognitive – affective suitability has a strong local character (GODINO, 2013). "Controlled experiments almost uniformly indicate that learners should be explicitly shown what to do and how to do it when dealing with new information" (KIRSCHNER; SWELLER; CLARK, 2006, p.79). The optimization of the didactic suitability of a mathematical study process requires a mixed instructional model in which both the student and the teacher play leading roles. "For the general to appear in the singular both the student and the teacher have to work together. The teacher and the student have to engage in the process of objectification" (RADFORD, 2013, p.35).

Case analysis: An instructional design in statistics with future teachers

Godino, Rivas, Arteaga, Lasa and Wilhelmi (2014) apply the OSA framework in the different phases of a training experience in statistics and elementary probability with future primary school teachers. The design of the formative experience is based on the resolution of three projects, two of them on elementary descriptive statistics and the third on elementary probability. The study was conducted over three weeks with 4.5 hours per week. In the first week (one 2-hour session in a large group, with 60 students) the first project (finding out the statistical characteristics of a "typical student") is presented and developed in the classroom. The teacher presents this project as a fully developed example but with mixed dynamics in trying to engage students in remembering and applying the statistical concepts and techniques they studied in high school. The analysis of the session observation shows that the moments of students' inquiry –where they work alone or in groups to respond to the issues raised- are interleaved by teacher's clearly transmissive moment; which can be directed to the whole class, a student group or individual students.

In the project above, the students, organized in pairs, are asked to respond to the questions using a data set provided by the teacher:

– What are the characteristics of a typical student, which is representative of the group?

– How representative is the typical student regarding the group?

Initially, and for about 15 minutes, the students work in trying to answer the questions. During the development of the activity, the teacher is guiding the work of the groups; while evaluating their progress, providing guidance and addressing specific questions.

However, during the teacher interaction with the students some conflicts arise when the students have to answer the first question (conflict with the meaning of “typical student”):

> E6: [...] What should we consider to select a typical student?
>
> P: You have to find out how many women and men are there, and then ... if there are more women it would be more representative to take a woman.
>
> E6: And then, do we do the same with all the variables?
>
> Q: For other variables, such as sport, you have to see how many people do not practice sport, and how many practice little or a lot of sport. Then you see which of these values is more representative ...

The teacher’s performance is motivated by the observation that some students attribute no meaning to the phrase “typical student”; a semiotic conflict is recognized because he implements a dialogic interaction format (which is a positive indicator of interactional suitability). However, the teacher solves the question, which is negative from the point of view of cognitive suitability (prevents the student himself to investigate the meaning of a typical student). Regarding the epistemic and ecological dimension, it is positive to recognize the conflict between the meanings of an expression in everyday language versus the statistical meaning.

In Godino et al. (2014) other didactical facts and interpretations that illustrate some features of the instructional model implemented in this experience are shown and; in particular the way in which, within a data analysis project based teaching, it is necessary to include some transmissive moments. Such moments of the study process are determined by the observation of blocking situations that prevent the students learning progression.

SYNTHESIS AND IMPLICATIONS

In this paper, we argued that instructional models based only on inquiry, or only on transmission are simplifications of an extraordinarily complex reality: the teaching and learning processes. As Hiebert and Grouws (2007) write, “classrooms are filled with complex dynamics, and many factors could be responsible for increased student learning. […] This is a very central and difficult question to answer” (p.371).

Although we need to establish instructional designs based on the use of rich problem – situations, which guide the learning and decision-making at the global and intermediate level, local implementation of didactical systems also requires special attention to managing the students’ background needed for solving the problems, and to the systematization of emerging knowledge. Decisions about the type of help needed essentially have a local component, and are mainly the teacher's responsibility; he /she needs some guidance in making these decisions to optimize the didactical suitability of the study process.

Hudson, Miller & Butler (2006) justify the implementation of mixed instructional models that adapt and mix explicit instruction (teacher-focused) with that based on problem solving (learner focused) taking into account the need for curricular adaptations given by the diversity of students' abilities. Similar conclusions were reached by Steele (2005), for whom, "The best teaching will often integrate ideas from constructivist and behaviorist principles" (p.3).

We also have supplemented the cognitive arguments of Kirschner, Sweller, and Clark (2006) in favour of models based on the transmission of knowledge in the case of mathematical learning, with reasons of onto-semiotic nature: What students need to learn are mainly, *mathematical rules*, the circumstances of their application and the required conditions for a proper application. The learners start from known rules (concepts, propositions, and procedures) and produce others rules that should be shared and compatible with those already established in the mathematical culture. Such rules (knowledge) must be stored in the subject's long-term memory and put to work at the right time in the short-term memory.

The scarce dissemination of IBE models in actual classrooms and the persistence of models based on the transmission and reception of knowledge can be explained not only by the teachers' inertia and lack of preparation, but by their perception or experience that the transmission models may be more appropriate for the specific circumstances of their classes. Faced with the dilemma that a majority of students learn nothing, get frustrated and disturb the class, it may be reasonable to diminish the learning expectations and opt for most students to learn something, even only routines and algorithms, and some examples to imitate. This may be a reason to support a mixed instructional model that articulates coherently, locally and dialectically inquiry and transmission.

The teacher can be convinced that direct instruction, verbal communication, supported by the collective display on the board of mathematical notation, is effective, provided that it is accompanied by an active students' reception. Can we ensure it is wrong when a lot of knowledge we learn in our lives is learned this way? Ausubel (2002) reasons and insists that learning can be significant even when it is based on oral presentation and reception.

Students should be given a chance to implement the mathematical activity, but also to know and master the mathematical cultural products that other people have developed as a result of their own activity. In addition, remembering and interpreting mathematical rules previously learned form part of the mathematical activity, and that are essential for the activity taking place (GODINO; CONTRERAS; FONT, 2006).

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